

Optimal Acquisition and Sorting Policies for Remanufacturing

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Managing Variable Used Product Condition: Optimal Sorting Policies for Remanufacturing

ABSTRACT

The condition of the used items acquired by remanufacturers is often highly variable, and sorting is an important aspect of remanufacturing operations. Sorting policies – the rules specifying which used products should be remanufactured and which should be scrapped – have received limited attention in the literature. In this paper, we examine the case of a remanufacturer who acquires unsorted used products as needed from third party brokers. As more used items are acquired for a given demand, the remanufacturer can be more selective when sorting. Thus, two related decisions are made – how many used items to acquire and how selective to be during the sorting process. We derive optimal acquisition and sorting policies in the presence of used product condition variability for a remanufacturer facing both deterministic and uncertain demand. We show the existence of a single optimal acquisition and sorting policy with a very simple structure and show that this policy is independent of production amount when acquisition costs are linear.

key words: remanufacturing, product acquisition management, used product sorting

Section 1: Introduction

In remanufacturing, used products can range in condition from slightly used with only minor cosmetic blemishes to significantly damaged and requiring extensive rework. For a remanufacturer, one of the critical operational decisions is the establishment of a sorting policy – given variable condition, which used products should be remanufactured and which should be scrapped? This important aspect of remanufacturing operations has received limited attention in the literature. In this paper, we extend earlier work by deriving and analyzing optimal sorting policies in the presence of used product condition variability. Our results, while motivated by the remanufacturing industry, are applicable to other manufacturing organizations that face variable raw material condition.

In this paper, the term “remanufacture” refers to restoring a used product to acceptable condition for resale. Other equivalent terms common in literature and practice are “recondition” and “refurbish.” Our analysis of sorting policies considers two possible dispositions of the used product – remanufacture or scrap – which are common in remanufacturing. Although some remanufacturing environments include other potential product dispositions, such as recovery of components, we do not consider these options in this study.

We analyze production to meet both deterministic and uncertain demands. In either case, remanufacturing follows a “make from stock” (Fleischmann et al. 2004) model, where used products are acquired and available as needed to meet remanufacturing needs. A common make from stock situation is that of an independent remanufacturer who obtains used products from third party brokers. Consider, for example, the cellular phone industry (Guide et al. 2003b), in which used products are purchased from brokers as needed to fulfill specific demands.

When condition is variable, some of the units acquired are likely to be too costly to remanufacture, and scrapping may be the appropriate disposition decision for these units. This implies that the number of used products acquired should be greater than the number of remanufactured products required. As the acquisition amount is increased, sorting can be made more stringent – only products with lower remanufacturing costs are actually remanufactured. Thus, the sorting policy should be driven by how many “extra” used products are acquired. Of course, the cost of acquiring more used products offsets (to some extent) the remanufacturing cost savings enabled by increased selectivity. The interaction of these two effects should drive the acquisition amount and corresponding sorting policy. In this paper, we analyze acquisition/sorting policies using a total cost model which incorporates used product condition variability. We show the existence of unique optimal policies which minimize expected total costs for a remanufacturer.

The rest of this paper is organized as follows. In section 2, we review the relevant research on remanufacturing. In section 3, we describe the impact of acquisition and sorting policies on expected remanufacturing costs. In section 4, we examine optimal acquisition and sorting policies to meet a fixed demand. In section 5, we extend these results to the case of uncertain demand. Section 6 summarizes the contribution of the analysis and suggests directions for future research.

Section 2: Remanufacturing Literature

While remanufacturing activities are often motivated by environmental concerns or demands from customers or government authorities (Thierry et al. 1995, Toktay et al. 2000, Field 2000, Fleischmann et al. 2001, Ginsberg 2001, Seitz and Peattie 2004, van Nunen and Zuidwijk 2004), the processing of returns has increasingly been viewed not simply as a cost of doing

business (Padmanabhan and Png 1997), but as a profitable business model and source of competitive advantage (Dowlatshahi 2000, Klausner and Hendrickson 2000, Stock et al. 2002, Anzel and Aichlmayr 2002). In fact, remanufacturing is a \$50 billion/year industry in the U.S. alone (Corbett and Kleindorfer 2001, Majumder and Groenevelt 2001). Realizing the potential for profitable remanufacturing, many independent businesses have emerged to exploit specific remanufacturing opportunities (Wells and Seitz 2004).

Reverse logistics/remanufacturing has many similarities to its traditional forward logistics/manufacturing counterpart. At the most basic level, both involve supply, production and distribution. The major difference between the two environments involves the supply side (Fleischmann et al. 1997, Fleischmann 2001). In a remanufacturing system, supply is largely exogenous, and the timing, quantity, and quality of supply are much more uncertain than in traditional production-distribution systems. A significant consequence of this uncertainty is the inclusion of an inspection stage and a corresponding system of quality-dependent routing of supply in a reverse logistics network (Trebilcock 2002). With rare exceptions, traditional supply chains do not include such a focus on inspection and supply quality.

Many studies have acknowledged the problem of variable supply quality for remanufacturing systems (Bloemhof-Ruwaard et al. 1999, Guide and Jayaraman 2000, Fleischmann et al. 2000, Stanfield et al. 2004). Guide (2000) discusses the additional complexity in scheduling and planning caused by the high variability of remanufacturing processing times, and he presents results of a survey of managers that provide insights on how these issues can be addressed. Toktay et al. (2000) incorporate random processing times for the returned products into their queuing network model of a closed-loop system. Guide et al. (2000) highlight the operational concern caused by highly variable processing times, which are a function of returned

product condition. The authors point to the need for remanufacturing firms to estimate used product condition to determine the appropriate disposition. This assessment of condition is an important step in determining the optimal recovery action (Bloemhof-Ruwaard et al. 1999, Van Nunen and Zuidwijk 2004, Blackburn et al. 2004). Several techniques for condition assessment and grading have been described in recent publications (Krikke et al. 1999, Rudi et al. 2000).

Simulation has been used by some researchers to model variable used product condition in remanufacturing operations. Fleischmann (2001) builds a simulation model which incorporates uncertainty in the quality of returned products and uses this model to evaluate several reverse logistics configurations. Humphrey et al. (1998) use a simulation which incorporates variable repair requirements to analyze a reverse logistics network for a U.S. Army depot. Guide and Srivastava (1997) incorporate stochastic processing times into their simulation model to evaluate various order release strategies in a naval aviation depot, and they acknowledge the high level of variation in work content that can be present in the remanufacturing setting.

Several models have been proposed for managing the problem of variable used product quality. Klausner and Hendrickson (2000) show how incentives can be used to increase the quality of returned goods. They assert that an increase in returned goods quality will lead to a higher remanufacturing yield. Guide and Van Wassenhove (2001) propose a model to manage the quality of cellular phone returns to help reduce variation in processing times. Guide et al. (2003b) expand the analysis in Guide and Van Wassenhove (2001) to include the consideration of optimal acquisition policies and pricing. They point out the limited amount of research that has addressed areas such as used product acquisition, testing, and disposition. Their research focuses on the criticality of product acquisition management in maximizing profits for a

remanufacturer, and it highlights the need to move beyond management intuition in managing these processes. Guide et al. assume that used products are acquired from third-party brokers who have sorted the products into condition categories, where remanufacturing cost for a given category is known.

In contrast to Guide et al., we take remanufacturing cost of acquired products to be unknown. This reflects many potential situations, including the case in which sorting does not eliminate remanufacturing cost variability and the case where third party brokers offer only unsorted products to the remanufacturer. In addition, our analysis could help remanufacturers quantify the price they are willing to pay brokers for the service of sorting.

Several recent papers have presented mathematical models of remanufacturing operations which do not incorporate variable used product condition into the analysis. In their 2004 paper, Savaskan et al. mention the growing interest in research that considers the quality uncertainty in return flows. They analyze three collections options and discuss the conditions under which different collections processes and supply chain structures are appropriate. However, their model assumes homogeneous quality of returned products for each collection option and that all returned products are remanufactured for resale (100% yield) at a fixed unit remanufacturing cost. Krikke et al. (1999) also acknowledge variability in returned product quality, but their MILP formulation of a reverse logistics network uses a constant unit cost of remanufacturing. Jayaraman et al. (1999) and Majumder and Groenevelt (2001) also model unit remanufacturing cost as a constant.

Section 3: Acquisition and sorting policies for deterministic demand

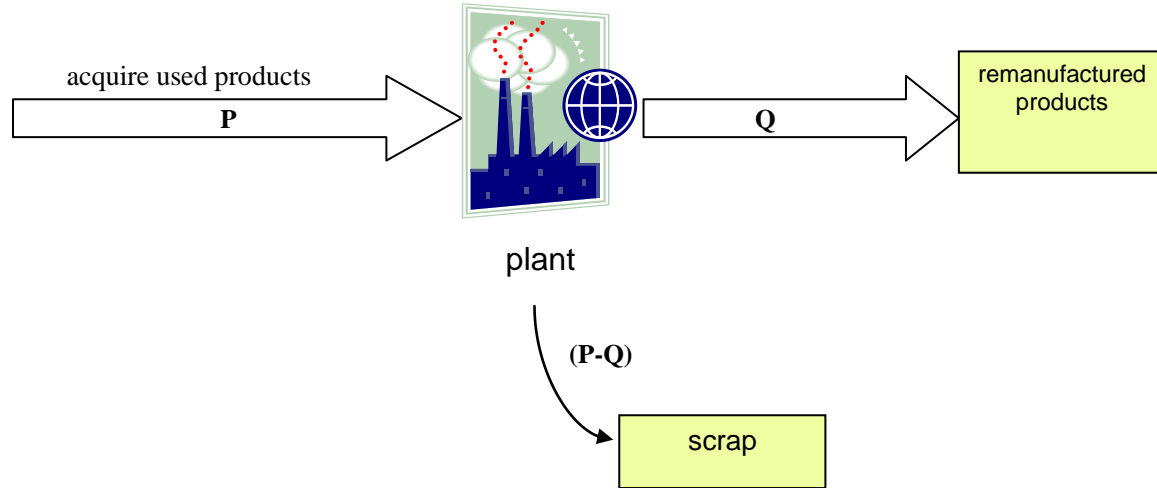
We develop a model which explicitly considers variable used product condition and use this model to examine how acquisition and sorting decisions affect remanufacturing costs. In

this analysis, we make the following assumptions, similar to Guide et al. (2003b): perfect testing, no capacity constraints, and no fixed costs. In our context, perfect testing means that remanufacturing cost depends on condition, but is known at the time of sorting. These assumptions result in a model that allows us to accurately depict the problem without trivializing it.

Our research is motivated by an independent remanufacturer who serves both the imaging supplies and cellular telephone markets. In many of these markets, the supply of used items from third party brokers is essentially unconstrained. The acquired products have not been sorted, and their condition is highly variable. When the unsorted items arrive at the remanufacturing facility, the firm must sort each item and assign it to one of two categories – remanufacture or scrap. Scrap items are disposed of at negligible cost, and the remaining items are immediately processed. The result is a remanufacturing “yield,” defined as the percentage of the used products acquired which are actually remanufactured. The firm acknowledges that increasing selectivity in its sorting (i.e. accepting fewer products for remanufacture) will have two effects: 1) yield will decrease, requiring the acquisition of more used products to meet a given demand, and 2) remanufacturing cost will decrease, since the used products sorted for remanufacture will be, on average, in better condition. Decreasing sorting selectivity will have opposite effects. Thus, the firm must make two related decisions given demand or demand forecast – how many used items to acquire and how selective to be during sorting. In this research, we develop a quantitative model for determining optimal acquisition and sorting policies. We first analyze the problem of deterministic demand and then extend our analysis to the stochastic demand setting.

Given that some used products might not be remanufactured, it follows that a firm facing a demand Q should acquire P used products, where $P \geq Q$. As each product is processed, it is sorted into one of two categories – remanufacture or scrap – such that Q products are remanufactured and $(P-Q)$ are scrapped (see Figure 1).

Figure 1: Acquiring and sorting used products



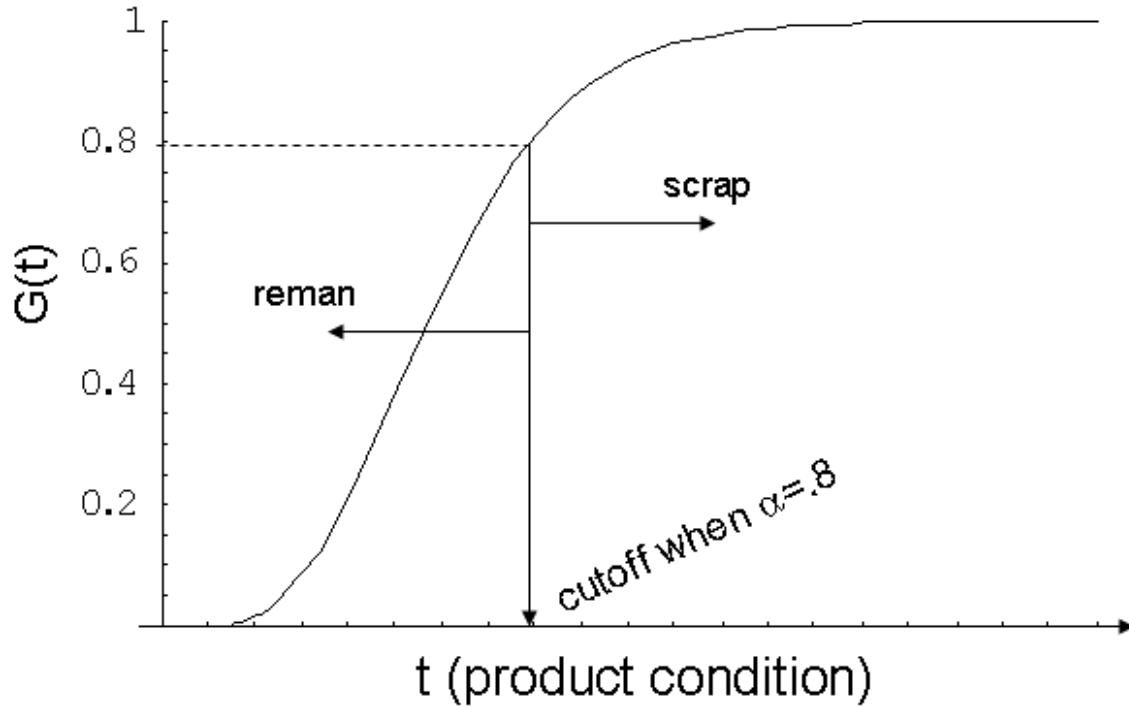
To translate a given P into a specific sorting policy, we use the distribution of condition of the used products, where *condition is defined as cost to remanufacture*. First, note that when Q remanufactured products are needed and P used products are acquired, the required yield, α , – that is, the fraction of used products that must be successfully remanufactured – is Q/P . Sorting policy should be set such that the expected yield from remanufacturing equals α . Given a distribution of used product condition $G(\cdot)$, we define the “cutoff” condition – that is, the worst condition acceptable to justify remanufacturing – to be some remanufacturing cost t such that:

$$G(t) = \alpha, \text{ or } G(t) = \frac{Q}{P}$$

This condition t defines the sorting policy – products with remanufacturing cost above t are scrapped, and those with cost below t are remanufactured.

In practice, used products might only be classified into discrete categories (e.g. excellent, good, poor, scrap). However, in this paper we examine the more general approach in which product condition (or remanufacturing cost) is defined on a continuum. Figure 2 demonstrates the relationship between α and expected remanufacturing cost and illustrates how the cutoff condition is determined given a desired yield of 80%.

Figure 2: Translating required yield into sorting policy



Since sorting policy must be set such that $G(t)=\alpha$, as α decreases (or, equivalently, as P increases for a fixed Q), a firm is able to be more selective when processing used products – higher-cost items are not remanufactured. Therefore, those products that are selected for remanufacture have lower remanufacturing cost as α decreases.

Given Q and $G(\cdot)$, we can express sorting policy t as a function of P :

$$t = G^{-1}\left(\frac{Q}{P}\right) \tag{1}$$

Since only the Q used products with a cost of t or less will be remanufactured, we have the following expression for expected remanufacturing cost:

$$R(P) = Q \frac{\int_0^t xg(x)dx}{\int_0^t g(x)dx} \quad (2)$$

Remanufacturers also incur acquisition costs, which we define as all costs to acquire, transport, and sort used products. Adding an acquisition cost function $z(\cdot)$ to (2) gives us the following expression for the total cost, defined as acquisition plus remanufacturing cost, of acquiring P and remanufacturing Q products:

$$TC(P) = z(P) + Q \frac{\int_0^t xg(x)dx}{\int_0^t g(x)dx} \quad (3)$$

Although most remanufacturing literature assumes linear acquisition costs, we start by examining the case where acquisition costs are nonlinear. In these cases, we assume increasing marginal cost (i.e. convex increasing cost) of used products because of scarcity; Guide et al. (2003b) make the same assumption in their analysis. We define $z(\cdot)$ in (3) as any non-negative convex increasing acquisition cost function.

To prove that $TC(P)$ is convex on $[Q, \infty)$, and therefore is minimized at a single critical number P^* , we first prove the following proposition:

PROPOSITION 1. For a given production amount, expected remanufacturing cost is a convex monotonically decreasing function of P , the number of used products acquired.

Proof: see Appendix A

Proposition 1 holds for all continuous distributions of remanufacturing cost. Given Proposition 1 and the fact that (3) is simply the sum of two terms which are convex in P on $[Q, \infty)$, we can state the following:

PROPOSITION 2. $TC(P)$ is convex in P on $[Q, \infty)$, therefore *given any convex acquisition cost function, there is a single optimal acquisition amount P^* (and corresponding sorting policy as defined by evaluating (1) at P^*) which will minimize total expected costs to meet a fixed demand, Q .*

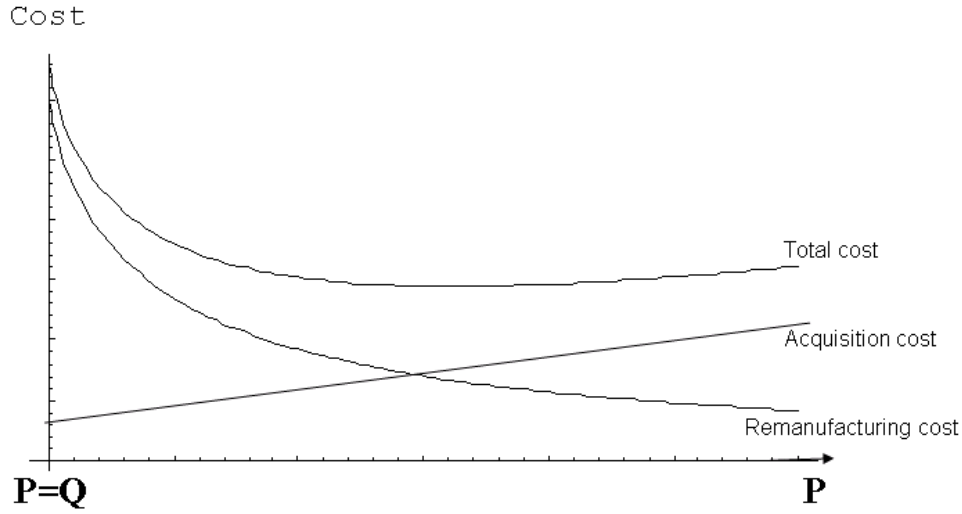
This is a strong result because it shows that, given a known distribution of product condition, a single optimal acquisition and sorting policy exists which has a very simple structure.

Section 4: Results for linear acquisition costs

We now examine the special case where the acquisition cost function is linear. Linearity is a reasonable assumption in many remanufacturing environments, particularly when the market is large and well-defined. Linear acquisition, transportation, and handling costs are commonly assumed in the literature (e.g., Majumder and Groenevelt 2001, Fleischmann et al. 2001, Savaskan et al. 2004). Klausner and Hendrickson (2000) provide a detailed justification, grounded in data from a German remanufacturer, of the use of a constant unit cost of acquired products in their model.

Given linear acquisition costs, total cost is a convex function of P on $[Q, \infty)$ as shown in Figure 3.

Figure 3: Total cost



We now introduce a new parameter, γ , which we define as $\frac{P}{Q}$. Rewriting (1) using γ gives:

$$t = G^{-1}\left(\frac{1}{\gamma}\right) \quad (4)$$

Let $u =$ unit acquisition cost. Then, the total cost of acquiring P (or, equivalently, γQ) used products at unit cost u is expressed in (5) as a function of γ . Note that since total cost is convex in P , it is also is convex in γ for a fixed Q .

$$TC(\gamma) = u\gamma Q + Q \frac{\int_0^t xg(x)dx}{\int_0^t g(x)dx} \quad (5)$$

The following proposition establishes that, under linear acquisition costs, the optimal sorting policy can be determined independently of the production amount:

PROPOSITION 3. When acquisition costs are linear, the value of γ which minimizes total cost is independent of the production amount Q .

Proof. Since $\int_0^t g(x)dx = G(t) = 1/\gamma$, we can simplify (5) to the following:

$$TC(\gamma) = Q * \gamma \left[u + \int_0^{G^{-1}(1/\gamma)} xg(x)dx \right] \quad (6)$$

By inspection of (6) we see that the optimal value for γ is independent of Q . Dividing (6) by Q results in the following expression for average total cost per unit:

$$UTC(\gamma) = \gamma \left[u + \int_0^{G^{-1}(1/\gamma)} xg(x)dx \right] \quad (7)$$

Since (7) is not dependent on Q , we can see that the optimal average total cost per unit is separable from the production amount. \square

Proposition 3 has strong implications for both practice and theory. From a practical view, since γ fully describes the acquisition amount (γQ) and the sorting policy ($G^{-1}(1/\gamma)$), the fact that the optimal γ is independent of Q means that a single acquisition and sorting policy is optimal for any production amount. From a theoretical standpoint, the fact that the optimal average total cost per unit is independent of Q allows us to easily extend our analysis to cases of uncertain demand.

Section 5: Acquisition and sorting policies for stochastic demand

Ferrer and Whybark (2001) point out that some remanufacturers have limited advance knowledge of demand, and overproduction of remanufactured items can expose them to obsolescence risk. In these cases, the classic newsvendor problem can be used to set a production amount which minimizes the sum of expected shortage and overage costs. We now

examine the applicability of the newsvendor problem given the variable production costs in remanufacturing.

We start by presenting a basic newsvendor formulation given uncertain demand with distribution $f(\cdot)$:

$$N(Q) = c_o \int_0^Q (Q - x) f(x) dx + c_s \int_Q^{\infty} (x - Q) f(x) dx \quad (8)$$

Where overage cost, c_o , is defined as the cost of producing an unsold unit; shortage cost, c_s , is defined as the lost margin per unit from producing fewer units than the actual demand (plus any penalty for disappointing customers). The newsvendor solution is the production quantity Q^* which minimizes $N(Q)$. In the context of the general remanufacturing problem (without the assumption of linear acquisition costs), c_o and c_s are as follows:

$$c_o = \frac{TC(P^*)}{Q}, c_s = A - \frac{TC(P^*)}{Q} + b$$

where $TC(P^*)$ is the optimal solution to (3), b is unit shortage penalty, and A is unit sales revenue.

In the general case of nonlinear acquisition costs, $TC(P^*)$ is not independent of Q . Thus, when the newsvendor approach is applied to the general remanufacturing problem to minimize the function $N(Q)$ as given in (8), the c_o and c_s are also functions of Q , and the problem becomes more difficult to solve.

However, by Proposition 3 the optimal average total cost per unit is independent of Q given linear acquisition costs. In this case we can define c_o and c_s as follows:

$$c_o = UTC(\gamma^*), c_s = A - UTC(\gamma^*) + b$$

where $UTC(\gamma^*)$ is the optimal solution to (7). Given that these expressions for c_o and c_s do not depend on Q , the newsvendor problem can be solved using standard techniques. The expression

defining the optimal newsvendor production quantity Q^* , $F(Q^*) = \frac{c_s}{c_o + c_s}$, can be written as

follows in this context:

$$F(Q^*) = \frac{A - UTC(\gamma^*) + b}{A + b} \quad \text{or} \quad Q^* = F^{-1}\left(\frac{A - UTC(\gamma^*) + b}{A + b}\right) \quad (9)$$

Since (7) does not depend on Q , $UTC(\gamma^*)$ is invariant with respect to the ultimate determination of Q^* . Thus, remanufacturers can set acquisition and sorting policies (as defined by γ) a priori and still use the standard newsvendor model to optimize production amounts when facing uncertain demands. This implies the following 3-step procedure:

1. find γ^* and $UTC(\gamma^*)$ using (7), and define the sorting policy by evaluating (4) at γ^* ;
2. find Q^* using (9);
3. acquire γ^*Q^* and remanufacture Q^* products (using the sorting policy defined in step 1).

The above procedure will result in minimum expected costs to meet an uncertain demand in a remanufacturing setting (see the following example).

Example Problem:

Assume used products are acquired for a unit cost of \$3.00. Condition follows a gamma distribution with parameters (5,2), implying an average cost to remanufacture of \$10.00. Unit sales revenue is \$15.00 and unit shortage penalty is \$4.00. In step 1 of the above procedure, we can use (7), with $u=3$ and $G(\cdot)=\text{gamma}(5,2)$, to find γ^* . In this example γ^* equals 1.4 (so the optimal acquisition policy is 1.4 * the number of remanufactured products needed). Using (4), sorting policy should be set to scrap all units with a remanufacturing cost greater than \$11.99

(this is equivalent to sequencing the items by condition, processing the best $1/\gamma^*$ (or 71%) of them, and scrapping the remaining 29%). In this case, the average remanufacturing cost of the products that are not scrapped is \$7.75, and \$4.20 is spent on used product acquisition for every remanufactured item. Therefore, the average total cost of products that are actually remanufactured, $UTC(\gamma^*)$, is \$11.95.

Note that the optimal acquisition and sorting policy, as defined by γ^* , is known *before* demand is estimated. We now have all the information required to determine optimal policies when faced with any uncertain demand. For example, if expected demand $f(\cdot)$ is normally distributed with mean 1000 and standard deviation 150, then in step 2 we have

$$Q^* = F^{-1}\left(\frac{15 - 11.95 + 4}{15 + 4}\right) = 951. \text{ In step 3 we calculate the optimal acquisition quantity for this}$$

production amount to be $951 * 1.4 = 1331$. By acquiring 1331 used products and processing them according to the sorting policy determined in step 1, we can expect to remanufacture the optimal quantity of 951 units at minimum cost.

Section 6: Conclusions and Future Research

In this paper we have provided a detailed analysis of optimal acquisition and sorting policies for remanufacturers facing variable used product condition. Our work provides a foundation for continued research in this area, which has been identified as having received little previous attention from researchers (Guide et al. 2003b). The analysis in the previous two sections has shown that, given reasonable assumptions, a single optimal acquisition and sorting policy exists for a remanufacturer. Given linear product acquisition costs, the optimal sorting policy can be defined independently of the production amount, and the optimal acquisition quantity can be calculated by applying a simple ratio, γ , to demand. Because the optimal average

total cost per unit does not vary by production amount, our results are easily extended to the case of uncertain demand. For remanufacturers that follow a make from stock model for their products – such as imaging supplies and cellular phones – this paper provides algorithms to find cost-minimizing acquisition amounts and sorting policies.

While our results provide useful insights into this emerging area of remanufacturing research, some of our assumptions could be relaxed for wider applicability. For example, although we impose no restrictions on scrapping, a term for scrapping penalty can be added to our total cost function to reflect situations in which scrapping is constrained. If penalties are assessed based on the percentage of acquired products which are scrapped, then the scrapping penalty is independent of Q . These penalties are also likely to be convex (increasing penalties as scrapping increases). Therefore, we expect that, for most cases involving scrapping restrictions, the total cost will remain a convex function of γ , and all results in sections 4 and 5 would remain valid.

The model presented in this paper assumes that yield (i.e. α) is deterministic. We believe that this assumption preserves the essence of the problem and allows us to present some interesting insights into this new research area. In reality, however, the number of items that meet a specified condition cutoff would be stochastic. Optimal sorting policies under random yield would be a useful extension of this research.

Appendix A: Proof of Proposition 1

To prove monotonically decreasing convexity, we show that the first derivative of the remanufacturing cost function is always negative and the second derivative is always positive:

Note that the denominator of (2) equals $\frac{Q}{P}$. We simplify (2) to the following expression for remanufacturing cost as a function of P:

$$R(P) = P \int_0^{G^{-1}\left(\frac{Q}{P}\right)} xg(x)dx \quad (\text{A1})$$

Recall that the derivative of an inverse function is defined as follows:

$$\frac{df^{-1}(x)}{dx} = \frac{1}{\frac{df}{dx}\left(f^{-1}(x)\right)}$$

Applying the above definition, as well as Leibnitz's Rule and the product rule, we have:

$$\frac{dR}{dP} = PG^{-1}\left(\frac{Q}{P}\right)g\left(G^{-1}\left(\frac{Q}{P}\right)\right)\frac{1}{g\left(G^{-1}\left(\frac{Q}{P}\right)\right)}\left(-\frac{Q}{P^2}\right) + \int_0^{G^{-1}\left(\frac{Q}{P}\right)} xg(x)dx$$

which simplifies to:

$$\frac{dR}{dP} = \int_0^{G^{-1}\left(\frac{Q}{P}\right)} xg(x)dx - \frac{Q}{P} * G^{-1}\left(\frac{Q}{P}\right) \quad (\text{A2})$$

Factoring out $\frac{Q}{P}$ from (A2) gives:

$$\frac{dR}{dP} = \frac{Q}{P} \left[\frac{P}{Q} \int_0^{G^{-1}\left(\frac{Q}{P}\right)} xg(x)dx - G^{-1}\left(\frac{Q}{P}\right) \right] \quad (\text{A3})$$

Replacing $\frac{P}{Q}$ in the first term of (A3) with the equivalent expression $\frac{1}{G^{-1}\left(\frac{Q}{P}\right) \int_0^{G^{-1}\left(\frac{Q}{P}\right)} g(x)dx}$:

$$\frac{dR}{dP} = \frac{Q}{P} \left[\frac{\int_0^{G^{-1}\left(\frac{Q}{P}\right)} xg(x)dx}{G^{-1}\left(\frac{Q}{P}\right) \int_0^{G^{-1}\left(\frac{Q}{P}\right)} g(x)dx} - G^{-1}\left(\frac{Q}{P}\right) \right] \quad (A4)$$

Observe that (A4) is always negative. The term in the brackets is the difference of two terms.

The first term is equivalent to the expected value of g on $\left[0, G^{-1}\left(\frac{Q}{P}\right)\right]$, and the expected value

of g is less than its upper limit, $G^{-1}\left(\frac{Q}{P}\right)$. Therefore, the bracket term is always negative, and

thus $\frac{dR}{dP}$ is always negative.

We now derive the second derivative, $\frac{d^2 R}{d P^2}$. Rewrite (A4) as follows:

$$\frac{dR}{dP} = \frac{1}{P} * \frac{Q \int_0^{G^{-1}\left(\frac{Q}{P}\right)} xg(x)dx}{G^{-1}\left(\frac{Q}{P}\right) \int_0^{G^{-1}\left(\frac{Q}{P}\right)} g(x)dx} - \frac{1}{P} * QG^{-1}\left(\frac{Q}{P}\right) \quad (A5)$$

Note that the first term of (A5) is (2) multiplied by $\frac{1}{P}$, and, applying the product rule, the

derivative of the first term of (A5) is

$$-\frac{R}{P^2} + \frac{dR}{dP}$$

or

$$-\frac{1}{P^2} * P \int_0^{G^{-1}(\frac{Q}{P})} xg(x)dx + \frac{1}{P^2} \left(P \int_0^{G^{-1}(\frac{Q}{P})} xg(x)dx - QG^{-1}\left(\frac{Q}{P}\right) \right)$$

which simplifies to

$$-\frac{QG^{-1}\left(\frac{Q}{P}\right)}{P^2} \tag{A6}$$

Differentiating the second term of (A5) gives:

$$\frac{Q^2}{P^3 g\left(G^{-1}\left(\frac{Q}{P}\right)\right)} - \frac{QG^{-1}\left(\frac{Q}{P}\right)}{P^2} \tag{A7}$$

Combining (A6) and (A7) we have

$$\frac{d^2 R}{d P^2} = -\frac{QG^{-1}\left(\frac{Q}{P}\right)}{P^2} - \left[\frac{Q^2}{P^3 g\left(G^{-1}\left(\frac{Q}{P}\right)\right)} - \frac{QG^{-1}\left(\frac{Q}{P}\right)}{P^2} \right] \tag{A8}$$

which simplifies to:

$$\frac{d^2 R}{d P^2} = \frac{Q^2}{P^3 g\left(G^{-1}\left(\frac{Q}{P}\right)\right)} \tag{A9}$$

which, by inspection, is always positive. \square

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