Decision Support for Lead Time and Demand Variation Reduction

Abstract

Companies undertaking operations improvement in supply chains face many alternatives. This work seeks to assist practitioners to prioritize improvement actions by developing analytical expressions for the marginal values of three parameters-- (i) lead time mean, (ii) lead time variance, and (iii) demand variance– which measure the *marginal* cost of an incremental change in a parameter. The relative effectiveness of reducing the lead time mean versus lead time variance is captured by the ratio ofthe marginal value of lead time mean to the marginal value of lead time variance. We find that the value of this ratio strongly depends on whether the lead time mean and variance are independent or correlated. We illustrate the application of the results with a numerical example from an industrial setting. These insights can help managers make tradeoffs among investment decisions to modify demand and supply characteristics in their supply chain, e.g., by switching suppliers, factory layout, or investing in information systems.

Keywords

Supply Chain Management, Inventory, Decision Analysis, Lead Time, Marginal Value

1 Introduction

Global competition creates a need for firms to collaborate with supply chain partners as well as leverage their performance by reducing demand variability(e.g., Lee et al. [1]; Lee et al. [2]) and supply variability (e.g., Fu and Piplani [3]; Lim [4]). Supply chain research has proposed a variety of models for potential supply chain improvements (Ganeshan et al. [5]; Swaminathan and Tayur [6]; Flynn et al. [7]). However, these models have limitations that restrict them from being fully exploited by practitioners.

The first limitation of existing models is that they target an optimal solution subject to a given set of supply and demand parameters (Silver [8]). However, in practice managers can often improve supply chain performance by changing these parameters. For instance, firms may be able to determine the cost and benefit of switching or consolidating suppliers, new market development, in-house capacity investment, or global outsourcing. These initiatives can result in significant changes to lead time as well as demand and supply variability. An analytical framework is needed for firms to understand the systematic influence of changed parameters and, consequently, to make better decisions on how to invest in and adapt to changing supply chain settings.

Second, given limited resources, firms must often choose among alternative investment decisions, e.g., between focusing on reducing demand variance, lead times, or lead time varoamce (Smith and Lockamy [9]). These alternatives, such as lead time mean and variance, are often correlated, and this makes the tradeoff decision more complicated because the evaluation of only one improvement at a time is insufficient. For example, Ryu and Lee [10] and Hayya et al. [11] use exponentially distributed lead times for which means and variances cannot be changed independently. Furthermore our observations from industry suggest that the relationship between lead time mean and variance exists in more general settings. The following figure shows the predicted lead times and standard deviation of the actual lead times of 7653 orders placed by a major steel distributor on 22 domestic and international suppliers during a one year period.

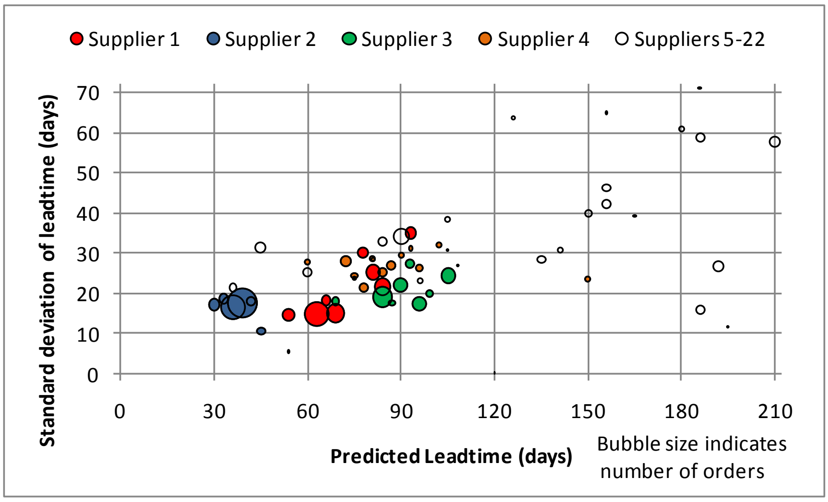

Figure 1: Correlation between lead time mean and variation

Each bubble in Figure 1 represents a particular supplier and a particular predicted lead time (horizontal axis). The number of orders is indicated by bubble size, and the standard deviation of the actual lead times is indicated on the vertical axis. The median coefficient of variation (the standard deviation divided by the predicted lead time) is 0.30. More than 60% of orders are in categories with coefficient of variation within the range 0.2-0.4. The figure illustrates positive correlation between the standard deviation of lead time and expected lead time, both overall, and for a given supplier. As some products are sourced from multiple suppliers, the figure suggests that assuming one can adjust the lead time mean and variance separately may not always be reasonable. A switch to a supplier with shorter lead time may reduce lead time variance. For other inventory improvements, such as removing outliers, information sharing or publicizing supplier performance, the mean and variance of the lead time may be reduced simultaneously.

To bridge these gaps between existing models and practice, this paper aims to help firms determine how to allocate investment to reduce demand and supply variation. The study analyzes the marginal effect on cost of reductions in (i) lead time mean, (ii) lead time vvariance, and (iii) demand variance, in cases where the lead time mean and variance are either independent or correlated through a functional relationship. We find the results in the correlated case to be quite different from those in the independent case.

Our study is motivated by the supply chain concerns of a supplier of components in the truck and automotive industry. To meet the demand of its US customers, the firm supplies on a JIT basis from a distribution center in the Southeastern US, but sources components domestically, from Mexico and from offshore. Consequently, the component supply chain faces significant variability in lead time mean and lead time variance. It faces difficulties in choosing among various alternatives — using local production to reduce lead time and lead time variance, improving supplier relationship management to provide more stable quality and supply, or moving more production offshore. The firm does not have a good model for judging how to rank these alternatives and make trade-offs in its supply chain design.

We develop a general analytical model incorporating demand and supply variation and validate our approach using industry data. To make the model more applicable in practice, we analyze the marginal effects in the situation where parameters may not be optimal -- firms can and do deviate from optimality for reasons related to the cost of implementation or the fulfillment of other, often strategic, business objectives. The results should assist organizations making practical decisions relating to changes in suppliers, factory layout, process improvement, and information systems investment, thereby improving supply chain management performance.

The remainder of the paper is structured as follows. First, we review the literature on demand variation, supply variation, and comparisons of the effects of the two types of variation. We then present our research model and assumptions for a general supply chain with both demand and supply variation. This is followed by an analysis of the total inventory cost and marginal values of changing demand variation and supply variation patterns. Based on the analytical results, we provide a numerical example using operations data from a truck component manufacturer. The paper concludes by discussing the findings, limitations, directions for future research, and implications for both research and practice.

2 Literature Review

Supply chain improvement alternatives can be classified into three categories: reducing demand variability (e.g., Lee et al. [1]), reducing supply variability (e.g., Zhang et al. [12]; Song et al. [13]; Qi and Shen [14]) and, reducing demand and supply variability simultaneously (e.g., Gerchak and Parlar [15]; Das [16]). Existing literature provides a foundation for our development of a general model for the analysis of the cost impacts of these categories of variation.

The research on demand variability mainly deals with mechanisms to reduce it and an evaluation of the benefits of such solutions. According to Lee et al. [1], demand variability can be moderated through the involvement of point-of-sale (POS) systems, third party logistics (3PL), and other forms of sharing demand information. Wu et al. [17] considered the incentives for firms to share their demand information. Hosoda and Disney [18] studied the setting where only delayed demand information is available and show that different levels of the supply chain benefit differently from shorter time delays. These results indicate the potential benefits of methods to reduce demand variation.

Supply uncertainty has long been identified as a fundamental factor influencing inventory decisions. Research has focused on inventory models with stochastic lead times (Bashyam and Fu [19], Bookbinder and Cakanyildirim [20]). In addition to these works, which are foundational to our current paper, other studies have focused on methods to reduce supply variability, such as order splitting - the partitioning of an order between two or more vendors (Hayya et al. [21]).

Two concerns emerge in reducing supply variability: one focuses on reducing the length of the supply lead time while the other focuses on reducing the variance of the supply lead time. Focusing on reducing average supply lead time, Blumenfeld et al.

[12] developed a queuing model to analyze how manufacturing response time affects the inventory required at retailers. Fisher et al. [22] considered the problem of determining retailer replenishment order quantities to minimize the total cost of lost sales, back orders, and obsolete inventory. The results can be used to quantify the benefit of lead time reduction and thus select the best replenishment contract. Blackburn [23] assessed the effect on cost of both increased and decreased (deterministic) lead time. Recently, Garcia et al. [24] proposed a method for the online identification of lead times based on a multi-model scheme. Chandra and Grabis [25] consider the trade-off between benefits of lead-time reduction and increases in procurement cost. On the other hand, research dealing with the effects of lead time variance includes Song et al. [13] which demonstrates that ignoring lead time variability can be costly, but relatively simple heuristics that include lead time variance perform quite well. Gerchak and Parlar [15] considered the joint optimization of lead time vvariance, lot size and reorder point in continuous review inventory models. Wang and Hill [26] investigated the effects of reducing lead time variance on safety stock when lead time is gamma distributed.

Lastly, several studies compare the relative importance of demand and supply variability. Paknejad et al. [27], observing the dedication of Japanese manufacturers to establishing long-term partnerships with their suppliers in order to reduce lead time variance, concluded that lead time variance was more costly than demand variance. Vinson [28] observed changes in optimum safety stock and inventory costs using different combinations of stockout cost, demand variance, lead time mean, and v lead time variance. He found lead time variance to be more important than either the lead time mean or the demand variance in explaining inventory cost behavior. However, Das [16] indicated that cost is more sensitive to lead time mean than it is to lead time variance. This apparent contradiction stems from the different inventory models and assumptions employed. More recently, Chopra et al. [29] indicated that there exists a threshold for service level, with the impacts of lead time variance and lead time mean differing above and below the threshold. According to He et al. [30], when demand rate is constant, it is the variance and not the mean of lead time that affects the total relevant cost in a stochastic lead time model.

The existing literature lacks an analysis of the marginal effects of simultaneous reductions in lead time mean, lead time variance, and demand variance in one general inventory system. This paper seeks to bridge this gap and help firms determine the best place to invest efforts in changing demand and supply parameters in their supply chain setting. We consider the non-optimal setting to provide a more generally applicable method for practical methods of inventory system improvement. Furthermore, the existing literature does not consider the situation when lead time mean and variance are explicitly correlated. The research on correlated lead time mean and variance is restricted to models in which lead time is exponentially distributed, e.g. Ryu et al. [10] and Hayya et al. [11]. Thus, they are not able to compare the correlated case with the case when the mean and variance are independent, and exponentially distributed lead time is only a special case of correlated lead time mean and variance. Here we explicitly compare the marginal values of lead time mean and lead time variance (i.e., the first derivatives of inventory cost with respect to lead time mean and variance) in the case when the mean and variance are correlated.

3 Model

We construct a model for a single product with both demand and supply variation. Demand per unit time is independent and identically distributed with mean μ_D and variance σ_D^2. The inventory system is managed by a conventional reorder quantity/reorder point (Q, r) policy—that is, when the inventory position (*inventory on hand* + *inventory on order*) falls below a level r, an order of size Q is released to the supplier. For organizations checking their inventory position regularly, with a short review period, their policy approximates that of a continuous (Q, r) policy. When the firm places an order, it receives products after a lead time L, which is a random variable with mean μ_L and variance σ_L^2. Finally, we specify that unfilled customer orders are backlogged, with each generating a one-time backlog cost dependent on the quantity ordered (but not the backlog period).

We evaluate the performance of the inventory system using total annual expected cost. We assume that demand over the lead time *L* denoted by X has mean μ_X and variance σ_X^2. The reorder point *r* is set equal to $\mu_X + k\sigma_X$ (mean + *k* standard deviations of demand during the lead time) for each product. Hereafter the inventory policy is denoted by (*Q*, *k*). For greater generality we assume that neither Q nor k is chosen optimally and that the firm maintains a consistent inventory policy (constant *k*) as *L* is changed (referring to change of either mean or variance). Product quality and productivity are assumed to be invariant with changes in lead time *L*. Additional notation for the model is introduced below:

A = Fixed cost component of each order;

D = Average annual demand;

h = Annual inventory carrying charge/unit;

B = Backorder cost/unit.

The expected quantity short during a lead time for reorder point level $\mu_X + k\sigma_X$ is $\sigma_X G(k)$ where $G(k) = \int_k^\infty (u-k) f(u)du$ refers to the unit linear loss integral, and *f(u)* refers to the distribution density function of $\frac{X - \mu_X}{\sigma_X}$ (Silver et al. [31]). We restrict our discussion to the set of distributions of the demand over lead time *X*, such that *G(k)* is not a function of the demand and lead time parameters. For example, *X* can be normally distributed (Silver and Bischak [32]).

Assuming that the order quantity *Q* has been selected, the expected annual total cost of a (*Q*, *k*) inventory policy is (Silver et al. [31]):

$$TC = \frac{AD}{Q} + (\frac{Q}{2} + k\sigma_X)h + \frac{D}{Q}BG(k)\sigma_X. \quad (1)$$

The total cost function is the sum of the annual ordering cost, the annual holding cost of cycle stock, safety stock, as well as the expected annual cost of backorders. Two terms in the *TC* expression represent the costs of a deterministic EOQ problem for the order quantity *Q*, *viz.* $\frac{AD}{Q} + \frac{Qh}{2}$. The remaining terms in (2) are the increase over the EOQ cost due to demand and supply variation. Then the decision of choosing the value of *k* can be viewed as a tradeoff between expected inventory holding and backorder costs (Blackburn [23]):

$$[kh + \frac{D}{Q}BG(k)]\sigma_X. \quad (2)$$

4 Analysis

In this section, we develop expressions for the marginal values of three parameters: demand variance, lead time mean, and lead time variance, as well as relationships between these marginal values. We consider incentives for the firm to change these parameters. Further, we analyze the case in which lead time mean and variance are correlated and show how the result differs from the case in which lead time and variance are independent.

4.1 Independent lead time mean and variance

To show how total inventory costs vary with changes in demand variance, lead time mean and lead time variance in the case of independent lead time mean and variance, we establish the marginal values from the partial derivatives of *TC* with respect to the variance of demand σ_D^2 (MVD), lead time mean μ_L (MML) and variance of lead time σ_L^2 (MVL). We express σ_X in (2) using the variables and assumptions defined earlier (Silver et al. [31]):

$$\sigma_X = \sqrt{\mu_L {\sigma_D}^2 + {\mu_D}^2 {\sigma_L}^2}. \quad (3)$$

Thus $[kh+\frac{D}{Q}BG(k)]\sigma_X = [kh+\frac{D}{Q}BG(k)]\sqrt{\mu_L\sigma_D{}^2+\mu_D{}^2\sigma_L{}^2}$.

Letting $\gamma(k)$ denote $kh+\frac{D}{Q}BG(k)$. Then for fixed *k*, we have

$$MVD = \frac{\partial TC}{\partial \sigma_D^2} = \frac{\mu_L}{2\sigma_X}\cdot\gamma(k),$$

$$MML = \frac{\partial TC}{\partial \mu_L} = \frac{\sigma_D^2}{2\sigma_X}\cdot\gamma(k), \tag{4}$$

$$MVL = \frac{\partial TC}{\partial \sigma_L^2} = \frac{\mu_D{}^2}{2\sigma_X}\cdot\gamma(k). \tag{5}$$

One may compare the ratios of the marginal values:

$$MML : MVL : MVD = \frac{\sigma_D^2}{2\sigma_X}\cdot\gamma(k) : \frac{\mu_D{}^2}{2\sigma_X}\cdot\gamma(k) : \frac{\mu_L}{2\sigma_X}\cdot\gamma(k) = \sigma_D^2 : \mu_D{}^2 : \mu_L. \tag{6}$$

Expression (6) indicates that $\delta_D^2 = \frac{\sigma_D^2}{\mu_D^2}$ (the coefficient of variation of demand) can be used to determine whether total cost is more sensitive to changes in lead time mean or lead time variance. Above, we show that δ_D^2 is a useful indicator in both optimal (the result of Das [16]) and non-optimal conditions. However, note that (6) does not apply when there are simultaneous changes in the parameters – as may well apply for various improvement initiatives.

It is clear from expressions (1) and (3) that both lead time mean and lead time variance influence the total cost only via σ_X . In (3), the coefficient of the lead time mean is $\sigma_D{}^2$ and the coefficient of the lead time variance is $\mu_D{}^2$. Thus we can use $(\Delta\mu_L)\sigma_D{}^2 + (\Delta\sigma_L^2)\mu_D{}^2$ ($\Delta\mu_L$ denotes the change in lead time mean and $\Delta\sigma_L^2$ denotes the change in lead time variance) to measure the marginal change in performance of any alternative which increases or decreases the two variables simultaneously, but in different degrees.

Incentive to Change We now examine some additional properties of the marginal values. Furthermore, we extend the incentives to improve lead time performance given by Blackburn [23] to the case of stochastic lead time.

The following proposition shows the effects of the changes of demand variance, lead time mean and lead time variance on the marginal values of the three, respectively.

Proposition 1: *The marginal value of demand variance* (*MVD) is a convex, decreasing function of* σ_D^2*, the marginal value of lead time mean* (*MML) is a convex, decreasing function of* μ_L*, and the marginal value of lead time variance (MVL) is a convex, decreasing function of* σ_L^2.

All the proofs are provided in the Appendix. For a manufacturer undertaking process improvements to reduce lead time mean, demand variance or lead time variance, Proposition 1 implies that there are increasing marginal cost savings from reductions of all three variables, creating greater incentives for further reductions. In other words, greater inventory savings can be achieved for cases with faster response, less variable demand and a more reliable supply process.

We next observe that these marginal values are minimized at the optimal value of *k*. Specifically, we make the following proposition:

Proposition 2: *Let k* be the value of k at which TC(k) is minimized, then k* is the value of k at which the marginal value of demand variance (MVD), the marginal value of lead time mean (MML) and the marginal value of lead time variance (MVL) are simultaneously minimized.*

Proposition 2 implies that, compared with a firm operating optimally, a firm managing inventories non-optimally gains more benefit from reducing lead time, demand variance and lead time variance. Furthermore, because *MVD, MML* and *MVL* are all convex in *k*, larger deviations from optimality imply larger marginal values for the changes. This is not, of course, a benefit of non-optimal behavior; it only suggests that organizations not managing inventories optimally have a greater incentive to reduce their lead time mean, lead time variance and demand variance.

Note that in our model, the marginal value of lead time is minimized at *k**. This result differs from that of Blackburn [23] who finds that with the assumption of constant lead time and normally distributed demand, the marginal value is minimized *near, but not exactly at, k**. This difference arises since in the model of Blackburn [23] total cost includes the expected number of backorders outstanding.

4.2 Correlated lead time mean and variance

We now consider the case where lead time mean and variance change simultaneously. Industry data on lead time show the ratio between lead time standard deviation and mean differing little between suppliers of similar products (Silver and Robb [33]). In such cases it would not be reasonable to calculate the marginal value of lead time mean and variance separately. In general, we assume that the mean and variance (the standard deviation to be precise) of lead time are correlated according to the following relationship:

$$\sigma_L = \beta\mu_L^{\alpha} \quad \text{where} \quad \alpha \neq 0 \quad \text{and} \quad \beta > 0\,. \tag{7}$$

We then substitute (7) into (3) to derive the marginal value of lead time mean (*MML*) and the marginal value of variance of lead time (*MVL*) in a manner similar to that in the previous section, viz.,

$$MML = \frac{\partial TC}{\partial \mu_L} = \frac{\sigma_D^2 + 2\alpha\mu_L^{2\alpha-1}\beta^2\mu_D^2}{2\sigma_X}\cdot\gamma(k), \tag{8}$$

$$MVL = \frac{\partial TC}{\partial \sigma_L^2} = \frac{\mu_D^2 + \dfrac{1}{2\alpha\beta^{1/\alpha}}(\sigma_L^2)^{\frac{1}{2\alpha}-1}\sigma_D^2}{2\sigma_X}\cdot\gamma(k). \tag{9}$$

The marginal values include terms additional to those in (4) and (5). When α is positive, these additional terms are positive, signaling the extra benefits a firm can gain. When the firm reduces the lead time mean, it simultaneously reduces lead time variance.

The ratio of the marginal values is as follows,

$$MML : MVL = \frac{\delta_D^2 + 2\alpha\mu_L^{2\alpha-1}\beta^2}{\frac{1}{2\alpha\beta^{1/\alpha}}(\beta^2\mu_L^{2\alpha})^{\frac{1}{2\alpha}-1}\delta_D^2 + 1} = 2\alpha\mu_L^{2\alpha-1}\beta^2 = 2\alpha\frac{\sigma_L^2}{\mu_L}. \qquad (10)$$

This result is quite different from expression (6). When lead time mean and variance can be changed independently, the relative effect on cost depends only on the ratio of demand variance to demand mean. However, when the lead time mean and variance are correlated, the ratio becomes linear in α and is independent of β. Thus, β, which may be determined empirically by regressing lead time variance against the mean, has no bearing on the relative importance of the mean and variance.

Now we consider the validity of Propositions 1 and 2 with correlated lead time mean and variance. Proposition 2 still holds, as $\gamma(k)$ is the only term containing *k*. Proposition 1 also holds, provided some constraints are placed on the value of α (which are sufficient).

Proposition 3: *(a) if* $\alpha \in (0, \frac{1}{2}]$, *the marginal value of lead time mean (MML) is a convex, decreasing function of* μ_L; *(b) if* $\alpha \in [\frac{1}{2}, \infty)$, *the marginal value of lead time variation (MVL) is a convex, decreasing function of* σ_L^2.

When $\alpha = \frac{1}{2}$, the first and second derivatives of both *MML* and *MVL* have the same sign as in the case in which lead time mean and variance are independent. For other values of α, *MML* or *MVL* may not be monotone decreasing. Unlike the case in which lead time mean and variance are independent, the reduction in total cost may not increase with the reduction of lead time mean and variance. In fact, as Proposition 4 shows below, critical points exist for lead time mean and variance at which the monotonicity of marginal values changes.

Proposition 4: *(a) For* $\alpha \in (0,1]$ *and all* $\mu_L \geq 0$, *the marginal value of lead time mean (MML) is a decreasing function of* μ_L; *For* $\alpha < 0$ *or* $\alpha > 1$, *there exists a critical point* μ_L^* *at which the monotonicity of MML changes.*

(b) For $\alpha \geq \frac{1}{4}$ *and all* σ_L^2, *the marginal value of lead time variance (MVL) is a decreasing function of* σ_L^2; *For* $\alpha < 0$ *or* $\alpha \in (0, \frac{1}{4})$, *there exists a critical point* σ_L^{2*} *at which the monotonicity of MVL changes.*

Proposition 4 implies that in the correlated case, evaluating the 'incentive' for firms seeking to improve their inventory performance by reducing the lead time mean or variance is a more complex function of lead time mean. Specifically, we have the monotonicity of marginal values as shown in Table 1.

<table>
<tr><th rowspan="2"></th><th rowspan="2">Independent</th><th colspan="3">Correlated</th></tr>
<tr><th>Value of α</th><th>Below the critical point</th><th>Above the critical point</th></tr>
<tr><td rowspan="3">MML</td><td rowspan="3">Decreasing</td><td>$\alpha < 0$</td><td>Increasing</td><td>Decreasing</td></tr>
<tr><td>$\alpha \in (0,1]$</td><td>Decreasing</td><td>Decreasing</td></tr>
<tr><td>$\alpha > 1$</td><td>Decreasing</td><td>Increasing</td></tr>
<tr><td rowspan="3">MVL</td><td rowspan="3">Decreasing</td><td>$\alpha < 0$</td><td>Increasing</td><td>Decreasing</td></tr>
<tr><td>$\alpha \in (0,1/4)$</td><td>Decreasing</td><td>Increasing</td></tr>
<tr><td>$\alpha \geq 1/4$</td><td>Decreasing</td><td>Decreasing</td></tr>
</table>

Table 1: Changes of monotonicity of marginal values

One may expect decreasing returns at first when seeking to reduce the lead time mean if the variance is very sensitive to the change of mean (i.e., with large α). On the other hand, firms seeking to reduce lead time variance may obtain decreasing returns at first if the variance is not very sensitive to the change of mean (with small α). However, as illustrated by Proposition 1, the incentive for further reductions still exists for the correlated case in the sense that increasing returns will always be observed when the mean or variance is small enough. When the mean and variance are negatively correlated, e.g., the supplier offers slower but more reliable performance (it is easier for the supplier to deliver on time if it promises a longer lead time), they should be mindful of decreasing marginal values and potentially negative marginal values (as seen in the numerical example below) when the mean or variance is small.

5 Numerical Examples

In this section, we use a numerical example to illustrate the above results in a practical situation. We consider the case of Springfield Manufacturing, the truck and automotive component supplier first described in Section 1. In sourcing their components both in North America and offshore, Springfield's supply chain has both demand and lead time (supply) variability. Their main customer—an assembler of large over-the-road trucks—provides a general forecast of the overall level of component demand but requires JIT shipments of components of an amount equal to one or two days' demand. Springfield must make to stock as these components are manufactured in Mexico and shipped to a distribution site in the Southeastern US.

We examine the effect of lead time on the cost of managing the inventory for one of their highest demand truck components. The cost of production and distribution for the component was \$25/unit. Daily demand for the item was about 90 units with a standard deviation of 28 units (Weekly demand was 450 units with a standard deviation of 62.6). A normal distribution provided an adequate fit to demand for the component and no autocorrelation was found in historical data . Production order quantities (Q) equaled about 4 weeks demand or about 1800 units. The setup cost of a production lot (A) was about \$250, and the annual cost of carrying a unit in stock (h) was \$3.75 (or about 15% of the cost of the unit). The penalty cost per unit for a backorder (B) was estimated to be \$25. The sum of supply and production lead time (also approximately normally distributed) was about 30 days with a standard deviation of 0.3.

In Table 2, we show the marginal values of lead time mean and lead time variation, for k from 1.9 to 2.7, obtained by substituting the Springfield data given above into equations (4) and (5).

k	*MML*	*MVL*
1.9	26.94	278.29
2.0	25.83	266.91
2.1	25.14	259.76
2.2	24.78	255.98
2.3	24.74	255.59
2.4	24.86	256.89
2.5	25.24	260.73
2.6	25.77	266.26
2.7	26.31	271.78

Table 2: Marginal values in the independent case

Taking one row from Table 2, we can calculate the ratio of marginal value of the lead time mean to that of lead time variance. For example, when *k=1.9*, we have $\frac{26.94}{278.29} \approx 0.097$ which equates to $(\frac{\sigma_D}{\mu_D})^2 = (\frac{28}{90})^2 \approx 0.097$ as in expression (6).

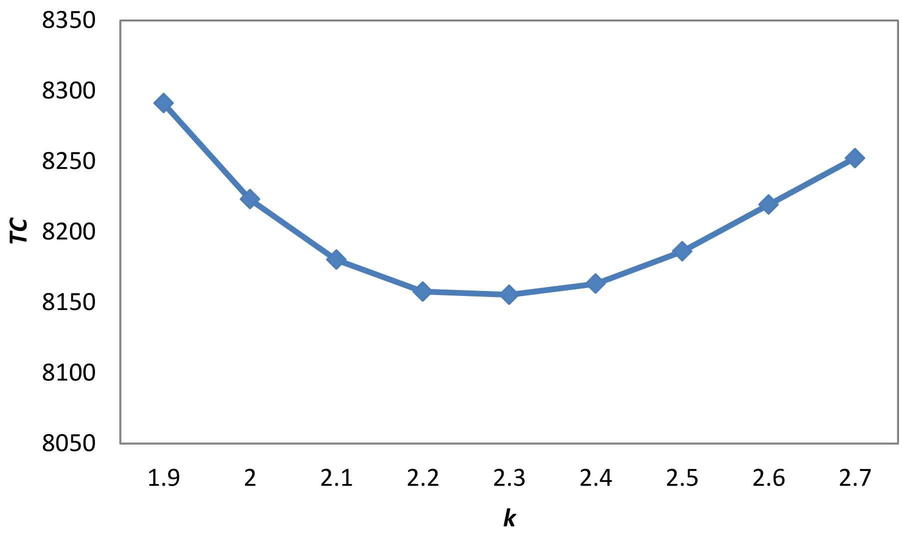

Figure 2: Total inventory cost for different values of *k*

Figure 2 shows how the total inventory cost changes with respect to different k values (from 1.9 to 2.7). As we can see, cost is minimized with k near 2.3 and, a point at which *MML* and *MVL* are also minimized (from Table 2). Thus, an equivalent lead time reduction would yield greater benefit in a firm managing its inventory with non-optimal *k* than one managing its inventory optimally.

Next we examine the case when lead time mean and variance are correlated through the relationship $\sigma_{LS} = \beta\mu_{LS}^{\alpha}$. We fix α at 1 and 0.5 and select β so that the original lead time mean and variance satisfy the correlation functions $\beta = \frac{\sigma_L}{\mu_L^{\alpha}} = \frac{0.3}{30^1} = 0.01$ and $\frac{0.3}{30^{0.5}} \approx 0.055$. Table 3 is then obtained using equations (8) and (9).

k	α =1, β =0.01		α =0.5, β =0.055	
	MML	*MVL*	*MML*	*MVL*
1.9	28.61	4767.51	27.78	9182.54
2.0	27.44	4572.58	26.64	8807.09
2.1	26.70	4450.05	25.93	8571.09
2.2	26.31	4385.44	25.55	8446.65
2.3	26.27	4378.76	25.51	8433.78
2.4	26.41	4401.04	25.64	8476.69
2.5	26.80	4466.76	26.02	8603.27
2.6	27.37	4561.44	26.58	8785.63

2.7	27.94	4656.12	27.13	8968.00

Table 3: Marginal values in the correlated case

When α is positive, for a given k the marginal value of lead time mean or variance is greater than is the case in Table 2. Again setting k equal to 1.9, the ratio of *MML* with respect to *MVL* is 28.61/4767.51=0.006 and 27.78/9182.54 =0.003, for $\alpha=1$ and $\alpha=0.5$, respectively. The ratios are proportional to α and could also be calculated using expression (10), viz., $2\alpha\frac{\sigma_L^2}{\mu_L}=2\times\frac{0.3^2}{30}=0.006$ and $\frac{0.3^2}{30}=0.003$.

Finally, in Figure 3 we show that when lead time mean and variance are correlated the incentive for the firm to reduce lead time mean is a more complex function of lead time mean. Assuming that lead time mean and variance are correlated through $\alpha=-1$, 1 or 2 and β is chosen such that the original lead time mean and variance satisfy $\sigma_L=\beta\mu_L^\alpha$ as above, we consider the marginal value of the mean reduction of the lead time.

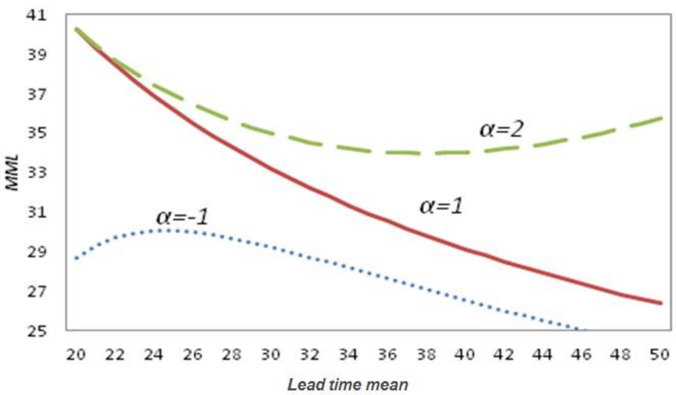

Figure 3: Marginal value of lead time mean for different values of α

The results are consistent with Proposition 4. When lead time mean and variance are correlated, the monotonicity of the marginal values depends on the value of α. When $\alpha\in(0,1]$ we still have increasing marginal value. When $\alpha<0$, we have increasing marginal value at first then decreasing marginal value after a critical point. This indicates that the firm should be aware of the changes in monotonicity of the marginal value when deciding whether to accept a proposal from its supplier for a more stable but longer lead time. When $\alpha > 1$, we have decreasing marginal value at first then and increasing marginal value after a critical point.

6 Conclusions and Discussion

This paper considers the marginal values of demand variance, lead time mean, and lead time variance. We constructed a general model dealing with both demand and supply variability. The model does not require an optimized inventory policy. We find that benefits from improving demand or supply processes are greater for firms with non-optimal inventory policies (in practice, this may refer to firms that manage their inventories without standard policies, or by *ad hoc* decision making) than for firms that manage their inventories optimally.

We compare the case in which lead time mean and variance are independent with the case in which they are correlated through a functional relationship. We find that in the independent case, the ratio of the marginal values of lead time mean and lead time variance is determined by the coefficient of variation of demand. This is because the impact of lead time mean and variance on the total inventory cost is determined by the demand during the lead time. On the other hand, when lead time mean and variance are correlated, the difference between the marginal values of the two can no longer be determined by the coefficient of variation of demand, but is a function of the coefficient of variation of the lead time. Furthermore, the benefit from a reduction in lead time mean or variance is more complex. Firms cannot expect increasing returns from reductions in lead time mean or lead time variance under all conditions. They should be aware of the potential for decreasing or even negative returns (in the case where the two are negatively correlated).

The results of this paper can help decision makers and managers with limited resources choose among a number of inventory improvement alternatives and determine where to invest their resources, even when they face poorly managed inventory or no standard inventory policy. Employing data on the demand or lead time distributions, the results concerning marginal values provide managers with guidance about the effects of changing demand variation, lead time mean, and lead time variation. At the same time, the incentive to change reminds managers of the importance of inventory management improvement, especially for firms that have paid little attention to their inventory policies - since they stand to gain even more.

We contend that many assumptions, such as requiring an optimal inventory policy, may not be essential for modeling purposes. As non-optimal policies are common in practice (e.g., the cases mentioned by Blackburn, [23]), it may prove useful to relax the optimality assumptions and to compare the differences. The current paper has made a step in this direction.

The paper has some limitations that may warrant extensions. For example, we assume that lead time mean and variance are correlated through a deterministic equation, whereas in practice firms may be faced with various discrete improvement plans reflected by a different α and β, in which case our continuous model may not be a good fit. We plan to test this issue in future research.

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Appendix

Proof of Proposition 1:

$$\frac{\partial MVD}{\partial \sigma_D^2} = -\frac{\mu_L{}^2}{4\sigma_X^3}\cdot\gamma(k) < 0 \quad \text{and} \quad \frac{\partial^2 MVD}{\partial \sigma_D^4} = \frac{3\mu_L{}^3}{8\sigma_X^5}\cdot\gamma(k) > 0;$$

$$\frac{\partial MML}{\partial \mu_L} = -\frac{\sigma_D^4}{4\sigma_X^3}\cdot\gamma(k) < 0 \quad \text{and} \quad \frac{\partial^2 MML}{\partial \mu_L^2} = \frac{3\sigma_D^6}{8\sigma_X^5}\cdot\gamma(k) > 0;$$

$$\frac{\partial MVL}{\partial \sigma_L^2} = -\frac{\mu_D^4}{4\sigma_X^3}\cdot\gamma(k) < 0 \quad \text{and} \quad \frac{\partial^2 MVL}{\partial \sigma_L^4} = \frac{3\mu_D^6}{8\sigma_X^5}\cdot\gamma(k) > 0.$$

Since the first and second partial derivatives of *MVD*, *MML* and *MVL* with respect to σ_D^2, μ_L and σ_L^2 are, respectively, negative and positive, the convex and decreasing characters of the functions are established. Q.E.D.

Proof of Proposition 2:

Since *TC*(*k*) is minimized at *k**, $\left.\frac{\partial \mathrm{TC}(k)}{\partial \mathrm{k}}\right|_{k=k^*} = 0$ and $\left.\frac{\partial^2 \mathrm{TC}(k)}{\partial \mathrm{k}^2}\right|_{k=k^*} > 0$. As *k* is only contained in expression (2), $\left.\frac{\partial \gamma(k)}{\partial \mathrm{k}}\right|_{k=k^*} = 0$ and $\left.\frac{\partial^2 \gamma(k)}{\partial \mathrm{k}^2}\right|_{k=k^*} > 0$. Furthermore,

$$\left.\frac{\partial \mathrm{MVD}}{\partial \mathrm{k}}\right|_{k=k^*} = \frac{\mu_L}{2\sigma_X}\cdot\left.\frac{\partial \gamma(k)}{\partial \mathrm{k}}\right|_{k=k^*} = 0 \quad \text{and} \quad \left.\frac{\partial^2 \mathrm{MVD}}{\partial \mathrm{k}^2}\right|_{k=k^*} = \frac{\mu_L}{2\sigma_X}\cdot\left.\frac{\partial^2 \gamma(k)}{\partial \mathrm{k}^2}\right|_{k=k^*} > 0 \quad .$$

Therefore, *MVD* is minimized at *k**. For *MML* and *MVL*, the proof is identical. Q.E.D.

Proof of Proposition 3:

Define $\Omega_M = \sigma_D^2 + 2\alpha\mu_L^{2\alpha-1}\beta^2\mu_D^2$. Then for $\alpha \in (0, \frac{1}{2}]$, we have

$$\frac{\partial MML}{\partial \mu_L} = [\frac{\alpha(2\alpha-1)\mu_L^{2\alpha-2}\beta^2\mu_D^2}{\sigma_X} - \frac{\Omega_M^2}{4\sigma_X^3}]\cdot\gamma(k) < 0,$$

$$\frac{\partial^2 MML}{\partial \mu_L^2} = [\frac{\alpha(2\alpha-1)(2\alpha-2)\mu_L^{2\alpha-3}\beta^2\mu_D^2}{\sigma_X} - \frac{\alpha(2\alpha-1)\mu_L^{2\alpha-2}\beta^2\mu_D^2\Omega_M}{2\sigma_X^3}$$
$$-\frac{\alpha(2\alpha-1)\mu_L^{2\alpha-2}\beta^2\mu_D^2\Omega_M}{\sigma_X^3} + \frac{3\Omega_M^3}{8\sigma_X^5}]\cdot\gamma(k) > 0.$$

Similarly, we define $\Omega_V = \mu_D^2 + \frac{1}{2\alpha\beta^{1/\alpha}}(\sigma_L^2)^{\frac{1}{2\alpha}-1}\sigma_D^2$ and for $\alpha \in [\frac{1}{2}, \infty)$

$$\frac{\partial MVL}{\partial \sigma_L^2} = \frac{\frac{1}{2\alpha\beta^{1/\alpha}}(\frac{1}{2\alpha}-1)(\sigma_L^2)^{\frac{1}{2\alpha}-2}\sigma_D^2}{2\sigma_X} - \frac{\Omega_V^2}{4\sigma_X^3}\cdot\gamma(k) < 0,$$

$$\frac{\partial^2 MVL}{\partial \sigma_L^4} = [\frac{\frac{1}{2\alpha\beta^{1/\alpha}}(\frac{1}{2\alpha}-1)(\frac{1}{2\alpha}-2)(\sigma_L^2)^{\frac{1}{2\alpha}-3}\sigma_D^2}{2\sigma_X} - \frac{\frac{1}{2\alpha\beta^{1/\alpha}}(\frac{1}{2\alpha}-1)(\sigma_L^2)^{\frac{1}{2\alpha}-2}\sigma_D^2\Omega_V}{4\sigma_X^3}$$
$$-\frac{\frac{1}{2\alpha\beta^{1/\alpha}}(\frac{1}{2\alpha}-1)(\sigma_L^2)^{\frac{1}{2\alpha}-2}\sigma_D^2\Omega_V}{2\sigma_X^3} + \frac{3\Omega_V^3}{8\sigma_X^5}]\cdot\gamma(k) > 0.$$

Q.E.D.

Proof of Proposition 4:

(a) Consider the first derivative of *MMLE* to calculate the critical point for μ_L:

$$\frac{\partial MML}{\partial \mu_L} = [\frac{\alpha(2\alpha-1)\mu_L^{2\alpha-2}\beta^2\mu_D^2}{\sigma_X} - \frac{(\sigma_D^2 + 2\alpha\mu_L^{2\alpha-1}\beta^2\mu_D^2)^2}{4\sigma_X^3}]\cdot\gamma(k) = 0.$$

After simplification, we have

$$4\alpha(\alpha-1)\mu_L^{4\alpha-2}\beta^4\mu_D^4 + 8\alpha(\alpha-1)\mu_L^{2\alpha-1}\beta^2\mu_D^2\sigma_D^2 - \sigma_D^4 = 0.$$

If $\alpha = 1$, we have $\frac{\partial MML}{\partial \mu_L} < 0$ for all $\mu_L \geq 0$.

For $\alpha \neq 1$, define $\omega = \mu_L^{2\alpha-1}\beta^2\mu_D^2$. Then one can view the above as a quadratic

function of ω with $\omega_1 + \omega_2 = -2\sigma_D^2$, $\omega_1\omega_2 = \frac{-\sigma_D^4}{4\alpha(\alpha-1)}$ and discriminant

$64\alpha^2(\alpha-1)^2\sigma_D^4 + 16\alpha(\alpha-1)\sigma_D^4$, where ω_1, ω_2 are the two real roots of the function.

Then, if $\alpha \in (0,1)$, the function has no positive roots and $\frac{\partial MML}{\partial \mu_L} < 0$ for all $\mu_L \geq 0$.

If $\alpha < 0$ or $\alpha > 1$, the function has one positive root ω^* (μ_L^* accordingly). Then $\frac{\partial MML}{\partial \mu_L}$ changes sign at that point.

(b) The proof is similar, with the simplified equation:

$$\frac{1}{\alpha}\left(\frac{1}{4\alpha}-1\right)(\sigma_L^2)^{1/\alpha-2}\beta^{-2/\alpha}\sigma_D^4+\frac{2}{\alpha}\left(\frac{1}{4\alpha}-1\right)(\sigma_L^2)^{1/2\alpha-1}\beta^{-1/\alpha}\sigma_D^2\mu_D^2-\mu_D^4=0.$$

Then we can define $\omega' = (\sigma_L^2)^{1/2\alpha-1}\beta^{-1/\alpha}\sigma_D^2$ and obtain the required result. Q.E.D.