Early Childhood Research Quarterly 66 (2024) 124–134

Contents lists available at ScienceDirect

Early Childhood Research Quarterly

journal homepage: www.elsevier.com/locate/ecresq

Parental early math support: The role of parental knowledge about early math development

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ARTICLE INFO

Keywords:
Home math environment
Parent math support
Repeating patterning knowledge
Parent knowledge
Parent math beliefs
Socioeconomic status

ABSTRACT

Parents vary substantially in the frequency and complexity of the math support that they provide to their children, and this variability is often related to their children's math knowledge. We hypothesized that parents' knowledge about the development of two critical early math topics would help explain some of this variability in their early math support. U.S. parents of 3- and 4-year-olds ($N = 196$ mothers and 148 fathers, 94% identified as the child's primary caregiver and 77% as White; 79% had at least a bachelor's degree) reported on their knowledge about the development of early numeracy and repeating patterning skills, numeracy and repeating patterning beliefs related to their children, numeracy and repeating patterning support, and education, income, and employment status via a survey. Parents' knowledge about early repeating patterning development was positively related to all the measured child-specific repeating patterning beliefs and both were predictive of the frequency and complexity of their reported repeating patterning support. Their knowledge about early numeracy development was also positively related to most of their child-specific numeracy beliefs, but while their child-specific numeracy beliefs uniquely predicted their reported numeracy support, their knowledge did not. Parents' knowledge about early numeracy and repeating patterning development was not consistently related to their education, income, nor employment status, but their education and employment status uniquely predicted their numeracy and repeating patterning support. Implications of these findings for research, theory, and parent-based interventions are discussed.

Children's mathematics knowledge prior to formal schooling or kindergarten (which they typically begin at age 5 in the US) is predictive of their later academic achievement as well as other life factors including their socioeconomic status (SES) in adulthood (e.g., Duncan et al., 2007; Ritchie & Bates, 2013). The most widely studied component of early mathematics knowledge is numeracy knowledge (i.e., the meaning of whole numbers, number relations, and number operations; Jordan et al., 2009; National Research Council, 2009). Children's early numeracy knowledge is the focus of most early mathematics standards, instruction, assessments, and theories of early mathematics development, and is the most robust predictor of their long-term academic achievement. For instance, preschool-aged children's numeracy knowledge uniquely predicted their math knowledge three to seven years later (Fyfe et al., 2019; Nguyen et al., 2016).

Much of mathematics also involves patterning (i.e., identifying, extending, describing and using predictable sequences in objects and numbers; Charles, 2005; Rittle-Johnson et al., 2013; Sarama & Clements, 2004). Preschool children most often engage with *repeating patterns* or linearly arranged sequences of elements with a unit that repeats (e.g., red-blue-red-blue-red-blue). Knowledge of repeating patterns at the end of preschool uniquely predicted children's math knowledge at the end of kindergarten and in 4th through 6th grade (e.g., Fyfe et al., 2019). We refer to "repeating patterning" as "patterning" throughout the paper for brevity. There is also growing evidence that improving children's numeracy and patterning knowledge leads to improvements in their future math achievement (e.g., Baroody et al., 2009; Kidd et al., 2014). Thus, we focus on these two areas of early mathematics knowledge, as they are the two math topics with the more robust theory and evidence for their influence on developing school mathematics knowledge.

1. Parents' early math support

The variability in children's math knowledge before formal schooling points to the potential role of their engagement in and exposure to math at home. Indeed, the *frequency of parents' numeracy support at home* is positively associated with children's early and later math

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https://doi.org/10.1016/j.ecresq.2023.10.003
Received 27 January 2023; Received in revised form 22 September 2023; Accepted 6 October 2023
Available online 11 October 2023

knowledge (see Daucourt et al., 2021 and Mutaf-Yıldız et al., 2020 for reviews). The *complexity of parents' early numeracy support* (the extent to which they provide support focused on more advanced numeracy skills within preschoolers' typical zone of proximal development) seems particularly important for children's numeracy development (e.g., Skwarchuk et al., 2014). However, past research suggests that parents of preschoolers tend to miss opportunities to provide support for more advanced ways of understanding and using numbers that can help push their children's numeracy development (e.g., Ramani et al., 2015; Susperreguy et al., 2020; Vandermaas-Peeler et al., 2012; Zippert & Ramani, 2017). There is also some evidence from a few small-scale studies that the *frequency of parents' patterning support* is positively related to their children's patterning knowledge (e.g., Rittle-Johnson et al., 2015). In these studies, parents infrequently reported providing complex support (i.e., support focused on more advanced, developmentally-appropriate patterning skills), perhaps indicating that they have limited knowledge about the range of early patterning skills that preschoolers can learn. Thus, it is important to understand why and how parents provide early numeracy and patterning support.

Previous research has highlighted the role of parents' SES and math beliefs in shaping their math support as reflected in existing theoretical models of parent socialization of math (Douglas et al., 2021; Eccles et al., 1983). The current study aimed to understand the nature and role of an understudied factor - parents' knowledge about early numeracy and patterning development - in shaping the numeracy and patterning support they provide to their preschool-aged children. Thus, we propose and test aspects of an expanded model of parent socialization of early math development that includes parents' knowledge about early math development and focuses on both numeracy and patterning (see Fig. 1). Specifically, we propose that *parents' knowledge about early math development* is related to their (a) *SES* and helps explain variability in their *(b) math beliefs* and *(c) early math support*.

2. The potential role of parents' knowledge about math development

Although the Home Mathematics Environment (HME) has been defined as "a multidimensional construct that consists of parents' values, attitudes, and beliefs about mathematics, as well as their *knowledge*, experience, and the resources they have to promote children's mathematics development" (Cosso et al., 2023), little research has examined parents' knowledge about early math development. We hypothesize that parents' knowledge about early math development- their awareness of the various math skills that are developmentally appropriate and academically important for preschoolers to develop - is related to their SES, helps explain variability in their math support and beliefs, and will be an important addition to models of the HME. Our hypotheses about parents' knowledge about early math development align with research on the role of parents' knowledge about *child* development in the home literacy environment which we describe next.

2.1. Parents' knowledge about child development and the home literacy environment

Parents' knowledge about child development, their awareness of developmental norms and milestones and strategies for promoting children's growth, is predictive of the home literacy environment that they facilitate and their children's literacy skills (Rowe et al., 2016; Sonnenschein & Sun, 2017). Importantly, parents' knowledge about child development is malleable and is causally related to their beliefs, support, and their children's language skills (Albarran & Reich, 2014; Auger et al., 2014). For instance, providing mothers with information about child development via baby books during their child's first year improved their maternal self-efficacy and their children's later language skills (Albarran & Reich, 2014). Notably, parents' knowledge about child development helped explain differences in the frequency of their academic support and their children's academic skills that were

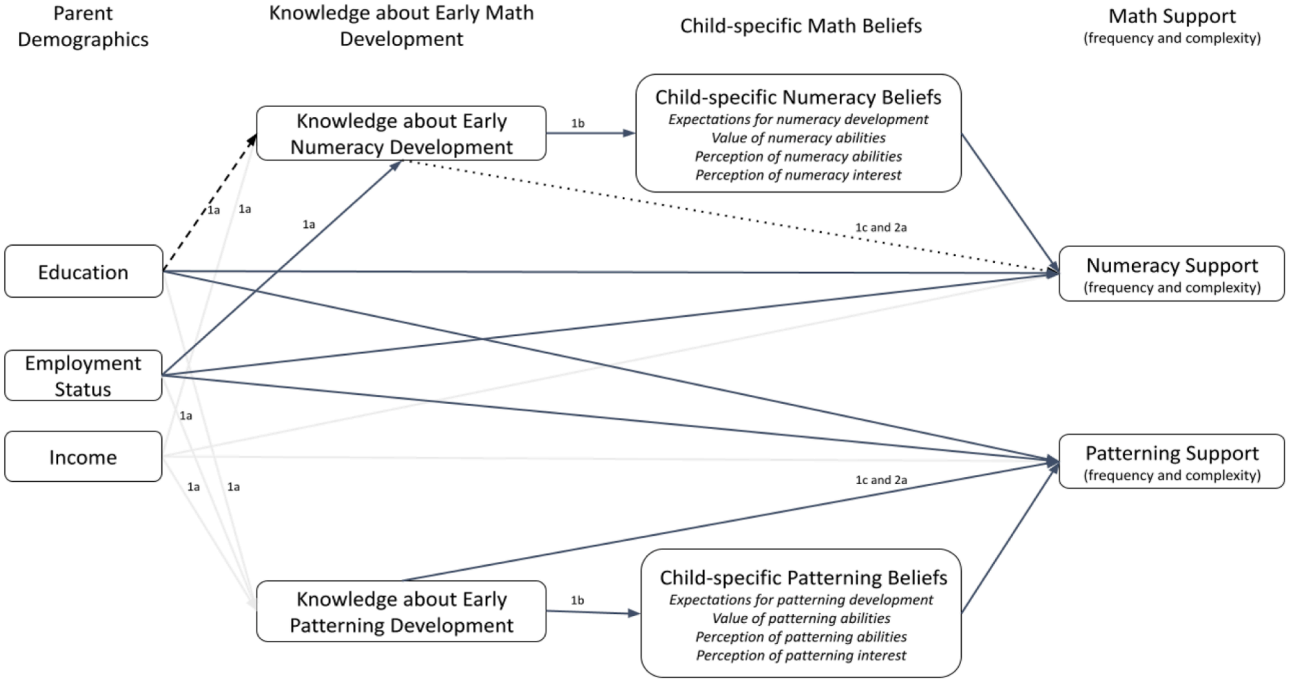

Fig. 1. Model summarizing current study findings regarding research questions 1 and 2.
Notes. The labels 1a-1c identifies the relations associated with research question 1 while the label 2a identifies the relations associated with research question 2. The broken line indicates a negative relation. The dotted line indicates a relation for which there was mixed evidence (i.e., knowledge about early numeracy development was positively related to the frequency of numeracy support but was unrelated to the complexity of numeracy support and was not a unique predictor of numeracy support). The gray lines indicate non-significant evidence for relations. The absence of an arrow does not necessarily indicate an absence of a relationship (e.g., analyses were not conducted for relations between patterning and numeracy variables nor how parent demographic variables related to beliefs).

associated with parents' highest educational attainment, an aspect of their SES (Rowe et al., 2016).

2.2. Past research on parents' knowledge about math development

Three studies have examined the nature of knowledge about early math development among parents in the US, Canada, and the UK and have found that parents have some knowledge about early math development but tend to not know much about its nuances (DeFlorio & Beliakoff, 2015; Fluck et al., 2005; Skwarchuk, 2009). For instance, most parents incorrectly anticipated that their child understood aspects of the numeracy skill of cardinality (i.e., understanding that the last count word used after accurately counting a set represents the quantity of items in the set) irrespective of their child's age and counting abilities (Fluck et al., 2005). In a second study, parents rated almost all activities that are appropriate for preschoolers, including ones that are not viewed as critical to math like "large muscle play", as being "important'' through "essential'' in promoting math development (Skwarchuk, 2009). In a third study, parents' accuracy at classifying math skills as ones that most children typically develop by age 5 and ones that are beyond most preschoolers' typical zone of proximal development varied substantially depending on the skill, ranging from a mean accuracy of 0.21 to 0.93 (DeFlorio & Beliakoff, 2015). Additionally, parents' knowledge about early math development was a unique, positive predictor of their child's math skills (DeFlorio & Beliakoff, 2015).

2.3. Parents' knowledge and their early math support

Previous research has not considered how parents' knowledge about early numeracy development relates to their numeracy support nor examined parents' knowledge about early patterning development. Thus, the current study examined how parents' knowledge about early numeracy and patterning development relate to their numeracy and patterning support, respectively (illustrated as 1c and 2a in Fig. 1). We anticipated that parents who had more accurate knowledge of the numeracy and patterning skills that preschoolers can learn would put more concerted effort towards helping their children develop these skills by engaging their children in related activities more frequently than parents with less knowledge. Relatedly, we anticipated that parents with more accurate knowledge of the range of numeracy and patterning skills that preschoolers can be developing would be more likely to engage their children in activities that focus on or support more complex, developmentally appropriate numeracy and patterning skills in comparison to parents with less knowledge (e.g., parents who think that counting and numeral recognition are the only developmentally appropriate numeracy skills for preschoolers).

2.4. Parents' knowledge and their math beliefs

Parents' knowledge about early math development might also help explain variability in their child-specific math beliefs. Parents hold various types of math beliefs, with past evidence focused primarily on numeracy beliefs. Parents' numeracy beliefs about their preschoolers (i.e., child-specific beliefs) are the most consistently related to the frequency and complexity of their early numeracy support (see Douglas et al., 2021 for a review). For example, parents who reported higher *value* of their children's numeracy development also reported more frequent and more complex numeracy support (Napoli et al., 2021; Skwarchuk et al., 2014). Similarly, parents who believed their children had better *numeracy abilities* tended to also report more frequent and more complex numeracy support (Uscianowski et al., 2020; Zippert & Ramani, 2017; Zippert & Rittle-Johnson, 2020). There is also anecdotal evidence that parents' beliefs about their children's *numeracy interests* are related to the frequency of their numeracy support (Cannon & Ginsburg, 2008). According to one study, parents' *expectations* for their child's numeracy development (how well they think their child will do in math in kindergarten) uniquely predicted their children's math skills but was unrelated to parents' math support (Kleemans et al., 2012). Unfortunately, only one study has examined the relations between parents' child-specific patterning beliefs and their early patterning support (Zippert & Rittle-Johnson, 2020). The study did not find evidence for a relation between any of the measured child-specific patterning beliefs and parents' patterning support, but it only had sufficient power to detect a large effect.

Further, little is known about factors that shape their numeracy or patterning beliefs. We hypothesize that parents' knowledge about early math development might be an important factor (illustrated as 1b in Fig. 1). For instance, knowing which math skills preschoolers typically learn might inform parents' expectations for their preschoolers' math development which may lead to them providing their preschoolers with more frequent or targeted support to meet those expectations. Likewise, parents with more accurate knowledge about early math development may value their preschoolers' development of these skills more than other parents and may therefore provide more frequent or more complex math support to their preschoolers.

2.5. Parents' knowledge and their SES

Parents' knowledge about early math development might also be related to their SES and help explain differences in their math support and beliefs associated with their SES. Specifically, parents with more financial resources reported more positive numeracy beliefs and more frequent numeracy support than parents with fewer financial resources (e.g., Casey et al., 2018; Vandermaas-Peeler et al., 2009; Zippert & Rittle-Johnson, 2020). Parents with more financial resources also demonstrated more accurate knowledge about early math development than parents with fewer financial resources (DeFlorio & Beliakoff, 2015). Similarly, parents with more advanced education demonstrated more positive child-specific numeracy beliefs and more frequent support (Gaylord et al., 2020; Thompson et al., 2017), but it is unknown whether their knowledge about early math development also varies with their highest educational attainment. Given past research suggesting that parental education is sometimes a better predictor of cognitive and educational attainment than other indicators of SES (see Bradley & Corwyn, 2002 for a review), we examine whether parents' knowledge about early math development also varies with their highest educational attainment (illustrated as 1a in Fig. 1). Overall, parents' knowledge about early patterning and numeracy development might be related to their SES and might help explain variability in their early patterning and numeracy beliefs and support.

3. Current study

The current study aimed to understand the nature and role of parents' knowledge about early patterning and numeracy development in their efforts to support their children's patterning and numeracy development. We propose and test aspects of an expanded model of parent socialization of early math development that includes parents' knowledge about early math development (see Fig. 1). More specifically, we aimed to validate measures of parents' knowledge about early numeracy and patterning development and to answer two main questions. We made the same hypotheses for numeracy and patterning for both research questions

First, how does parents' knowledge about early numeracy and patterning development relate to components of the Home Math Environment, specifically (1a) their SES, (1b) their child-specific numeracy and patterning beliefs, and (1c) the frequency and complexity of the numeracy and patterning support they provide? As shown in Fig. 1, we hypothesized that

a. Parents with more accurate knowledge about early numeracy and patterning development would have higher educational attainment

given that parents with higher educational attainment tend to provide more frequent numeracy support than parents with lower educational attainment (Gaylord et al., 2020; Thompson et al., 2017) and tend to have children with better early patterning skills (e.g., Zippert & Rittle-Johnson, 2020). We also explored whether a two additional indicators of parents' SES, their household income and employment status, were related to their knowledge to better understand which aspects of parents' SES are most relevant.

b. We hypothesized that parents' knowledge about early numeracy and patterning development would be positively related to their expectations for and value of their 3-year-old or 4-year-old child's *future* numeracy and patterning development respectively. We anticipated that parents' knowledge about early numeracy and patterning development would also be related to their beliefs about their child's *current* numeracy and patterning abilities and interests, but these hypotheses were more exploratory.

c. We hypothesized that parents' knowledge about numeracy and patterning development would be positively related to the frequency and complexity of their numeracy and patterning support in light of research findings that there is wide variability in parents' numeracy and patterning support and that parents report that they do not know what math concepts and skills their preschool-aged child should be learning or how to support their children's numeracy development (e.g., Cannon & Ginsburg, 2008; Ramani et al., 2015).

Our second research question was: to what extent does parents' knowledge about numeracy and patterning development uniquely predict their early numeracy and patterning support? While prior research has examined various parental factors as potential predictors of their numeracy and patterning support, little is known about the role of parents' knowledge about early numeracy and patterning development. We hypothesized that parents' knowledge about early numeracy and patterning development would positively predict the frequency and complexity of their numeracy and patterning support (2a), above and beyond the predictive role of their child-specific numeracy and patterning beliefs and their SES which have been shown to relate to their support (e.g., Douglas et al., 2021; Thompson et al., 2017). We hypothesized that parents' knowledge about numeracy and patterning development would be unique predictors as we view parents' knowledge as distinct from (though possibly related to) their beliefs and SES. Relatedly, we explored the extent to which the relations between parents' knowledge about early numeracy and patterning development and their numeracy and patterning support are mediated by their child-specific numeracy and patterning beliefs.

4. Method

4.1. Participants

Parents of preschoolers ($N = 344$) participated in the study. The study was sufficiently powered to detect up to five small bivariate correlations ($R = 0.20$, $p = .05$) as needed to test the first hypothesis according to Gatsonis and Sampson (1989) power analyses for multiple correlations. The study was also sufficiently powered for linear multiple regression analyses as needed to test the second hypothesis. Specifically, an a priori power analysis conducted using G*power indicated that a sample of 343 participants would have 80% power to detect small to medium effects ($f^2 = 0.057$) of seven predictors. The effect size was estimated from previous research on the relation between parents' value of their child's numeracy development and their numeracy support (Skwarchuk et al., 2014; Susperreguy et al., 2020). Notably, the power analysis was conducted with a Bonferroni-corrected alpha of 0.0125 (i.e., 0.05/4), given that one regression would be conducted for each of the four dependent variables. The analytic plan was preregistered (https://aspredicted.org/2YG_39S).

4.2. Demographics

Participants reported about their 3-year-old (52%) or 4-year-old (48%) child, a majority of whom were their sons (61%) and who heard English at home (99%). About half of their children had attended preschool during the previous school year (2020–2021; 58%). Most parents identified as the child's primary caregiver (94%) and almost half were fathers (43%) and had more than one child (55%), including 20% with children who were 5 years or older.

Over half of the parents' highest educational attainment was a bachelor's degree (55%), while 24% had at least some graduate education, and 21% had some college education or less education. Parents also reported the highest educational attainment of the child's other parent or legal guardian if applicable, with over half having a bachelor's degree (see Table S1). About two-fifths of the parents reported a household income of $45,000-$89,999 (41%), while 31% reported $90,000 or more, and 28% reported $44,999 or less (see Table S2). Most were employed either full-time (79%) or part-time (11%). Chi-square tests indicated that participating parents' highest level of educational attainment was related to their household income, $X2$ (4, $N = 344$) = 49.01, $p < .001$, and their employment status, $X2$ (2, $N = 344$) = 75.02, $p < .001$, with most parents with some college education or less education reporting a household income of $44,999 or less and being employed part-time or being unemployed (see Table S3). Most parents reported receiving financial assistance for their child's preschool attendance (65%).

Most parents were White (77%) while 8% were Black or African American, 5% were Asian or Pacific Islander, 4% were Biracial or Multiracial, and 3% were American Indian or Alaska Native. A few parents indicated that they did not identify as any of the previously described races or ethnicities (2%) or were unsure about or preferred not to share their race (1%). Additionally, 20% of parents identified as Hispanic or Latine. Some were pre-K or elementary school (36%) teachers. About half of the parents reported that they had previously (47%) or were currently (48%) participating in a program where they receive information about family engagement in academics. Parents were from 46 states across the United States (see Table S4).

4.3. Measures

4.3.1. Knowledge about early math development survey

A previously used measure was adapted to assess parents' knowledge about early numeracy and patterning development i.e., before formal schooling which begins around age 5 (DeFlorio & Beliakoff, 2015). The measure was adapted to measure a wider variety of numeracy and patterning skills across five rounds of pilot data collection, analysis, and revision with 288 parents not included in the current sample (Douglas et al., 2023). Parents were asked, "Which of these academic skills do you think that most children in the United States develop by their 5th birthday?" (similar to DeFlorio and Beliakoff (2015)). Parents in the current study were presented with a list of 10 numeracy and 12 patterning skills that typically developing children in the United States develop between ages 3 and 8 years. Seven of the 10 numeracy and nine of the 12 patterning items were about skills that children usually develop by age 5 (i.e., skills that are *within* the typical developmental range for preschool-aged children). Examples include "Counts out the correct number of things when asked for a specific number of things up to 10 (for example gives 6 cookies when asked for 6 cookies)" and "Copy a pattern someone else makes in the same way (for example, your child beats a drum in a loud-soft pattern just like do). See Tables S5 and S6 for a list of all items on skills that are within the typical developmental range. In contrast, three numeracy and three patterning items were about skills that most typically developing children do not develop by age 5. Examples of these three items that were beyond the typical developmental range for children younger than 5 are "Name the written numbers from 1 to 100 (for example, says the word "ninety-three" when

shown the written number 93)" and "Use number patterns to solve problems (for example, fills in the blanks when presented with 26, 22, 18, _, 10, _)". As in Douglas et al. (2023), the items about skills that are beyond the typical developmental range for children younger than 5 were excluded from final analyses given unacceptable reliability when they were included (numeracy α = 0.33; patterning α = 0.41). Parents also reported on 10 spatial skills which served as distractors given the study's focus on parents' numeracy and patterning support.

Subscales were created with parents' responses to the items about numeracy and patterning skills that a majority of children develop by age 5 (i.e., skills that are *within* their developmental range; Claessens et al., 2014; Clements & Sarama, 2014; Litkowski et al., 2020). Their affirmative answers to these items were scored as 1 and negative responses were scored as 0. Thus, the subscales measured parents' awareness of or accuracy with identifying numeracy and patterning skills that are within the typical developmental range for preschool-aged children. We report on descriptive statistics and the reliability and validity of these two subscales in the results.

4.3.2. Child-specific math and literacy beliefs survey

The parental beliefs survey was composed of scales from previous instruments with the addition of a few items (LeFevre et al., 2009; Skwarchuk et al., 2014; Zippert & Rittle-Johnson, 2020). Parents reported their child-specific numeracy and patterning beliefs, and a distractor topic (literacy skills) using 7-point Likert scales. For all types of math beliefs except for their beliefs about the importance of their child's future math abilities, parents reported about two numeracy items (i.e., "Counting and naming numbers" and "Comparing the magnitudes (size) of numbers") and two patterning items (i.e., "Noticing and making patterns" and "Figuring out what should come next in patterns"). All items are presented Table S7.

4.3.2.1. Values of numeracy and patterning abilities. Parents' value of their child's development of numeracy and patterning skills for their future was measured by their responses to a question about the utility of numeracy and patterning skills and a question about the importance of numeracy and patterning skills. As in Zippert and Rittle-Johnson (2020), parents were asked "How useful do you think each of these kinds of skills will be to your child in the future?" and rated the two previously described numeracy and two previously described patterning skills. They were also asked "How important is it for your child to achieve each of the following benchmarks before first grade?" (which children typically begin at age 6 in the US) and rated three different numeracy and three different patterning skills (e.g., "Know simple sums (for example, 2 + 2)" and "Identify the part that repeats in a pattern"), as in Skwarchuk et al. (2014) and LeFevre et al. (2009). Their ratings of the numeracy skills and the patterning skills were averaged as measures of their value of their child's numeracy abilities (α = 0.65) and patterning abilities (α = 0.79) respectively. We screened inter-item correlations and did not identify any items that could be dropped to improve reliability.

4.3.2.2. Expectations for numeracy and patterning development. As in Zippert and Rittle-Johnson (2020), parents were asked, "How well do you think your child will do in each of these areas in Kindergarten?" (which children typically begin at age 5 in the US). Their ratings of the two numeracy and two patterning items were averaged as measures of their expectations for their child's numeracy (α = 0.71) and patterning development (α = 0.87) respectively.

4.3.2.3. Perception of numeracy and patterning abilities. As in Zippert and Rittle-Johnson (2020), parents were asked, "How good is your child currently in each area listed below?" . Their ratings of the two numeracy and two patterning items were averaged as measures of their perception of their child's numeracy (α = 0.70) and patterning abilities (α = 0.85).

4.3.2.4. Interest in numeracy and patterning activities. As in Zippert and Rittle-Johnson (2020), parents were asked, "How much does your child like each of the following activities?" . Their ratings of the two numeracy and two patterning items were averaged as measures of their perception of their child's numeracy (α = 0.67) and patterning interests (α = 0.90).

4.3.3. Parent-reported math support survey

An adapted version of a survey used in previous research (Zippert & Rittle-Johnson, 2020; see Tables S8 and S9) served as a measure of parents' support of their preschoolers' numeracy and patterning development at home. See the supplemental materials for a detailed description of revisions. Parents reported how frequently they engaged their preschoolers in numeracy (n = 15) and patterning (n = 15) activities in the past month using a 6-point Likert-type scale, where 0 = never, 1 = once a month or less, 2 = few times a month, 3 = about once a week, 4 = few times a week, 5 = almost daily or daily. As distractors, they also reported how frequently they engaged their preschooler in spatial activities (n = 7) using the same scale.

To measure the frequency of parents' support, their ratings of their engagement in all numeracy activities (α = 91) and all patterning activities (α = 0.96) were averaged. To measure the complexity of parents' numeracy support, the frequency of their engagement in activities focused on more advanced early numeracy skills i.e., symbolic magnitude comparison and simple arithmetic were averaged (n = 4; α = 0.84). This was similar to subscales used in previous research (referred to as "advanced", "advanced formal". and "operational"; del Río et al., 2017; Skwarchuk et al., 2014; Susperreguy et al., 2018; Zippert & Ramani, 2017). We created a patterning complexity subscale that was similar (i.e., included skills that children usually develop later/ after they have developed more foundational skills). Specifically, children learn to extend and abstract repeating patterns and identify pattern units after they learn how to duplicate, recognize, and fill in the missing part of patterns (Clements et al., 2008; Kaufman et al., 2021; Papic et al., 2011; Rittle-Johnson et al., 2015; Starkey et al., 2004). Thus, to measure the complexity of their patterning support at home, the frequency of parents' reported engagement in more advanced activities focused on extending patterns, abstracting patterns, and identifying pattern units were averaged (n = 4; α = 0.91).

4.4. Procedure

Parents of 3- and 4-year-olds were recruited to participate in the study using CloudResearch and were paid $10. CloudResearch, formerly known as TurkPrime, is an internet-based research platform that integrates with Amazon's crowdsourcing platform Mechanical Turk (MTurk; Litman et al., 2017). After providing informed consent, parents completed surveys on their child-specific math beliefs, knowledge about early math development, math support at home, and their demographics. They also completed attention checks that were embedded in the survey such as "To show that you are paying attention, please select the 'none of the above' option as your answer". Participants who failed at least one attention check (n = 121) were not included in the final sample of 344 participants.

5. Results

5.1. Validating the measure of parents' knowledge about early math development

5.1.1. Reliability

We examined the internal consistency of items focused on patterning skills that are within the developmental range for most typically developing preschool-aged children (n = 9) and separately for items focused on numeracy skills that are within the developmental range using item-total correlations (n = 7). A patterning item ("Sort a set of objects into 3 groups based on color such as red, blue, and green") and a

numeracy item ("Count a row of 15 objects") had low item-total correlations (< 0.2) with their respective scales and were excluded from the scales. We found evidence of acceptable reliability (α = 0.65 and KR20 = 0.66) for the final patterning scale ($n = 8$). Cronbach alpha and KR20 revealed poor but not unacceptable internal consistency (α = 0.59 and KR20 = 0.60) for the final numeracy scale ($n = 6$). Our interpretation of these statistics were based on existing conventions for interpreting Cronbach alpha such as Cooper et al. (2003) and Hinton et al. (2004) which recommend that α < 0.5 be interpreted as "unacceptable/ very low" or "poor/ low", respectively.

5.1.2. Validity

Confirmatory Factor Analyses (CFA) models provide evidence for construct validity. All numeracy items loaded significantly onto a 1-factor model (see standardized factor loadings in Table S5) suggesting that the items measure the same theoretical construct and the same was true for patterning items (see Table S6). Importantly, both models fit the data well according to indices such as Adjusted Goodness of Fit > 0.9. We found evidence of convergent validity for both subscales (i.e., that parents' knowledge about early patterning and numeracy development are related to their beliefs about their child's patterning and numeracty abilities respectively). Specifically, parents' knowledge about early patterning and numeracy development were significantly correlated with their perception of their child's patterning, $r(342) = 0.24$, $p < .001$, and numeracy abilities, $r(342) = 0.11$, $p = .036$, respectively. We also found evidence of discriminant validity for both subscales (i.e., that parents' knowledge about early patterning and numeracy development are not related to their beliefs about their child's literacy abilities given evidence that the home math environment and home liteacy environment are separate constructs; Napoli and Purpura (2018). Specifically, parents' knowledge about early numeracy development was not significantly correlated with their perception of their child's literacy abilities (i.e., "How good is your child currently" at "learning to read and write"), $r(343) = 0.05$, $p = .342$. The same was true for the correlation between parents' knowledge about patterning development and their perceptions of literacy abilities, $r(343) = 0.09$, $p = .112$.

5.1.3. Descriptive analyses

Parents accurately classified 79% of items about numeracy skills that are within the developmental range for most 5-year-olds ($M = 0.79$, $SD = 0.23$). Their knowledge about early numeracy development was substantially left-skewed as indicated by a skewness < −1 and a ratio of skewness to standard error < 2 (skew = −1.09, SE = 0.13; ratio = −8.35, kurtosis = 0.94). Notably, the Item Response Theory (IRT) score of parents' knowledge about early numeracy development did not show substantial skew or kurtosis (skew = −0.75, SE = 0.13, kurtosis = −0.29), suggesting that it is a more statistically sound measure of parents' knowledge about numeracy development. Parents accurately classified 74% of items about patterning skills that are within the developmental range for most 5-year-olds ($M = 0.74$, $SD = 0.24$). Neither measure of parents' knowledge about patterning development showed substantial skew or kurtosis (total score: skew = −0.63, SE = 0.13, kurtosis = −0.31; IRT score: skew = −0.31, SE = 0.13, kurtosis = −0.89). For consistency, parents' IRT scored knowledge was used in analyses.

5.2. How does parents' knowledge relate to their SES, beliefs, and support?

We report on how parents' knowledge about early numeracy and patterning development relates to other components of the HME namely their (1a) SES, (1b) numeracy and patterning beliefs, and (1c) numeracy and patterning support below. These results are illustrated in Fig. 1, with relations labeled as 1a-1c. See Table S10 for exploratory analyses on the relations between parents' knowledge about numeracy and patterning development and other parent-child factors.

5.2.1. Parents' knowledge about early numeracy development

First, contrary to our hypothesis (1a), parents' highest level of educational attainment was negatively, rather than positively, related to their knowledge about early numeracy development. Specifically, parents with at least some graduate education had less accurate knowledge than other parents. Parents with a bachelor's degree did not have significantly different knowledge from parents with less education. See Table 1 for statistical information. Follow-up analyses on the individual items revealed that only parents' knowledge about preschoolers' ability to identify written numerals up to 10 and to solve simple arithmetic problems using manipulatives varied significantly with their education (see Table S5). Additionally, parents' knowledge was no longer significantly related to their educational attainment after controlling for their employment status, $F(2, 340) = 2.69$, $p = .069$. Parents' knowledge was significantly related to their employment status in that parents who were employed part-time or were unemployed had more accurate knowledge than parents who were employed full-time. Parents' knowledge was unrelated to their income.

Second, in partial support of our hypothesis (1b), parents' knowledge was positively related to each measure of their child-specific numeracy beliefs except for their expectations for their children's future numeracy development. Third, in partial support of our hypothesis (1c), parents' knowledge was positively related to the frequency but not the complexity of their reported numeracy support. See Table 2 for correlations among numeracy measures.

5.2.2. Parents' knowledge about early patterning development

First, contrary to our hypothesis (1a), parents' knowledge about early patterning development did not vary significantly with their highest level of educational attainment. Similarly, parents' knowledge was also unrelated to their employment status and income. See Table 1 for statistical information. Second, as hypothesized (1b), parents' knowledge about early patterning development was positively related to each of their child-specific patterning beliefs. Third, as hypothesized (1c), parents' knowledge was positively related to both the frequency and complexity of their patterning support. See Table 3 for correlations among numeracy measures.

Table 1
Parents' knowledge about early numeracy and patterning development by their socioeconomic status.

Variables	N	Knowledge about early math development			
		Numeracy		Patterning	
		M (SD)	F	M (SD)	F
Education			−3.93		−1.96
Less than a bachelor's degree	73	0.84 (0.20)		0.75 (0.24)	
Bachelor's degree	189	0.83 (0.18)		0.75 (0.24)	
More than a bachelor's degree	82	0.76 (0.25)[a]		0.70 (0.22)	
Income					
Less than $45,000	95	0.81 (0.21)	0.13	0.71 (0.24)	0.69
$45,000-$89,999	140	0.82 (0.18)		0.75 (0.21)	
$90,000 or more	109	0.81 (0.23)		0.74 (0.26)	
Employment Status			−7.23		−1.48
Part-time or unemployed	72	0.85 (0.18)		0.76 (0.22)	
Full time	272	0.78 (0.24)		0.73 (0.24)	

[a] Parents with more than a bachelor's degree versus parents with less education, $p < .05$.

Table 2
Correlations among parents' numeracy support, knowledge about early development, and their child-specific beliefs.

Variable	1	2	3	4	5	6	7	*M(SD)*
1. Knowledge about development	–	.09	.23***	.17**	.22***	.16**	.13**	0.79(0.23)
2. Expectation for child's development	.08	–	.49***	.51***	.50***	.28***	.20***	5.61(1.23)
3. Value of child's abilities	.22***	.49***	–	.46***	.43***	.41***	.35***	4.75(0.66)
4. Perception of child's abilities	.11*	.51***	.49***	–	.75***	.58***	.55***	5.27(1.29)
5. Perception of child's interest	.19***	.51***	.44***	.76***	–	.50***	.45***	5.48(1.22)
6. Frequency of support	.12*	.30***	.42***	.62***	.54***	–	.88***	3.39(0.93)
7. Complexity of support	.07	.22***	.35***	.61***	.49***	.90***	–	3.15(1.30)

Notes. Correlations use parents' IRT-scored knowledge, but we report the average proportion of items that parents categorized correctly based on the raw score for ease of understanding. Beliefs and support are the average ratings on 7-point scales. Partial correlations are above the diagonal.
*** $p < .001$.
** $p < .01$.
* $p < .05$.

Table 3
Correlations among parents' patterning support, knowledge about early development, and their child-specific beliefs.

Variable	1	2	3	4	5	6	7	*M(SD)*
1. Knowledge about development	–	.11*	.33***	.28***	.25***	.34***	.34***	0.74(0.24)
2. Expectation for child's development	.11*	–	.43***	.54***	.52***	.19***	16**	5.74(1.06)
3. Value of child abilities	.32***	.44***	–	.58***	.57***	.46***	.44***	4.56(0.82)
4. Perception of child's abilities	.24***	.53***	.59***	–	.79***	.55***	.54***	5.14(1.38)
5. Perception of child's interest	.23***	.52***	.58***	.80***	–	.54v	.54***	5.31(1.39)
6. Frequency of support	.30***	.20***	.48***	.59***	.57***	–	.96***	3.12(1.15)
7. Complexity of support	.30***	.17**	.46***	.59***	.57***	.96***	–	2.95(1.36)

Notes. Correlations use parents' IRT-scored knowledge, but we report the average proportion of items that parents categorized correctly based on the raw score for ease of understanding. Beliefs and support are the average ratings on 7-point scales. Partial correlations are above the diagonal.
*** $p < .001$.
** $p < .01$.
* $p < .05$.

5.3. To what extent does parents' knowledge predict their support?

5.3.1. Preliminary analyses

First, we examined descriptive statistics for parents' reported numeracy and patterning support. Parents reported providing numeracy support ($M = 3.39$, $SD = 0.93$) and patterning support ($M = 3.12$, $SD = 1.15$) about once per week. Similarly, they reported providing support *focused on more complex numeracy skills* ($M = 3.15$, $SD = 1.30$) and *complex patterning skills* ($M = 2.95$, $SD = 1.36$) about once per week. There was no substantial skewness or kurtosis for the frequency nor complexity of parents' support, with values between −1.00 and 1.00.

Next, we determined which of the preregistered predictors would be included as covariates in the main analyses. We also examined whether employment status should be included as a covariate given its relation to parents' knowledge about early numeracy development. Parents' numeracy and patterning support were significantly (and positively) correlated with all four child-specific numeracy and patterning belief variables (see Tables 2 and 3). Additionally, parents' numeracy and patterning support varied significantly and positively with their highest educational attainment,household income, and employment status (see Table S13). Finally, numeracy and patterning support did not vary significantly among parents of 3-year-olds and parents of 4-year-olds (see Table S13), so child age was not included.

5.3.2. Main analyses

To determine the extent to which parents' knowledge uniquely predicts their support, four linear regression analyses were performed with one of the four math support measures as the dependent variable in each analysis. The first regression block included parents' educational attainment,household income, and employment status. Next, their child-specific numeracy or patterning beliefs were entered into the second regression block. Finally, parents' knowledge about early numeracy or patterning development was entered in the third block. We tested for multicollinearity by screening for correlations above 0.8 between predictor variables and estimating variance inflation factor (VIF) scores for all predictor variables. All VIF scores were less than 4, indicating that multicollinearity was not biasing the results (Forthofer et al., 2007). However, parents' expectations for their child's numeracy and patterning development were negative predictors of their support when (and only when) their other child-specific beliefs were included in the model as separate variables. As such, we averaged parents' ratings of their child-specific beliefs to create composite variables which we use in the final analyses ($\alpha = 0.81$ for the numeracy variable; $\alpha = 0.84$ for the patterning variable). Table 4 includes statistics for each predictor in the final models and the results are illustrated in Fig. 1, with the relations labeled as 2a.

5.3.2.1. Parents' numeracy support. The final regression models explained almost 40% of the variance in the frequency, adjusted $R^2 = 0.39$, $F(5, 338) = 45.67$, $p < .001$, and complexity, adjusted $R^2 = 0.37$, $F(5, 338) = 40.17$, $p < .001$, of parents' numeracy support. Parents' highest educational attainment, employment status, and child-specific numeracy beliefs were unique, positive predictors of both the frequency and complexity of their numeracy support (see Table 4). However, contrary to our hypothesis (2a), parents' knowledge about early numeracy development did not uniquely predict their reported numeracy support.

5.3.2.2. Parents' patterning support. The final regression models explained about 40% of the variance in the frequency, adjusted $R^2 = 0.40$, $F(5, 338) = 45.98$, $p < .001$, and complexity, adjusted $R^2 = 0.42$, $F(5, 338) = 50.02$, $p < .001$, of parents' patterning support. Parents' educational attainment, employment status, and child-specific patterning beliefs were unique, positive predictors of both the frequency and complexity of their patterning support. Additionally, as hypothesized (2a), their knowledge about early patterning development was a unique, positive predictor of their patterning support. In particular, for each standard deviation increase in knowledge score, the

Table 4
Linear regression predicting parents' numeracy and patterning support.

Variables (final block)	Numeracy support				Patterning support			
	Frequency [a]		Complexity [b]		Frequency [c]		Complexity [d]	
	β	t	β	t	β	t	β	t
Educational attainment	0.14	2.75**	0.20	3.96***	0.16	3.31***	0.18	3.69***
Household income	0.04	0.78	−0.03	−0.72	−0.02	−0.45	−0.02	−0.50
Employment Status	0.16	3.28***	0.20	4.16***	0.15	3.13***	0.20	4.37***
Child-specific math beliefs	0.51	11.55***	0.44	9.68***	0.46	10.17***	0.43	9.78***
Knowledge about early math development	0.07	1.54	0.05	0.23	0.20	4.52***	0.21	4.92***

Notes. Parents' beliefs and knowledge about numeracy and patterning were used in numeracy and patterning models respectively. ***$p < .001$. **$p < .0125$. †$p < .05$.

frequency and complexity of their patterning support increased by 0.20 and 0.21 respectively.

5.3.3. Exploratory analyses

To explore the extent to which the relation between parents' knowledge about early math development and their math support was mediated by their child-specific math beliefs, four simple mediation analyses were conducted using the PROCESS macro for SPSS (Hayes, 2022). The frequency and complexity of their patterning or numeracy support was the dependent variable in each model. The statistics are reported in Table 5. The results revealed that parents' child-specific numeracy beliefs mediated the relation between their knowledge about early numeracy development and their numeracy support. Specifically, there was a significant indirect effect of parents' knowledge about early numeracy development on the frequency and complexity of their numeracy support. The results were similar for patterning.

5.4. Summary of results

The study findings are summarized in Fig. 1. Parents' knowledge about early numeracy development, but not early patterning development, was negatively related to their highest educational attainment. Both types of math knowledge were positively related to parents' child-specific math beliefs and to parents' math support. Parents' knowledge about early patterning development also uniquely predicted their patterning support, but parents' knowledge about early numeracy development did not uniquely predict their numeracy support when their highest educational attainment, income, employment status, and child-specific numeracy beliefs were accounted for. Additionally, parents' child-specific beliefs mediated the relations between both types of knowledge and their support.

Table 5
Summary of mediation analyses for indirect effect of knowledge on support via beliefs.

Outcome	Total effect	Direct effect	Indirect effect			
			Estimate	Lower bound CI	Upper bound CI	T statistics
Frequency of patterning support	.30	.16	.14	.08	.21	4.29
Complexity of patterning support	.30	.17	.14	.08	.20	4.30
Frequency of numeracy support	.12	.02	.10	.03	.17	2.76
Complexity of numeracy support	.07	−0.02	.09	.03	.16	2.74

Notes. Models were Knowledge of Early Patterning/Numeracy Development -> Composite of Child-specific Patterning/Numeracy Beliefs -> Patterning/Numeracy Support, with knowledge and belief measures matched to the type of support (patterning or numeracy). Statistics are standardized.

6. Discussion

The current study contributes to a more comprehensive theory of early math development as it is the first study to integrate how parents' knowledge about early math development relates to the math support that they provide to their preschool-aged children. The current study also provides insight into how models of the home math environment (HME) can be expanded to include parents' early patterning support, child-specific patterning beliefs, and knowledge about early patterning development, allowing for a parent socialization model for early math development that moves beyond the numeracy subdomain. In the next sections, we discuss our findings and their implications for supporting preschoolers' math development.

6.1. Parents' knowledge about early math development and their math support

Parents' knowledge about early patterning and numeracy development was positively related to their numeracy and patterning support respectively, but only their knowledge about early patterning development remained a unique predictor after other parent-child variables were controlled for. This suggests that parents who know more about which patterning skills most typical preschoolers develop also engage their preschoolers in related activities and discussions more frequently than parents with less knowledge. Notably, this includes activities and discussions around more complex, developmentally appropriate early patterning skills such as identifying the pattern unit, which parents tend to focus on infrequently (Zippert & Rittle-Johnson, 2020). The match hypothesis (Hunt, 1961) holds that parents who are more accurate at estimating their child's skills are better at supporting their child's growth and development. Similarly, the current findings may indicate that parents who are more knowledgeable about which patterning skills preschoolers typically learn are better at identifying and engaging their child in developmentally appropriate activities that can further support their child's patterning development.

There are several potential explanations for the differences in findings regarding the role of parents' knowledge about early patterning versus early numeracy development. One potential reason, for which we found supporting evidence, is that parents' child-specific numeracy beliefs mediated the relation between their knowledge and their numeracy support. The differences in findings could also be because the measure of parents' knowledge about numeracy development was only somewhat reliable (less so than knowledge about patterning development), making it more difficult to detect a relation. Overall, parents' knowledge about patterning development, which has not been studied previously, uniquely predicts and potentially influences their efforts to support their children's patterning development.

6.1.1. The role of parents' socioeconomic status

The current study also helps disentangle the role of parents' educational attainment and income level in the home math environment,

allowing for specific theorization about the role of SES in parents' efforts to support their children's math development. Previous studies that have examined the role of parents' education or income separately have found that parents' support is positively related to their educational attainment (e.g., Gaylord et al., 2020; Thompson et al., 2017) and their financial resources (e.g., Casey et al., 2018; DeFlorio & Beliakoff, 2015). However, we found that parents' educational attainment, but not their unadjusted household income level, uniquely predicts the frequency and complexity of their support. Additionally, we found that parents' employment status uniquely predicted the frequency and complexity of their support.

Further, the current study is the first to examine which aspects of parents' SES relate to and may potentially be sources of variability in their knowledge about early math development. Parents' knowledge about early numeracy and patterning development was unrelated to an indicator of their financial resources (unadjusted household income), and knowledge about patterning development was also unrelated to their educational attainment. An unexpected finding was that parents' knowledge about early numeracy development was negatively related to their educational attainment, although this difference was no longer present after controlling for employment status. At the same time, parents' support was positively related to their educational attainment, as found in past research (e.g., Gaylord et al., 2020; Thompson et al., 2017). This suggests that factors such as access and attention to educational materials and resources from the media and preschool teachers, and/or parents' beliefs might be more influential in explaining why parents with greater educational attainment seem to provide more frequent and more complex home math support.

6.1.2. *The role of parents' child-specific math beliefs*

As proposed in the Parent Early Numeracy Support model (Douglas et al., 2021), parents' child-specific numeracy beliefs were differentially related to their numeracy support and accounted for a substantial percentage of the variance in the current study. This was also true of parents' patterning beliefs, providing supportive evidence that parents' patterning beliefs are important components of the home math environment. Further, the current study was the first to examine whether parents' knowledge about early math development relates to their child-specific math beliefs and might potentially be a source of variability in their beliefs. Indeed, parents' knowledge about early numeracy and patterning development was often positively correlated with their child-specific numeracy and patterning beliefs in the current study, similar to previous findings about parents' knowledge about child development and their parenting beliefs like maternal self-efficacy (e.g., Albarran & Reich, 2014). Further, parents' child-specific beliefs mediated the relation between their knowledge about early math development and their support. Thus, parents' knowledge about early math development may be an important source of variability in their child-specific math beliefs. For instance, parents with more accurate early math knowledge may value their preschoolers' development of these skills more than other parents. Likewise, knowing which math skills preschoolers typically learn might lead parents to ask their children to demonstrate these skills and may lead to them having a more accurate perception of their children's math abilities. At the same time, parents' beliefs may influence their knowledge. Future research should examine the effect of changing parents' knowledge to test for causal relations.

6.2. *Limitations and future directions*

Despite five rounds of measurement development and pilot testing, the measures of parents' knowledge were only somewhat reliable indicating that parents do not have a uniformed sense of early numeracy and patterning development. Another limitation is that although the sample consisted of participants from 46 states across the US and a large percentage of fathers (43%), its SES was not representative of the US population and only a few parents were at the ends of the income spectrum, potentially reducing the study's ability to detect income-related differences. Additionally, the survey question about income could have been improved to be more specific (the ranges that participants were asked to select from were large and unequal). Relatedly, the majority of the participating parents were White and so the findings could be different with a more racially diverse sample.

A third limitation is that the study relied only on parent self-reports which could be susceptible to bias. Fourth, the current study did not explore how parents' child-specific beliefs are related to their socioeconomic status. Future research should examine this to capture a more complete picture of the home math environment. Finally, the current study only provides correlational evidence about the potential role of parents' knowledge about early math development in their efforts to support their children's math development. Future research should examine the malleability of parents' knowledge about early patterning development and the effect of increasing this knowledge on their math support. Increasing parents' patterning support might be important given previous findings that parents rarely mentioned patterning spontaneously when asked about their math support and reported supporting their children's patterning development less often than other aspects of their math development (Cannon & Ginsburg, 2008; Zippert & Rittle-Johnson, 2020). Previous findings that parents are interested in receiving information about math (Sonnenschein et al., 2021) suggest that parents might benefit from receiving information about early math development, especially patterning development.

7. Conclusion

The current study provides insight into the nature and role of parents' knowledge about early math development. Parents' knowledge about early repeating patterning development may play an important role in their efforts to support their children's math development, given that it uniquely predicted their repeating patterning support and was related to their child-specific repeating patterning beliefs. Theoretically, models of the home math environment should be expanded to include repeating patterning and to include parents' knowledge about early math development. Practically, interventions to improve parents' home math support should integrate information on early patterning development.

CRediT authorship contribution statement

Ashli-Ann Douglas: Conceptualization, Methodology, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization, Funding acquisition. **Bethany Rittle-Johnson:** Methodology, Resources, Writing – review & editing, Visualization, Supervision, Funding acquisition.

Data availability

Data will be made available on request.

Acknowledgments

The research was supported by Heising-Simons Foundation grant #2021-2772 to Bethany Rittle-Johnson and Ashli-Ann Douglas. The authors thank Camille Msall, Rachael Kim, and Aarushi Rohila for their assistance with data collection. We also thank Dr. Lisa Fazio, Dr. Georgene Troseth and Dr. Anita Wager for their feedback on an early draft of the paper.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ecresq.2023.10.003.

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