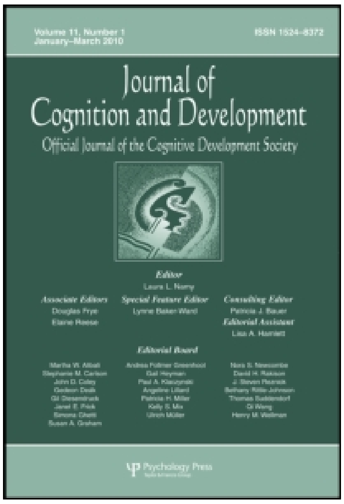

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Continuity in Representation Between Children and Adults: Arithmetic Knowledge Hinders Undergraduates' Algebraic Problem Solving

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This study examined if solving arithmetic problems hinders undergraduates' accuracy on algebra problems. The hypothesis was that solving arithmetic problems would hinder accuracy because it activates an operational view of equations, even in educated adults who have years of experience with algebra. In three experiments, undergraduates ($N = 184$) solved addition facts or participated in one of several control conditions. Those who solved addition facts were less likely to solve prealgebra equations (e.g., $6 + 8 + 4 = 7 + \_\_$) correctly under speeded conditions. In a fourth experiment, the negative effects of solving arithmetic problems extended to undergraduates ($N = 74$) solving algebra problems with no time pressure. Taken together, results suggest that arithmetic activates knowledge that hinders performance on algebra problems. Thus, an

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operational view of equations, which is prevalent in children, does not seem to be revised or abandoned, even after years of experience with algebra.

Many aspects of cognition improve with time and with experience. Adults and experts tend to think in more advanced ways than do children and novices (Chase & Simon, 1973; Larkin, 1983). However, research suggests that more advanced ways of thinking do not simply replace or subsume less advanced ways of thinking (Kuhn, Garcia-Mila, Zohar, & Andersen, 1995; Ohlsson, 2009; Siegler, 1996). This phenomenon is particularly striking in the domain of mathematics where educated adults often use less advanced strategies for solving problems than might be expected (e.g., Clement, Lochhead, & Monk, 1981; LeFevre, Sadesky, & Bisanz, 1996). These continuities in cognition are important to understand because they provide clues about the nature of learning and cognitive development.

In this study, we investigated possible continuities in mathematical thinking between children and adults. The equations of interest were prealgebra equations with operations on both sides of the equal sign, such as $6+8+4=7+\_\_$. These target equations tap a fundamental concept in algebra — mathematical equivalence (i.e., the concept that the two sides of an equation are interchangeable). In the United States, they are rarely included in the traditional K–8 mathematics curriculum (Li, Ding, Capraro, & Capraro, 2008; McNeil et al., 2006; Seo & Ginsburg, 2003), and few children (ages 7–11) generate a correct strategy for solving the equations in the absence of specially designed interventions (Carpenter & Levi, 2000; De Corte & Verschaffel, 1981; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Saenz-Ludlow & Walgamuth, 1998).

One might assume that educated adults would not have difficulties with mathematical equivalence because they have learned algebra, and pilot data indicated that most undergraduates solve equations like $6+8+4=7+\_\_$ correctly under nonspeeded conditions. However, in McNeil and Alibali's (2005b) study, some undergraduates solved the equations incorrectly under speeded conditions (1,500 ms display period). Surprisingly, these undergraduates did not merely make calculation errors but rather solved equations using incorrect strategies typically used by children. When striking performance deficits such as these occur, it provides a window into the content and organization of knowledge (Diamond & Kirkham, 2005; Sophian, 1997). Speeded conditions can break down adults' performance on complex cognitive tasks, leading to reversion to immature strategies (Diamond & Kirkham, 2005). Such reversions have implications for theories of learning and development because they are consistent with the idea that old representations continue to coexist alongside newer, more advanced representations, and new or old may be activated, depending on the task and context (e.g., Barsalou, 1982; Munakata, McClelland, Johnson, & Siegler, 1997; Siegler & Stern, 1998; Thelen & Smith, 1994).

According to this account, individuals who learn arithmetic in the United States may not revise or abandon their original view of equations, even after they learn algebra and understand mathematical equivalence. Before learning algebra, children in the United States develop an *operational* view of equations, in which they: a) assume the equal sign is always at the end of equations (Alibali, Phillips, & Fischer, 2009; Cobb, 1987; McNeil & Alibali, 2004); b) interpret the equal sign as an operator that signals to "add up the numbers" (Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980; Kieran, 1981; McNeil & Alibali, 2005a); and c) solve equations like $8+4=5+\_\_$ by adding $8+4+5$ (Falkner, Levi, & Carpenter, 1999; Matthews & Rittle-Johnson, 2009; Perry, Church, & Goldin-Meadow, 1988; Rittle-Johnson & Alibali, 1999). This operational view is thought to be constructed from experience with arithmetic in school (Baroody & Ginsburg, 1983; Jacobs et al., 2007; McNeil & Alibali, 2005b; Seo & Ginsburg, 2003). In the United States, teachers and textbooks rarely reference the meaning of the equal sign or the concept of mathematical equivalence (Baroody & Ginsburg, 1983; Behr et al., 1980; Kieran, 1981; Seo & Ginsburg, 2003). Rather, children predominantly see equations with operations to the left of the equal sign and the "answer" to the right (e.g., $3+4=7$, Cobb, 1987; McNeil et al., 2006; Seo & Ginsburg, 2003). This "operations on left side" format does not highlight the interchangeable nature of the two sides of an equation (Weaver, 1973). Moreover, children learn to solve arithmetic problems by performing all of the given operations on all the given numbers (e.g., they add up all the numbers in addition problems; McNeil & Alibali, 2005b). Children consistently encounter problems that reinforce their operational view of equations when learning arithmetic, and they do not learn to reason about equations relationally (as expressions of mathematical equivalence) until they start to learn prealgebra or algebra later in school.

We hypothesized that the operational view of equations continues to coexist alongside the relational view, even in adulthood, and either can be activated depending on the context (McNeil & Alibali, 2005a). Although educated adults who have learned algebra well are likely to activate the relational view in most mathematics and science contexts (e.g., McNeil & Alibali, 2005a), they may be likely to activate the operational view in the context of solving arithmetic problems because that is the context in which the operational view was originally established. In support of this account, research has shown that undergraduates tend to define the equal sign as a relational symbol in many contexts, but some define the equal sign operationally in the context of a typical arithmetic problem (McNeil & Alibali, 2005a). This context effect is even stronger for children in middle school who are just learning algebra (McNeil et al., 2006).

We predicted that solving arithmetic problems would activate an operational view of equations and hinder undergraduates' performance on equations. However, there are at least two reasons to predict the opposite. First, solving arithmetic problems might lead to increased fluency with basic arithmetic facts, thus "freeing" cognitive resources for higher-level problem solving. This account is based on the Decomposition Thesis (Anderson, 2002; Lee & Anderson, 2001), which suggests that complex skills (e.g., equation solving) can be decomposed into many small component skills (e.g., encoding the numbers and operators, performing arithmetic computations, etc.) and that increased fluency with one of the component skills is an effective way to improve execution of the more complex skill (Anderson, Corbett, Koedinger, & Pelletier, 1995; Kotovsky, Hayes, & Simon, 1985). Because arithmetic computation is a component skill necessary for solving the target equations, this account suggests that increased fluency with the arithmetic facts relevant to solving the target equations should improve equation-solving performance (cf., Carnine, 1980; Haverty, 1999; Kaye, 1986).

Second, solving arithmetic problems might activate representations in educated adults that are helpful to solving equations. Undergraduates have been solving arithmetic problems for many years, and they also have learned how to think about equations relationally in algebra. Those who have learned algebra well may abandon or revise their old, operational view of equations. Their knowledge of algebra may subsume their knowledge of arithmetic, and they may come to think of arithmetic problems relationally (e.g., $3 + 4$ is equivalent to 7). If this is the case, then arithmetic problems would activate undergraduates' knowledge of mathematical equivalence and would facilitate performance on equations.

Surprisingly, no study to date has examined how solving arithmetic problems affects performance on algebraic equations. We performed four experiments to test the hypothesis that solving arithmetic problems hinders undergraduates' accuracy on equations. In the first three experiments, we tested if solving arithmetic problems would hinder undergraduates' accuracy solving prealgebra equations under speeded conditions. In the fourth experiment, we tested if the negative effects of solving arithmetic problems would extend to algebra problems solved under nonspeeded conditions. The findings have implications for whether an operational view of equations persists into adulthood and is activated by solving arithmetic problems.

EXPERIMENTS 1–3

The target problems in the first three experiments were equations of the form $a + b + c = d + \_\_$. Our goal was to test if performance solving these equations under speeded conditions would be hindered by solving arithmetic problems beforehand. We predicted that solving arithmetic problems would hinder performance by activating participants' operational view of equations.

Method

Participants. Participants were 124 undergraduates in Experiment 1, 44 in Experiment 2, and 43 in Experiment 3. Across the experiments, 27 participants were excluded because they did not attend elementary school in the United States (but see supplemental analysis). Thus, the final sample consisted of 184 undergraduates (50 men, 134 women; 5 African American or Black, 10 Asian, 5 Hispanic or Latino, 164 White) from a public university in the Midwestern United States. Participants received one extra credit point for participating. Participants' scores on the quantitative portion of the ACT/SAT fell between the 39th and 99th percentiles ($M = 85$th percentile).

Apparatus. Stimuli were presented on iMac G4 computers using PsyScope 1.2.5 (Cohen, MacWhinney, Flatt, & Provost, 1993).

Procedure. Participants in the experiments were seated at computers situated in individual cubicles. They were told: a) to record all answers on an answer sheet; b) to use the answer sheet exclusively for writing answers, not as scratch paper (to ensure memory-intensive conditions); and c) to go as quickly as possible throughout the experiment while still maintaining accuracy.

Each experiment had two distinct phases: an activation phase followed by an equation-solving phase. During the activation phase, participants were randomly assigned to the arithmetic condition or to one of three control conditions. In the arithmetic condition (Experiments 1 and 2), participants solved simple addition facts (e.g., $8 + 4$). We used addition facts, rather than subtraction, multiplication, or division facts, for two main reasons. First, addition is the arithmetic operation most relevant to solving the target equations. Second, addition is the operation that is learned first in school, so it is the operation upon which all the other operations are built. Each addition fact was presented as two addends (without the equal sign) because we wanted to test the effect of solving addition facts per se, rather than exposure to the "operations on left side" format.

Our primary control condition (Experiments 1–3) was the color-mixing condition, based on the control condition used by McNeil and Alibali

(2005b). In this condition, instead of adding numbers together, participants mixed colors together. The problems contained color pairs (e.g., yellow and blue), and participants' goal was to figure out what color the color pairs made when mixed together. For example, the correct answer for the color pair "yellow and blue" was "green" because yellow and blue mix to make green.

To account for potential alternative explanations of our predicted effects, we included two additional control conditions: the no-input condition (Experiment 2) and the algebra condition (Experiment 3). In the no-input condition, participants did not solve any problems prior to solving the equations. We included this condition so we could attribute differences between the arithmetic and color-mixing conditions to negative effects of solving addition facts, rather than to positive effects of mixing colors together.

In the algebra condition, participants solved problems that were constructed by converting the addition facts from the arithmetic condition to simple algebra problems of the form $a = b + x$, where a and b were the numbers from a problem in the arithmetic condition (e.g., $8 = 4 + x$). Thus, participants in this condition saw the same numbers in the same order as participants in the arithmetic condition. This condition addressed two alternative explanations of the predicted negative effects of the arithmetic condition. First, without this condition, any negative effects of the arithmetic condition could be attributed to exposure to numbers, rather than to arithmetic per se. Second, without this condition, negative effects of the arithmetic condition could be attributed to participation in an effortful task, rather than to arithmetic per se. Solving addition facts may have been more mentally tiring than mixing colors together, and mental fatigue is often accompanied by deterioration in cognitive performance (Lorist et al., 2000). The algebra condition exposed participants to numbers, and it was (at least) as effortful as solving addition facts. Thus, if participants in the algebra condition performed similarly to or better than participants in the other control conditions, then we could not reasonably attribute poor performance in the arithmetic condition to exposure to numbers or participation in an effortful task.

Participants in all conditions completed eight problem sets, each of which contained 12 problems (addition facts, color pairs, or simple algebra problems). Each set appeared on the screen until participants solved all 12 problems and pressed a key on the keyboard to move on to the next set. We included a large number of problems in the activation phase for two reasons. First, we wanted to make sure that we activated undergraduates' operational view of equations enough to compete with the relational view that had been learned more recently in algebra. Second, it allowed us to compare our predictions to the predictions of the Decomposition Thesis (Anderson, 2002; Lee & Anderson, 2001), which suggests that increased fluency with the addition facts relevant to solving the target equations should improve equation-solving performance (see Haverty, 1999). Prior research indicates that undergraduates sometimes compute (rather than retrieve from memory) single-digit addition facts (Campbell & Timm, 2000; Campbell & Xue, 2001; LeFevre et al., 1996), and solving a large number of single-digit addition facts leads to a significant reduction in undergraduates' solution times (Frensch & Geary, 1993). Thus, in contrast to our predictions, the Decomposition Thesis predicts that undergraduates will perform better on equations after solving a large number of addition facts needed to solve the target equations.

After the activation phase, participants were presented with a new screen of instructions to indicate that they would be moving on to a new phase of the experiment. They were informed that they would be shown a new set of mathematics problems one at a time and that each problem would appear on the screen for a brief period of time. They were told to record solutions on the answer sheet. During this equation-solving phase, participants solved three equations: $7 + 9 + 6 = 7 + \_\_$, $6 + 8 + 4 = 7 + \_\_$, and $9 + 7 + 8 = 5 + \_\_$. Before each equation was presented, participants' gaze was directed to the center of the screen at the location where the equation would be presented. Each equation was presented for a brief period of time (1,000 ms for a subset of participants in Experiment 1 and 1,500 ms for all other participants; time of presentation did not affect performance).

Results

The same procedure was used in all three experiments, so we collapsed the data across experiments for efficient presentation (conclusions are unchanged when each experiment is analyzed separately).

Correctness. Performance on the equations was poor overall, and not normally distributed, with 112 of the 184 participants (61%) solving zero of three equations correctly. Low levels of performance like this tend to occur in studies of *children's* performance on the equations (e.g., Alibali, 1999; Falkner et al., 1999; McNeil, 2008). Given this distribution of scores, a parametric analysis would be inappropriate. Based on the shape of the distribution, we categorized participants into two groups: those who solved zero equations correctly ($N = 110$) and those who solved at least one equation correctly ($N = 74$). We then used binomial logistic regression to predict the log of the odds of solving at least one equation correctly (see Agresti, 1996). Conclusions were unchanged when we used other analysis strategies (e.g., multinomial logistic regression, Weighted Least Squares analysis). Predictor variables included condition (arithmetic, algebra, color mixing, no practice) and national percentile rank on the quantitative portion of the

ACT/SAT (continuous, centered) as a control variable. Conclusions were unchanged when ACT/SAT was removed from the model. To test our predictions, we used three Helmert contrast codes to represent the four levels of condition: 1) arithmetic versus all control conditions, 2) algebra versus the other two control conditions, and 3) color mixing versus no practice.

Table 1 displays the percentage of participants in each condition who solved at least one equation correctly by condition. As predicted, participants in the arithmetic condition were less likely than those in the control conditions to solve at least one equation correctly, $\widehat{\beta} = -1.64$, $z = -4.15$, $Wald(1, N = 184) = 19.78$, $p < .001$. The model estimates that the odds of solving at least one equation correctly are more than five times lower after participating in the arithmetic condition versus a control condition. Separate pairwise comparisons indicated that participants in the arithmetic condition were significantly less likely than participants in any given control condition to solve at least one equation correctly. Moreover, when each equation was analyzed separately, participants in the arithmetic condition were less likely than those in the control conditions to solve each equation correctly, Equation 1: $\widehat{\beta} = -1.06$, $z = -1.88$, $Wald(1, N = 184) = 3.51$, $p = .06$; Equation 2: $\widehat{\beta} = -1.07$, $z = -2.72$, $Wald(1, N = 184) = 7.43$, $p = .006$; Equation 3: $\widehat{\beta} = -1.54$, $z = -3.89$, $Wald(1, N = 184) = 15.10$, $p < .001$. Thus, the arithmetic condition hindered equation-solving performance relative to the control conditions.

Considering only the control conditions, participants in the algebra condition were more likely than participants in the other two control conditions to solve at least one equation correctly, $\widehat{\beta} = 1.67$, $z = 2.35$, $Wald(1, N = 98) = 5.50$, $p = .02$, and there was no statistical difference between the color-mixing and no-practice conditions, $\widehat{\beta} = 0.31$, $z = 0.47$, $Wald(1, N = 79) = 0.22$, $p = .64$. The log of the odds of solving at least one

TABLE 1
Percentage of Undergraduates who Solved at Least One Equation Correctly by Condition, and Average Number of Equations Solved With the Add-All Strategy by Condition

	Full sample		*High-achieving subsample*	
	% at least one corr.	*Add-all M (SD)*	*% at least one corr.*	*Add-all M (SD)*
Arithmetic	24% (21 of 86)	1.94 (1.23)	21% (8 of 38)	1.97 (1.26)
Algebra	84% (16 of 19)	0.68 (1.06)	83% (10 of 12)	0.75 (1.06)
Color mixing	46% (31 of 68)	1.66 (1.28)	48% (15 of 31)	1.74 (1.21)
No input	54% (6 of 11)	1.36 (1.29)	67% (4 of 6)	1.00 (1.26)

Note. The numbers are presented for the full sample and the high-achieving subsample from Experiments 1 through 3.

equation correctly was not associated with national percentile rank on the quantitative portion of the ACT/SAT, $\widehat{\beta}=0.11$, $z=0.48$, $Wald(1, N=183)=0.23$, $p=.63$.

Thus, educated adults who solved addition problems were less likely than those who participated in control conditions to solve equations correctly under speeded conditions. Importantly, the poor performance seemed to be due to solving arithmetic problems specifically, not to facilitative effects of mixing colors together (practice mixing colors was no different than no practice) or exposure to numbers or practice with an effortful task (practice with simple algebra problems was helpful to solving equations).

Use of the add-all strategy. According to our account, participants in the arithmetic condition were less likely than those in the control conditions to solve the equations correctly because solving arithmetic problems activated participants' operational view of equations. Activation of the operational view should lead participants to solve the target equations by adding all the numbers (i.e., the add-all strategy). Thus, we predicted that participants in the arithmetic condition would use the add-all strategy more than would participants in the control conditions.

Use of the add-all strategy was mixed overall, with 56 of the 184 participants (30%) using it on zero of three equations, 22 (12%) using it on one equation, 32 (17%) using it on two equations, and 74 (40%) using it on all three equations. Table 1 displays the average number of equations solved with the add-all strategy by condition. We performed an analysis of covariance (ANCOVA) with condition as the independent variable, national percentile rank on the quantitative portion of the ACT/SAT as the covariate, and number of equations solved with the add-all strategy (out of three) as the dependent variable. There was a significant main effect of condition, $F(3, 179)=5.39$, $p=.001$, $\eta_p^2=.08$. We again used the set of orthogonal Helmert coefficients to test our predictions. As predicted, participants in the arithmetic condition solved more equations with the add-all strategy than did participants in the control conditions, $F(1, 179)=10.55$, $p=.001$. Participants in the algebra condition solved fewer equations with the add-all strategy than did participants in the other two control conditions, $F(1, 179)=5.62$, $p=.02$. There was no statistical difference between use of the add-all strategy in the color-mixing and no-practice conditions, $F(1, 179)=0.52$, $p=.46$. Use of the add-all strategy was not associated with national percentile rank on the quantitative portion of the ACT/SAT, $F(1, 179)=0.14$, $p=.71$, and conclusions were unchanged when ACT/SAT was not included as a covariate in the analysis. These results indicate that the arithmetic condition increased undergraduates' use of the incorrect strategy most often used by elementary-school children—the add-all strategy.

Supplemental analysis 1. Results of Experiments 1–3 are important because they suggest that arithmetic activates knowledge that is detrimental to solving equations, even in educated adults. But what if participants had never developed an operational view of equations? Children in high-achieving countries in Asia, such as China and Taiwan, typically solve equations such as $6+8+4=7+__$correctly (Capraro et al., 2009; Watchorn, Lai, & Bisanz, 2009) and analysis of elementary-school textbooks and teachers' manuals from China suggests that a relational view of equations is well supported (Li et al., 2008), so children in these countries may never develop an operational view of equations. If this is true, then undergraduates who were educated in these countries should solve equations correctly, regardless of whether they solve arithmetic problems or not. To explore this possibility, we compared the performance of undergraduates who attended elementary school in countries in Asia ($N=22$, 8 in Korea, 4 in Singapore, 3 in China, 3 in Hong Kong, 2 in India, and 2 in Taiwan) to that of undergraduates who attended elementary school in the United States matched for university, condition, quantitative SAT score, and gender. Seventy-three percent of undergraduates educated in countries in Asia solved at least one equation correctly (regardless of condition), but only 32% of undergraduates educated in the United States did the same. The model estimates that the odds of solving at least one equation correctly are nearly six times higher for undergraduates educated in countries in Asia versus those educated in the United States. Results were similar when we considered use of the add-all strategy. Undergraduates educated in countries in Asia solved fewer equations with the add-all strategy ($M=0.54$, $SD=1.06$) than did undergraduates educated in the United States ($M=2.04$, $SD=1.25$), $t(42)=-4.29$, $p<.001$, $d=-1.29$.

The difference between the two groups was even greater when we limited the analysis to the arithmetic condition ($n=13$ in each group). The percentage of participants who solved at least one equation correctly was 70% for those educated in countries in Asia compared with only 8% for those educated in the United States. The average number of equations solved with the add-all strategy was only 0.54 ($SD=1.05$) for those educated in countries in Asia compared with 2.61 ($SD=0.87$) for those educated in the United States.

Although factors other than differences in early experience with arithmetic may account for the differences in performance between the two groups, the results nonetheless show that undergraduates who were not expected to have an operational view of equations performed well on equations under speeded conditions and rarely resorted to adding all the numbers, even if they participated in the arithmetic condition. This result makes it difficult to attribute the poor equation-solving performance of undergraduates in the arithmetic condition in Experiments 1 through 3 to some aspect of the method that "tricked" participants into solving the equations incorrectly. As we have argued, solving arithmetic problems can only activate an operational view of equations in individuals who are predisposed to think about arithmetic in an operational way.

Supplemental analysis 2. Given the superior performance of undergraduates educated in countries in Asia, one might wonder if the results of Experiments 1 through 3 would have been different if we had tested a select subset of the undergraduates educated in the United States—particularly those with high levels of algebra achievement. According to the account we have been advancing in this article, even high-achieving undergraduates should be negatively affected by solving arithmetic problems because once the operational view of equations is established in elementary school, it is not revised or abandoned, even when algebra is learned in the later school years. However, an alternative account might suggest that undergraduates who were educated in countries in Asia have learned algebra well enough to revise or abandon their old, operational view. According to this account, undergraduates who were educated in countries in Asia may have fully integrated their knowledge of arithmetic with their knowledge of algebra, thus making them immune to the negative effects of solving arithmetic problems. To rule out this alternative, it would be ideal to show that the results of Experiments 1 through 3 hold for a group of "algebra experts" who attended elementary school in the United States. Because algebra is one of the primary content areas tested in the SAT and ACT, it was possible for us to get a rough estimate of participants' algebra achievement through their quantitative SAT/ACT scores. We operationalized high-achieving undergraduates as those who scored at or above the 90th percentile on the quantitative SAT/ACT ($N = 87$), and we re-ran the analyses from Experiments 1 through 3 to test if the results held for this select subset of undergraduates. Conclusions held regardless of the criterion we used to define high achievement (e.g., at or above 90th, above 90th, above 95th).

Table 1 displays the percentage of high-achieving undergraduates who solved at least one equation correctly by condition. Consistent with the results from the full sample, high-achieving undergraduates in the arithmetic condition were less likely than those in the control conditions to solve at least one equation correctly, $\widehat{\beta} = -2.11$, $z = -3.74$, $Wald(1, N = 87) = 13.81$, $p < .001$. The model estimates that the odds of solving at least one equation correctly are more than eight times lower after participating in the arithmetic condition versus a control condition. High-achieving undergraduates in the algebra condition were not statistically more likely than those in the other two control conditions to solve at least one equation correctly, $\widehat{\beta}=1.23$, $z=1.36$, $Wald(1,\ N=49)=1.85$, $p=.17$. There was no statistical difference between the color-mixing and no-practice conditions, $\widehat{\beta}=-0.63$, $z=0.67$, $Wald(1,\ N=37)=0.67$, $p=.41$.

Results were similar for use of the add-all strategy. Table 1 displays the high-achieving undergraduates' average number of equations solved with the add-all strategy by condition. High-achieving undergraduates in the arithmetic condition solved more equations with the add-all strategy than did those in the control conditions, $F(1,\ 83)=7.64$, $p=.007$. There was not a statistical difference in use of the add-all strategy between the algebra condition and the other two control conditions, $F(1,\ 83)=1.96$, $p=.16$, nor between the color-mixing and no-practice conditions, $F(1,\ 83)=1.87$, $p=.18$. On the whole, results were consistent with the results from the full sample and suggest that arithmetic hinders the equation-solving performance of undergraduates educated in the United States, even those with high levels of mathematics achievement.

EXPERIMENT 4

We performed a final experiment to test if the negative effects of solving arithmetic problems extend beyond prealgebra equations solved under speeded conditions to algebra problems solved under nonspeeded conditions. Even though the arithmetic condition hindered performance relative to control conditions on all three equations, it is possible that the negative effects of solving arithmetic problems may occur only for a few minutes or only for problems with perceptual features that overlap substantially with typical arithmetic problems.

Method

Participants. Seventy-six undergraduates participated. Two were excluded because they did not attend elementary school in the United States. The final sample consisted of 74 undergraduates (35 men, 39 women; 2 African American or Black, 6 Asian, 8 Hispanic or Latino, 1 Native American, 56 White, 1 "Other") from a private university in the Midwestern United States. Participants received one extra-credit point toward a psychology class for participating. Participants' scores on the quantitative portion of the ACT/SAT fell between the 74th and 99th percentiles ($M=93$rd percentile).

Procedure. The target problems for this experiment were algebra problems that require coordination of relationships of equivalence involving multiple unknown values or double references to a single unknown value (e.g., John bought three shirts and two caps for \$58. Sue bought two shirts and three caps for \$52. What is the cost of one shirt?). Six problems were selected from previous studies (Booth & Koedinger, 2007; Koedinger, Alibali, & Nathan, 2008; Landy & Goldstone, 2007; Rosnick & Clement, 1980). Participants had unlimited time to solve the problems; they took 18.68 minutes on average ($SD = 10.52$). The procedure was similar to that in Experiments 1 through 3: Participants solved algebra problems after solving arithmetic problems or participating in one of two control conditions (solving nonarithmetic problems or no input). The arithmetic problems included single-digit addition problems, as well as arithmetic problems that were generated from a pilot study in which we determined which arithmetic facts undergraduates tend to use when solving the target algebra problems. The nonarithmetic problems were magnitude comparison problems in which participants had to determine whether a number is greater than, less than, or equal to another number by filling in the appropriate symbol to make a statement true (e.g., the correct symbol for "13 O 16" is "<"). We did not expect performance in the two control conditions to differ because research has shown that educated adults already tend to activate the relational view in the context of algebra problems (McNeil & Alibali, 2005a).

Results

Performance on the algebra problems was mediocre ($M = 2.92$ out of 6, $SD = 1.19$), and scores were normally distributed. We performed an ANCOVA with condition (arithmetic, magnitude comparison, or no input) as the independent variable, national percentile rank on the quantitative portion of the ACT/SAT as the covariate, and number of algebra problems solved correctly (out of six) as the dependent variable. The effect of condition was significant, $F(2, 68) = 4.14$, $p = .02$, $\eta_p^2 = .11$. We again used Helmert coefficients to test our hypothesis, and as expected, participants in the arithmetic condition solved fewer algebra problems correctly ($M = 2.43$, $SD = 1.20$) than did participants in the control conditions ($M = 3.14$, $SD = 1.13$), $F(1, 68) = 7.15$, $p = .01$. Participants in the magnitude comparison and no-practice control conditions solved about the same number of problems correctly ($M = 2.96$, $SD = 1.21$ vs. $M = 3.32$, $SD = 1.03$), $F(1, 68) = 1.73$, $p = .20$. Performance on the algebra problems was associated with national percentile rank on the quantitative portion of the ACT/SAT, $F(1, 68) = 4.42$, $p = .04$, $\eta_p^2 = .06$. However, the significant effect of condition did not depend on the inclusion of this covariate in the analysis, nor did it interact with ACT/SAT scores.

DISCUSSION

We examined how solving arithmetic problems affects undergraduates' equation solving. We found that solving addition facts such as $8+4$ hinders accuracy solving equations such as $6+8+4=7+\_\_$ under speeded conditions. We also found that solving simple algebra problems such as $8=4+x$ improves accuracy solving equations such as $6+8+4=7+\_\_$, which suggests that the negative effects of solving arithmetic problems are not due to exposure to numbers or practice with a mentally tiring task but rather are due to the knowledge activated by the problems. Finally, we showed that the negative effects of solving arithmetic problems extend to performance solving algebra problems under nonspeeded conditions. Taken together, results suggest that arithmetic activates knowledge that hinders educated adults' performance on problems that require relational thinking. This supports Diamond and Kirkham's (2005) claim that adults do not ever fully outgrow the naïve representations and biases they construct in childhood. In the following paragraphs, we first discuss how these results extend classic findings on mental set. Then, we consider potential mechanisms underlying the effects, highlight educational implications, and propose future directions.

The present results extend classic findings on mental set. For example, consider Luchins's seminal water jar experiments (Luchins, 1942; Luchins & Luchins, 1950; see also Chen, 1999; Chen & Mo, 2004; Crooks & McNeil, 2009). In these experiments, both children and adults who solved several problems that required the same complicated strategy persisted in using that strategy on target problems that could be solved by a much simpler strategy. They did so even when the more complicated strategy did not lead to a correct solution and were said to be operating according to an *Einstellung*, or mental set.

Consistent with Luchins's results, the present results suggest that experience with a single strategy can reduce problem-solving flexibility and hinder performance solving more complex problems. However, the present results go beyond Luchins's in several ways. First, they suggest that a "mental set" can be constructed not only within a single problem-solving experience but also through experience that is distributed across many years of schooling. Second, they suggest that the negative effects of experience with a single strategy occur not only in the context of highly constrained puzzle problems but also in the context of educationally relevant problems taught in school. Finally, they suggest that experience with a single strategy can be detrimental not only when the strategy needs to be abandoned and replaced with a novel strategy but also when the strategy is a component skill necessary for carrying out a more complex task. Indeed, even though arithmetic computation was a component skill necessary for solving the target equations, solving arithmetic problems beforehand did not help—but hindered—participants' accuracy. These findings suggest that it may be necessary for accounts based on the Decomposition Thesis (e.g., Haverty, 1999; Kotovsky et al., 1985) to incorporate constraints for when experience with component skills harms or facilitates performance on a complex task.

The present study also provides some clues about the possible processes involved in the negative effects of solving arithmetic problems. We have argued that arithmetic activates undergraduates' operational view of equations. We know that children in the United States construct an operational view of equations from their experience with arithmetic in elementary school, and this view may continue to be activated and maintained by exposure to arithmetic, even after years of experience with algebra (McNeil & Alibali, 2005b). Activation of the operational view, in turn, may interfere with accurate encoding of, strategy selection on, and conceptualization of problems that require relational thinking (cf., Bruner, 1957; McGilly & Siegler, 1990). This account implies continuities in representations between children and adults.

However, it is possible that other processes could be involved in the negative effects of solving arithmetic problems. For example, instead of activating the old, operational view of equations, it is possible that undergraduates could have constructed a "response set" for the very first time during the arithmetic condition in Experiments 1 through 3 and then persisted in using it when they were presented with equations. Although an impromptu response set may have contributed to the negative effects of arithmetic found in Experiments 1 through 3, this account has difficulty explaining why undergraduates who were educated in countries such as China and Korea would not be expected to construct a similar response set. It also has difficulty explaining why solving arithmetic problems hindered accuracy on algebra problems in Experiment 4.

Another related process that could be involved in the negative effects of arithmetic practice is *mindlessness*, which is defined as being committed to a "single, rigid perspective and ... oblivious to alternative ways of knowing" (Langer, 2000). Mindless problem solvers rely on knowledge that has been retrieved many times in the past at the expense of taking into account what is actually present in the external environment (cf., Gray & Fu, 2001). For example, after solving addition facts, undergraduates in the current study may have persisted in using an add-all strategy when it was no longer appropriate, simply because it was the strategy that had been retrieved most often in the recent past. One variant of Luchins's (1942) water jar experiments provides support for this view. Participants were warned, "Don't be blind," before solving the target problems, and were, as a result, less likely to persist in using the original strategy. If the current results are due to mindlessness, then the negative effects of arithmetic might be overcome by reminding undergraduates to be mindful.

In terms of educational implications, the present experiments were far removed from children learning in classrooms; thus, it would be premature to suggest that the processes underlying undergraduates' performance are the same as the processes underlying children's difficulties with algebra. Nonetheless, findings suggest that the representations constructed early on in mathematics may not be abandoned, even after years of instruction. Thus, it may be especially important for educators to pay attention to the ways in which children's early experience is structured.

For example, results support a growing body of research suggesting that children should not practice arithmetic facts in a single-problem format as they do year after year in traditional mathematics classrooms in the United States (Seo & Ginsburg, 2003; Valverde & Schmidt, 1997). Arithmetic problems are almost always presented in the format $a + b = c$ in elementary- and middle-school mathematics textbooks in the United States (McNeil et al., 2006; Seo & Ginsburg, 2003). This finding is troublesome, given that equations with operations on both sides of the equal sign are more likely than other equation formats to elicit correct, relational interpretations of the equal sign (McNeil & Alibali, 2005a; McNeil et al., 2006).

Unvarying practice in a single context often leads to inflexible knowledge that is not applied appropriately to new contexts (Chen, 1999; Chen & Mo, 2004; Cognition and Technology Group at Vanderbilt, 1997; Gick & Holyoak, 1983). However, exposure to a variety of problem types facilitates broader knowledge representations (Chen, 1999; VanderStoep & Seifert, 1993; Tennyson & Tennyson, 1975). Thus, instead of presenting arithmetic problems in a single-problem format, textbooks and educators should expose children to a more varied, richer set of problem formats from the beginning of formal schooling (Baroody & Ginsburg, 1983; Blanton & Kaput, 2005; Carpenter, Franke, & Levi, 2003; Denmark, Barco, & Voran, 1976; Hiebert et al., 1996; Kaput, 1998; National Council of Teachers of Mathematics, 2000). Learning and practicing arithmetic facts in a variety of formats seems like a prudent, and simple, change to instructional practice, especially because it recently has been shown to facilitate children's understanding of mathematical equivalence (McNeil, 2008).

Despite the present study's strengths, several aspects warrant additional research. For example, it is unclear how long the negative effects of solving arithmetic problems persist. In undergraduates, where the operational view of equations coexists with the more advanced, relational view, negative effects of arithmetic may persist for minutes, not hours. However, in people whose relational view of equations is fundamentally weaker (e.g., noncollege adults, middle-school children), negative effects may last longer. How long negative effects of arithmetic last, and whether or not this differs for children and adults, merits additional research.

Future research should also explore if undergraduates' old, operational view of equations can ever be revised or abandoned. In this study, arithmetic activated the operational view of equations, even in undergraduates who achieved highly in algebra. These results support theories of learning and development that suggest that old representations continue to coexist alongside newer, more advanced representations, and new or old may be activated, depending on the task and context (e.g., Barsalou, 1982; Munakata et al., 1997; Siegler & Stern, 1998; Thelen & Smith, 1994). However, it remains possible that some adults (e.g., mathematicians, scientists, engineers) may have learned algebra so well that their knowledge of algebra subsumes their knowledge of arithmetic. If we can identify individuals who have overcome the operational view, then we may not only gain clues about the nature of cognitive change but also uncover ways to help individuals whose operational view is still strong enough to hinder their success with algebra.

Finally, future research should explore the features of more and less effective forms of experience with arithmetic. For example, experience with arithmetic tasks that share the same conceptual structure as the target problem (e.g., the algebra condition in the present study) can facilitate performance. Similar positive effects might be observed for experience solving a single type of arithmetic problem presented in multiple formats or experience solving multiple types of arithmetic problems (Gick & Holyoak, 1983; VanderStoep & Seifert, 1993). If we can ascertain which forms of arithmetic experience are most effective at promoting flexible knowledge structures, we may be able to tailor early arithmetic instruction to help ease children's transition from arithmetic to algebra.

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