It's a Pattern! The Importance of Early Pattern Knowledge for Mathematics

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Pattern: a predictable sequence
E.g., alternating sequence of shapes or sounds
E.g., functional relationships between two variables

Why pattern knowledge?

- Young children spontaneously engage in patterning activities (Ginsburg, Inoue, & Seo, 1999; Ginsburg, Lin, Ness, & Seo, 2003).
- Parents and preschool teachers report engaging children in patterning activities many times a week (Rittle-Johnson, Fyfe, Loehr & Miller, 2014)
- Patterning is a core skill for mathematical thinking (Charles, 2005; Sarama & Clements, 2004; Steen, 1988).
- Knowledge of patterns is included as a central algebraic topic in consensus documents in mathematics education (NCTM, 2000; NAEYC, 2014).

NCTM National Council of Teachers of Mathematics naeyc

Why *not* pattern knowledge?

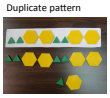
- The National Mathematics Advisory Panel (2008) concluded: "In the Major Topics of School Algebra set forth in this report, **patterns are not a topic of major importance**. The prominence given to patterns in PreK–8 is not supported by comparative analyses of curricula or mathematical considerations" (p. 59).
- The Common Core State Standards followed this recommendation, not including patterns as a math content standard at any grade level (CCSS, 2010).

My goals

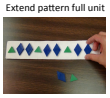
1. Review recent evidence indicating that patterns are a topic of major importance for mathematics
   a. Longitudinal evidence
   b. Experimental evidence
2. Suggest how early patterning activities can be re-designed to better bring out the mathematical nature of patterns

Sample Pattern Activities

Duplicate pattern



Extend pattern full unit



(e.g., Clements & Sarama, 2009; Klein & Starkey, 2004; Papic et al., 2011; Warren & Cooper, 2006)

1a. Early Pattern Knowledge Predicts Later Mathematics Achievement

- Children (n = 517) from low-income homes assessed across time
- Early predictors from pre-k, kindergarten, and first-grade
  - **Pattern knowledge**: Pattern items (n = 7) from Research-based Early Maths Assessment (REMA; Clements, Sarama, & Liu, 2008)
  - **General math knowledge**: Quantitative concepts and Applied Problem Solving subtests from the Woodcock Johnson (WJ III) Tests of Achievement
  - **Reading knowledge**: Letter-Word Identification subtest from WJ-III
  - **Controls: General cognitive skills**, such as oral language, and demographics
- Math outcome measures in fifth-grade
  - KeyMath 3 Diagnostic Assessment subtests: Numeration, Algebra, Geometry
  - Quantitative concepts subtest from WJ-III
  - Created composite score by summing z-scores across the outcome measures because effects consistent across measures

(Rittle-Johnson, Fyfe, Hofer & Farran, 2015; under review)

1a. Early Pattern Knowledge Predicts Later Mathematics Achievement

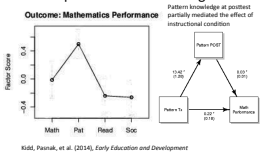
Early Measure	5th Grade Composite Math Score		
Academic Skills	**End PreK**	**End K**	**End 1st**
Pattern	.16 (.04)***	.11 (.04)**	.11 (.03)**
Math: Quant. Concepts	.30 (.05)***	.19 (.05)***	.16 (.04)***
Math: Applied Problems	.17 (.05)***	.25 (.04)***	.32 (.04)***
Reading	.02 (.04)	.07 (.04)	.12 (.04)**
Controls	Inc.	Inc.	Inc.

Standardized regression coefficients (standard errors), **p < .01, *** p < .001

1b. Improving Pattern Knowledge Improves Math Knowledge

- Instruction on repeating patterns supports:
  - Knowledge of growing patterns (Papic et al., 2011)
  - Knowledge of ratios (Warren & Cooper, 2007)
  - General math achievement (Kidd et al., 2013; Kidd et al., 2014)
- For example:
  - First-grade students (n = 120) with poor pattern skills were randomly assigned to receive supplemental instruction in their classrooms throughout the school year
    - Instruction conditions: Math, pattern, reading, social studies
  - Outcome measures at end of school year:
    - Woodcock Johnson Quant Concepts and KeyMath 3 Diagnostic Assessment (Kidd, Pasnak, et al., 2014)

1b. Improving Pattern Knowledge Improves Math Knowledge



Kidd, Pasnak, et al. (2014), *Early Education and Development*

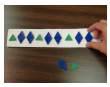
2. Bringing Out the Mathematical Nature of Patterns

- One solution may be to focus on the more advanced skills that highlight underlying structure and are accessible to preschoolers.

Easier Pattern Activities

Duplicating
- Making an exact copy of a model pattern

Extending
- Continuing a model pattern
  - Not just what comes next- require extending *by one pattern unit*



1. Summary

- Patterns are a topic of major importance for early mathematics learning and instruction
  1. Early pattern knowledge predicts later mathematics achievement
  2. Improving pattern knowledge improves math knowledge

Current Practices

- Teachers and past math standards often focused on simple pattern tasks: duplicating and extending simple patterns (Rittle-Johnson et al., 2014; NAEYC, 2014; NCTM, 2006)
- Are these tasks mathematically meaningful? (Economopoulos, 1998; NMAP, 2008; Threlfall, 1999)
  - Research evidence that supports the importance of pattern knowledge has included more sophisticated patterning activities

Abstract Pattern Task

Advanced Pattern Activities

Abstracting
- Recreating a model pattern using different materials

Unit Identification
- Identify the unit of repeat in reference to a model pattern

2a. Repeating Pattern Knowledge in Preschool

- 66 preschool children
  - Ages 4.0 years to 5.3 years
- Assessed in Fall and Spring of pre-k
- Developed and validated assessment and construct map of repeating pattern knowledge
  - Adapted from Clements and Sarama (2009, 2010)
  - Used Wilson (2005) Construct Modeling Approach

Rittle-Johnson, Fyfe, McLean & McEldoon (2013) *Journal of Cognition and Development*
Rittle-Johnson, Fyfe, Loehr & Miller (2014) *Early Childhood Research Quarterly*

2a. Construct Map for Repeating Pattern Knowledge

Level	Skill	Sample task
Level 4: Pattern unit recognition	Identifies the pattern unit	"What is the smallest tower you could make and still keep the same pattern as this?"
Level 3: Pattern abstraction	Translates patterns into new patterns with same structural rule.	"I made a pattern with these blocks. Please make the same kind of pattern here, using these cubes" (using new colors and shapes).
Level 2: Pattern extension	Extends patterns at least one pattern unit.	"I made a pattern with these blocks. Finish my pattern here the way I would."
Level 1: Pattern duplication	Duplicates patterns.	"I made a pattern with these blocks. Please make the same kind of pattern here."

IRT analyses provided strong support for construct map. Continuum - Not Stages!

2a. Improvements in Pattern Knowledge During PreK

Level	Time 1 (Fall) Proportion Correct	Time 2 (Spring) Proportion Correct
Level 4: Pattern unit recognition	.12	.16
Level 3: Pattern abstraction	.31	.60*
Level 2: Pattern extension	.47	.71*
Level 1: Pattern duplication	.77	.95*

n = 65; *improved from fall to spring

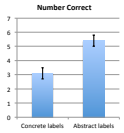
2b. Sources of Individual Differences in Pattern Knowledge

- Large individual differences in pattern knowledge in preschool
  - Low-income children have lower pattern knowledge than their middle-income peers (Rittle-Johnson et al., 2013; Starkey, Klein & Wakeley, 2004).
- Potential sources of individual differences:
  - Analogical reasoning skill (Miller, Rittle-Johnson, Loehr & Fyfe, 2015)
  - Executive function skills:
    - Working memory (Collins & Laski, 2015; Miller et al., 2013; Rittle-Johnson et al., 2013)
      - Helps to explain why lower-income children have lower pattern knowledge (Rittle-Johnson et al., 2013)
    - Cognitive flexibility (Bennett & Müller, 2010; Miller et al., 2015)
    - Visual-spatial short-term memory (Collins & Laski, 2015)



2c. Supporting Pattern Knowledge: Abstract Pattern Labels

- **Abstract label**: "The part that repeats in my pattern is A-B-B…"
- **Concrete label**: "The part that repeats in my pattern is purple-blue-blue…"
- "Can you make the same kind of pattern using these blocks?"
- 62 preschool children ages 3.6 – 4.9



Fyfe, McNeil & Rittle-Johnson (2015) *Child Development*

2. Summary: Bringing Out the Mathematical Nature of Patterns

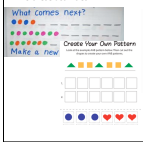
- 4-year-olds can go beyond simple pattern tasks
  - Many move beyond duplicating and finding what comes next
  - Can extend patterns and abstract patterns to recreate with new materials, although pattern unit recognition is difficult
    - Improves over prek year, although teachers not spending much time on abstracting patterns
    - Using abstract labels helps children abstract patterns
- Individual differences in cognitive skills impact pattern knowledge.

Conclusion


- Patterns are a topic of major importance for early mathematics learning and instruction
  1. Early pattern knowledge predicts later mathematics achievement
  2. Improving pattern knowledge improves math knowledge
- Early patterning activities can be re-designed to better bring out the mathematical nature of patterns
  - Young children are paying attention to structure in the world – build on this!
  - Label and discuss the unit of repeat
  - And…

Bring Out the Mathematical Nature of Patterns

Do these activities: **Not just these:**



Final Note:

- Pattern activities are a useful context for studying developmental issues:
  - Concrete vs. abstract labels
  - Analogical reasoning

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Slides, Paper and Materials available at:

vanderbi.lt/earlyalgebra

2b. Individual Differences in Preschool Pattern Knowledge

Variable	*B*	*SE*	β	ΔR^2
Block 1				.31
Age	0.72	0.21	.31**	
Analogical reasoning	0.27	0.06	.37**	
Block 2				.15
Working memory	0.22	0.07	.26**	
Inhibitory control	-0.06	0.03	-.13	
Cognitive flexibility	0.14	0.04	.29**	

** $p < .01$.

Miller, Rittle-Johnson, Loehr, & Fyfe (2015) *Journal of Cognition and Development*

Extra Slides

2b. Individual Differences in Preschool Pattern Knowledge

- 124 preschoolers, ages 4.0 to 5.8 years
- Pattern knowledge
- Analogical reasoning: Match-to-Sample task (Kotovsky & Genter, 1996)
- Executive Function:
  - Working memory: Backward digit span
  - Inhibitory control: Hand game (Hughes, 1996)
  - Cognitive flexibility: Flexible item selection task (FIST; Jacques & Zelazo, 2001)

Miller, Rittle-Johnson, Loehr, & Fyfe (2015) *Journal of Cognition and Development*