Learning from Explanation:
The Timing and Source of Explanations For Learning Early Algebra

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Different Perspectives on the Role of Explanation and Exploration in Learning

Theoretical Perspective. Some theories of learning focus on how much children learn through exploration and self-discovery of their environment without explicit instruction from a more knowledgeable other (Hirsh-Pasek, Golinkoff, Berk, & Singer, 2009; Piaget, 1973; Schulz & Bonawitz, 2007; Sylva, Bruner, & Genova, 1976). Other theories focus on how children learn through guidance and instruction from more knowledgeable others such as parents and teachers (Csibra & Gergely, 2009; Kirschner, Sweller, & Clark, 2006; Vygotsky, 1978). Both exploration and instruction are thought to benefit learning in numerous ways. For example, allowing learners to explore a new environment or topic area may increase their motivation, encourage broad hypothesis testing, and improve depth of understanding (Bonawitz, Shafto, Gweon, Goodman, Spelke, & Schulz, 2011; Piaget, 1973; Sylva et al., 1976). At the same time, children learn extensively from social partners, and teaching children new information directly can lessen the burden on cognitive resources and support the development of accurate knowledge (Kirschner et al., 2006; Klahr & Nigam, 2004; Sweller, van Merrienboer, & Paas, 1998; Tomasello, Carpenter, Call, Behne, & Moll, 2005).

Contemporary learning theorists have begun to integrate exploration and instructional guidance rather than contrast the two (e.g., Mayer, 2004; Schwartz & Bransford, 1998). This stems in part from the fact that learners left to discover knowledge on their own often fail to invent correct concepts and procedures (Klahr & Nigam, 2004; Rittle-Johnson, 2006). At the same time, explicit instruction alone often leads to rote memorization that is easily forgotten and is not integrated with learners' prior knowledge (Fisher, Hirsh-Pasek, Newcombe, & Golinkoff, 2013; Schwartz & Bransford, 1998). By combining instructional guidance with exploratory learning, one can avoid unfruitful dichotomies and capitalize on the strengths of each (Lorch, Lorch, Calderhead, Dunlap, Hodell, & Freer, 2010; Mayer, 2004; Schwartz & Bransford, 1998).

In this paper, I focus on combining exploration and direct instruction in the context of explanation and mathematics problem solving. Explanation is an important source of knowledge, and explanations can be generated by the learner (i.e., self-explanation) or provided by experts (i.e., instructional explanations). *Self-explanation* is a constructive activity – learners must construct the explanation based on their prior knowledge, features of the current example and recently encountered material (Chi, 2009). *Instructional explanations* provided by more knowledge others, such as teachers and parents, are meant to elucidate underlying reasons or pattern. Both types of explanations can improve learning but also have limitations (Renkl, 2002; Rittle-Johnson, 2006; Wittwer & Renkl, 2010).

Objective. The objective of this paper is to synthesize three of our recent studies on exploration and explanation. Across studies, all children explored unfamiliar mathematics problems and received instructional explanations. We manipulated the order of exploration and instruction to evaluate the impact of the timing of instructional explanations on exploration and learning. Studies varied in whether children were prompted to self-explain during the explore phase.

Empirical evidence comes from elementary school children learning about mathematical equivalence. Mathematical equivalence is the idea that two sides of an equation represent the same quantity. Math equivalence is foundational for arithmetic and algebra and requires knowledge of concepts (e.g., the meaning of the equal sign) and procedures (e.g., for solving problems with operations on both sides of the equal sign) (Kieran, 1981). Yet, elementary curricula do not typically include definitions of the equal sign or math equivalence problems— problems with operations on both sides of the equal sign (Powell, 2012). Children in Western countries have a persistent misconception about the meaning of the equal sign, often interpreting it as an operator symbol meaning "get the answer," rather than as a relational symbol that indicates two equal amounts (e.g., Baroody & Ginsburg, 1983; McNeil & Alibali, 2005). Further, this misconception often leads to poor performance on math equivalence problems (e.g., McNeil & Alibali, 2005).

Research Evidence. In the first study, 159 2nd-4th graders learned about mathematical equivalence during a one-on-one tutoring session (DeCaro & Rittle-Johnson, 2012). Children explored unfamiliar math equivalence problems and received instructional explanations in one of two orders: explore-instruct or instruct-explore. During the explore phase, we also manipulated whether children were prompted to self-explain or were given additional problems to solve (to control for time on task). Thus, children participated in one of four conditions.

See Table 1 for an overview of findings from the intervention for the explore-instruct vs. instruct-explore conditions. As expected, children in the explore-instruct conditions solved fewer problems correctly in the explore phase than those in the instruct-explore conditions. In contrast, strategy variability was greater in the explore-instruct condition. Both groups tried the same number of correct strategies; however, the explore-instruct condition tried more incorrect strategies. Surprisingly, differences in self-explanation quality (among those who were prompted to self-explain) were small and unreliable. Instructional explanations did not seem to impact self-explanation quality. Finally, the explore–instruct group was more likely to encode the structure of math equivalence problems correctly on a mid-test. The mid-test was given between the explore and instruct phases. We measured children's encoding of the problem structures by asking children to reproduce equivalence problems from memory. Exploring the problems allowed children in the explore–instruct condition to better notice the structure of the equations than children who had received explicit instruction. Overall, the timing of instruction impacted exploration, but not explanation, during the explore phase.

On both an immediate posttest and a two-week retention test, children in the explore-instruct conditions demonstrated greater conceptual knowledge than children in the instruct-explore conditions (see Figure 1). This was true for both explicit knowledge of the equal sign and knowledge of equation structures. The two groups did not differ in procedural knowledge – in accuracy at solving math equivalence problems with familiar problem features (learning) or

with novel problem features (transfer). Contrary to expectations, self-explanation prompts did not impact performance relative to solving additional problems.

The results of Study 1 highlight potential consequences of providing instruction prior to problem exploration. Instructional explanations can reduce exploration and learning. However, we had expected instructional explanations to impact self-explanation quality, as it had in past research (Matthews & Rittle-Johnson, 2009). In Study 2, we worked to improve the connection between the instructional explanations and the self-explanation prompts. Different explanation prompts can trigger different cognitive processes and lead to different learning outcomes (Nokes, Hausmann, VanLehn, & Gershman, 2011). Thus, in Study 2, we used conceptual self-explanation prompts to facilitate knowledge integration. We also employed two techniques thought to activate and engage misconceptions better: inclusion of familiar problem types in line with a common misconception and side-by-side contrast of the familiar problem with a novel problem type (Vosniadou & Vamvakoussi, 2006).

In Study 2, we worked with 122 second- and third-grade students (Fyfe, DeCaro, & Rittle-Johnson, 2014). Once again, children explored unfamiliar math equivalence problems and received instructional explanations in one of two orders: explore-instruct or instruct-explore. During the explore phase, all children solved familiar and unfamiliar math problems and were prompted to self-explain.

See Table 1 for an overview of findings from the intervention. As expected, children in the explore-instruct conditions solved fewer problems correctly in the explore phase than those in the instruct-explore conditions. Unlike Study 1, strategy variability was similar in the two conditions. However, this was because the explore-instruct group tried more incorrect strategies but fewer correct strategies. Children in the explore-instruct condition were less likely to discover a correct strategy. Unlike Study 1, there were also differences in self-explanation quality. Instructional explanations increased conceptual self-explanations. Finally, children in the instruct-explore group were more likely to encode the structure of the problems correctly at mid-test, unlike in Study 1. Overall, there were no advantages to exploration prior to instructional explanations during the intervention; rather, instruction first aided learning.

As would be expected from intervention behavior, children in the instruct-explore condition demonstrated greater knowledge across the posttest and retention test (see Figure 1, panel B). They had greater success on procedural knowledge items, both learning and transfer problems. They also had greater conceptual knowledge of problem structure. Conceptual knowledge of the equal sign was similar. Explanation quality during the intervention partially mediated the effect of condition on learning outcomes. Accuracy during the intervention was also an important mediator.

The findings of Study 1 and 2 highlight how relatively minor changes can impact the relation between instructional explanations and exploration, both for exploration behaviors and for learning outcomes. In a final study, we were interested in testing these effects in a classroom setting. Effective self-explanation seemed hard to accomplish in classrooms. Effective exploration may be easier to achieve.

In Study 3, we worked with 47 2nd grade students in small groups during their math class. Children were not prompted to self-explain given the difficulties of implementing this consistently in small groups. During the explore phase, we included more familiar problems, but not ones that activate misconceptions. We also did not provide immediate feedback, both because it was no longer practical and because evidence indicates that immediate feedback can harm learning for children with some knowledge of a correct procedure (Fyfe, Rittle-Johnson, & DeCaro, 2012). The posttest was given the day following the intervention. A retention test was not given.

Because data was collected in classrooms, we had less evidence on behavior during the intervention. Accuracy in the explore phase was higher in the instruct-explore condition (88% vs. 60% correct). However, at posttest, those in the explore-solve condition had greater knowledge (see Figure 1, panel C). In particular, their procedural knowledge was higher for learning and transfer problems. Conceptual knowledge was similar across conditions.

Significance. Overall, the timing of instructional explanations relative to problem solving impacted learning. However, the optimal timing seemed to vary based on whether learners were supported in generating self-explanations that built on the instructional explanations and in whether they were confronted with misconceptions during the explore phase.

The benefits of exploring prior to instruction converge with broader research on the benefits of an explore-instruct approach. For example, Schwartz and colleagues suggest that problem exploration should be used to prepare students for future instruction because explicit instruction often presupposes a level of prior knowledge that novices lack (Schwartz & Bransford, 1998; Schwartz, Chase, Chin, & Oppezzo, 2011; Schwartz & Martin, 2004). Exploring a set of domain-relevant problems can help build up their knowledge and thus prepare them to learn more from instruction. Prior exploration can also create opportunities for productive failure (Kapur, 2011, 2012), in which learners experience difficulty discovering correct solutions, but ultimately process subsequent instruction at a deeper level. Finally, instruction is criticized for constraining future exploration (Bonawitz et al., 2011), which can have negative consequences for learning untaught information. The proposed solution is "delaying instruction until the learner has had a chance to investigate on her own" (Bonawitz et al., 2011, p. 328). This perspective is also in line with Dewey's (1910) vision of thinking and education, which incorporated learning through active inquiry and facilitated guidance. Finally, teachers in mathematically precocious countries, such as Japan, endorse teaching in line with the explore-instruct approach. They believe that "students learn best by first struggling to solve mathematics problems [and] then participating in discussions about how to solve them" (Stigler & Hiebert, 1998, p. 3).

The current findings also highlight the need to identify boundary conditions for the benefits of exploration prior to instruction. There is a time for providing instruction before exploration. Providing instructional explanations, promoting concept-based self-explanations and activating misconceptions during problem solving is one promising time. More generally, the studies illustrate the need to consider both exploration and explanation in learning and instruction. Both the source of explanations (i.e., self or instructional explanation) and their timing can impact learning.

Table 1:
*Performance During the Intervention For Explore-Instruct vs. Instruct-Explore Conditions*

	Study 1 (DeCaro & Rittle-Johnson, 2012)		Study 2 (Fyfe, DeCaro & Rittle-Johnson, 2014)	
	Explore-Instruct	Instruct-Explore	Explore-Instruct	Instruct-Explore
Solution accuracy in explore phase	52%*	65%	34%*	55%
# correct strategies used	1.2	1.3	0.8 *	2.2
# incorrect strategies used	0.74* (out of 2)	0.47 (out of 2)	3.0* (out of 5)	1.6 (out of 5)
Frequency of concept-based explanations	26%	35%	26%*	46%
Accuracy encoding problem structure at mid-test	54%*	44%	37%*	52%

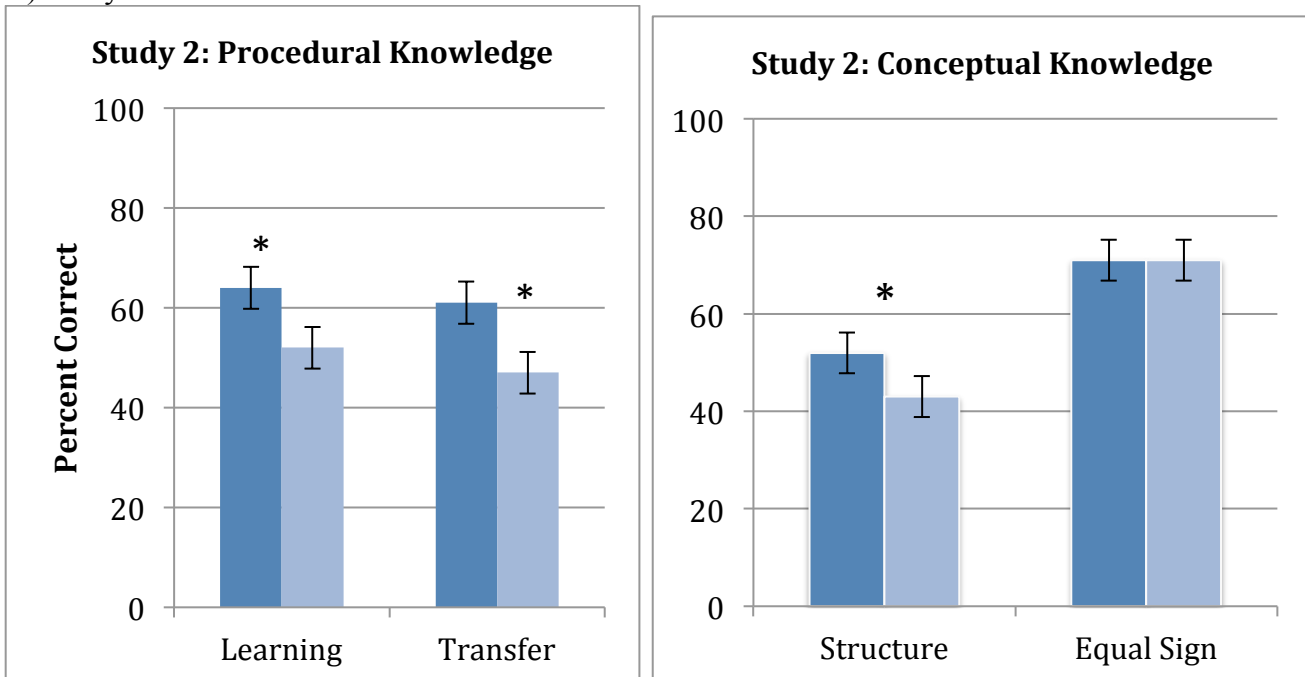Note: * Conditions differ at p < .05

Figure 1: Accuracy Across Posttest and Retention Study for Procedural and Conceptual Knowledge for Instruct-Explore (darker bars) and Explore-Instruct (lighter bars) Conditions
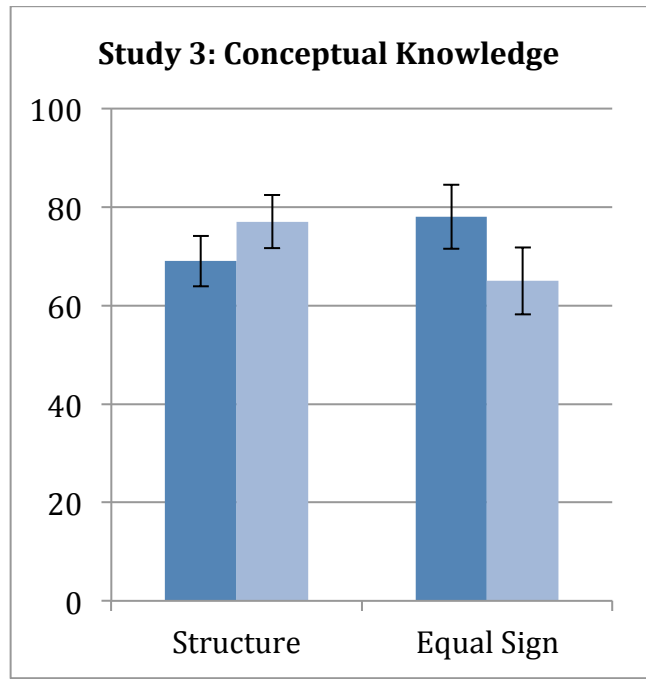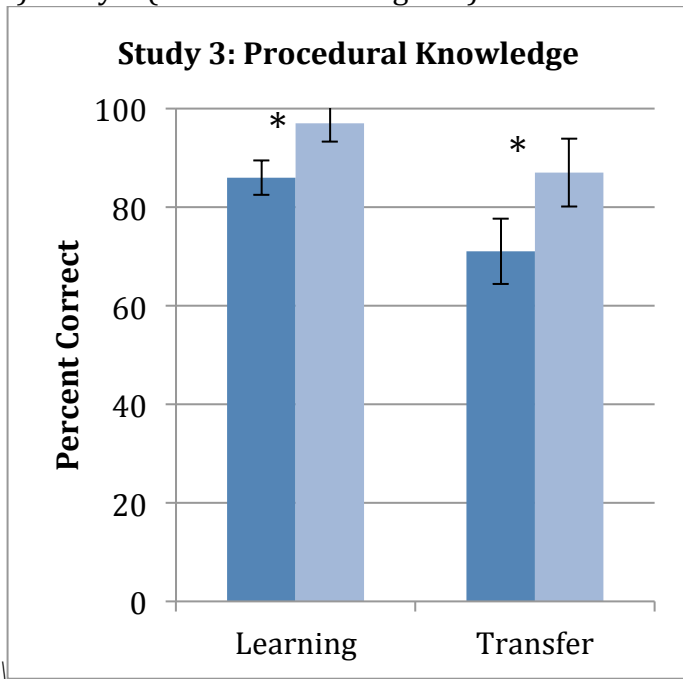**Indicates Condition Difference at p < .05*

A) Study 1
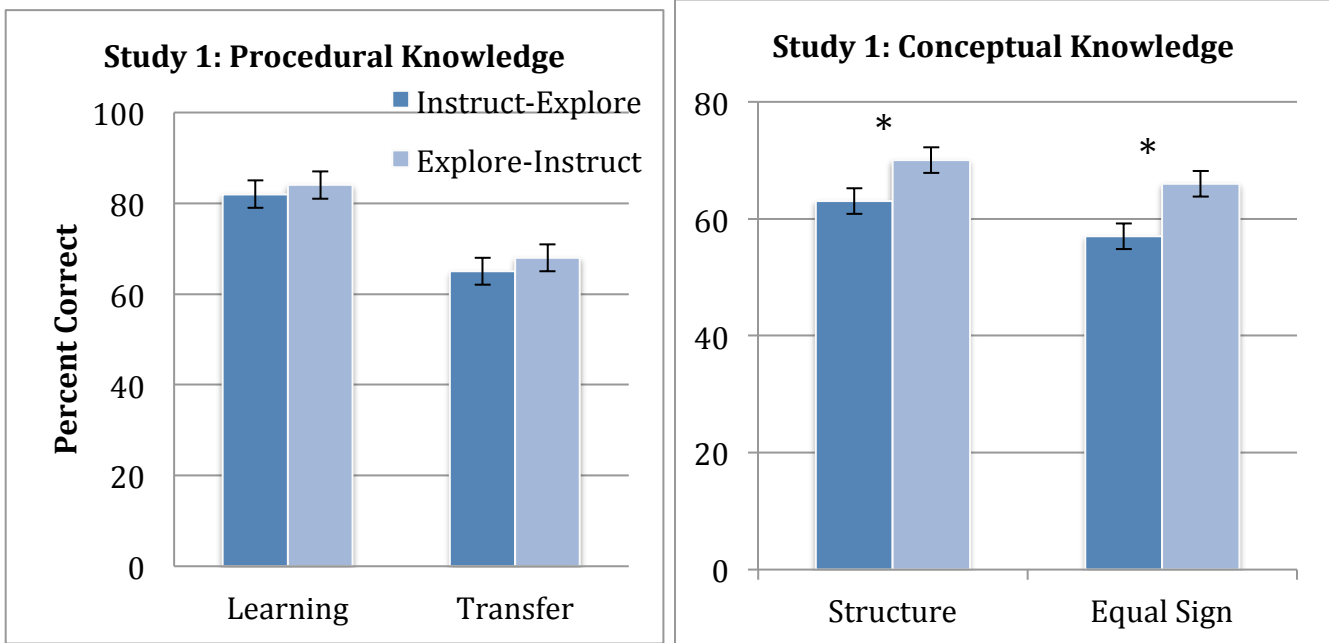
B) Study 2

C) Study 3 (no retention test given)

Study 3: Procedural Knowledge

Study 3: Conceptual Knowledge

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