

# Verbal Imprecision in Mathematical Explanations as an Indicator of Learning

Katherine L. McEldoon, Ran Liu & Bethany Rittle-Johnson  
Vanderbilt University

## Current Focus

Are disfluencies in self-explanation responses indicators of learning?

## Verbal Disfluency & Learning

As students learn a new domain, they must construct a schema or mental model of the domain and problem solving procedures. These must be iteratively revised as learning occurs.

- An impasse occurs when the step the learner believes should be executed next cannot be preformed (Van Lehn, 1988)
- Speakers use *uh* and *um* to implicate when they are searching or deciding about what to utter next (Clark & Fox Tree, 2002)
- Learning is thought to occur at these impasses (Van Lehn, 1988)

## Method

•**DOMAIN:** Mathematical equivalence is the principle that two sides of an equation represent the same value

• Foundational for algebra (Falkner, Levi, & Carpenter, 1999)

$$3 + 5 + 6 = \_ + 6$$

117 2<sup>nd</sup> through 4<sup>th</sup> graders with less than 75% correct at pretest on conceptual and procedural knowledge of mathematical equivalence



### Three Conditions

**Control (n = 39)** Matched for amount of practice  
Solve 6 problems

**Self-Explain (n = 39)** Matched for amount of time on task  
Solve 6 problems & explain

**Additional-Practice (n = 39)**  
Solve 12 problems

### Intervention Problems

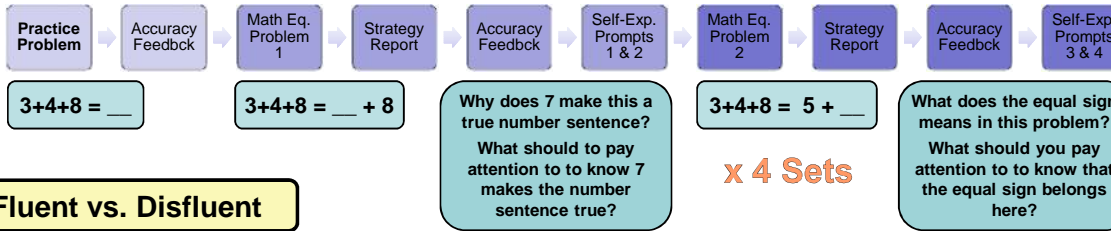
$$6 + 3 + 4 = 6 + \_ \quad 3 + 4 + 8 = \_ + 8$$

### Self-Explanation Prompts

Why does [7] make this a true number sentence?  
What should you pay attention to to know that [7] makes the number sentence true?

What does the equal sign mean in this problem?  
What should you pay attention to to know that the equal sign belongs here?

## Procedure



## Fluent vs. Disfluent

16 explanation response. Coded as disfluent if:

**Contained a pause, 'umm', or required additional experimenter prompting**

Of the 39 students in the self-explanation condition, 19 were fluent (21sec) and 13 were disfluent (27sec).

**Disfluent Explainers**  
≥ 4 Disfluencies

The numbers. [Exp. "Can you tell me more?"] The numbers... um... when you subtracting problems... you know where it goes.

[Long struggling] It equals 5. [Exp: Okay, can you tell me a little bit more?] The equal sign means the sum.

Umm... it means you, umm, you kinda, umm... added those... and it kinda, you kinda made it a 5.

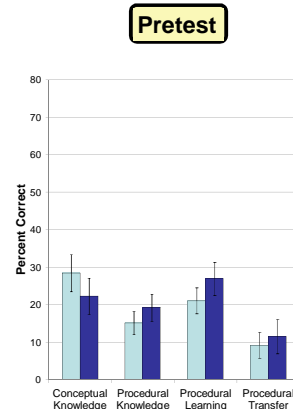
**Fluent Explainers**  
< 3 Disfluencies

Because I know that 10 plus 7 equals 17 just like 5 plus 3 plus 9.

The first side equals the second side, that is what an equal sign is.

The equal sign means they would be the same sum, the answer. It has to be the same answer, but it doesn't have to be the same number as the beginning...The equal sign means that

## Results



## Assessment Components

Knowledge of equivalence is typically assessed through (e.g., Rittle-Johnson, Matthews, Taylor & McEldoon, 2011; Behr, Erlwanger, & Nichols, 1980; Falkner, Levi, & Carpenter, 1999; McNeil, 2007; Rittle-Johnson & Alibali, 1999)

### Conceptual Knowledge

**Explicit-** Equal Sign Knowledge

What does the equal sign mean?

**Implicit-** Equation Structure Knowledge

3 + 5 = 5 + 3 True or False

### Procedural Knowledge

**Learning Items-** Same as those practiced during the intervention

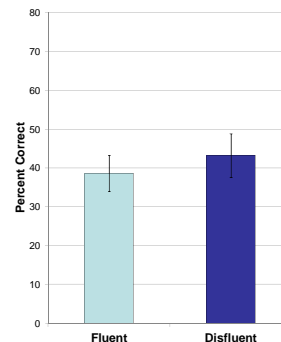
7 + 6 + 4 = 7 + \_\_\_

**Transfer Items-** Different from those practiced during the intervention

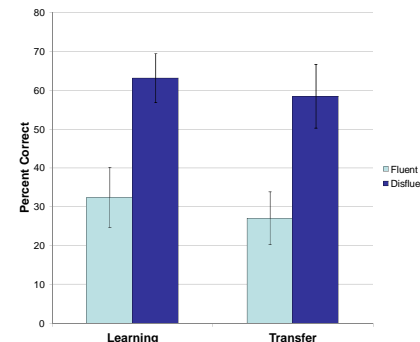
8 + \_\_\_ = 8 + 6 + 4    6 - 4 + 3 = \_\_\_ + 3

## Post & Retention Tests

### Conceptual Knowledge



### Procedural Knowledge



## Summary

Elementary students who were verbally imprecise have better problem solving skills at post and retention tests than those who had precise explanations.

When students are prompted to self-explain, they engage in a learning process where they move from an initial imperfect understanding of equivalence to a correct understanding. Verbal imprecisions, such as saying 'umm', pausing, or requiring addition prompting from the experimenter, may indicate a conflict between their initial imperfect mental model and the correct one. Processing this conflict, as evidenced by verbal imprecision, may help them perform better on solving learning items and more difficult items after intervention.

**"When children are imprecise while explaining ... [we presume they are] in the process of rejecting old, presumably inefficient or ineffective problem-solving approaches." (Perry & Lewis, 1999)**

## References

Behr, M., Erlwanger, S., & Nichols, E. (1980). How children view the equals sign. *Mathematics Teaching*, 92, 13-15.

Clark, H., & Fox Tree, J. (2002). Using uh and um in spontaneous speaking. *Cognition*, 84, 73-111.

McNeil, N. M. (2007). U-shaped development in math: 7-year-olds outperform 9-year-olds on equivalence problems. *Developmental Psychology*, 43(3), 687-695.

Perry, M., & Lewis, J. (1999). Verbal imprecision as an index of knowledge in transition. *Developmental Psychology*, 35(3), 749-759.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology*, 91(1), 175-189.

Rittle-Johnson, B., Matthews, P. G., Taylor, R. S., & McEldoon, K. L. (2011). Assessing knowledge of mathematical equivalence: A construct-modeling approach. *Journal of Educational Psychology*, 103(1), 85-104.

Van Lehn, K. (1988). Towards a theory of impasse-driven learning. In H. Mandl & A. Lesgold (Eds.), *Learning Issues for Intelligent Tutoring Systems* (pp. 19-41). New York, NY: Springer.

## Contact

Katherine L. McEldoon  
K.McEldoon@Vanderbilt.edu  
Psychology & Human Development, Peabody College, Vanderbilt University, Nashville, TN

## Acknowledgments

The first author is supported by a predoctoral training grant provided by the Institute of Education Sciences, U.S. Department of Education, through Vanderbilt's Experimental Education Research Training (EXPERT II) grant (David S. Cordray, Director; grant number R305B080025). The second author was supported by Vanderbilt University Summer Research Program. The opinions expressed are those of the authors and do not represent views of the U.S. Department of Education. This work was also supported by an NSF CAREER Grant (#DRL0746656) awarded to Dr. Bethany Rittle-Johnson.

