Abstract Title Page

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Title: An Alternative Time for Telling: When Conceptual Instruction Prior to Exploration Improves Mathematical Knowledge

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Abstract Body

Limit 4 pages single spaced.

Background / Context:

An emerging consensus suggests that guided discovery, which combines discovery and instruction, is a more effective educational approach than either one in isolation (e.g., Alfieri et al., 2011; Mayer, 2004; Schwartz & Bransford, 1998). However, guided discovery is a broad construct, making it difficult to determine the optimal combination of discovery and instruction in a given situation. In the current study, we examine two specific forms of guided discovery, testing if conceptual instruction should precede or follow exploratory problem solving. We compare these results with previous, contrasting findings to better understand the timing of instruction and discovery.

There are several reasons to suggest that providing instruction prior to exploratory problem solving is more effective than the converse. Prior instruction can familiarize learners with domain principles and give them the opportunity to discover how the principles apply during problem solving (Wittwer & Renkl, 2008). That is, prior instruction can facilitate the integration of the provided information with the problem solving activity. Prior instruction may also help reduce the cognitive demands of problem solving, by narrowing the problem space and limiting subsequent exploration (Bonawitz et al., 2011). Finally, prior instruction may facilitate the discovery of correct procedures (e.g., Chen & Klahr, 1999). Learners often fail to discover correct procedures on their own (e.g., Klahr & Nigam, 2004) and sometimes invent incorrect ones (Rittle-Johnson, 2006). Past research suggests that prior instruction can facilitate discovery of other domain knowledge, including correct procedures (e.g., Perry, 1991).

However, a number of researchers suggest delaying instruction until after exploration (e.g., Hiebert & Grouws, 2007; Schwartz et al., 2009). Learning from instruction often requires a level of knowledge that novices do not have. Providing prior exploration can facilitate the development of this knowledge and thus better prepare them to learn from instruction (Schwartz & Bransford, 1998). Further, delaying instruction may allow learners to explore more broadly (DeCaro & Rittle-Johnson, 2012) and to extract knowledge about problem structure, which is key for transfer (Schwartz et al., 2011). Growing evidence has accumulated in support of delaying instruction (e.g., Kapur, 2011; Schwartz & Martin, 2004). For example, middle school students who explored a set of density problems before hearing a lecture exhibited better transfer than students who heard the lecture first and practiced the problems after (Schwartz et al., 2011). In a recent experiment, we also found support for delaying conceptual instruction (DeCaro & Rittle-Johnson, 2012).

Initial studies on the timing of instruction raise several issues that warrant further consideration. For example, the majority of them provide instruction on a specific procedure. In this case, subsequent problem solving is no longer exploratory because students are told how to complete it. In contrast, prior conceptual instruction does not transmit ready-made solutions that may undermine the process of discovery, but rather guides the discovery activity (Wittwer & Renkl, 2008). Relatedly, the instruction should not remove the need for struggle during exploration, but rather provide guidance so that the struggle is productive (e.g., Hiebert & Grouws, 2007). If the exploratory activity is too simple, any prior guidance may prevent struggle and thus render it less helpful. Finally, the instruction and exploration should be well-matched to promote knowledge integration. In DeCaro and Rittle-Johnson (2012), evidence suggested that children did not sufficiently link the conceptual instruction and subsequent problem-solving, which potentially hindered the integration of knowledge. Thus, in the current study, we employed conceptual instruction, created a challenging problem solving task to maintain struggle, and had children explain the problems as a way to facilitate integration of the learned information with the exploratory activity. This study adds to a growing body of literature seeking to identify specific forms of guided discovery and the conditions under which they are effective.

Purpose / Objective / Research Question / Focus of Study:

The goal of this study was to examine two specific forms of guided discovery, testing whether conceptual instruction should precede or follow exploratory problem solving. In both cases, the problem solving is considered exploratory because children have not been told how to solve the problems; rather, children are expected to generate or apply their own problem solving procedures. The difference is in the timing of conceptual instruction. Providing instruction first offers more initial guidance and should support the integration of knowledge. Solving problems first offers less initial guidance, but may better prepare students to learn from the instruction. The focus of instruction was math equivalence—the idea that two sides of an equation represent the same quantity. Elementary curricula rarely include explicit instruction on math equivalence (Rittle-Johnson et al., 2011) and children often exhibit difficulties with the relevant concepts and procedures. Thus, math equivalence is an apt domain in which to investigate guided discovery.

Setting:

We worked with children from 12 second- and third-grade classrooms in two urban, public schools in Tennessee.

Population / Participants / Subjects:

Participants were 183 second- and third-grade children. Forty-seven were excluded from participation for scoring above 75% on a pretest designed to assess children's prior knowledge of math equivalence. Data from 14 additional children were excluded due to incomplete data (n = 5) or diagnosed learning disabilities (n = 9). The final sample consisted of 122 children (M age = 8 yrs, 2 mo; 57% female; 31% ethnic minorities).

Intervention / Program / Practice:

The intervention had a conceptual instruction phase and an exploratory problem solving phase. The two phases were identical for all children; the only difference between conditions was the timing of the two phases (instruct-solve vs. solve-instruct). During instruction, children were taught the concept of math equivalence. Specifically, in the context of five nonstandard number sentences (e.g., $3 + 4 = 3 + 4$), the experimenter provided a relational definition of the equal sign and explained how the two sides were equal. No procedures were discussed.

During problem solving, children solved 12 problems, presented in four sets. Each set contained three problems with similar addends. The first problem in each set was a standard arithmetic problem and the last two were math equivalence problems with operations on both sides of the equal sign (e.g., $5 + 3 + 9 = \square$, $5 + 3 + 9 = \square + 7$, $5 + 3 + 9 = 5 + \square$). Problems in a set were displayed on the same screen to facilitate spontaneous comparison. After each problem, children received accuracy feedback. On the math equivalence problems, children were prompted to self-explain. Specifically, after the correct answer was given, children were asked to explain it (e.g., "Why does x make this a true number sentence?"). Finally, on the last problem in each set children were asked to describe their problem solving procedure before receiving feedback. The problem-solving phase was intended to be difficult so that all children experienced a certain amount of struggle.

Research Design:

We used a pretest – intervention – posttest design followed by a two-week retention test. Children were randomly assigned to one of two conditions: instruct-solve or solve-instruct.

Data Collection and Analysis:

Children completed a pretest in their classrooms in one 25-minute session. Within 1 week, they completed a one-on-one tutoring intervention and posttest in a single session lasting approximately 50 minutes. Two weeks later they completed the retention test in small groups. A previously developed math equivalence assessment (Rittle-Johnson et al., 2011) was given at pretest, posttest, and retention test. It included conceptual (10 items) and procedural (8 items) knowledge scales. The procedural scale included both learning and transfer items. The conceptual scale assessed two key concepts: the meaning of the equal sign and the structure of equations. Items and scoring criteria are presented in Table 1. We also examined performance during the intervention by coding children's problem-solving procedures and the quality of their self-explanations. Coding is currently completed for half of the sample, (interrater agreement is high; kappas = 89% - 92%), and the remaining coding is currently underway.

Findings / Results:

To analyze performance on the posttest and retention test, we examined procedural and conceptual knowledge using two mixed-factor ANCOVAs. The model included time (posttest and retention test) and knowledge subscale as the within-subject effects and condition (instruct-solve and solve-instruct) as the between-subject effect. For procedural knowledge, the subscales were learning and transfer, and for conceptual knowledge, the subscales were equal sign and structure. To control for differences in prior knowledge and cognitive functioning, we included procedural and conceptual knowledge at pretest, age, working memory capacity, and retrieval fluency as covariates.

Procedural Knowledge. Children's procedural knowledge was moderate at posttest and remained similar two weeks later. There was a main effect of condition, $F(1, 115) = 4.69$, $p = .03$, $\eta_p^2 = .04$. As shown in Figure 1, children in the instruct-solve condition exhibited higher procedural knowledge than children in the solve-instruct condition. No other effects were significant, Fs < 2.3. Thus, the advantage of instruct-solve over solve-instruct was maintained two weeks later and was consistent across the procedural learning and transfer subscales.

Conceptual Knowledge. Children's conceptual knowledge was moderate at posttest and dropped slightly at retention, as indicated by a marginal effect of time, $F(1, 115) = 2.93$, $p = .09$, $\eta_p^2 = .03$. A main effect of subscale, $F(1, 115) = 16.67$, $p < .001$, $\eta_p^2 = .13$, was qualified by a condition by subscale interaction, $F(1, 115) = 3.52$, $p = .06$, $\eta_p^2 = .03$. No other effects were significant, Fs < 2. To follow up the interaction, we examined the effect of condition for each subscale (see Figure 2). There was a main effect of condition for the structure items, ($p = .03$), but not for the equal sign items, ($p = .84$). Children in the instruct-solve condition exhibited higher knowledge of structure than children in the solve-instruct condition at both time points.

Procedure Use. We next explored procedure use during the intervention. Relative to children in the solve-instruct condition, children in the instruct-solve condition used a greater number of correct procedures ($M = 2.2$, $SE = 0.3$ vs. $M = 0.8$, $SE = 0.3$, $F(1, 50) = 13.4$, $p = .001$,

$\eta_p^2 = .21$) and fewer incorrect procedures ($M = 1.5$, $SE = 0.3$ vs. $M = 3.0$, $SE = 0.3$, $F(1, 50) = 12.7$, $p = .001$, $\eta_p^2 = .20$). To test if correct procedure use predicted knowledge at posttest and retention test, we conducted our primary ANCOVAs with number of correct procedures as an added covariate. Correct procedure use predicted procedural, $F(1, 49) = 27.9$, $p < .001$, $\eta_p^2 = .36$, and conceptual knowledge, $F(1, 49) = 17.8$, $p < .001$, $\eta_p^2 = .27$. Further, its inclusion in the model eliminated the effect of condition for procedural knowledge ($p = .61$), as well as the condition by subscale interaction for conceptual knowledge ($p = .75$). Thus, promoting the use of correct procedures is one mechanism by which prior conceptual instruction improves learning.

Self-Explanations. We also explored the quality of children's explanations during the intervention. Children's explanations of correct answers included a conceptual rationale on nearly 40% of all trials (see Table 2), though this differed significantly by condition, $F(1, 50) = 14.9$, $p < .001$, $\eta_p^2 = .23$. Children in the instruct-solve condition generated more conceptual explanations ($M = 53\%$, $SE = 6\%$) than children in the solve-instruct condition ($M = 20\%$, $SE = 6\%$). To test if explanation quality predicted knowledge at posttest and retention test, we conducted our primary ANCOVAs with frequency of conceptual explanations as an added covariate. Conceptual explanations predicted both procedural, $F(1, 49) = 20.8$, $p < .001$, $\eta_p^2 = .30$, and conceptual knowledge, $F(1, 49) = 17.8$, $p < .001$, $\eta_p^2 = .27$. Further, its inclusion in the model eliminated the effect of condition for procedural knowledge ($p = .56$), as well as the condition by subscale interaction for conceptual knowledge ($p = .75$). Thus, enhancing the quality of explanations is another mechanism by which instruct-solve improves learning.

Conclusions:

Evidence suggests that guided discovery promotes deeper learning than discovery or instruction alone (Alfieri et al., 2011). We examined two forms of guided discovery, testing whether conceptual instruction should precede or follow exploratory problem solving. Providing conceptual instruction prior to exploration resulted in higher procedural learning, transfer, and conceptual knowledge of structure than delaying instruction—immediately and two weeks later.

Our results also suggest two potential explanatory mechanisms for the benefits of prior instruction. Children who received instruction prior to problem solving generated a greater number of correct strategies than children who solved the problems first. This is consistent with research indicating that prior instruction can facilitate the discovery and generalization of other, relevant domain knowledge (e.g., Chen & Klahr, 1999; Perry, 1991). Prior conceptual instruction also enhanced the quality of children's explanations. It seems the self-explanation prompts helped children integrate the provided information with the problem-solving activity (e.g., Chi, 2000), thus resulting in more conceptual explanations. Consistent with prior research, higher-quality explanations were associated with greater learning outcomes (e.g., Chi et al., 1989).

The primary results are in contrast to several recent studies (e.g., Schwartz et al., 2011), including one of our own (DeCaro & Rittle-Johnson, 2012). Comparisons across these studies can help refine our conclusions and elucidate the boundaries for specific forms of guided discovery. For example, our problem-solving activity was more challenging and also elicited more information from the instruction relative to that in DeCaro and Rittle-Johnson (2012). Thus, when struggle is greater, but knowledge integration is supported, prior conceptual instruction may be more beneficial than delayed conceptual instruction. Other studies suggest that when the instruction is procedural in nature, exploring problems first may be more effective (e.g., Kapur, 2011; Schwartz & Martin, 2004). These results continue to push our understanding of different forms of guided discovery learning and the conditions under which they work.

Appendices

Not included in page count.

Appendix A. References

References are to be in APA version 6 format.

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Appendix B. Tables and Figures

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Table 1

Example Items from the Math Equivalence Assessment

Item Type	Example Items	Scoring Criteria
Procedural		
Learning Items (α = .87) (*n* = 4)	Solve problem with operation on right side (8 = 6 + □)	Use correct strategy (if ambiguous, must be ±1 of correct answer)
	Solve problem with operations on both sides, blank on right (7 + 6 + 4 = 7 + □)	Same as above
Transfer Items (α = .89) (*n* = 4)	Solve problem with operations on both sides, blank on left (□ + 6 = 8 + 6 + 5)	Same as above
	Solve problem with operations on both sides, includes subtraction (5 – 2 + 4 = □ + 4)	Same as above
Conceptual		
Equal Sign Items (α = .66) (*n* = 5)	Define equal sign	Provide relational definition (e.g., the same amount as)
	Rate definitions of equal sign as good, not good, or don’t know	Rate “two amounts are the same” as a good definition
Structure Items (α = .73) (*n* = 5)	Reproduce math equivalence problems from memory	Reconstruct numerals, operators, equal sign, and blank in correct location
	Indicate whether equations such as 3 = 3 are true or false	Correctly recognize nonstandard equations as true or false

Note. Cronbach alphas are for retention test. Alphas were similar at posttest, but somewhat lower at pretest largely due to floor effects on some items.

Table 2

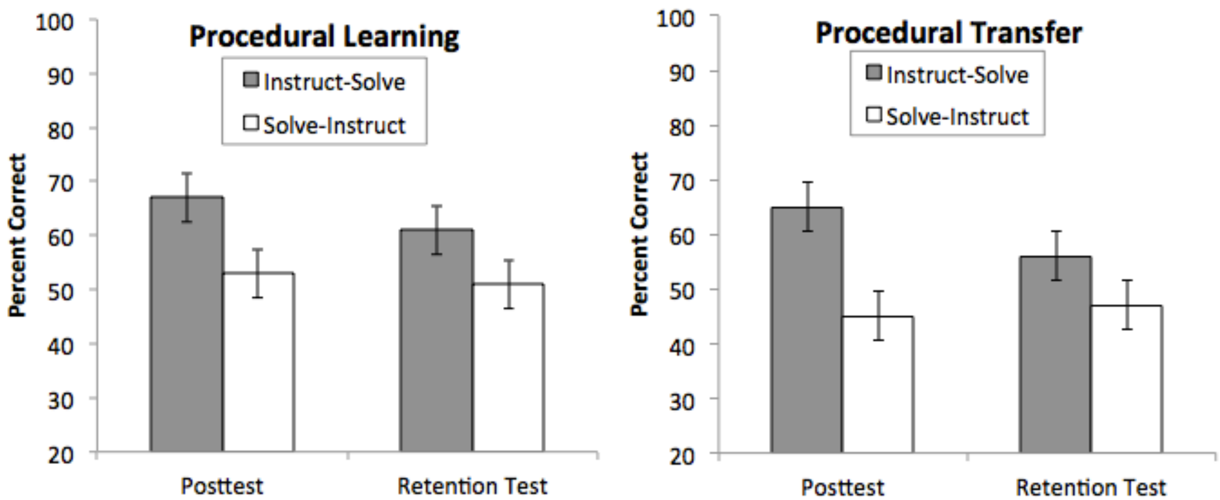
Children's Explanations for Why an Answer Was Correct (e.g., "Why does x make this a true number sentence?")

Explanation Type	Sample Explanation	% Trials		
		Instruct-Solve	Solve-Instruct	Total
Equal Sides (Conceptual)	They both have to equal the same	45	19	32
Equal Sign (Conceptual)	Because there's an equal sign	8	2	5
Answer	Because that's the answer	8	20	14
Procedure	Because you need to add	26	31	29
Other	That's how you do it	13	28	20

Note. Based on half the sample that has been coded thus far.

Figure 1

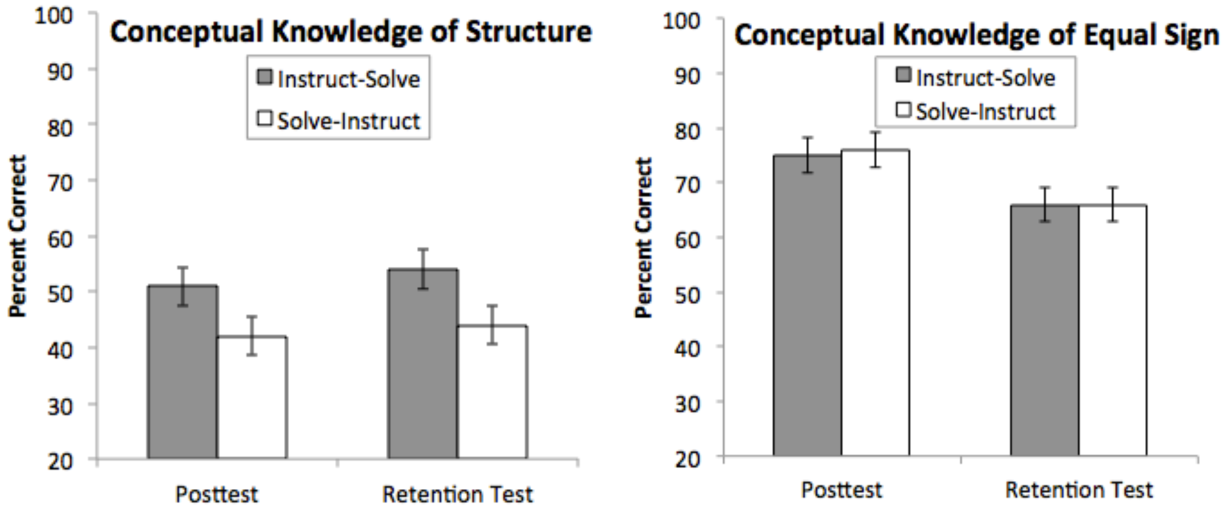
Procedural Learning and Transfer by Condition at Posttest and Retention Test



Note. Scores are estimated marginal means. Error bars represent standard errors.

Figure 2

Conceptual Knowledge of Structure and Equal Sign by Condition at Posttest and Retention Test



Note. Scores are estimated marginal means. Error bars represent standard errors.