OVERCOMING MISCONCEPTIONS: INSIGHTS FROM RESEARCH ON UNDERSTANDING THE EQUAL SIGN

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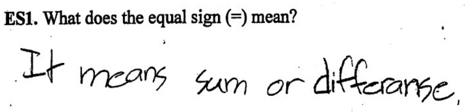
Project website

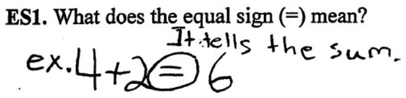
vanderbi.lt/earlyalgebra

A Common Misconception



ES1. What does the equal sign (=) mean?

It means sum or differanse,



ES1. What does the equal sign (=) mean?

ex. $4+2 = 6$ It tells the sum.

- Apparent when asked to solve equations with operations on both sides of the equal sign.
  - $3 + 7 = \square + 6$

$3 + 7 = \square + 6$

- “because 3 plus 7 equals 10.”

Reflects an ***Operational View*** of equal sign

Misconceptions Galore!

- In Science
  - Force as a property of physical objects (Chi, Slotta, & de Leeuw, 1994).
  - Sun and moon revolve around the earth (Vosniadou & Brewer, 1992, 1994).
- In Mathematics
  - Incorrectly apply natural number concepts to rational numbers (Durkin, 2012; Hartnett & Gelman, 1998; Merenluoto & Lehtinen, 2002; Stafylidou & Vosniadou, 2004).
    - E.g., .25 is bigger than .7 because 25 is bigger than 7
- Immature understanding

Misconceptions Necessitate Conceptual Change

- "The term *conceptual change* is used to characterize the kind of learning required when the new information to be learned comes in conflict with the learners' prior knowledge …. a major reorganization of prior knowledge is required—a conceptual change." – Vosniadou & Verschaffel (2004) p. 445
- Contrast with *additive* knowledge change – enrichment of prior knowledge.

Talk Themes

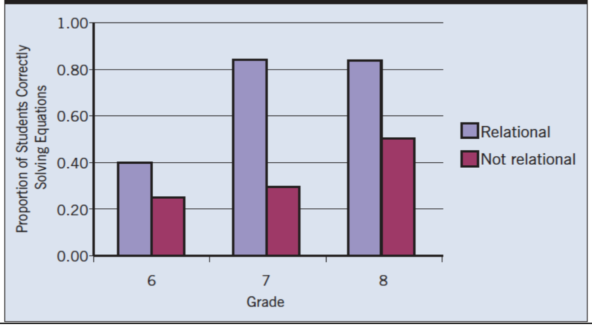
1. Conceptual change happens *gradually over time* as learners adjust their knowledge to integrate particular instances that contradict their current knowledge.
    - A construct modeling approach is useful to assessing, and thus helping understand, this change process.
2. Instructional methods that support additive knowledge change can also be appropriate for supporting conceptual change.
    - Focus on *self-explanation* & *problem exploration*.

- Illustrate for understanding ***mathematical equivalence.***
    - Principle that two sides of an equation represent the same value (also called equality). Symbolized by "="
        - ***Relational*** view of equal sign

Why Math Equivalence?

- Mathematical equivalence is an early developing & foundational concept in algebra
    - Provides the foundation for key algebra proficiencies (e.g., Carpenter et al., 2003; Kieran, 1992; Knuth, Stephens, McNeil, & Alibali, 2006; MacGregor & Stacey, 1997)

Fig. 4 Proportion of sixth-, seventh-, and eighth-grade students in each equal sign understanding category that solved the linear equations correctly ($n = 177$)

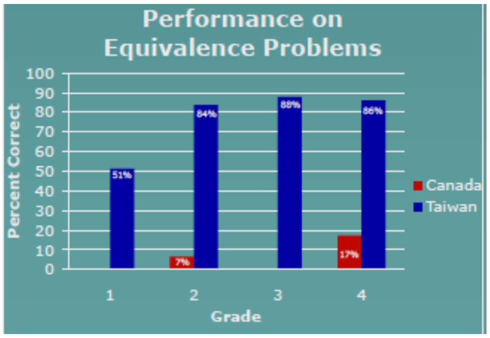


From Knuth, Alibali, Hattikuder, McNeil & Stephens (2008)

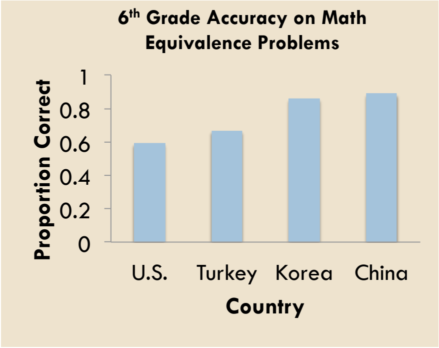
Prevalence of Misconception

- 35 years of research indicates that a majority of first through sixth graders in the U.S. have operational view of equal sign (e.g., Weaver, 1973, Behr, Erlwanger & Nichols, 1980; Perry, 1991; Alibali, 1999; Powell & Fuchs, 2010)

Operational View is Not Universal



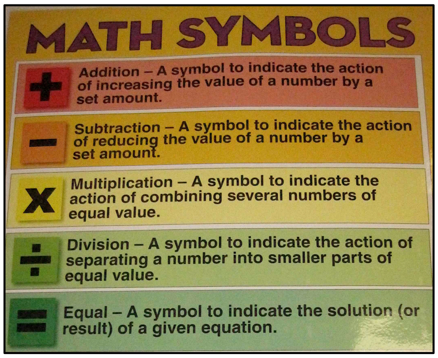
Watchorn, Lai & Bisanz (2009)



Capraro,Yetkiner, Ozel & Capraro (2009)

- Misconception not based on interactions in natural world; rather product of experience with mathematics instruction

Potential Source of Misconception



MATH SYMBOLS

Addition – A symbol to indicate the action of increasing the value of a number by a set amount.

Subtraction – A symbol to indicate the action of reducing the value of a number by a set amount.

Multiplication – A symbol to indicate the action of combining several numbers of equal value.

Division – A symbol to indicate the action of separating a number into smaller parts of equal value.

Equal – A symbol to indicate the solution (or result) of a given equation.

- Explicit definition of equal sign is rare: not present in textbook series we analyzed

Potential Source of Misconception

- In U.S., overwhelming exposure to equations in ***operations* = *answer*** format (e.g., 2 + 2 = 4)
  - *Textbook Analysis: Percentage of Instances of the Equal Sign in Each Equation Structure* (Rittle-Johnson et al., 2011)

	Grade						
	1	**2**	**3**	**4**	**5**	**6**	Average
Operations = Answer	97	82	70	52	38	31	62
Operations both sides	0	1	6	6	3	8	4

- Instruction on meaning of equal sign less effective if given with problems in operations = answer format (McNeil, 2008)
- Teachers overestimate children's knowledge of equivalence (e.g., Bisanz, Sherman & Watchorn, 2010)

Research Question:

How does children's knowledge of math equivalence develop?

Talk Theme 1

- Conceptual change happens gradually over time as learners adjust their knowledge to integrate particular instances that contradict their current knowledge
  - A construct modeling approach is useful to assessing, and thus helping understand, this change process.

Construct Modeling Approach

- Core idea (Wilson, 2005):
  - Develop and test a *construct map* – a representation of the continuum of knowledge that people are thought to progress through.

Equivalence Construct Map*

Sophistication

Level	**Description**	**Core Equation Structure(s)**
Level 4: Comparative Relational		
Level 3: Basic Relational		
Level 2: Flexible Operational		
Level 1: Rigid Operational		

* For children in U.S. & Canada

Knowledge change is gradual and dynamic, not stages.

Equivalence Construct Map

Level	Description	Core Equation Structure(s)
Level 4: Comparative Relational		
Level 3: Basic Relational		
Level 2: Flexible Operational		
Level 1: Rigid Operational	Define equal sign operationally. Only successful with equations with an *operations = answer* structure.	*Operations = answer* structure: a + b = c

Equivalence Construct Map

Level	Description	Core Equation Structure(s)
Level 4: Comparative Relational		
Level 3: Basic Relational	Successful with operations on both sides of the equal sign. Recognize and generate relational definition of the equal sign.	Operations on both sides: e.g., a + b = c + d
Level 2: Flexible Operational		
Level 1: Rigid Operational	Define equal sign operationally. Only successful with equations with an *operations = answer* structure.	*Operations = answer* structure: a + b = c

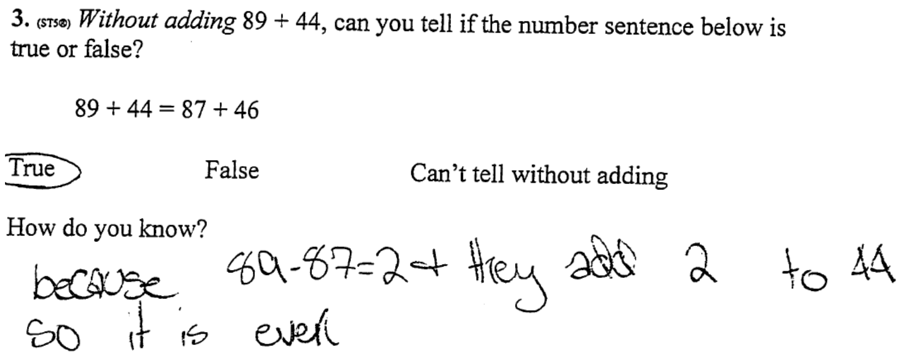
Equivalence Construct Map: Transitional Knowledge

Level	Description	Core Equation Structure(s)
Level 4: Comparative Relational		
Level 3: Basic Relational	Successful with operations on both sides of the equal sign. Recognize and generate relational definition of the equal sign.	Operations on both sides: e.g., a + b = c + d
Level 2: Flexible Operational	Successful with atypical equation structures that remain compatible with an operational view of the equal sign.	Operations on right *or* no operations: c = a + b & a = a
Level 1: Rigid Operational	Define equal sign operationally. Only successful with equations with an *operations = answer* structure.	*Operations = answer* structure: a + b = c

Equivalence Construct Map: Advanced Knowledge

Level	Description	Core Equation Structure(s)
Level 4: Comparative Relational	Compares the expressions on the two sides of the equal sign. Recognizes relational definition as the best definition.	Operations on both sides with multi-digit numbers or multiple instances of a variable.
Level 3: Basic Relational	Successful with operations on both sides of the equal sign. Recognize and generate relational definition of the equal sign.	Operations on both sides: e.g., a + b = c + d
Level 2: Flexible Operational	Successful with atypical equation structures that remain compatible with an operational view of the equal sign.	Operations on right *or* no operations: c = a + b & a = a
Level 1: Rigid Operational	Define equal sign operationally. Only successful with equations with an *operations = answer* structure.	*Operations = answer* structure: a + b = c

Comparative Relational Thinking: Level 4 Example



3. (ST5⊗) *Without adding* 89 + 44, can you tell if the number sentence below is true or false?

89 + 44 = 87 + 46

[✓ True] False Can't tell without adding

How do you know?

because 89-87=2 + they add 2 to 44 so it is even

(modified from Jacobs, et al, 2007)

Tasks

1. Solving Equations items: abilities to solve open equations.
   - 8 + 4 = ☐ + 5
2. Structure of Equations items: knowledge of valid equation structures.
   - 3 + 5 = 5 + 3 True or False
3. Defining the Equal Sign items: explicit knowledge of equal sign.
   - What does the equal sign mean?

(e.g., Alibali, 1999; Behr, Erlwanger, & Nichols, 1980; Falkner, Levi, & Carpenter, 1999; Li, Ding, Capraro, & Capraro, 2008; McNeil, 2007; Rittle-Johnson & Alibali, 1999; Weaver, 1973)

Assessment

- 31-item written assessment, using items from past research. (available on project website)
- Selected items so at least two per construct map level for each of the three common item types.
- Created **2 parallel forms** (to use as pretest and posttest in future research).

Data Source

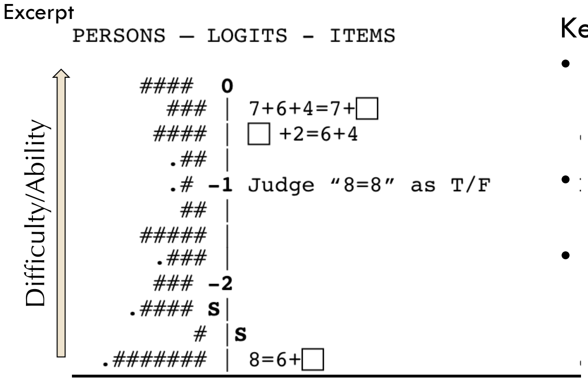
- Study 1: Assessment administered to 174 students in 2nd-6th grade classrooms.
  - Administered twice in the fall, two weeks apart.
  - For details, see Rittle-Johnson, Matthews, Taylor & McEldoon (2011)
- Study 2: Assessment administered to 224 students in 2nd-6th grade classrooms.
  - Administered once in spring
  - For details, see Matthews, Rittle-Johnson, Taylor & McEldoon (2012)

Construct Validity

- Compare empirical data to construct map
  - Rasch model – type of Item Response Theory (IRT) model.
    - Estimates the difficulty of each item and the ability of each student simultaneously.
  - Wright Map - graphical display of the results that helps us evaluate our construct map.

Interpreting the Wright Map

Excerpt



```
PERSONS — LOGITS - ITEMS

     ####   0
       ###  |  7+6+4=7+☐
      ####  |  ☐ +2=6+4
       .##  |
       .# -1  Judge “8=8” as T/F
        ##  |
     #####  |
      .###  |
      ### -2
    .#### S|
         #  |S
  .#######  |  8=6+☐
```

Key:

- On left: Each "#" is 2 students. Each "." is 1 student.
- On right: Individual items
- Center: Interval measurement scale in Logits, with 0 set to mean difficulty

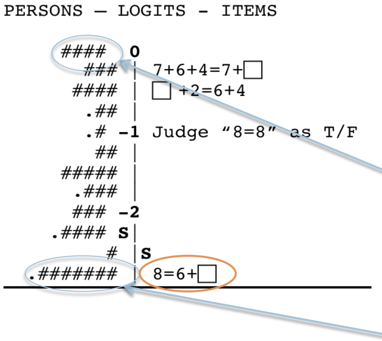
Data from Matthews et al (2012)

```
PERSONS — LOGITS - ITEMS

          . |
            4
      .#### |
            | If 76+45=121,does 76+45-9=121-9?
        .## |
          # 3 Judge “89+44=87+46” as T/F
       .### | If 56+85=141,does 56+85-7=141-7?       n+n+n+2=17. Find the value of n.
          # |S
    ####### S|
      .#### 2
            | c+c+4=16. Find the value of c
    .###### | What does the equals sign mean?
    ####### |
    ####### 1
      .#### |
         .# |
       .### |
      #### MOM
        ### | 7+6+4=7+c
       #### | □ +2=6+4               Judge “7+6=6+6+1” as T/F
        .## | 13=n+5. Find the value of n.
         .# -1 Judge “8=8” as T/F   Rating “The equal sign means two amounts are same”
         ## |
      ##### |
       .### |
       ### -2
     .#### S|
          # |S
   .####### | 8=6+□             Judge “4=4+0” as T/F
       .## -3
       #### |
        ### |
          # |
        ## -4
            |
          # |
           T|
           -5T
            | □+5=9
          . |
          . |
           -6

     EACH "#" IS 2 PEOPLE. EACH "." IS 1 PERSON.
```

Hypothesized Item Level
Level 4
Level 3
Level 2
Level 1

- Most items are at expected level of difficulty
- 2 items harder than expected:
- “What does the equals sign mean?” clustered with level 4 items
- “Judge 8 = 8 as true” clustered with Level 3 items

Interpreting the Wright Map: Probability of Success

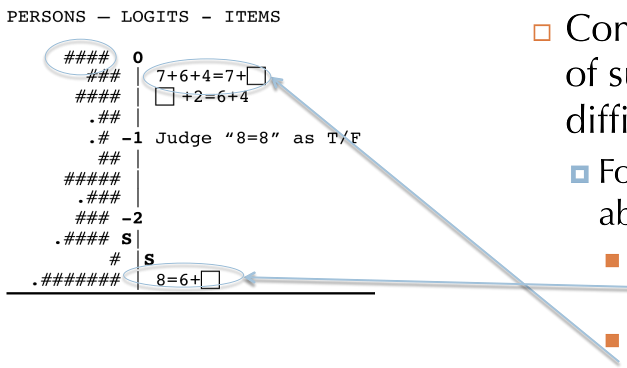


```
PERSONS — LOGITS - ITEMS

   ####  0
    ###  |  7+6+4=7+□
   ####  |  □ +2=6+4
    .##  |
     .# -1  Judge “8=8” as T/F
     ##  |
  #####  |
   .###  |
    ### -2
  .#### S|
      #  |S
.#######  |  8=6+□
```

$$\Pr(success) = \frac{1}{1+e^{-(\theta-d)}}$$

- Based on ability of student & difficulty of item.
- For Level 2 item 8=6+☐
  - Student of average ability expected to get correct 94% of time
  - Student with low ability estimate expected to get correct 50% of time

Interpreting the Wright Map



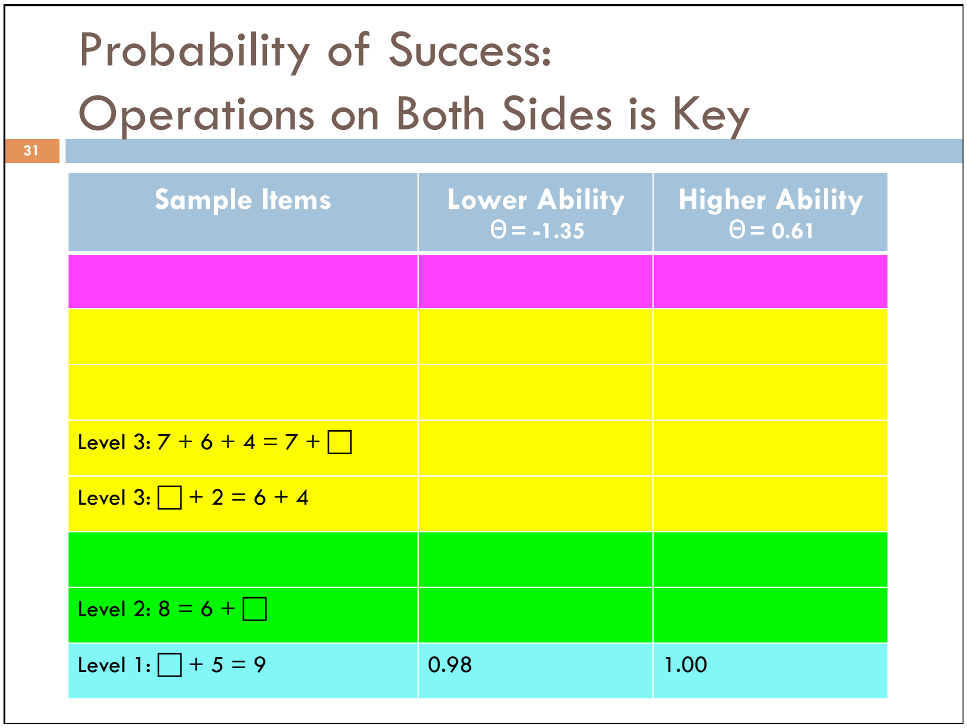
```
PERSONS — LOGITS - ITEMS

     ####  0
      ###  | 7+6+4=7+□
     ####  | □ +2=6+4
      .##  |
      .# -1 Judge "8=8" as T/F
       ##  |
    #####  |
     .###  |
     ### -2
   .#### S |
        #  |S
 .#######  |  8=6+□
```

- ☐ Compare probability of success based on difficulty of item:
  - For student of average ability:
    - This level 2 item correct 94% of time
    - This level 3 item correct 50% of time

$$\Pr(success) = \frac{1}{1+e^{-(\theta-d)}}$$

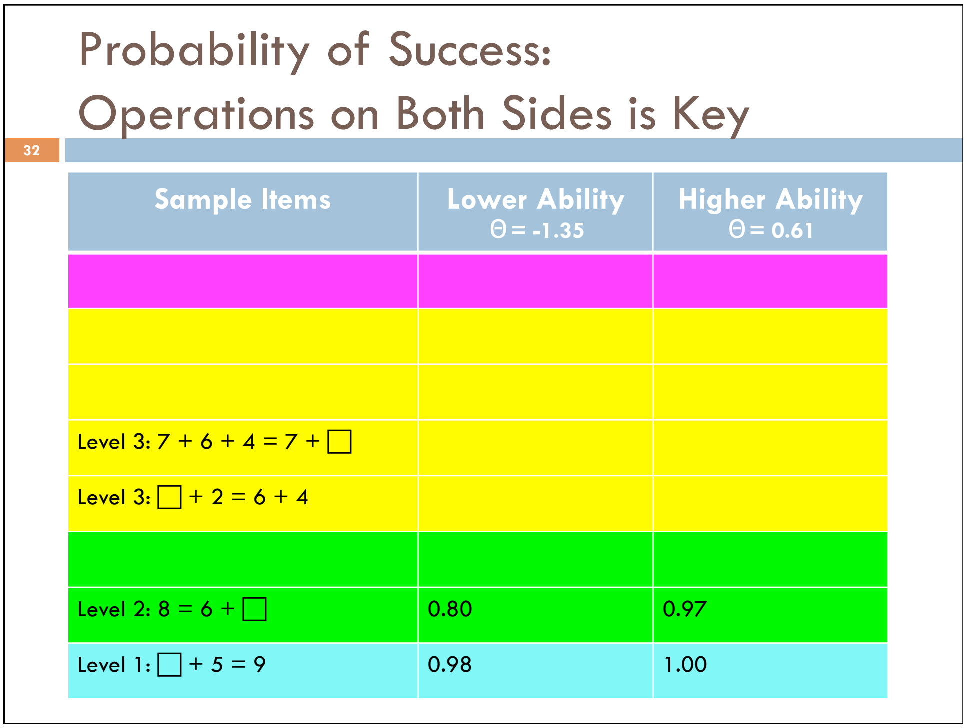
Probability of Success: Operations on Both Sides is Key

Sample Items	Lower Ability Θ = -1.35	Higher Ability Θ = 0.61
Level 3: 7 + 6 + 4 = 7 + □		
Level 3: □ + 2 = 6 + 4		
Level 2: 8 = 6 + □		
Level 1: □ + 5 = 9		



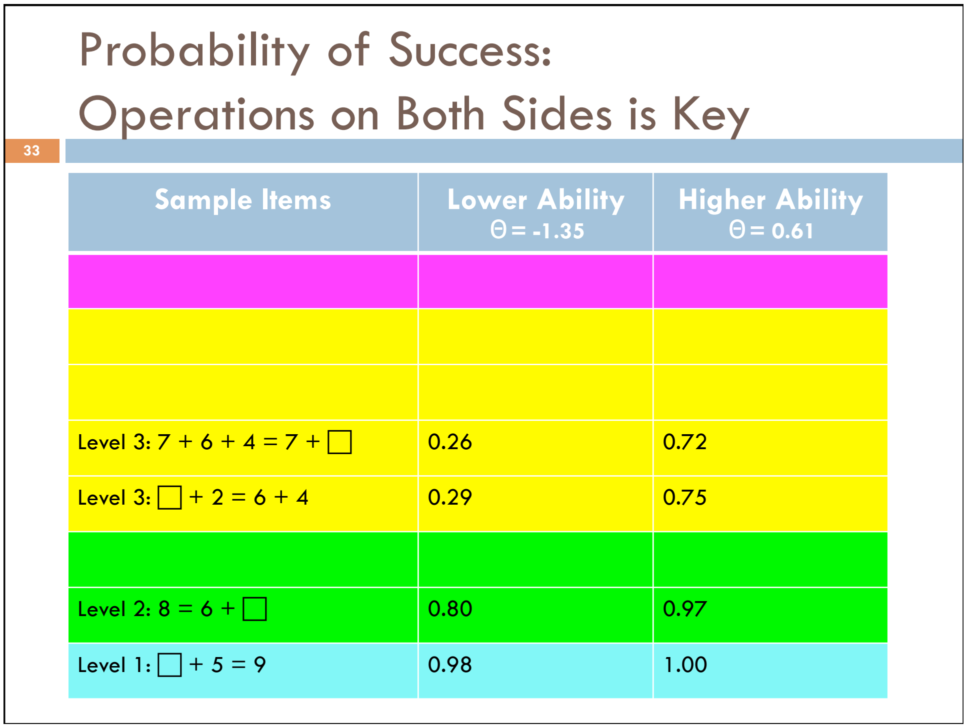
Probability of Success: Operations on Both Sides is Key

Sample Items	Lower Ability Θ = -1.35	Higher Ability Θ = 0.61
Level 3: 7 + 6 + 4 = 7 + ☐		
Level 3: ☐ + 2 = 6 + 4		
Level 2: 8 = 6 + ☐		
Level 1: ☐ + 5 = 9	0.98	1.00



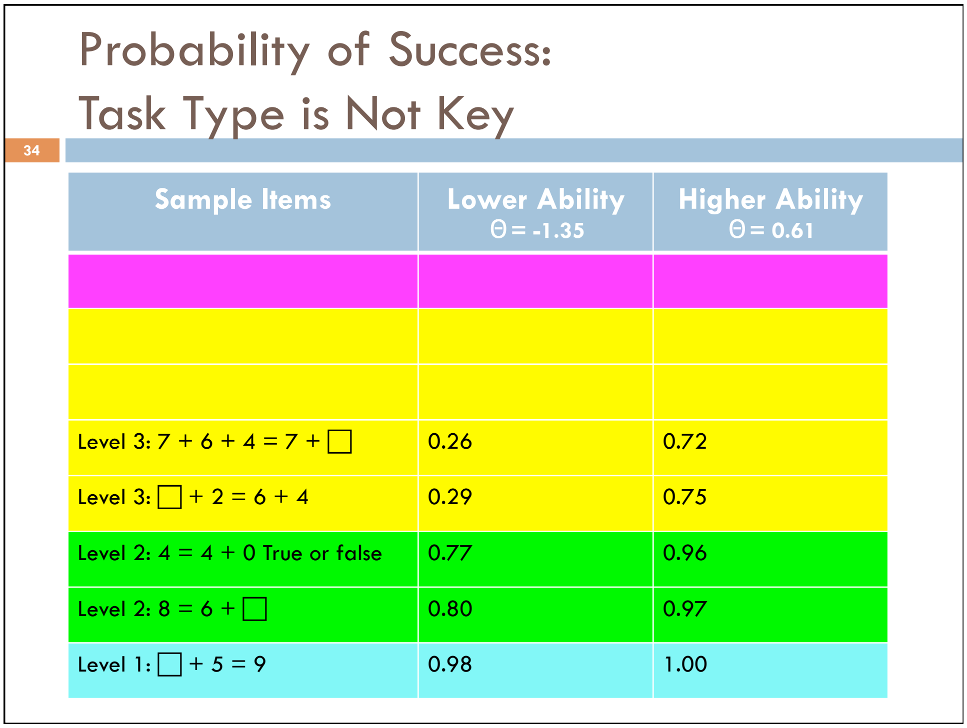
Probability of Success: Operations on Both Sides is Key

Sample Items	Lower Ability Θ = -1.35	Higher Ability Θ = 0.61
Level 3: 7 + 6 + 4 = 7 + ☐		
Level 3: ☐ + 2 = 6 + 4		
Level 2: 8 = 6 + ☐	0.80	0.97
Level 1: ☐ + 5 = 9	0.98	1.00



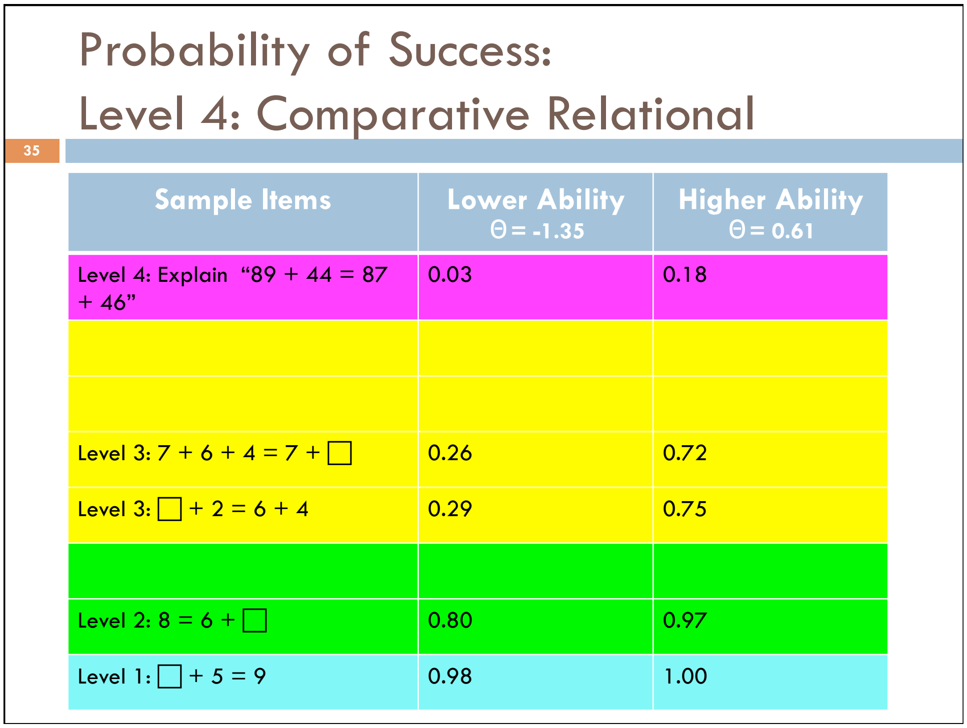
Probability of Success: Operations on Both Sides is Key

Sample Items	Lower Ability Θ = -1.35	Higher Ability Θ = 0.61
Level 3: 7 + 6 + 4 = 7 + ☐	0.26	0.72
Level 3: ☐ + 2 = 6 + 4	0.29	0.75
Level 2: 8 = 6 + ☐	0.80	0.97
Level 1: ☐ + 5 = 9	0.98	1.00



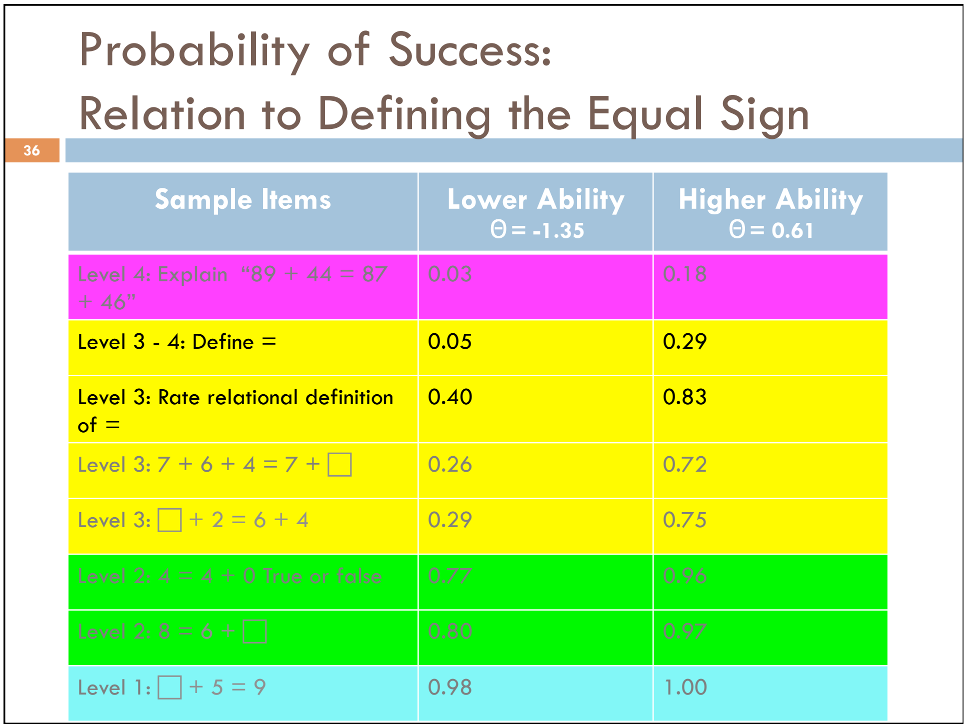
Probability of Success: Task Type is Not Key

Sample Items	Lower Ability Θ = -1.35	Higher Ability Θ = 0.61
Level 3: 7 + 6 + 4 = 7 + ☐	0.26	0.72
Level 3: ☐ + 2 = 6 + 4	0.29	0.75
Level 2: 4 = 4 + 0 True or false	0.77	0.96
Level 2: 8 = 6 + ☐	0.80	0.97
Level 1: ☐ + 5 = 9	0.98	1.00



Probability of Success: Level 4: Comparative Relational

Sample Items	Lower Ability Θ = -1.35	Higher Ability Θ = 0.61
Level 4: Explain "89 + 44 = 87 + 46"	0.03	0.18
Level 3: 7 + 6 + 4 = 7 + ☐	0.26	0.72
Level 3: ☐ + 2 = 6 + 4	0.29	0.75
Level 2: 8 = 6 + ☐	0.80	0.97
Level 1: ☐ + 5 = 9	0.98	1.00



Probability of Success: Relation to Defining the Equal Sign

Sample Items	Lower Ability Θ = -1.35	Higher Ability Θ = 0.61
Level 4: Explain "89 + 44 = 87 + 46"	0.03	0.18
Level 3 - 4: Define =	0.05	0.29
Level 3: Rate relational definition of =	0.40	0.83
Level 3: 7 + 6 + 4 = 7 + ☐	0.26	0.72
Level 3: ☐ + 2 = 6 + 4	0.29	0.75
Level 2: 4 = 4 + 0 True or false	0.77	0.96
Level 2: 8 = 6 + ☐	0.80	0.97
Level 1: ☐ + 5 = 9	0.98	1.00

Equivalence Construct Map*

Level	Description	Core Equation Structure(s)
Level 4: Comparative Relational	Compares the expressions on the two sides of the equal sign. *Generates relational definition* and recognizes it as the best definition.	Operations on both sides with multi-digit numbers or multiple instances of a variable
Level 3: ~~Basic~~ Implicit Relational	Successful with operations on both sides of the equal sign. Recognize ~~and generate~~ relational definition of the equal sign.	Operations on both sides: e.g., a + b = c + d
Level 2: Flexible Operational	Successful with atypical equation structures that remain compatible with an operational view of the equal sign.	Operations on right *or* no operations: c = a + b & a = a
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* For children in U.S. & Canada

Summary of Construct Map

- Developed a valid and reliable measure of students' knowledge of equivalence.
  - Replicated findings across 2 studies.
- Construct map captures shifts in knowledge of equivalence over grade levels.
  - Incorporate flexible operational view as transition.
  - Distinguish implicit from explicit relational knowledge. Capture developing comparative thinking.
- Construct modeling approach is a useful tool for understanding conceptual change.

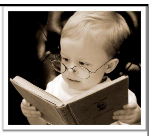
Benefits of Construct Modeling Approach

- ☐ Captures incorrect ways of thinking as well as correct ways.
- ☐ Permits testing of whether performance on specific items fit expectations.
- ☐ Probabilistic approach captures variability in individuals' thinking & performance.
- ☐ Produces a criterion-referenced measure that is particularly appropriate for assessing the effects of an intervention (Wilson, 2005).

Talk Themes

1. Conceptual change happens *gradually over time* as learners adjust their knowledge to integrate particular instances that contradict their current knowledge.
    - A construct modeling approach is useful to assessing, and thus helping understand, this change process.
2. Instructional methods that support additive knowledge change can also be appropriate for supporting conceptual change.
    - Focus on *self-explanation* & *problem exploration.*

Importance of Explanation



- Children often try to explain the world around them (Gopnik, 1998).
- Prompting people to generate explanations improves their learning:
  - For 4-year-olds to college students
  - In domains ranging from number conservation to human circulatory system

(e.g., Aleven & Koedinger, 2002; Atkinson, Renkl, & Merrill, 2003; Bielaczyc, Pirolli, & Brown, 1995; Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Neuman & Schwarz, 1998; Pine & Messer, 2000; Renkl, Stark, Gruber, & Mandl, 1998; Rittle-Johnson, Saylor & Swygert, 2008; Siegler, 1995, 2002; Wong, Lawson, & Keeves, 2002).

Explanation and Mathematics

- Unfortunately, children spend very little time explaining mathematical ideas
  - in school (Pianta et al., 2007)
  - at home (Smith-Chant et al., 2009)

Self-Explanation & Conceptual Change

- Prompting people to generate explanations improves their learning
  - Effective in domains without strong misconceptions to overcome, such as Geometry (i.e., additive learning).
  - Also effective at promoting conceptual change?
    - Explain correct *and incorrect* examples

Self-Explain Correct and Incorrect Answers to Math Equivalence Problems

When kids at another school solved it, Jane got 7, which is the right answer.

$$3 + 7 = 3 + \boxed{7}$$

Jill got 13, which is a wrong answer:

$$3 + 7 = 3 + \boxed{13}$$

- Tell me how you think she got 7, which is the *right* answer? Why do you think 7 is the right answer?
- Tell me how you think she got 13, which is the *wrong* answer? Why do you think 13 is the wrong answer?

(DeCaro & Rittle-Johnson, 2012; Matthews & Rittle-Johnson, 2009; McEldoon, Durkin & Rittle-Johnson, in press; Rittle-Johnson, 2006; Siegler, 2002)

Sample Explanations

$3 + 7 = 3 + \boxed{7}$

Why is 7 the right answer?

“Because, um 3 plus 7 is 10. And then on the other side, it shows that 3 plus, and you're trying to find out what, what other number equals 10. And 7 was the answer.”

$3 + 7 = 3 + \boxed{13}$

Why is 13 a wrong answer?

“Um it's the wrong answer because if 3 plus 13 that would be 16, and that equals 10 <pointing to left side>. And so she's basically kind of way off of the answer.”

Key Self-Explanation Findings

Self-explanation:

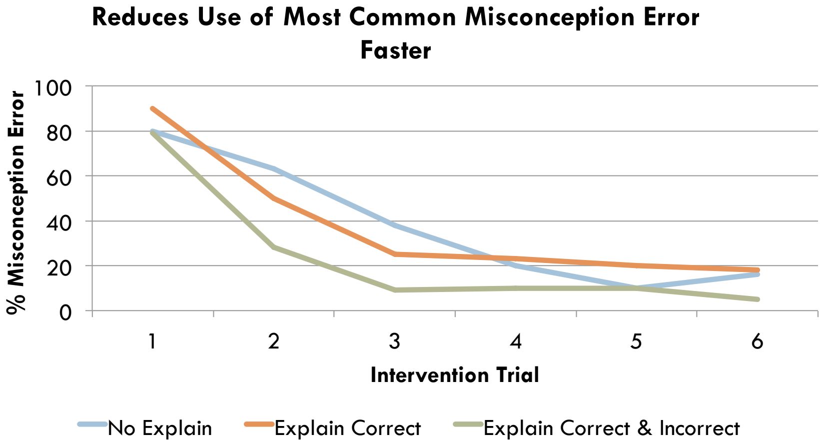
1. Of correct and incorrect examples more effective than explaining correct only (Siegler, 2002).
2. Does more than increase time on task (McEldoon, Durkin & Rittle-Johnson, in press).
3. Enhances learning with or without instruction on correct procedures (Rittle-Johnson, 2006).
4. May be redundant with instruction on correct concepts (Matthews & Rittle-Johnson, 2009).

Available on project website: vanderbi.lt/earlyalgebra

1. Explaining Correct & Incorrect Examples: Design

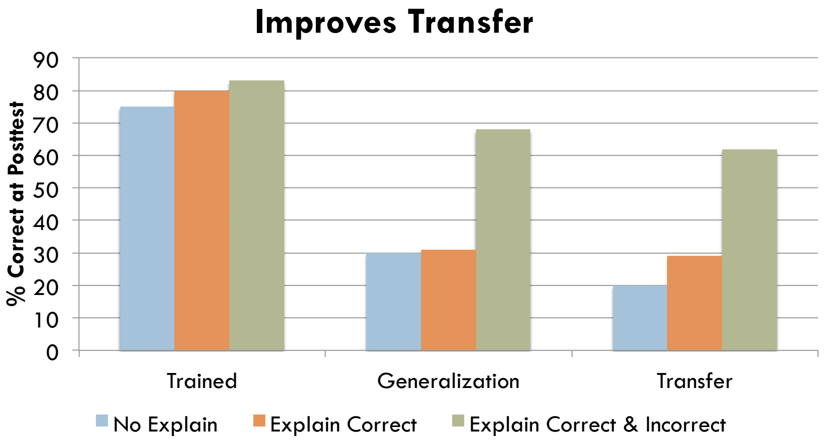
- Siegler (2002)
- 3 conditions:
  - No explain
  - Self-explain correct answers
  - Self-explain correct & incorrect answers
- Solve 6 math equivalence problems with feedback – very brief!

1. Explaining Correct & Incorrect Examples: Results



Siegler, 2002

1. Explaining Correct & Incorrect Examples: Results



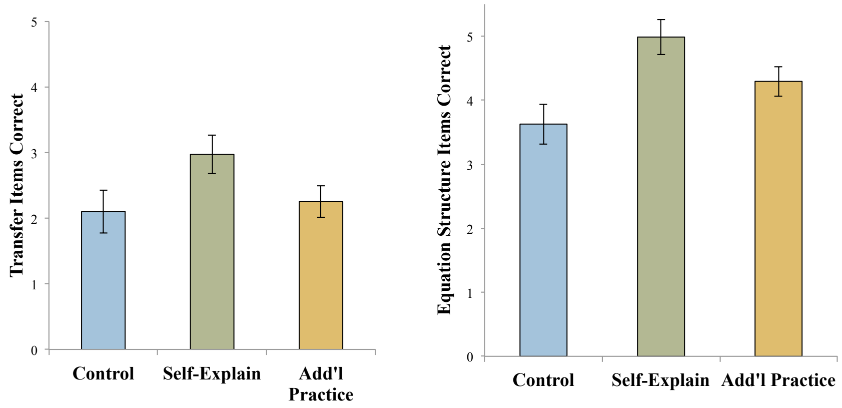
Siegler, 2002

2. Time on Task: Design

- Self-explaining greatly increases time on task
  - Evaluate relative to an alternative use of time: solving additional practice problems
- McEldoon, Durkin & Rittle-Johnson (in press)
- 3 conditions
  - No Explain Control (solve 6 problems)
  - Self-explain correct and incorrect answers (solve & explain 6 problems)
  - Additional Practice: solve twice as many problems (12) to help control for time on task

2. Time on Task: Results

Self-Explanation Does More Than Increase Time on Task

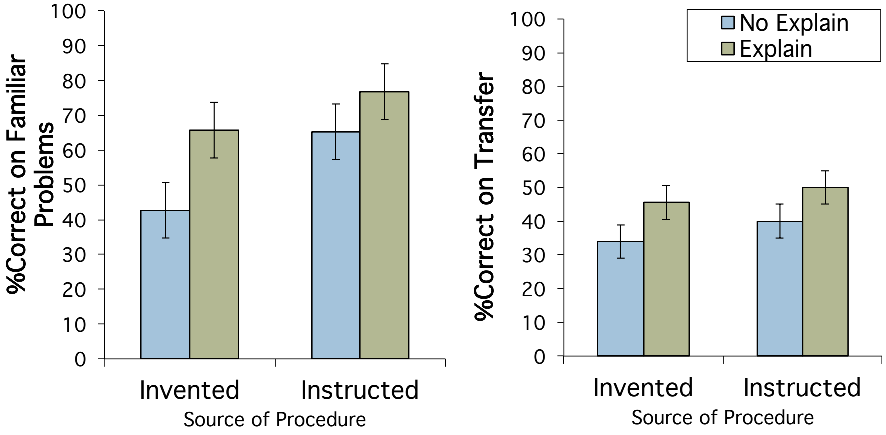


3. Instruction on A Correct Procedure: Design

- Impact of instructional context on self-explanation effects: varied in past research
  - No instruction (e.g., Siegler, 2002)
  - Instruction on correct procedure (McEldoon, Durkin & Rittle-Johnson, in press)
- Rittle-Johnson (2006)
- 4 conditions, crossing 2 factors:
  - Instruction on procedure – Instructed vs. Invented
  - Self-explanation prompts – No explain vs. explain
    - All solved 8 problems

3. Instruction on A Correct Procedure: Results

Explanation Enhances Learning With or Without Instruction on Procedure



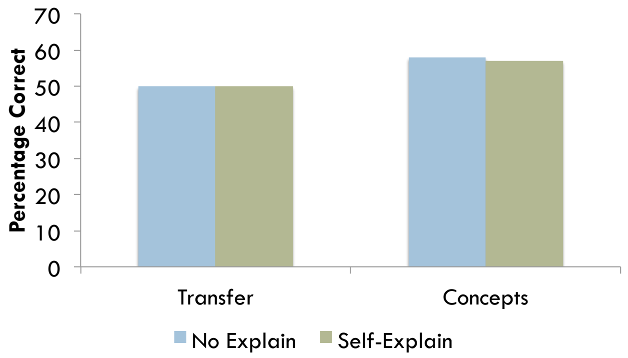
Rittle-Johnson (2006)

4. Instruction on Underlying Concepts

- Are self-explanation prompts also effective in combination with instruction on underlying concepts?
- Matthews & Rittle-Johnson (2009)
- 2 Conditions
  - All first received instruction on underlying concepts
  - No explain (with additional practice – solved 12 problems)
  - Explain (solved and explained 6 problems)

4. Instruction on Underlying Concepts

Self-Explanation May Be Redundant with Instruction on Correct Concepts in Combination With Additional Practice



Matthews & Rittle-Johnson (2009)

Key Self-Explanation Findings

Self-explanation:

1. Of correct and incorrect examples more effective than explaining correct only (Siegler, 2002)
2. Does more than increase time on task (McEldoon, Durkin & Rittle-Johnson, in press)
3. Enhances learning with or without instruction on correct procedures (Rittle-Johnson, 2006)
4. May be redundant with instruction on correct concepts (Matthews & Rittle-Johnson, 2009)

Example Instructional Methods

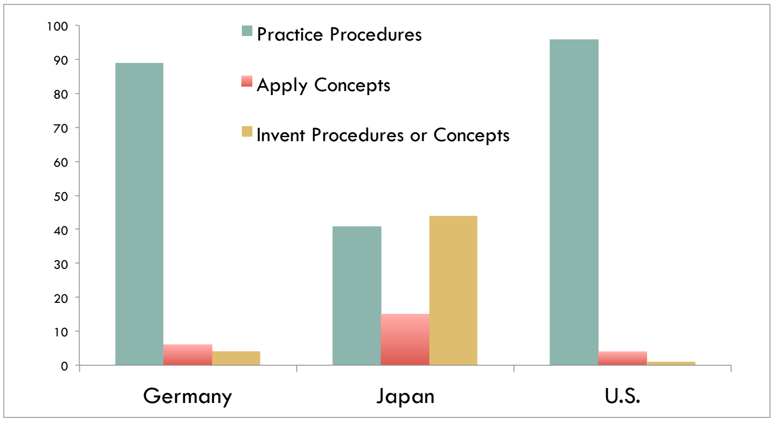
1. Self-explanation
2. Problem exploration

Problem Exploration

- ☐ A key feature of teaching for understanding is to allow students to *struggle* some: try to figure something out that is not immediately apparent. (Hiebert & Grouws, 2007)

Opportunities to Struggle Seem Rare in U.S. & German Lessons

Percentage of Seatwork Time 8th Grade Students Spent Doing Each Activity



Stigler & Hiebert (1998)

When should children be taught new concepts directly…

and when should they discover these ideas for themselves?

When should children be taught new concepts directly…

Explicit Instruction

How can aspects of both approaches be combined to improve learning?

Discovery Learning

and when should they discover these ideas for themselves?

Combing Explicit Instruction and Problem-Solving Activities

- Explicit instruction and problem-solving: Which should come first?
  - Explicit instruction followed by problem solving
    - Method used in my past self-explanation research
  - Problem solving followed by explicit instruction
    - Solve unfamiliar problems before instruction as a discovery activity

Exploratory Activities May Help Children Learn from Instruction

Evidence

- College students who explored examples learned more deeply from a psychology lecture than those who summarized a text

(Schwartz & Bransford, 1998)

- 9th graders who explored datasets before instruction and practice on descriptive statistics learned more from new instructional resources than those who received extended instruction followed by practice

(Schwartz & Martin, 2004)

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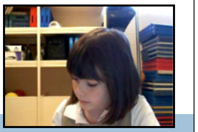
1. Problem exploration in Math Equivalence



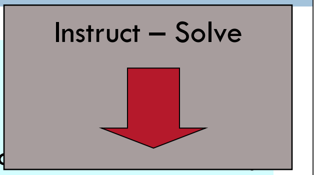
Instruction on Concept

3 + 4 = 3 + 4

There are two sides to this problem...

*What the **equal sign** means is that the things*

*sign are **equal** or **the same**…*

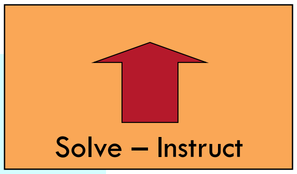


Instruct – Solve

Problem Solving

3 + 4 + 8 = ☐ + 8

<after solve> *7 is the correct answer.*



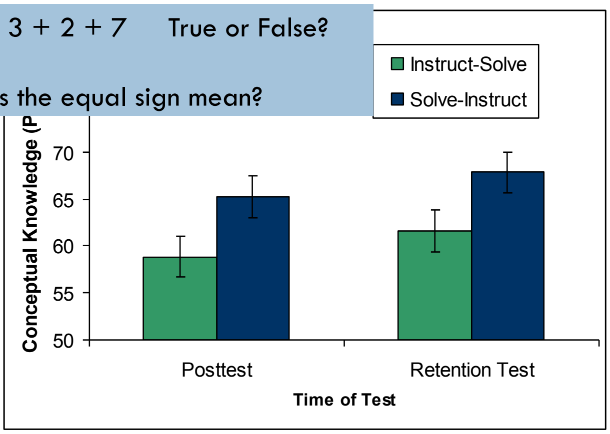
Solve – Instruct

(DeCaro & Rittle-Johnson, 2012)

Problem Exploration Results

Is 4 + 8 = 3 + 2 + 7 True or False?

What does the equal sign mean?


Conceptual Knowledge

*Solve-Instruct order led to greater learning

(DeCaro & Rittle-Johnson, 2012)

Problem Exploration Highlights

- Sometimes the best *time for telling* is after students explore (Schwartz & Bransford, 1998)
- During exploration, students do not need to figure out correct ideas. Rather, exploration:
  - Activates their prior knowledge
  - Helps them notice important features of problems
  - Helps them recognize misconceptions
  - Preparing them to learn from the instruction
- Easy for teachers to implement

Problem Exploration Follow-Up: Role of Feedback

- Opportunity to explore problems prepared children to learn from instruction.
  - Received accuracy feedback during exploration. Was this important?
- Feedback touted as one form of guidance that may be particularly effective during problem solving (e.g., Alfieri et al., 2011).
  - Learners with low domain knowledge benefit from feedback; learners with high knowledge may not (e.g., Hofer, Nussbaumer & Schneider, 2011)

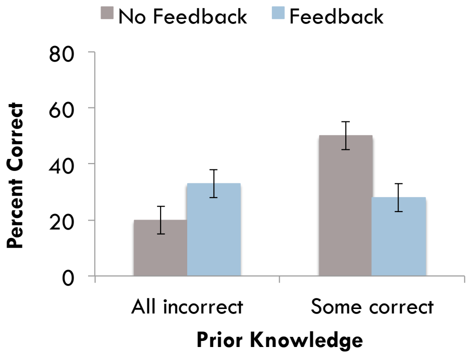
(Fyfe & Rittle-Johnson, 2012)

Role of Feedback

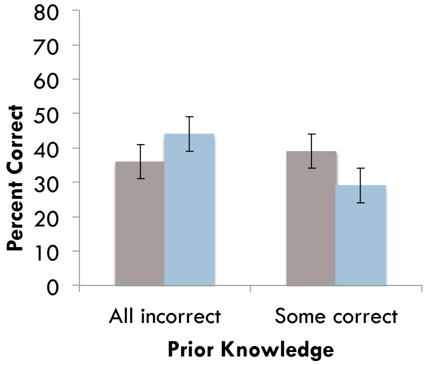
- All children solved 12 problems followed by instruction
- During solve phase, either:
  - No feedback
    - "OK, let's move on to the next problem."
  - Feedback
    - e.g., "Good try, but you did not get the right answer." or "Good try, but that is not a correct way to solve that problem."
- Prior knowledge:
  - All incorrect – never used correct procedure on pretest
  - Some correct – used correct procedure at least once on pretest

Feedback Results

Procedural Knowledge



Conceptual Knowledge



- If all incorrect at pretest, learned more if received feedback
- If some correct at pretest, learned more *without feedback*
- Replicated in a new sample

(Fyfe & Rittle-Johnson, 2012)

Feedback Summary

- Feedback during exploration is important for children with little prior domain knowledge, but may be harmful for children with some prior knowledge.

Conclusion

1. Conceptual change happens *gradually over time* as learners adjust their knowledge to integrate particular instances that contradict their current knowledge
   - A construct modeling approach is useful to assessing, and thus understanding, this change process.
2. Instructional methods that support additive knowledge change can also be appropriate for supporting conceptual change.
   - Self-explanation, esp. with incorrect examples, and problem exploration are valuable methods.

For More Information

Project website:

vanderbi.lt/earlyalgebra