Abstract Title Page.

Title:

Preparing to Learn from Math Instruction: Mastery-Oriented Students Benefit Most from Exploratory Activities

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Abstract Body

Background and Purpose

Should children be taught new concepts directly or discover the ideas for themselves? Although this question has been strongly polarizing within the educational and psychological communities (e.g., Kirschner, Sweller, & Clark, 2006; Tobias, 2009), both discovery learning and direct instruction are thought to benefit learning in numerous ways. For example, allowing learners to explore a new topic area may increase their motivation and depth of understanding (Mayer, 2004; Piaget, 1973; Schwartz, Lindgren, & Lewis, 2009; Tobias, 2009; Wise & O'Neill, 2009). Teaching children new information directly can lessen the burden on cognitive resources and support the development of accurate knowledge structures (Kirschner et al., 2006; Klahr & Carver, 1988; Klahr & Nigram, 2004; Sweller, van Merrienboer, & Paas, 1998).

Rather than favoring one approach to the exclusion of the other, it may be more productive to examine how beneficial aspects of direct instruction and discovery-learning methods can be used in conjunction to enhance children's learning (Mayer, 2004). For example, recent evidence supports the idea that students benefit from exploratory activities prior to receiving direct instruction, better preparing children to learn from the instruction (Schwartz & Bransford, 1998; Schwartz & Martin, 2004). These benefits of initial exploratory practice have been most apparent on complex problems requiring new insight, as opposed to traditional measures of memory or success solving familiar problem types.

However, because exploratory learning activities can be challenging to students, individual differences in how students respond to challenge during learning may impact whether students tend to learn better from exploratory experiences. Specifically, a number of studies in social psychology and education demonstrate that students approach learning events with different goals in mind and different conceptions of what constitutes "ideal" learning performance (Hidi & Renninger, 2006). Specifically, students who are high in *mastery-orientation* are strongly motivated by a desire for personal growth and tend to view challenge (e.g., confusion and difficult problems) as a signal that there is an opportunity to learn something new and, thereby, grow. As a result, highly mastery-oriented learners generally not only respond to apparent setbacks with increased effort and persistence, but also seek out challenge (e.g., new problems and problem-solving strategies; Diener & Dweck, 1980). In contrast, students who lack such mastery-orientation do not especially appreciate the learning process itself (they want the answers), and they may even interpret challenge as signaling failure, causing them to withdraw from the learning activity (cf. Dweck & Leggett, 1988). The current examined whether exploratory experiences—specifically, solving novel mathematics problems—prepared children to learn from instruction at a deeper level, compared to a more traditional tell-then-practice approach. Specifically, given differences in how students respond to challenge during learning, we examined whether the benefits of such exploration are more pronounced for more mastery-oriented children.

We examined these ideas within the domain of mathematical equivalence—the concept that operations on both sides of the equal sign represent the same quantity. Based on early experiences solving problems in which the equal sign is at the end of the problem (e.g., 3 + 5 = _), children often incorrectly infer that the equal sign is a symbol which means to "sum" or "get the answer" (e.g., Carpenter et al., 2003). Thus, when solving problems requiring a relational understanding of the equal sign (e.g., 3 + 5 = _ + 2), children often add the numbers to the left of the equal sign (e.g., answering "8") or add all the numbers (e.g., answering "10"), consistent with their misconceptions about the meaning of the equal sign (e.g., Perry et al, 1988; McNeil &

Alibali, 2004, 2005a). A relational understanding of the equal sign is an important indicator of children's flexibility in representing and using basic arithmetic ideas and is an important prerequisite for understanding algebra (Carpenter et al., 2003; Kieran 1992; Knuth, Stevens, McNeil, & Alibali, 2006; Rittle-Johnson et al., 2011).

We hypothesized that reversing the traditional tell-then-practice approach would benefit learning. Indeed, rather than inducing too great of a cognitive load (as would be the case if children never received instruction), we anticipated that asking children to solve problems before instruction would produce a desirable demand on processing. We further anticipated that higher mastery orientation would enhance this effect. We reasoned that, based on their typically-positive reaction to challenge, children who score higher on a measure of mastery orientation towards mathematics would be better equipped motivationally to deal with the potential confusion and intellectual obstacles posed by exploratory activities. Because prior work suggests that the benefits of exploration occur on measures of deeper understanding (Schwartz et al., 2009), we expected the benefit of exploring problems prior to instruction to selectively occur on the conceptual knowledge measure.

Such findings would not only demonstrate the utility of combining aspects of both discovery learning and direct instruction, but would also identify an important constraint to using this methodology in the classroom. Specifically, use of such exploratory learning activities may be best suited for the students that are the most motivated for this type of learning challenge or after mastery orientation has been promoted.

Setting & Participants

The study was conducted in second through fourth grade classrooms at a suburban public school serving a middle-class population. Children who scored below 80% on the pretest were selected for the study and randomly assigned to experimental condition. The final sample (N = 78, 50% female) consisted of 39 second-graders, 26 third-graders, and 13 fourth-graders. The average age was 8.6 years (range 7.3-10.8). Approximately 16% of the participants were ethnic minorities (10% African-American, 6% Asian).

Research Design

We used a pretest-intervention-posttest and delayed retention test design.

Assessment. The mathematical equivalence assessment was adapted from past research (e.g., McNeil & Alibali, 2004, 2005b; Rittle-Johnson, 2006; Rittle-Johnson et al., 2011). Two parallel forms of the assessment were used, with Form 1 administered at pretest and Form 2 at posttest and retention test. The two forms differed primarily in the specific numbers presented in the items. The assessment included a conceptual and a procedural knowledge subscale (10 points each). *Conceptual knowledge* items assessed two key concepts of mathematical equivalence: (a) the meaning of the equal sign as a relational symbol, and (b) the structure of equations (see Table 1). The *procedural knowledge* problems were eight math equivalence problems, which have operations on both sides of the equal sign (e.g., 4 + 5 + 8 = ☐ + 8), and two easier, but non-standard, problems (e.g., 7 = ☐ + 5).

Mastery Orientation Measure. At the end of the pretest session, the children were asked two questions to assess their mastery orientation (e.g., "In math class, I prefer course material that really challenges me so I can learn new things"). The children responded on Likert scales ranging from 1 (*strongly disagree*) to 6 (*strongly agree*). A mastery orientation score was later created for each student by averaging his or her responses for the two items (cf. Elliot, 1999). We also assed students' performance orientation, but this measure was not related to outcomes.

Procedure. Children completed the written pretest and the motivation questionnaire in their classrooms in one 30-minute session. Children who scored below 80% on the pretest completed a one-on-one tutoring intervention and immediate posttest in one session lasting approximately 45 minutes. The tutoring intervention consisted of two components, a problem-solving block and an instruction block. Children were randomly assigned to one of two intervention order conditions: solve-instruct (n = 40) or instruct-solve (n = 38). Approximately two weeks after the tutoring intervention, children completed the written retention test in group sessions.

Intervention

Conditions. During the intervention, all children completed problem-solving and instruction blocks, but the order in which they completed these blocks varied. Specifically, children given the *instruct-solve order* received instruction first, followed by the problem-solving block. Children given the *solve-instruct order* did the opposite, receiving the problem-solving block first, followed by instruction.

Problem-Solving. During the problem-solving block, children saw mathematical equivalence problems on a computer screen and were asked to try to figure out the number that went in the box to make the number sentence true. After each problem, children were asked to explain how they derived their answer. Then they were shown the correct answer.

Instruction. During instruction (adapted from Matthews & Rittle-Johnson, 2009), children were taught about the relational meaning of the equal sign. Specifically, number sentences (e.g., 3 + 4 = 3 + 4) were shown on the computer screen, and the experimenter explained both the structure of these number sentences (i.e., that there are two sides) and the explicit meaning of the equal sign (i.e., that the equal sign means that both sides are equal, or the same).

Data Analysis

On the conceptual knowledge assessment, each item was scored using the criteria in Table 1. Two raters independently coded 20% of the items requiring an explanation, and inter-rater agreement was high (kappas = 89% - 96%). On the procedural knowledge assessment, answers to problems were scored as correct if they came within one of the correct answer, to reduce false negatives due to arithmetic errors. Scores were converted to percentage correct and analyzed using regression.

Results

We report retention test results here, because we were ultimately interested in whether students retained mathematical equivalence knowledge over time (i.e., retention) and because posttest and retention test findings were similar. We first discuss the results for procedural knowledge; then we discuss the results for conceptual knowledge.

Procedural Knowledge. As expected, procedural knowledge at retention test was not impacted by tutoring order. Mastery orientation also did not impact procedural knowledge, and an order × mastery orientation interaction did not reach significance, p>.05.

Conceptual Knowledge. For conceptual knowledge at retention test, there were no main effects of order or mastery orientation. However, as predicted, there was a significant order ×

mastery orientation interaction, *B*=.08, *t*=1.98, *p*=.05. The effect of order depended on students' mastery orientation. To better understand this interaction, we examined the relationship between mastery orientation and conceptual knowledge at retention test separately for each instructional order. As shown in Figure 1, higher mastery orientation was associated with better conceptual knowledge at retention in the solve-instruct order, *B*=.09, *t*=2.90, *p*=.005, but not in the instruct-solve order, *B*=.01, *t*=.18, *p*=.86. Simple-effects tests (Cohen et al., 2003), further revealed that students lower in mastery orientation (1 SD below the mean) demonstrated similar knowledge levels across both instructional orders, *B*=-.008, *t*=-.17, *p*=.87. In contrast, students higher in mastery orientation (1 SD above the mean) showed better conceptual knowledge in the solve-instruct order than in the instruct-solve order, *B*=.12, *t*=2.67, *p*=.008.

Conclusions

As predicted, using problem solving as an exploratory activity helped students higher in mastery-orientation learn from conceptual instruction, compared to a learning condition in which this problem-solving activity followed instruction instead. In contrast, students lower in mastery orientation did not benefit from these exploratory activities—they performed at the same, lower level, regardless of instructional order.

Recent discussions on the discovery learning versus direct instruction debate in the educational and psychology literatures have concluded that there may be benefits of combining aspects of both approaches (e.g. Mayer, 2004). The current findings demonstrate that using exploratory problem-solving activities prior to instruction can have a benefit—but namely for children who have a mastery-orientated approach to learning mathematics. Discovery learning activities can be challenging, as children must often attempt new approaches that are not always successful. Mastery-oriented children tend to be more resilient to such challenges—approaching such activities with the goal to master and understand the content. In contrast, children with less mastery-oriented goals may be quicker to feel frustrated in the face of challenge. Importantly, these children did not perform worse when given exploratory activities relative to the more traditional instruct-then-practice approach. Thus, overall, implementing a discovery-learning component into math instruction seems to be a better approach than the traditional tell-then-practice method. Specifically, something as simple as asking students to solve novel math problems prior to giving instruction may boost subsequent conceptual understanding. However, it may be important to support mastery orientation goals in students when using discovery-oriented activities. Moreover, it remains possible that a more complicated or challenging exploratory activity could have worsened less mastery-oriented children's learning, if they became overly burdened by the cognitive demand and challenge of the activity (cf. Dweck & Leggett, 1988). Future research is needed to extend these findings to the classroom, with other domains, and with other types of exploratory activities.

Appendices

Appendix A. References

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Appendix B. Tables and Figures

Table 1. *Conceptual Knowledge Assessment Items*

Concept	Item	Scoring criteria
Structure of Equations	1) Correct encoding: Reproduce 3 equivalence problems, one at a time, from memory after a 5 sec delay	1 Point if child put numerals, operators, equal sign and blank in correct respective positions for all 3 problems
	2) Recognize correct use of equal sign in multiple contexts	
	(a) Indicate whether 7 equations in non-standard formats, such as 8 = 5+3 and 5+3 = 3+5, are true or false	1 point if 75% of equations correctly identified as "true" or "false"
	(b) Explain why 2 equations are true	1 Point per explanation if child shows through words or math that both sides of the equation are the same
Meaning of Equal Sign	1) Define the equal sign	1 Point if defined relationally (e.g., "both sides are the same")
	2) Identify the pair of numbers from a list that is equal to another pair of numbers (e.g. 6+4)	1 Point if identified correct pair of numbers
	3) Identify the symbol from a list that, when placed in the empty box (e.g. "10 cents ☐ one dime"), will show that the two sides are the same amount	1 Point if chose the equal sign
	4) Rate definitions of the equal sign: Rate 3 definitions (2 fillers) as "good," "not good," or "don't know"	1 Point if rated the statement "The equal sign means two amounts are the same" or "The equal sign means the same as" as a good definition.
	5) Which (of the above) is the best definition of the equal sign	1 Point if chose the relational definition (see above)
	6) Define the equal sign in the context of a money-related question (e.g., 1 dollar = 100 pennies)	1 point if defined relationally

Figure 1

Conceptual knowledge at retention test as a function of instruction order and mastery orientation. Lower and higher mastery orientation are plotted at one standard deviation below and above the mean.