Measuring Multidimensional Mathematical Equivalence: A Construct Modeling Approach

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Focus

This project investigates the dimensional structure of an elementary level mathematical equivalence construct

-How is mathematical equivalence knowledge change best measured?

-What are the benefits of a multidimensional model?

Mathematical Equivalence

- Mathematical equivalence is the principle that two sides of an equation represent the same value.
- Foundational concept is critical for learning algebra *(Carpenter, et al., 2003)*
- Provides the foundation for two key algebra proficiencies *(Kieran, 1992)*:
  - Understanding the equivalence of expressions
  - Competence at performing same operation on both sides of an equation
- Children's Understanding of Equivalence

Bad News: 35 years of research show that a majority of first through sixth graders treat the equal sign operationally *(e.g., Alibali, 1999)*

- **Operational View**
  - View "=" as a command to carry out arithmetic operations
  - 8 + 4 = _ + 5, most get 12 (add to equal) or 17 (add all)
- **Relational View**
  - View "=" as meaning two sides of an equation have the same value

We want to a) push kids' understanding of equivalence forward, and b) chart their progress

Method

Assessing Equivalence Knowledge

- Despite its critical importance, there is no standard measure of equivalence knowledge
- Instead, researchers often make up their own measures
  - -Our literature review found no study that reported validity of a particular measure of equivalence knowledge
- Potential Impact of valid measure
  - -Comparing research
  - -Evaluating interventions
  - -Charting developmental sequences
  - -Formative and summative assessment
  - -Informing differentiated instruction

This mathematical equivalence assessment was administered to 157 2nd through 4th graders as part of an instructional intervention study at three time points: pretest, post test, and retention test.

All analyses were completed with Item Response Theory methodology.

The Equivalence Construct

Knowledge of equivalence is typically assessed through 3 main types of tasks *(e.g., Alibali, 1999, Behr, Erlwanger, & Nichols, 1980; Falkner, Levi, & Carpenter, 1999 , McNeil, 2007; Rittle-Johnson & Alibali, 1999)*

1. **Solving Equations** – tap students' abilities to solve open equations
   - 8 + 4 = __ + 5
2. **Equation Structure Items** – probe students' knowledge of valid equation structures
   - 3 + 5 = 5 + 3 True or False
3. **Equal Sign Items** – probe students' explicit knowledge of the equal sign.
   - What does the equal sign mean?

- These components of mathematical equivalence knowledge can be considered as one construct (**unidimensional**), as three separate components (**three dimensional**), or by breaking them down into conceptual (structure and equal sign items) and procedural (solving equations) knowledge (**two dimensional**)

Unidimensional

Sample Tasks:	What does the equal sign mean? Decide if equations are true or false (e.g. 3 + 5 = 6 + 2) Solve equations such as 4 + 6 = 7 + __
Level 4: Relational without need to compute	Successful with equations with large numbers because can use relation between expressions, rather than computing. Understand principles of equality (doing same thing to both sides).
Level 3: Relational with computational support	Successful with equations with operations on both sides, by computing solutions, and knows a relational definition of the equal sign, although it co-exists with an operational definition.
Level 2: Flexible Operational	Successful with equations with operations on the right (c = a + b) because they are just "backwards" but continues to think of equal sign operationally, or in other non-relational ways.
Level 1: Rigid Operational	Only successful on equations in standard "a + b = c" format and think of equal sign operationally (e.g., it means "get the answer").

Two Dimensional

	Conceptual		**Procedural**
Level of Success	Equal Sign	Structure of equations	Equation Solving
Sample Task:	Define equal sign. Decide if closed equations are true or false (e.g. 8 = 8)		Solve open equations with operations on right side or on both sides
Level 4: Relational without need to compute	Relational definition dominates Uses relation between expressions to judge; Articulate compensation		Solve equations using relation between expressions
Level 3: Relational with computational support	Relational definition co-exists with operational definition Comfortable with most non-standard formats		Solve equations with operations on both sides (by calculating)
Level 2: Flexible Operational	Correct use of equal sign in non-numeric contexts Accept some non-standard formats (e.g. 8 = 8; 7 = 3 + 4)		Solve equations with operations on right side
Level 1: Rigid Operational	Operational definition Reject non-standard formats		Incorrect: solve by computing numbers before equal sign or all numbers

Three Dimensional

Level of Success	**Equal Sign**	**Structure of equations**	**Equation Solving**
Sample Task:	Define equal sign	Decide if closed equations are true or false (e.g. 8 = 8)	Solve open equations with operations on right side or on both sides
Level 4: Relational without need to compute	Relational definition dominates	Uses relation between expressions to judge; Articulate compensation	Solve equations using relation between expressions
Level 3: Relational with computational support	Relational definition co-exists with operational definition	Comfortable with most non-standard formats	Solve equations with operations on both sides (by calculating)
Level 2: Flexible Operational	Correct use of equal sign in non-numeric contexts	Accept some non-standard formats (e.g. 8 = 8; 7 = 3 + 4)	Solve equations with operations on right side
Level 1: Rigid Operational	Operational definition	Reject non-standard formats	Incorrect: solve by computing numbers before equal sign or all numbers

Ability Scores

Each student has an ability estimate for each dimension In the model. A multidimensional model provides a more detailed quantification of each students' ability.

Sub 189	Uni
Pre	-0.78
Post	0.28
Reten	0.89

Sub 189	Concept	Proced
Pre	-0.48	-0.71
Post	0.08	1.21
Reten	1.03	2.06

Sub 189	ST	ES	OE
Pre	-0.38	-0.44	-0.60
Post	0.19	-0.04	1.20
Reten	1.28	0.82	1.96

Correlational Structure of Dimensions

Unidimensional: No information

Two Dimensional:

Pre	**.71**
Post	**.89**
Retention	**.86**

Three Dimensional:

Pre Test	ST	ES	OE
ST			
ES	**.88**		
OE	**.72**	**.65**	

Post Test	ST	ES	OE
ST			
ES	**.93**		
OE	**.91**	**.82**	

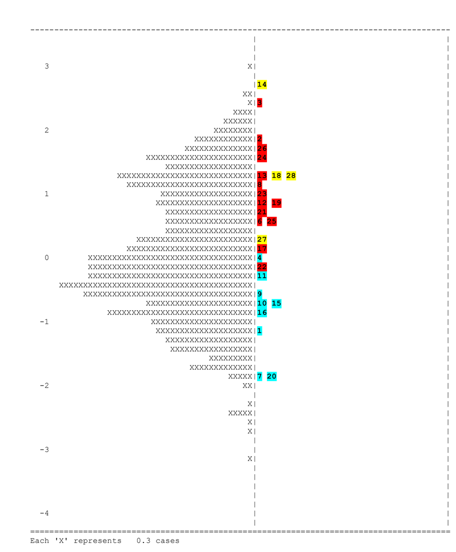
Retention Test	ST	ES	OE
ST			
ES	**.72**		
OE	**.86**	**.82**	

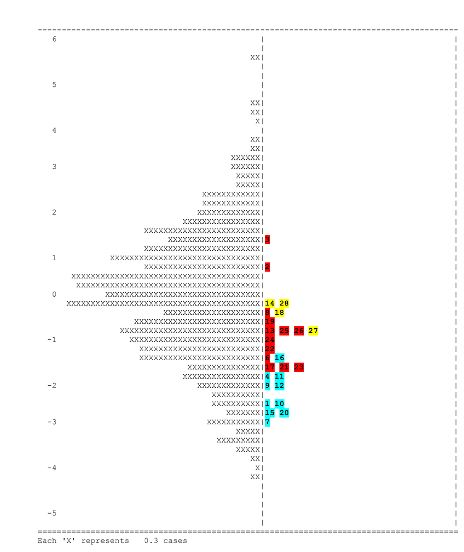
The correlational structure between dimensions can be examined using multidimensional models. This can help elucidate the nature of the construct being measured.

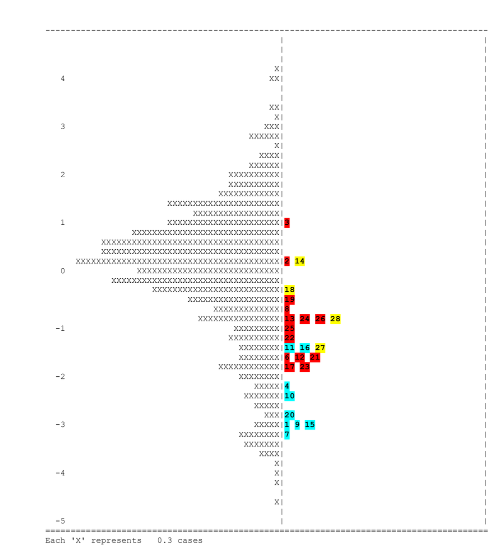
Pre Test

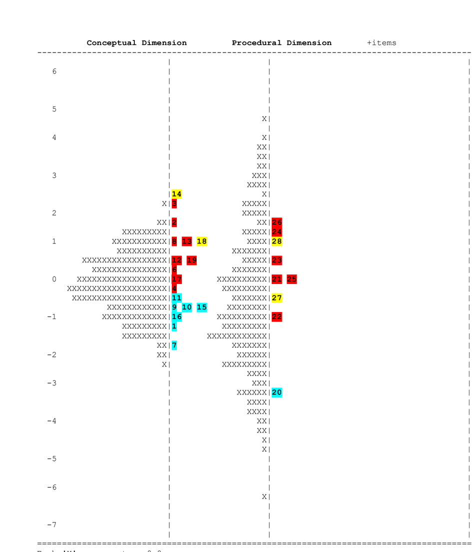
Post Test

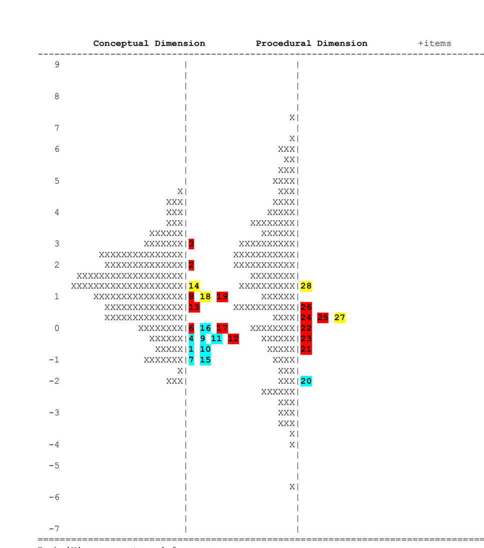
Retention Test

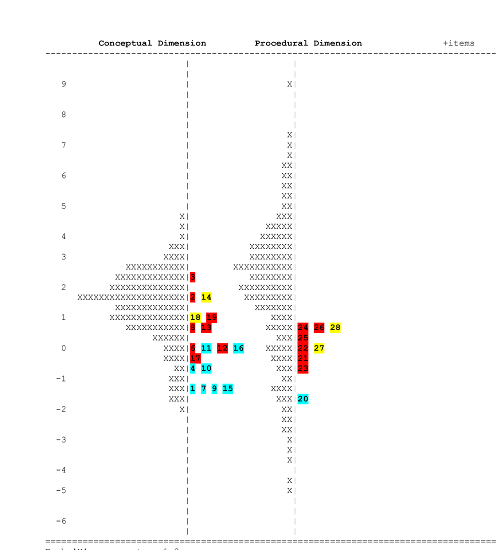


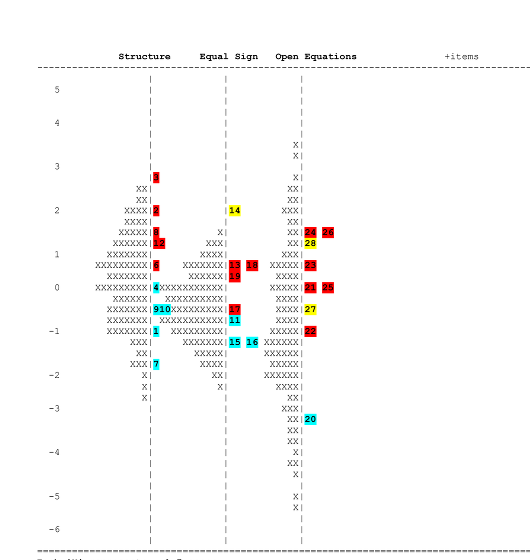


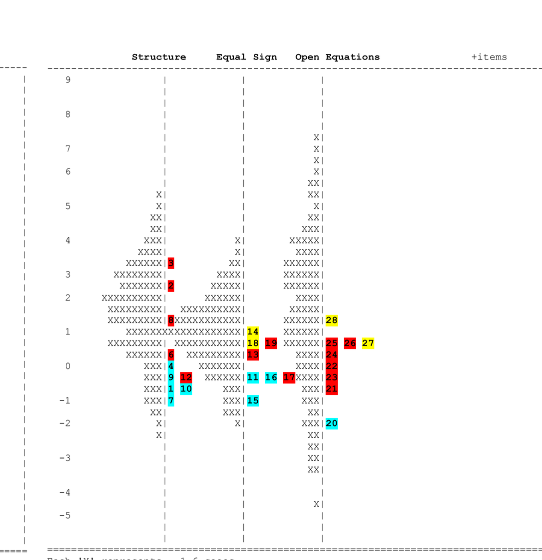


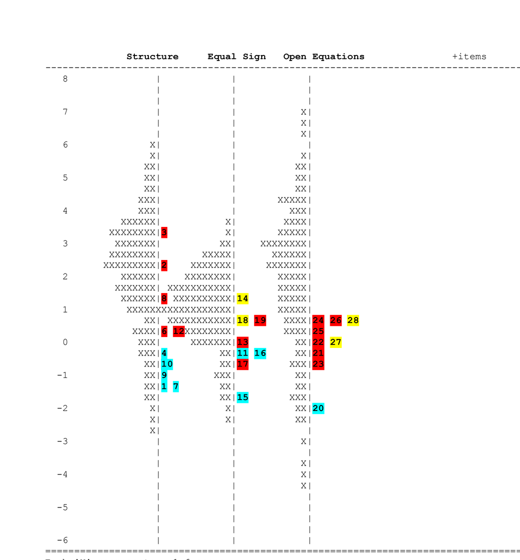












Benefits of Multi

-A mutlidimensional model allows us to have more detailed knowledge of components of children's understanding

- A multidimensional model gives us an ability estimate of each student for each knowledge component

-The relationship between the knowledge components can be examined

-This will allow for a more fine grained measure of knowledge change due to instructional intervention

- More rigorous measurement methodology will allow for more generalization and comparison across research studies

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