Mapping Children's Understanding of Functional Thinking

Katherine L. McEldoon & Bethany Rittle-Johnson
Vanderbilt University

Focus

To develop a criterion-referenced assessment and developmental trajectory of Functional Thinking; a type of algebraic reasoning appropriate for elementary students.

Abstract

We have developed an initial framework for assessing and interpreting students' level of understanding of Functional Thinking. Within this framework we explore and map which specific aspects of functions are easy and which are difficult for students. This framework was used to develop an assessment, which was administered to several 2nd through 6th grade classrooms (N=250). Both the developmental framework and assessment will be iteratively refined using Classical Test Theory and Item Response Modeling. The final product will be a criterion-referenced and developmentally leveled assessment of Functional Thinking. This will not only clarify the developmental path of functional and algebraic reasoning, it can be used to later test the efficacy of instructional interventions.

Functional Thinking

• Functional Thinking is a component of algebraic reasoning which deals with relations between two sets of numbers.

• Cultivating this reasoning is a fruitful way to develop algebraic reasoning because it transitions the focus from particular *instances* to *sets* of allowable values.

• In particular, we focus on the ability to use and identify a rule that defines the linear relationship between two sets of data presented in a table. **Y = X + 4**

X	2	3	4	5	6	14		41
Y	6	7	8	9			36	

Construct Map

Proposed Developmental Levels

1	**Recursive Thinking**
2	**Emergent Functional Thinking**
3	**Rule Recognition**
4	**Verbal Rule Generation**
5	**Symbolic Rule Generation**

Preliminary results support this developmental framework. (N= 23, students in grades 2-4)

Level One

Recursive Thinking **72% correct**

Extend numerical patterns to next instance based on previous instance, not x-y relationship. Can also apply function rule.

X:	2	3	4	5	6
Y:	6	7	8	9	10

Level Two

Emerging Functional Thinking 58% correct

Extend numerical pattern to far instances. Can think about relation between x and y for a particular instance.

X: 2	3	4	5	6	14
Y: 6	7	8	9	10	18

Level Three

Rule Recognition **49% correct**

Recognize function rule represented in a function table (for any input to any output)

Which equation represents the data in the table?

A. B = A + 2
B. B = A + 4
C. B = 2 x A
D. B = 2 x A + 1

A	B
2	6
3	7
4	8

Level Four

Verbal Rule Generation **51% correct**

Generalize pattern in function table by generating function rule in words.

What is a rule for figuring out what number belongs in column B?
"You add four to the A number to get the B number"

Level Five

Symbolic Rule Generation **43% correct**

Express the function rule in an equation

Performance on level 4 & 5 was higher than expected due to scaffolding during item piloting and only asking older children.

Write this rule as a number sentence, using "A" to stand for any number in column A and "B" to stand for any number in column B.

B = A + 4

Exploring Difficulty Factors

What elements affect the difficulty of function table problems?

- Indexicality of X values
- Lack of Story Context

Subjects saw one of three parallel versions of function table items

Indexical Function Tables

•*X values increase sequentially by one*
This format is conducive to a recursive strategy
(Carraher, Martinez, & Schliemann, 2008)

A	2	3	4	5	6	14		41
B	6	9	12	15			36	

What is a rule for figuring out what number belongs in *column B*?

Write this rule as a number sentence, using "A" to stand for *any number in column A* and "B" to stand for *any number in column B*.

Non-Indexical Function Tables

• *X values increase non-sequentially*
Encourages a functional approach, as recursive strategy cannot be applied
(Warren & Cooper, 2005)

A	2	4	5	7	8	14		41
B	6	12	15	21			36	

What is a rule for figuring out what number belongs in *column B*?

Write this rule as a number sentence, using "A" to stand for *any number in column A* and "B" to stand for *any number in column B*.

Story Context Function Tables

•*Indexical X values, but with a story context*
Helps understand the functional relation more intuitively, grounds the functional relationship
(Schliemann, Carraher, & Brizuela, 2003)

Cost of Present (A)	2	3	4	5	6	14		41
Cost of Present with Gift-Wrapping (B)	6	9	12	15			36	

At a gift shop, you can pay extra to have your present gift-wrapped, as shown in the table below. What is the *total cost of the present with gift-wrapping* if the *cost of the present* is $6? $14? What about $41? If the *total cost of a present with gift-wrapping* is $36, what was the *cost of the present* itself?

What is a rule for figuring out the total cost of the *present with gift-wrapping*?

Write this rule as a number sentence, using "A" to stand for *the cost of any present* and "B" to stand for the *total cost of a present with gift wrapping*.

Data Collection & Analysis

The revised assessment was administered to **250** 2nd through 6th grade students in a suburban public school district serving predominantly Caucasian middle class families.

Data analysis is currently in progress.

The performance data will be analyzed using

- **Classical Test Theory**
- **Item Response Theory**

These analyses will be used to refine the assessment and to evaluate our developmental trajectory.

References

Carraher, D., Martinez, M., Schliemann, A. (2008). Early algebra and mathematical generalization. *ZDM Mathematics Education*, 403-422.

Schliemann, A. D., Carraher, D.W., & Brizuela, B. (2003). *Bringing Out the Algebraic Character of Arithmetic: From Children's Ideas to Classroom Practice*. Mahwah, NJ: Lawrence Erlbaum Associates.

Warren, E., Cooper, T. (2005). Introducing Functional Thinking in Year 2: a case study of early algebra teaching. *Contemporary Issues in Early Childhood, 6*(2), 150-162.

Contact

Katherine L. McEldoon
K.McEldoon@Vanderbilt.edu
Dr. Bethany Rittle-Johnson
Bethany.Rittle-Johnson@Vanderbilt.edu
Psychology & Human Development, Peabody College, Vanderbilt University, Nashville, TN 37235

Acknowledgments

The first author is supported by a predoctorial training grant provided by the Institute of Education Sciences, U.S. Department of Education, through Vanderbilt's Experimental Education Research Training (ExpERT II) grant (David S. Cordray, Director; grant number R305B080025). The opinions expressed are those of the authors and do not represent views of the U.S. Department of Education.

VANDERBILT KENNEDY CENTER
FOR RESEARCH ON HUMAN DEVELOPMENT

This poster was printed by the Graphics Core of the Vanderbilt Kennedy Center, Vanderbilt University, supported in part by NICHD Grant P30 HD15052. The Vanderbilt Kennedy Center for Research on Human Development is devoted to unlocking the mysteries of the brain and child development, and improving the lives of people with disabilities. kc.vanderbilt.edu

VANDERBILT
PEABODY COLLEGE