Easy as ABCABC: Abstract language facilitates performance on a concrete patterning task

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Abstract

The labels used to describe patterns and relations can influence children's relational reasoning. In this study, 62 preschoolers (*M* age = 4.4 years) solved and described eight pattern abstraction problems (i.e., recreated the relation in a model pattern using novel materials). Some children were exposed to concrete labels (e.g., blue-red-blue-red) and others were exposed to abstract labels (e.g., A-B-A-B). Children exposed to abstract labels solved more problems correctly than children exposed to concrete labels. Children's correct adoption of the abstract language into their own descriptions was particularly beneficial. Thus, using concrete learning materials in combination with abstract representations can enhance their utility for children's performance. Further, abstract language may play a key role in the development of relational thinking.

Easy as ABCABC: Abstract language facilitates performance on a concrete patterning task

The ability to think relationally is the "*sine qua non* of human cognition" (Gentner & Loewenstein, 2002, p. 88). It allows us to detect patterns, make comparisons, and draw inferences between superficially different situations. One early form of relational thinking is pattern abstraction: recreating the relation in a model pattern using novel materials (Clements & Sarama, 2009). For example, a child might be shown blue and red tiles with an ABAB relation (e.g., blue-red-blue-red) and be asked to create the same kind of pattern using green squares and circles (e.g., square-circle-square-circle). Although research suggests that most preschoolers can *recognize* an abstracted pattern in a relational match-to-sample task (e.g., Kotovsky & Gentner, 1996; Son, Smith, & Goldstone, 2011), much less is known about preschoolers' ability to *generate* the abstracted pattern themselves. Some four-year-olds are capable of pattern abstraction, but large individual differences exist (Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013). Further, little is known about the mechanisms that support pattern abstraction. In the present study, we tested one hypothesized method for supporting pattern abstraction: using abstract, rather than concrete, language to describe the patterns. Abstract labels are generic, represent structure efficiently, and are arbitrarily linked to their referents. Concrete labels are familiar, reference perceptual features, and connect with learners' prior knowledge.

Research on relational thinking suggests language may play a critical role in children's pattern abstraction. Labels, in particular, have been shown to affect children's detection of the same relation across multiple instances (e.g., Gentner & Rattermann, 1991; Namy & Gentner, 2002). For example, four-year-olds can extend a relation illustrated in one situation (e.g., knife is the cutter for melon) to another situation (e.g., a pair of scissors is the cutter for paper) when the relation is labeled, but not in the absence of a label (Gentner et al., 2011). Similarly, providing

labels for target relations (e.g., "top-middle-bottom" for location) helps preschoolers transfer

those relations to novel materials (Kotovsky & Gentner, 1996; Loewenstein & Gentner, 2005).

Thus, the words used to label patterns and relations in the environment seem to play an important

role in children's ability to notice those patterns and relations.

Abstract labels, in particular, may be especially helpful for pattern abstraction because

they provide a way to link concrete materials (e.g., blue and yellow tiles) to the relation they are

intended to instantiate (e.g., A-B-A-B). Researchers in psychology and education suggest that

children's understanding is facilitated when there are explicit links between concrete learning

materials and the abstract ideas they are intended to represent (e.g., Brown et al., 2009).

Abstract labels may also facilitate pattern abstraction because they encourage encoding of

the relations among the elements in the pattern. According to the cognitive alignment framework

(Laski & Siegler, 2014), concrete learning materials are only effective to the extent that they and

their associated activities are aligned with the desired mental representation. Activities should be

designed to increase encoding of the structural features that are relevant for the desired mental

representation. In the case of patterns, the goal is for children to construct a representation of the

unit of repeat that captures the relations among the elements (e.g., AAB vs. ABA). Thus,

materials and techniques that facilitate encoding of those relations should be most effective.

There are at least three reasons why abstract labels may facilitate children's success in

encoding the relations among elements in the pattern. First, abstract labels may draw children's

attention to the structural aspects of the pattern materials as opposed to the surface features of the

materials themselves. Abstract representations limit extraneous perceptual features and make the

structure more apparent than concrete representations (e.g., Kaminski, Sloutsky, & Heckler,

2009; Son, Smith, & Goldstone, 2011). For example, four-year-olds are more successful at

mapping objects based on the relations among those objects when the objects are generic (e.g., simple line drawing of a pot) as opposed to perceptually rich (e.g., detailed drawing of bouquet; Gentner & Ratterman, 1991). Further, exposure to abstract labels (e.g., "one-two-three") facilitates four-year-olds' performance on a spatial mapping task relative to concrete labels (e.g., "dog-pig-sheep"; Gentner & Christie, 2006).

Second, the same abstract label can be shared across different materials, which may help children encode the same underlying structure in new materials. For example, the letter "A" can refer to a blue tile in one pattern and a green square in a second pattern. Indeed, a key benefit of abstract representations is their portability across various situations (e.g., Kaminski et al., 2009). Importantly, shared labels encourage children to treat objects similarly (e.g., Gelman & Markman, 1986; Graham et al., 2004) and to map sets of objects based on their relations, rather than their perceptual features. For example, four-year-olds who learn to label symmetric relations (i.e., ABA) as "even" show relational reasoning on a match-to-sample task that is otherwise too difficult (e.g., match light-dark-light with little-big-little; see Gentner & Medina, 1998).

Third, combining abstract labels and concrete materials provides children with multiple ways to encode the target relation. For example, consider a pattern of blue and red tiles (e.g., "blue-red-blue-red") that are labeled with arbitrary letters (e.g., "A-B-A-B"). The tiles provide a concrete, visual representation and the labels provide an abstract, verbal representation. Relabeling the concrete features with abstract language provides two distinct, but complementary representations, and researchers have long known that multiple representations can be beneficial (e.g., Ainsworth, 1999, 2006; Dienes, 1960; Mayer & Moreno, 2002).

Despite these potential benefits of abstract labels, some educators might expect concrete labels to be more effective for children's pattern abstraction. Indeed, concrete labels seem more

accessible for young children. Early theorists posited that children's thinking is inherently concrete (Montessori, 1964; Piaget, 1953) and that children construct abstract knowledge through their experience with concrete materials (e.g., Bruner, 1966). Concrete labels may ease children's understanding of the label's referent and allow for the construction of appropriate knowledge. Concrete labels are also more familiar (e.g., Rittle-Johnson et al., 2013), and learners often benefit from familiarity. For example, providing familiar labels to objects arranged in increasing order (e.g., "Daddy, Mommy, Baby") significantly improves 3-year-olds' ability to map the objects based on their relation (Gentner & Rattermann, 1991). Finally, concrete labels may communicate relevant features of the pattern ("relevant concreteness"). For example, in a blue-yellow-blue pattern, color is the feature that creates the pattern and can be used to identify its underlying structure. Past research indicates that relevant concreteness can facilitate learning and performance in the target domain (Kaminski, Sloutsky, & Heckler, 2005).

In the current study, we tested the impact of concrete versus abstract labels on four-year-olds' pattern abstraction. Given the importance of (a) forging connections between concrete materials and the abstract ideas they are intended to represent and (b) the cognitive alignment framework's focus on encoding of key structural features, we hypothesized that children exposed to abstract labels would outperform children exposed to concrete labels.

Method

Participants

Parent consent was obtained for 62 children (28 female) attending one of six preschools in an urban area, each of which primarily served Caucasian, middle-class children. The average age was 4.4 years ($SD = 0.4$, $range = 3.6$–4.9) and did not differ as a function of condition, $F<1$.

Design

Children were randomly assigned to one of two conditions: abstract (*n* = 30) or concrete

(*n* = 32). The only difference between conditions was the labels the experimenter used when

describing patterns. The primary outcome of interest was children's performance on problem-

solving items presented during the experimental session.

Task and Materials

The experimental task was *pattern abstraction*—recreating a model pattern using

different materials. Table 1 provides a list of each item, its pattern unit, and the materials used.

The model pattern for all items was constructed with colored shapes that had been glued to a

strip of cardstock in the desired linear pattern. For each item, the pattern was abstracted to new

materials that varied only in color or shape (see Figure 1 for example materials).

Procedure

Children met individually with one of two trained female experimenters in a quiet room

at their preschool in a single session lasting approximately 20 minutes. At the beginning of the

session, children completed a single pretest item (taken from a validated patterning assessment;

Rittle-Johnson et al., 2013). The experimenter made a pattern with blocks and asked the child to

make the same kind of pattern using different materials.

Next, children completed the experimental task, which consisted of three examples and

eight problems to solve. See Table 1 for the order in which the items were presented. For the

example items, children in both conditions were explicitly told that the two patterns were alike.

The experimenter described the model pattern, placed the abstracted pattern so that the two

patterns were aligned vertically on the table, and then stated how the two patterns were similar.

For the solve items, the experimenter described the model pattern and then asked the child to

make the same kind of pattern using the objects provided. After completing the pattern, children were prompted to describe their pattern. Children never received any feedback.

The labels used to describe the pattern elements depended on the child's assigned condition. In the *abstract* condition, the experimenter described the patterns using an arbitrary, but conventional naming system—letters of the alphabet. For instance, on the example item with an ABB pattern unit, the experimenter described the patterns as follows:

"The part that repeats in my pattern is A-B-B because it has one and then two that are different…The part that repeats in your pattern is also A-B-B because it has one and then two that are different…These patterns are alike because the secret code for both patterns is A-B-B (points to first unit in both patterns simultaneously)."

In the *concrete* condition, the experimenter described the patterns by referring to the changing perceptual dimension—the color or shape of the elements. For instance, on the example item with an ABB pattern unit, the experimenter described the patterns as follows:

"The part that repeats in my pattern is purple-blue-blue because it has one purple and then two blue ones…The part that repeats in your pattern is red-green-green because it has one red one and then two green ones…These patterns are alike. The secret code for my pattern is purple-blue-blue. The secret code for your pattern is red-green-green (points to first unit in both patterns simultaneously)."

On solve items, the experimenter only described the model pattern.

Coding

All solve items were scored dichotomously based on a system used in previous research (Rittle-Johnson et al., 2013). Responses were counted as correct if the child produced at least one full unit of the pattern and made no errors. Children's responses to the description prompt

("What is your pattern?") were also coded. Descriptions fell into one of six categories. A second

rater coded 30% of the responses and inter-rater agreement was high (kappa = .90).

Results

Performance

The percent of children solving the pretest item correctly was similar across the abstract

(37%) and concrete conditions (41%), $p = .75$. To analyze performance on the solve items, we

ran an ANCOVA with condition (abstract or concrete) as the independent variable, number

correct (out of 8) as the dependent variable, and age and pretest as covariates. There was an

effect of age, with older children performing better than younger children, $F(1, 58) = 5.08$, $p =$

.03, $\eta_p^2 = .08$. There was also an effect of pretest, with children who solved the pretest item

correctly performing better than children who solved it incorrectly, $F(1, 58) = 11.53$, $p = .001$,

$\eta_p^2 = .17$. Importantly, there was a large effect of condition, $F(1, 58) = 14.67$, $p < .001$, $\eta_p^2 = .20$.

Children in the abstract condition ($M = 5.4$, $SE = 0.4$) outperformed children in the concrete

condition ($M = 3.1$, $SE = 0.4$). The pattern was the same across all items (see Table 2).

Because scores were not normally distributed, with most children getting either 0 or 1

items correct (30%) or 7 or 8 items correct (34%), we also performed a nonparametric analysis to

ensure that results did not depend on the method of analysis. We used binomial logistic

regression to predict the odds of solving 7 or 8 items correctly. We included condition, age, and

pretest score in the model. Results were consistent with the ANCOVA. Children in the abstract

condition were more likely than children in the concrete condition to solve 7 or 8 items correctly

(17 of 30 [57%] vs. 4 of 32 [13%], $\hat{\beta} = 2.63$, $z = 3.40$, $Wald$ $(1, N = 62) = 11.56$, $p = .001$. The

model estimates that the odds of solving 7 or 8 items correctly are more than 13 times higher

after participating in the abstract condition than after participating in the concrete condition.

Pretest score was a marginal predictor in the model, $p = .06$, as was age, $p = .08$.

Pattern Descriptions

We examined children's descriptions of their patterns. Table 3 presents the six response

types. To test for condition differences, we performed a multivariate ANCOVA with condition

as the independent variable, frequency of using each description as the dependent variables, and

age and pretest as covariates. There was a main effect of condition, $F(1, 54) = 37.73$, $p < .001$,

$\eta_p^2 = .78$. As shown in Table 3, children in the abstract condition produced Correct AB and

Incorrect AB descriptions more frequently and Correct Concrete descriptions less frequently than

children in the concrete condition. Results were similar when we analyzed the percentage of

children who used each description type at least once (see Table 3).

Next, we examined the correlations between children's descriptions and their

performance on the eight solve items. Frequency of Correct AB descriptions was positively

correlated with number correct, $r(62) = .60$, $p < .001$, but frequency of Correct Concrete

descriptions was not related to performance, $r(62) = -.13$, $p = .32$. Finally, frequency of Incorrect

AB descriptions, $r(62) = -.24$, $p = .03$, and frequency of naming a concrete feature, $r(62) = -.42$,

$p < .001$, were both negatively correlated with number correct.

Finally, we examined whether children's adoption of Correct AB descriptions mediated

the relation between condition and performance. We used a bootstrapping technique

recommended by Preacher and Hayes (2008), which produced a 95% bias-corrected confidence

interval for the indirect effect. We included condition as the independent variable, total correct as

the dependent variable, frequency of Correct AB descriptions as the mediator, and age and

pretest as covariates. Frequency of correct AB descriptions significantly mediated the effect of

condition on performance (CI: 0.52, 3.69, *p* < .05). Conclusions were unchanged when we used

the dichotomous outcome variable (i.e., solved 7 or 8 items correctly; CI: 0.11, 4.71, *p* < .05).

Frequency of using any other description type did not mediate the effect of condition.

Discussion

In the current study, we focused on a key construct of cognitive development—children's

relational reasoning—and we tested the effect of using concrete versus abstract labels in

reference to patterns. We found that using abstract language to describe patterns facilitates

children's pattern abstraction. Four-year-olds exposed to abstract labels (e.g., "A-B-B-A-B-B")

solved more problems correctly than children exposed to concrete labels (e.g., "red-blue-blue-

red-blue-blue"). The effect was large, robust, and did not depend on our method of analysis.

Children's correct adoption of the abstract language was particularly beneficial.

Parents and teachers often use concrete learning materials, such as blocks or counters, to

help children learn abstract concepts. Yet, the mere use of these materials does not guarantee

success (Ball, 1992). The cognitive alignment framework suggests that learning materials are

only effective if both the materials *and* the ways in which the materials are used are aligned with

the mental representation we want learners to acquire (Laski & Siegler, 2014). The framework

specifies the need to promote attention to key, structural features. Previous research inspired by

this approach has demonstrated how cues in a number board game help children learn relations

among numbers (Laski & Siegler, 2014). Similarly, in the present study, we demonstrated how

abstract language in a patterning task helped children detect relations among the elements.

The cognitive alignment framework is consistent with recommendations to draw more

explicit links between concrete learning materials and the abstract ideas they are intended to

represent (e.g., Brown et al., 2009). Indeed, research indicates that a combination of concrete and

abstract examples is often more beneficial than either one in isolation (e.g., Goldstone & Son, 2005; McNeil & Fyfe, 2012). Here, children may have benefitted from the opportunity to physically manipulate the concrete pattern blocks, allowing for knowledge construction (e.g., Martin & Schwartz, 2005). At the same time, the use of abstract labels in connection with the blocks may have "decontextualized" children's knowledge, allowing for generalization across multiple instances (Fyfe, McNeil, Son, & Goldstone, 2014).

More generally, the findings contribute to a growing body of literature touting the benefits of abstract representations. There are two key aspects of the abstract labels that may have been particularly helpful. First, the abstract labels focused children's attention on the structure of the pattern (i.e., unit of repeat) rather than on the particular perceptual dimension that varied. This may have facilitated children's ability to recreate that structure using novel materials. Indeed, abstract examples have been shown to facilitate alignment of core features across examples, whereas concrete examples have not (Kaminski, Sloutsky, & Heckler, 2013). This view is consistent with arguments that favor abstract representation because they highlight structural or representational aspects, rather than surface features (e.g., Uttal et al., 2009).

Second, the abstract labels were shared across patterns, which provided additional cues to the "likeness" of the two patterns (e.g., your pattern is *also* A-B-B). This consistency helped children map elements from one pattern onto elements of another pattern. Indeed, shared labels encourage children to treat objects similarly and to generalize properties based on that label (e.g., Gentner & Medina, 1998). We specifically chose to use shared abstract labels in our experiment because the portability of abstract representations to multiple contexts is one of their key advantages (e.g., Kaminski et al., 2009; Son, Smith, & Goldstone, 2008). But abstract and shared labels *can* be separated. For example, one could use shared concrete labels if the elements were

identical across patterns. However, this would not necessarily inform our understanding of relational reasoning because correct performance could be achieved by simple object-matching, which is quite easy for four-year-olds (e.g., Gentner & Ratterman, 1991). One could also use unshared abstract labels (e.g., A-A-B and 1-1-2) to determine whether the abstract labels are beneficial in and of themselves. The current results attest to the benefits of shared abstract labels.

Although *exposure* to abstract language is beneficial, the results also point to *adoption* of abstract language as a mechanism in the development of pattern abstraction. Specifically, frequency of using correct abstract labels mediated the relation between condition and performance. This is consistent with reports on the role of children's explanations on patterning knowledge. In two previous studies, a tutor modeled abstract same-different language (but no other type of language) in reference to repeating patterns (e.g., "same-same-different"). Children's adoption of the same-different language predicted subsequent performance (Fyfe, Loehr, Rittle-Johnson, & Miller, 2014; Rittle-Johnson et al., 2013). Together, these results suggest that the use of shared, abstract labels may be one mechanism that underlies improvements in children's repeating pattern knowledge.

Although this study confirms that abstract labels *can* play a positive role, it also attests to the difficulties children experience using them. Several children used the abstract language incorrectly by repeating the labels used by the experimenter without referencing particular elements. Moreover, the frequency of incorrect abstract language was negatively correlated with performance. This supports the notion that abstract representations can be difficult to interpret, which can lead to rote use of them without understanding (e.g., Nathan, 2012). Thus, although the abstract labels had a positive effect overall, their ambiguity was a hindrance at times.

Finally, results provide insight into the role of language in the development of relational thinking. Previous research indicates that the use of labels promotes structural alignment and the ability to *detect* a common relation (e.g., Gentner & Rattermann, 1991). For example, exposure to relational labels (e.g., "top-middle-bottom") facilitates three-year-olds' performance on a spatial relational mapping task (Loewenstein & Gentner, 2005). Similarly, relational labels that specify direction (e.g., red is on left) help four-year-olds remember a color-pattern relation better than non-directional labels (e.g., red is touching green; Dessalegn & Landou, 2008). In the current study, the use of arbitrary, abstract labels facilitated four-year-olds' ability to *generate* the same relation using novel, perceptually dissimilar materials. Thus, shared, abstract labels can increase children's explicit knowledge of the relational structure.

These results may provide impetus for future research on patterns in early learning settings. Patterning is central to mathematics (Charles, 2005; Steen, 1988), and teaching children about patterns improves their mathematics knowledge (e.g., Kidd et al., 2014). However, a recent report by the National Mathematics Advisory Panel (2008) recommended reducing attention to patterns because the mathematical nature of patterns is often not made clear. Abstracting patterns, especially in combination with shared abstract labels, may help children attend to mathematically meaningful information: the unit of repeat. Some early mathematics curricula include patterns, and some even use abstract labels to reference them (e.g., *enVision* Math, 2014). Although the current results suggest that these labels may be beneficial, future research is needed to test the effects of abstract labels on a wider range of patterning tasks in more ecologically valid settings.

Several limitations of the present study suggest additional avenues for future research. For example, we tried to label the patterns as naturally as possible, so the labels in our two

conditions differed on a number of dimensions. The letters were abstract, shared, and created a gesture-speech mismatch (e.g., pointing to red cube but labeling it "A"). The color and shape labels were concrete, unshared, and created a gesture-speech match (e.g., pointing to red cube and labeling it "red"). Future studies could isolate which attributes of abstract labels matter most (e.g., contrast shared vs. unshared abstract labels). Also, we examined the performance of preschoolers from middle-SES backgrounds on a specific kind of patterning task in a scripted, one-on-one setting. Although this type of experiment provides a good tool for testing theoretical predictions about basic cognitive processes (Haeffel et al., 2009), future research should examine whether abstract labels function similarly for learning tasks with a diverse sample of children.

Despite the limitations, we have shown that the labels used to describe patterns affect children's ability to generate abstracted patterns. Children's performance benefitted after hearing a pattern labeled as "A-B-A-B," rather than as "red-blue-red-blue." Thus, one implication of this work is that minor differences in the language used to describe learning materials can have a large effect on how children think about those materials. These results support recommendations to draw explicit links between concrete materials and the abstract ideas they are intended to represent (e.g., Brown et al., 2009), and also provide support for theories that emphasize the role of language in the development of relational thinking (e.g., Gentner, 2003).

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Table 1

A list of each item in the order it was presented during the experimental session

Item-Unit	Model Materials	Abstract Materials
Pretest1-AAB	Orange square, blue diamond	Green/yellow squares
Example1-AB	Yellow/green cubes	Blue/red circles
Solve1-AB	Orange/purple cubes	Green/red triangles
Solve2-AB	Wooden triangles/stars	Blue circles/squares
Example2-AAB	Wooden stars/hearts	Yellow triangles/squares
Solve3-AAB	Wooden squares/hearts	Green triangles/circles
Solve4-AAB	Blue/green pompoms	Red/yellow triangles
Example3-ABB	Purple/blue pompoms	Red/green triangles
Solve5-ABB	Green/red pompoms	Purple/orange squares
Solve6-ABB	Wooden stars/hearts	Red triangles/squares
Solve7-AABB	Red/blue pompoms	Green/purple squares
Solve8-ABC	Wooden triangles/stars/hearts	Red circles/squares/triangles

Table 2

Percent of children who solved each item correctly by condition

Item	Abstract	Concrete
Solve1-AB	70	47
Solve2-AB	67	56
Solve3-AAB	63	38
Solve4-AAB	70	22
Solve5-ABB	80	44
Solve6-ABB	67	38
Solve7-AABB	77	31
Solve8-ABC	50	31

Table 3

*Children's descriptions of their abstracted patterns*

Type	Example	% use across all eight trials		% children who used at least once	
		Abstract	Concrete	Abstract	Concrete
Correct AB	It goes A-B-B-A-B-B (in reference to ABBABB sequence)	55*	0	77*	0
Incorrect AB	It goes A-B-B-A-B-B (in reference to ABBBBA sequence)	19*	0	47*	0
Correct Concrete	Red-blue-blue-red-blue-blue (in reference to ABBABB sequence)	12*	75	30*	94
Names Feature	Red triangles	8	20	17*	41
Random	Dinosaurs	6	1	7	3
No Response	I don't know (or silence)	1	4	7	19

Note. Differences are between the abstract and concrete condition for each code.*$p < .05$.

Figure 1

Example materials used during the experimental session
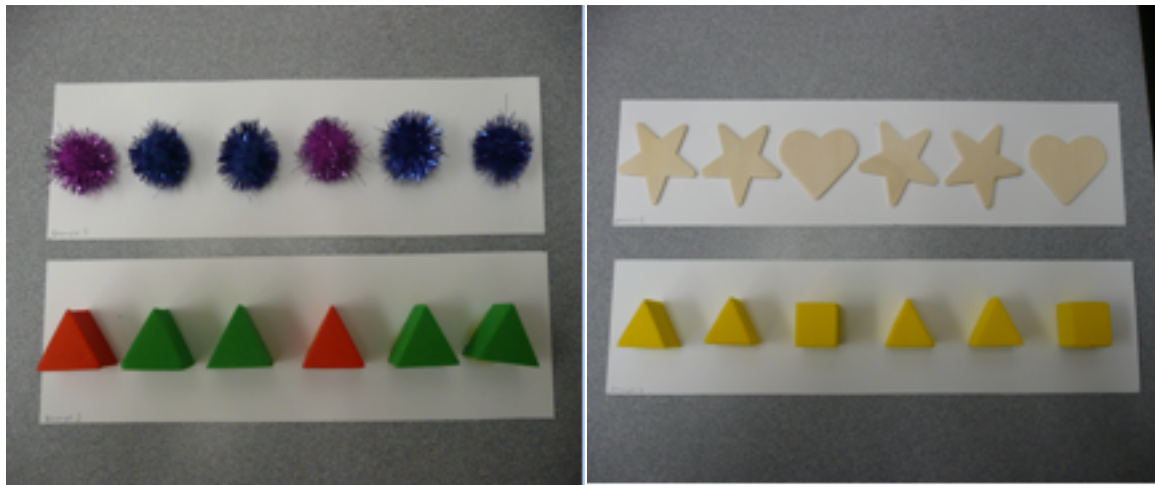
Note. The picture on the left shows materials for an abstract-to-color ABB item. The top pattern is made of purple and blue pompoms and the bottom pattern is made of red and green triangles. The picture on the right show materials for an abstract-to-shape AAB item. The top pattern is made of wooden stars and hearts and the bottom pattern is made of yellow triangles and squares.