Including Repeating Patterning Skills in Early Mathematics Education

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Influence of Research on Practice

- Common Core State Standards (2010) reflect influence of math development research

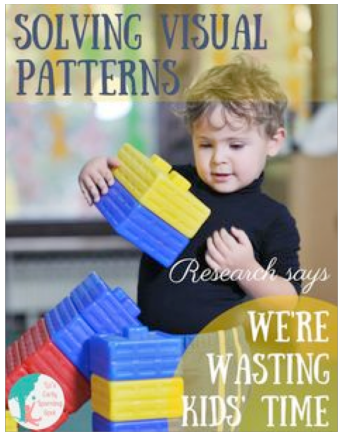
COMMON CORE STATE STANDARDS for MATHEMATICS

Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Influence of *Lack of* Research on Practice



www.lizs-early-learning-spot.com/

- The (U.S.) National Mathematics Advisory Panel (2008) concluded: "In the Major Topics of School Algebra set forth in this report, **patterns are not a topic of major importance**. The prominence given to patterns in PreK–8 is not supported by comparative analyses of curricula or mathematical considerations" (p. 59).
  - Paucity of evidence available at the time of the report.
  - U.S. Common Core State Standards followed this recommendation, dropping patterning as a content standard.

Goal Today

- Build case that newer evidence indicates that dropping patterning from content standards was likely a mistake
  1. Define patterning
  2. Briefly review past evidence that patterning skill is related to math knowledge
  3. Present new evidence for *unique* importance of early patterning skill for numeracy knowledge at end of Kindergarten
  4. Overview new training study designed to build theory of change and test causal relations

Image from:
https://www.lovelycommotion.com/blog/14-preschool-patterning-activities

Finding the pattern allows us to know what comes next (and next after that …)

***Pattern**: a predictable sequence (i.e., follows a rule)*

E.g., alternating sequence of shapes or sounds

E.g., functional relationships between two variables

Early Patterning: Repeating Patterns

- Repeating Patterns follow a rule that one part repeats over and over. The *core unit* is the part that repeats.

core unit

Development of Repeating Pattern Knowledge

- Age 3: Begins to develop. Children notice and fill in simple alternating AB patterns (e.g., black and white striped shirt) and notice patterns in songs.
- Ages 4-7: Expanding to
  - Increasing complexity of core unit (AB, ABB, ABC)
  - Increasing demands of patterning task: copying vs. extending pattern

(Clements & Sarama, 2009)

Example Patterning Tasks

- Copy pattern: “Please make the same pattern”

- Extend pattern: “Keep the pattern going”

nuttinbutpreschool.com

Recent Research Linking Patterning Skill to General Math Knowledge

1. Improving children's patterning skills can improve their math knowledge
   1. Children attending a preschool that focused on patterning did better on some patterning and numeracy tasks at end of school year than children attending a preschool that did not focus on patterning (Papic et al., 2011).
   2. Among first-grade students with low patterning skills, those randomly assigned to receive a patterning intervention had greater math achievement at end of school year than students receiving reading intervention (Kidd et al., 2013; Kidd et al., 2014).
2. Early patterning skill predicts end of pre-K math knowledge (Rittle-Johnson, Zippert & Boice, 2018).
3. Early patterning skill predicts middle-grades math achievement (Fyfe, Rittle-Johnson & Farran, 2018; Nguyen et al., 2016; Rittle-Johnson, Fyfe, Hofer & Farran, 2016).

What's Next?

- Given building evidence that patterning skill is related to math knowledge:
  - Are its contributions separable from other related skills, such as working memory and spatial skills?
  - *How* might patterning skills contribute to math development?

Is Patterning Skill Just a Proxy for Other Cognitive Skills?

- Working memory:
  - Capacity to actively maintain and regulate a limited amount of task-relevant information (Baddeley & Logie, 1999).
    - Information can be verbal or visual-spatial (Raghubar, Barnes, & Hecht, 2010)
  - Related to math (Bull et al., 2008; Geary, 2011) and to patterning (Miller, Rittle-Johnson, Loehr, & Fyfe, 2015; Rittle-Johnson, Zippert & Boice, 2018)
- Spatial visualization:
  - Ability to imagine and mentally transform spatial information (Uttal et al., 2013)
  - Related to math (Mix & Cheng, 2012) and to patterning (Rittle-Johnson, Zippert & Boice, 2018)

Preliminary Theory of Change

- Patterning is a core skill for mathematical thinking (Charles, 2005; Sarama & Clements, 2004; Steen, 1988).

Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics

Charles (2005)

BIG IDEA #9

PATTERNS: Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.

Examples of Mathematical Understandings:

Numbers
- Skip counting on the number line generates number patterns.
- The structure of the base ten numeration system produces many numerical patterns.
- There are patterns in the products for multiplication facts with factors of 0, 1, 2, 5, and 9.
- There are patterns when multiplying or dividing whole numbers and decimals by powers of ten.
- The difference between successive terms in some sequences is constant.
- The ratio of successive terms in some sequences is a constant.
- Known elements in a pattern can be used to predict other elements.

Geometry
- Some sequences of geometric objects change in predictable ways.

Preliminary Theory of Change for Link to Early Numeracy

- Big Idea: Numbers follow rules just like repeating patterns follow rules. When we find a pattern, we know what comes next.

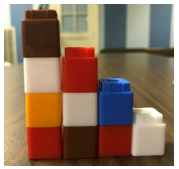
Repeating Patterns in Number System

- One's digits repeat in each decade:
  - Counting sequence above twenty
  - Written numerals above 10

Patterns in Numbers Cont.

Growing patterns: Items increase or decrease following a rule, such as add 1 or add 2.

2, 3, 4, 5, ?



– e.g., Successor principle –Cardinality for each count word is cardinality of previous count word plus one (Gelman & Gallistel, 1978; Sarnecka & Carey, 2008).

Current Study

- Help refine a theory of change by examining longitudinal relations between patterning skills in preschool and math knowledge, esp. numeracy knowledge, at the end of Kindergarten.
  - Use a fast, teacher-friendly patterning measure.
  - Control for early working memory and spatial skills, as well as verbal ability
  - Include assessments of 2 pattern-intensive numeracy knowledge in Kindergarten – count sequence to 100 and successor principle.

Study Hypothesis

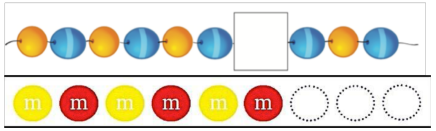
- Children's patterning skills in pre-K would predict their math knoweldge one year later, above and beyond their early spatial, working memory and verbal skills.
  - This will be true for *numeracy* knowledge, esp. for pattern-intensive numeracy knowledge (i.e., count sequence to 100, successor principle)

Participants

- 65 children assessed at three time points
  - Time points
    - Beginning of pre-K (M age = 4.6, SD = .3)
    - *End of pre-K (M age = 5.1, SD = .3)*
    - *End of Kindergarten (M age = 6.1, SD = .3)*
  - Recruited from 6 public and private schools
  - Sex: 32 boys and 33 girls
  - 52% Ethnic minorities
  - 55% received financial assistance for tuition

New Teacher-Based Patterning Assessment

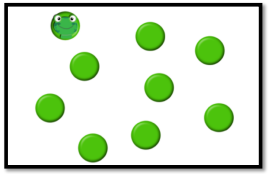
- Assessed pattern knowledge using worksheets with paper cut-outs
- Based on materials available for teachers on the internet
- Four tasks:
  - What comes next?
  - Filling-in missing item
  - Extending
  - Matching



- Reliable: Internal consistency: Alpha = .84
- Valid: Strong relation to existing patterning measure: $r(76) = .59$ (Rittle-Johnson, Zippert & Boice, 2018 ; Zippert, Douglas, Rittle-Johnson, 2019)

Cognitive Controls

- **Spatial visualization skill:** WPPSI Block design
- **Working memory** (WM): Ecorsi block tapping and backward digit span tasks. Created composite due to high correlation.
- **Verbal ability**: Receptive vocabulary



Math Knowledge Assessment

Research-Based Early Mathematics Assessment (REMA) – Short Form (Weiland, Wolfe, Hurwitz, Clements, Sarama & Yoshikawa 2012).

Numeracy

Math Topic	Sample Item
Non-symbolic Quantity	Shown two cards, with 4 dots and 3 dots: 'Which one has more?'
Counting	Shown 8 dinosaurs in a line: "' Please tell me how many toy dinosaurs I have."
Symbolic Mapping	Match the numerals 1-5 to the appropriate number of grapes.
Shape	Select all triangles from a collection of 24 shapes; some are prototypic shapes and some are not.

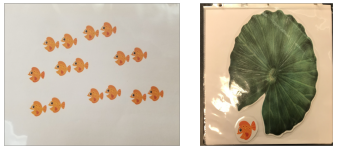
Specific Numeracy Knowledge

Count to 100

- Knowledge of the count sequence to 100
  - "How high can you count? Start at 1 and tell me… "
  - When stop: "What comes next? Can you go higher?"
- Because of ceiling effect, scored as count to 100 (75% of children) or not.

Successor Principle

- Fish Pond task: How many when add one? (Cheung et al., 2017)
  - 10 items with numbers ranging from 15 to 116, alpha = .83
- Because of ceiling effect, scored as mastery (all correct; 59% of children) or not.



"Fifteen fish are swimming under the lilypad. Now watch… another fish swims in! Now are there 16 or 17 fish?"

Results: Patterning Skills' Relation to Other Cognitive Skills

- Patterning skill measure was moderately to strongly related to other cognitive skill measures.
  - Stronger than in our past studies focused on beginning of preK (Rittle-Johnson, Zippert & Boice, 2018; Zippert, Clayback & Rittle-Johnson, 2019 Saturday at 9:45am)
- Screened for multi-collinearity. Ok for linear regression models.

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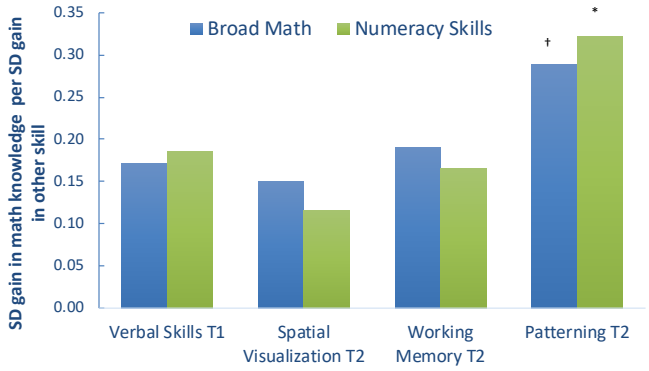
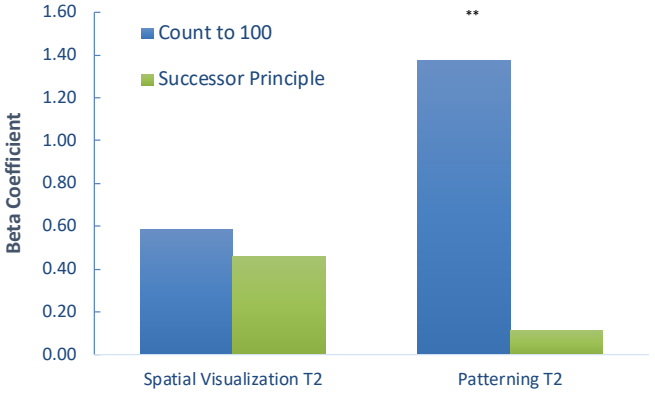
Spatial Visualization	.45
Working memory	.76
Verbal skill	.40

Results: Broad Math and Numeracy Knowledge



† $p < .1$ * $p < .05$

Results: Specific Numeracy Knowledge



Logistic Regression Models

** $p < .01$.

Results Summary

- Children's repeating patterning skills at the end of pre-K uniquely predicted children's math knowledge one year later, at the end of kindergarten for:
  - Broad math measure (marginal)
  - Numeracy subscale
  - Ability to count to 100
  - But not mastery of the successor principle
    - A ceiling effect limited our ability to detect differences
    - Ability to count to 100 related to concurrent successor principle knowledge in 5-6 year olds (Cheung et al., 2017)

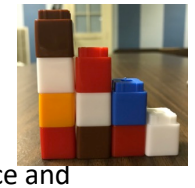
Implications

- Rather than removing patterning from early math standards, encourage high-quality instruction on patterning.
  - Particularly important because patterning is popular among children, teachers & parents.
    - Preschool children spontaneously engage in patterning activities (Ginsburg, Inoue, & Seo, 1999; Ginsburg, Lin, Ness, & Seo, 2003).
    - Some preschool teachers report engaging children in patterning activities many times a week (Rittle-Johnson, Fyfe, Loehr & Miller, 2014).
      - However, the mathematical nature of patterns are often not highlighted (Economopoulos, 1998).
    - Parents engage their children in patterning activities at home, although engage in numeracy activities more often (Zippert & Rittle-Johnson, in press)
- Need effective ways to use patterning to promote math development, which will also help refine a theory of change.

In-Progress Training Study

- Aim:
  - Evaluate if improving repeating patterning skills helps preschool children better develop numeracy and general math knowledge.

Hypothesis



- Repeating patterning skills prepare children to notice and learn about patterns in numbers, supporting their numeracy knowledge.
  - Focused on teaching the *successor principle*
    - Foundational numeracy knowledge that reflects a key conceptual insight about counting, integers and arithmetic (Gelman & Gallistel, 1978; Sarnecka & Carey, 2008).
    - Theoretically, successor principle knowledge may be learned via generalization of a pattern in the relation between the order of the count words and set size (Carey, 2004; Cheung, et al 2017).
- Study Hypothesis:
  - Improving children's repeating patterning skills prepare them to learn more from numeracy instruction on the successor principle, both on that skill and math more broadly.

Training Conditions

- Randomly assigned to 3 conditions:
  - Repeating Patterning + Numeracy Instruction
    - Repeating patterning activities combined with a numeracy activity (related to the successor principle) at end of each lesson
  - Literacy + Numeracy Instruction
    - Literacy activities so spend same time with experimenter combined with a numeracy activity at end of each lesson
  - Passive control group
    - Regular classroom instruction only

Stay Tuned!

Conclusion

- Deducing underlying rules core to repeating patterns and all of math.
- Common Core Mathematical Practice "Look for and make use of structure" hints at this, but value of repeating patterning activities needs to be highlighted.

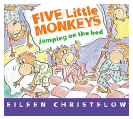
Need Patterning Tips for Teachers and Parents

- **Notice and make patterns with everyday objects and movements!**


- **Vary the pattern unit!** Start with easier pattern units, and make them harder as your child learns!

- **Do Different Patterning Activities!** Copy, extend and abstract patterns

- **Talk About the Patterns** "How do you know it's a pattern?"
- **Find Patterns in Numbers**



Acknowledgements

Children's Learning Lab

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- Slides available at:
- vu.edu/patterns-and-math-publications

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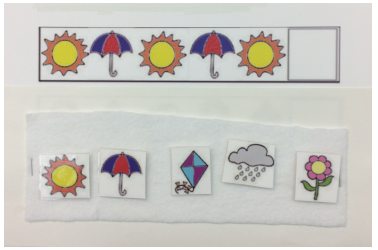
EXTRA SLIDES

Teacher Patterning Sample Item

- **What's Next Pattern AB**



- "What comes next in the pattern? Use one of these."

Teacher Patterning Sample Item
Match Pattern ABBB

"Can you make the same kind of pattern using your pictures?"

Example of What Was Dropped: National Council of Teachers of Mathematics (NCTM) Focal Points for Instruction 2006

Curriculum Focal Points and Connections for Kindergarten

The set of three curriculum focal points and related connections for mathematics in kindergarten follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Kindergarten Curriculum Focal Points	Connections to the Focal Points
Number and Operations: **Representing, comparing, and ordering whole numbers and joining and separating sets** Children use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set, creating a set with a given number of objects, comparing and ordering sets or numerals by using both cardinal and ordinal meanings, and modeling simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the number in a small set, counting and producing sets of given sizes, counting the number in combined sets, and counting backward.	***Data Analysis:*** Children sort objects and use one or more attributes to solve problems. For example, they might sort solids that roll easily from those that do not. Or they might collect data and use counting to answer such questions as, "What is our favorite snack?" They re-sort objects by using new attributes (e.g., after sorting solids according to which ones roll, they might re-sort the solids according to which ones stack easily). ***Geometry:*** Children integrate their understandings of geometry, measurement, and number. For example, they understand, discuss, and create simple navigational directions (e.g., "Walk forward 10 steps, turn right, and walk forward 5 steps"). ***Algebra:*** Children identify, duplicate, and extend simple number patterns and sequential and growing patterns (e.g., patterns made with shapes) as preparation for creating rules that describe relationships.
Geometry: **Describing shapes and space** Children interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of shapes, such as squares, triangles, circles, rectangles, (regular) hexagons, and (isosceles) trapezoids presented in a variety of ways (e.g., with different sizes or orientations), as well as such three-dimensional shapes as spheres, cubes, and cylinders. They use basic shapes and spatial reasoning to model objects in their environ	
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Algebra: Children identify, duplicate, and extend simple number patterns and sequential and growing patterns (e.g., patterns made with shapes) as preparation for creating rules that describe relationships.

Overview of Patterning + Numeracy Training Sessions

Session	Patterning Focus	Numeracy Focus (Successor Principle)
1	Duplicate and extend patterns	Add 1 (small numbers)
2	Duplicate and extend patterns	Add 1 (small numbers)
3	Identify core unit of patterns	Subtract 1 (small numbers)
4	Abstract patterns	Add 1 (large numbers and review small numbers)
5	Review core unit of patterns and abstract patterns	Add and subtract 1 (small and large numbers)

Study 2 Design

Pretest

Children are pretested in one 25-minute session
- Math (REMA)
- Pattern: Teacher-Based
- Successor Principle: Fish Pond

Randomized to Condition

Children are first randomized to condition, and then assigned to a group of 2 based on varying pattern ability

Intervention

Children in groups of 2 receive five 25-minute training sessions across 2-3 weeks

Post test Day 1

The day after session 5, children are assessed in one 20-minute session
- Pattern: Research-Based
- Successor Principle

Post test Day 2

Within a few days of post day 1, children are assessed in another 20-minute session
- Math (REMA)
- Pattern: Teacher-Based