Finding patterns in objects and numbers: Repeating patterning in pre-K predicts kindergarten mathematics knowledge

Keywords: Repeating pattern knowledge, mathematical development, numeracy knowledge, counting, successor function

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Abstract

Both recent evidence and research-based early mathematics curricula indicate that repeating patterns—predictable sequences that follow a rule—are a topic of major importance for mathematics development. The purpose of the current study was to help build a theory for how early repeating patterning knowledge contributes to early math development, focusing on development in children ages 4-6. The current study examined the relation between 65 preschool children's repeating patterning knowledge (via a fast, teacher friendly measure) and their end-of-kindergarten broad math and numeracy knowledge, controlling for verbal and visual-spatial working memory (WM) skills as well as end-of-pre-k broad math knowledge. Relations were also examined between repeating patterning and specific aspects of their numeracy knowledge–knowledge of the count sequence to 100 and the successor principle. Children's repeating patterning knowledge was significantly predictive of their broad math and general numeracy knowledge, as well as one specific aspect of their numeracy knowledge (counting to 100), even after controlling for verbal and visual-spatial WM skills. It remained a unique predictor of general numeracy knowledge and count to 100 after controlling for end-of-pre-K broad math knowledge. The relation between repeating patterning and mathematics may be explained by the central role that identifying predictable sequences based on underlying rules plays in both. Theories of math development and early math instruction standards should give greater attention to the role of children's repeating patterning knowledge.

Keywords: Repeating patterning knowledge, mathematical development, numeracy knowledge, counting, successor principle

Finding patterns in objects and numbers: Repeating patterning supports early mathematics knowledge

Both recent evidence and research-based early mathematics curricula indicate that repeating patterns—sequences that follow a rule that one part repeats over and over—are a topic of major importance for mathematics development. Repeating patterns range from alternating sequences of objects such as shapes or sounds to the repeating structure of the base-ten numeration system (e.g., the ones digits repeat in each decade). Thus, repeating pattern instruction can be introduced at various stages of mathematical development. Recent evidence indicates that repeating patterning knowledge in preschool predicts math knowledge concurrently, months later, as well as in first, fourth, fifth, and sixth grades (Fyfe et al., 2019; Nguyen et al., 2016; Rittle-Johnson et al., 2017, 2019; Zippert et al., 2019). Further, improving children's repeating patterning knowledge can improve their math knowledge in preschoolers (Papic et al., 2011) and in first-graders with low patterning knowledge (Kidd et al., 2013, 2014). However, recommendations from the National Mathematics Advisory Panel (2008) led to the removal of repeating patterning from the Common State Standards (2010) as a math content standard in the early primary school grades. This was likely due to a paucity of evidence at the time the suggestions were made; however, it means that early childhood teachers are left unclear about whether and how to use repeating patterning to facilitate mathematics instruction (Raber et al., 2017).

In order to make the case for emphasizing repeating patterning in regular mathematics instruction in the early grades (e.g., kindergarten), we must better understand how repeating patterning knowledge contributes to math development. The purpose of the current study was to help build a theory for how early repeating patterning knowledge contributes to early math development, focusing on development in children ages 4-6.

Developing a Theory of How Early Repeating Patterning Supports Early Math Knowledge

Young children, teachers, and parents all regularly work with repeating patterns (Economopoulos, 1998; Ginsburg et al., 2003; Rittle-Johnson et al., 2015; Zippert & Rittle-Johnson, 2018). By age 3, children notice and fill in simple alternating AB repeating patterns (e.g., a red and green striped shirt) and notice repeating patterns in songs (Sarama & Clements, 2009). Between the ages of 4 to 7, children are expanding their knowledge of repeating patterns to include increasingly complex core units (e.g., ABB, AABB, ABC) and increasingly demanding repeating patterning tasks (Papic et al., 2011; Rittle-Johnson et al., 2013; Sarama & Clements, 2009; Starkey et al., 2004). By the end of preschool, many children can *complete* (identifying the missing item in a repeating pattern), *duplicate* (make an exact replica of a model repeating pattern) and *extend* (continue an existing repeating pattern by at least one unit of repeat) repeating patterns (Papic et al., 2011; Rittle-Johnson et al., 2015; Sarama & Clements, 2009). Eventually, children learn to abstract repeating patterns (creating the same type of repeating patterns using new materials), and to identify the core unit of repeating patterns (Clements & Sarama, 2014; Rittle-Johnson et al., 2015).

Like repeating patterning, math inherently involves identifying, extending, and describing predictable sequences in objects and numbers (Charles, 2005; Sarama & Clements, 2004; Steen, 1988). For example, Charles (2005) proposed that one big idea in mathematics is “Patterns: Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways” (p. 17). Examples for number patterns include skip counting on a number line, the structure of the base ten numeration system (e.g., the ones digits repeat in each decade), multiplying or dividing whole numbers and decimals by powers of ten, and sequences in which numbers or their ratios differ by a constant (Charles, 2005). Similarly, we propose the Big Idea for early mathematics that "Numbers follow rules just like repeating patterns follow rules. When we find a pattern, we can create and use rules that underlie our number system." In line with this claim, the National Council of Teachers of Mathematics (NCTM) Focal Points for Instruction (2006) included this standard for kindergarten: "Children identify, duplicate, and extend simple number patterns and sequential [repeating] and growing patterns (e.g., patterns made with shapes) as preparation for creating rules that describe relationships" (p. 12).

Indeed, recent evidence indicates that repeating patterning knowledge is predictive of future numeracy knowledge. Numeracy knowledge encompasses children's understanding of whole numbers and number relations, including counting (knowledge of the number-word sequence and applying count words to quantify objects) and symbol-magnitude mappings (mapping written numerals and their number names to their respective non-symbolic quantities). Numeracy is often considered the foundation of early math knowledge (Jordan et al., 2006; National Research Council, 2009; Purpura et al., 2013). Children's repeating patterning knowledge at the beginning of preschool was predictive of their general numeracy knowledge concurrently and at the end of preschool (Rittle-Johnson et al., 2019; Zippert et al., 2019). Also their repeating patterning knowledge at the end of pre-K predicted numeracy knowledge at the end of first grade (Rittle-Johnson et al., 2017).

To guide our theory, we examined particular aspects of early numeracy knowledge that may serve as pathways through which repeating patterning knowledge supports numeracy knowledge. In particular, we propose that repeating patterning knowledge should help children learn: (a) the verbal and written numeration system, and (b) the successor principle, each of which are important for mathematical thinking (see our conceptual model in Figure 1). We define and discuss the importance of repeating patterning for both aspects of numeracy knowledge below.

Repeating patterning knowledge may be important for learning about counting through the verbal and written numeration system. This is because recognizing that the one's digits repeat in each decade greatly simplifies the task; and a core repeating pattern in the base ten numeration system is that the one's digits repeat in each decade, leading to a predictable count word sequence (above twenty in English) and to a predictable written numeral system. Noticing and using these repeating patterns greatly reduces what children need to memorize and can help them predict what number comes next in the counting sequence.

A second core pattern is captured in the successor principle, or the knowledge that the cardinality for each count word is the cardinality of the previous count word plus one. This concept is a foundational aspect of numeracy knowledge because it reflects a key conceptual insight about counting, integers and arithmetic, including how numerals represent the natural numbers (Gelman & Gallistel, 1978; Sarnecka & Carey, 2008). Repeating patterning might be important if learning the successor principle involves the generalization of a pattern in the relation between the order of the count words and set size (Carey, 2004; Cheung et al., 2017).

Distinguishing the Effect of Repeating Patterning, Working Memory on Math Knowledge

There are concerns that repeating patterning knowledge may be a proxy for other cognitive skills in how it relates to math knowledge (Burgoyne et al., 2017). Particularly, working memory (WM), or the ability to actively maintain and regulate a limited amount of task-relevant information (Baddeley & Logie, 1999), is related to early math (Bull et al., 2008; Geary, 2011) and to repeating patterning (Miller et al., 2016; Rittle-Johnson et al., 2015, 2019; Zippert et al., 2019). Verbal and visual-spatial WM are the strongest and most consistent predictors of later math performance, and are thought to support processing of information when solving math problems (Bull et al., 2008; Geary, 2011). Similarly, verbal and visual-spatial WM ability are related to repeating patterning performance, more so than short-term memory, inhibitory control or cognitive flexibility (Collins & Laski, 2015; Miller et al., 2016). Verbal WM ability also predicts growth in repeating patterning knowledge from instruction (Miller et al., 2016). Verbal and visual-spatial WM may support identifying and applying relations between pattern elements to recreate the core unit using the same or new materials. Given the demands of repeating patterning and math tasks on WM, controlling for individual differences in verbal and visual-spatial WM ability is important in evaluating how repeating patterning predicts later mathematics knowledge. Measures of WM were not available in several previous studies indicating that early repeating patterning knowledge predicted later math knowledge (Fyfe et al., 2019; Nguyen et al., 2016; Rittle-Johnson et al., 2017).

Only one previous study has considered the relation between repeating patterning knowledge and *later* math knowledge after controlling for WM abilities (Rittle-Johnson et al., 2019). In this study, repeating patterning knowledge at the beginning of pre-K (ages 4- to 5) predicted math knowledge at the end of the school year, beyond verbal and visual-spatial WM, spatial visualization, form perception, and verbal ability. Further, neither spatial visualization, form perception, nor verbal ability were unique predictors of later math knowledge beyond WM ability, although visual-spatial was marginally significant (Rittle-Johnson et al., 2019). Additionally, two recent single-timepoint studies in preK provide promising evidence that the repeating patterning-numeracy relation exists beyond both visual-spatial WM and spatial visualization (Wijns et al., 2019; Zippert et al., 2019). Whether this relation holds after formal mathematics instruction begins in kindergarten is unknown. Formal instruction may reduce or eliminate the impact of preschool repeating patterning knowledge on school-age math knowledge after accounting for WM ability. Overall, understanding the relation between repeating patterning and math knowledge requires confirmation that it is not driven by their shared reliance on verbal and visual-spatial WM ability.

Current Study

The current study extends past research by evaluating the relations between repeating patterning knowledge at the end of pre-K and math knowledge at the end of kindergarten, including several measures of numeracy knowledge. This included two types of pattern-intensive aspects of numeracy knowledge–the count sequence to 100 and the successor principle. We considered both verbal and visual-spatial WM so that the unique predictive value of repeating patterning knowledge could be isolated. We also aimed to establish the reliability and validity of a fast, teacher-friendly repeating patterning measure at the end of preschool (Rittle-Johnson et al., 2019). This measure captures early developing repeating patterning skills, is aligned with common classroom activities, and could easily and quickly be implemented by teachers.

We hypothesized that repeating patterning knowledge at the end of pre-K would predict math knowledge one year later, above and beyond verbal and visual-spatial WM skills. We predicted this would be true for numeracy knowledge, specifically knowledge of the count sequence to 100 and the successor principle. We also expected strong reliability (e.g., internal consistency) and validity for our end-of-pre-K teacher-friendly repeating patterning measure.

Method

Participants

Participants were 65 children who were recruited from six preschool programs (three public, one Head Start center, and two private). Of the original 79 children with parental consent, two children would not assent to participate in the study, one child was withdrawn from the study because of other commitments, and 11 children had incomplete data on multiple measures. In the final sample of 65 children (51% females), the average age was 6.14 years (*SD* = .29 years; range = 5.55 years – 6.72 years) at the end of kindergarten. Based on parent report, 39% of children were White, non-Hispanic, 46% of children were Black, 6% were biracial, 3% were Hispanic or Latino, 3% were Asian or Pacific Islander, and 3% were another race (Middle Eastern, Kurdish, Ethiopian). Most children (86%) spoke only English at home. A majority of families received either some (17%) or full (39%) financial aid for tuition to attend their preschool program, and only 8% of children were receiving special education services while in preschool. Public preschools used a sliding scale for tuition payment, with some families paying nothing, some families paying partial tuition, and some families paying full tuition. Children attended kindergarten at 24 different schools, with 92% attending public kindergarten programs. Past research has reported large effect sizes for the predictive relation between preK repeating patterning knowledge and later mathematics knowledge (Rittle-Johnson et al., 2017), but we considered a more conservative range of estimates effect sizes because we included working memory as a control variable in this study. A G*Power (Faul et al., 2007) a priori power analysis suggested that a sample size of 55 was sufficient to detect a medium effect size of $f^2 = .15$ (corresponding increase in $R^2 = .13$), $\alpha = .05$, with power of .80, via a linear multiple regression analysis with 1 tested predictor (4 total). An a priori power analysis for the logistic regression analyses suggested that a sample size of 34 was sufficient to detect a medium effect size of $f^2 = .15$ (corresponding increase in $R^2 = .13$), $\alpha = .05$, with power of .80, an odds ratio of 4.06, probability ($Y = 1 \mid X = 1$) of .70, and a normally distributed predictor ($\mu_x = 0$, $\sigma_x = 1$).

Procedure

During the final quarter of their pre-K year, children were assessed individually in two 20-minute sessions. They were given a verbal WM and a math assessment (which included a number knowledge component) in one 20-minute session, and then a repeating patterning, and a visual-spatial WM measure in a second 20-minute session. During the final quarter of their kindergarten year, children were individually given the same math and repeating patterning measures administered at the end of pre-K and two specific numeracy tasks in a single 30-minute session. The average delay between the two time points was 373.71 days (SD = 17.72 days). Institutional review board approval was obtained (IRB #151356, Exploring the Roles of Pattern and Spatial Skills in Early Mathematics Development).

Measures

Math and numeracy knowledge.

Broad math and general numeracy knowledge. The REMA Short-Form, comprised of 19 items (13 measuring numeracy knowledge and 6 measuring shape knowledge), was used to assess children's broad math knowledge (Weiland et al., 2012). Numeracy items included rote counting to 5, non-symbolic and number word magnitude comparison, enumerating, object counting, set size production, non-symbolic addition and subtraction, and numeral to non-symbolic quantity matching. A stop-criteria suggested and validated by past work was implemented (Rittle-Johnson et al., 2019; Weiland et al., 2012). Items were scored according to the authors' specification, although scores on the 4 polytomous items were collapsed into fewer categories due to the low incidences of some values. IRT ability estimates were generated using a partial credit model. We constrained the item parameters to improve the precision of ability estimates given our sample size by doing Empirical Bayes estimation (Baker & Kim, 2004) using WinBUGS 1.4.3 (Spiegelhalter, Thomas, Best, & Lunn, 2003). The informative prior distribution on the item difficulty parameters and the sum-to-zero constraints on the item location and threshold parameters were chosen based on results reported in Weiland et al. (2012). Internal consistency in our sample at the end of kindergarten was good ($\rho_{XX'}$ = .74). Considering the two sections separately, internal consistency was acceptable for the numeracy section ($\rho_{XX'}$ =.71), but unacceptable for the shape section ($\rho_{XX'}$ = .45). Separate IRT ability estimates for the numeracy section were used as a measure of children's general numeracy knowledge.

Count to 100. The rote counting item on the REMA Short-Form was used to measure children's ability to count to 100. Children were asked to count as high as they could starting with 1, and were prompted to continue counting if they stopped after counting correctly as many times as necessary to reach 100 or until a counting error was made. On average, children counted to 88 without error (*M* = 88.06, *SD* = 22.39, range 29-100), with a majority of children (75%) successfully counting to 100 without error.

Successor principle. The successor principle task with a fishpond theme was created by Barner and colleagues (based on Cheung et al., 2017). Children were quickly shown an initial number of fish before they were hidden behind a cut-out lily pad, and then one new cutout picture of a fish was shown. They were asked "*N* fish are swimming under the lily pad. Now watch… another fish swims in! Now are there *N+1* or *N+2* fish?" with the position of the correct response counterbalanced across trials (first or second). There were 10 items with numbers ranging from 15 to 116 (*N* = 15, 20, 34, 46, 51, 62, 73, 95, 107, 116). Internal consistency was strong (α = .83). Children solved 9 items correctly on average (*M* = 8.71, *SD* = 2.12, range = 2 to 10), with a majority of children (59%) solving all items correctly.

Repeating patterning skill. The Teacher-based Repeating Patterning Assessment described and validated with younger preschoolers in previous studies (Rittle-Johnson et al., 2019) was used to measure children's knowledge of repeating patterns. The 10-item assessment tested children's ability to complete, extend, and match patterns of pictures with various units (i.e. AB, ABB, ABC and AABB pattern units). See Table 1 for a list of items including the difficulty of the pattern unit as well as item-level statistics. See Figure 2 for example items. The assessment took approximately six minutes to administer. Children earned a point for each item answered correctly, and internal consistency in our sample using IRT scores was high ($\rho_{XX'}$ = .90), suggesting good measure reliability. We generated ability estimates for children using a Rasch model with a Laplace approximation and empirical Bayesian prediction method that has been shown to be stable for sample sizes around 50 (Cho & Rabe-Hesketh, 2011). Laplace approximation was implemented in R (http://www.r-project.org), using the glmer function of the lme4 package (Bates, Maechler, & Dai, 2008).

WM.

Verbal WM. We administered the forward and backward digit span task from the *Wechsler Intelligence Scale for Children* (Wechsler, 2003). Children were first trained on the task to ensure their comprehension (see Miller, Rittle-Johnson, Loehr, & Fyfe, 2015). The backward digit span was used as our measure of verbal WM, calculated by summing the number of total backward trials answered correctly. Some children were unable to complete any backward trials correctly (n = 24, 37% of sample), but internal consistency in our sample was acceptable, α = .71.

Visual-spatial WM. The Corsi Block Tapping Task was used to measure children's visual-spatial WM. Children completed a 3-minute task using the *PathSpan* program, an iPad version of the Corsi Block task appropriate for young children (available at https://hume.ca/ix/pathspan.html) and used in prior studies (LeFevre et al., 2010; Rittle-Johnson et al., 2019; Xu & LeFevre, 2016). Nine green circles in fixed positions were presented to children as lily pads. Children watched a frog "jump" to different lily pads, and had to touch the same lily pads in the same order. First, the experimenter demonstrated the task on one trial and children practiced on two trials with feedback. The number of lily pads that the frog jumped started at two and increased to a maximum of eight, with two trials for each span length. Testing discontinued when both trials within a given span length were completed incorrectly. Two additional two-span trials were provided if children failed the first two trials to give them adequate practice with the task. Internal consistency within our sample was high and acceptable ($\alpha = .73$).

Data Preparation and Screening

The verbal and visual-spatial WM measures were highly correlated with each other, $r(64) = .69$, $p < .001$, so we created a composite measure of WM by standardizing and averaging children's scores on these measures. All measures were screened for skew and kurtosis. The two specific measures of numeracy knowledge were highly skewed due to ceiling effects, so we dichotomized performance on each as *mastery* (count correctly to 100; answer all successor principle items correctly) or *non-mastery*.

We tested for non-independence in math scores that might arise from children being nested within different schools, controlling for general cognitive skills, but intra-class correlations were near 0, indicating that OLS regression analyses were appropriate.

We tested for multicollinearity by estimating variance inflation factors (VIF) for all independent variables, and all VIF scores for independent variables were less than 3, indicating multicollinearity was not biasing the results. We also tested whether children's broad math, general numeracy, and specific aspects of their numeracy knowledge differed by demographic factors (i.e., sex, financial assistance, race; see Table S1). Children's specific numeracy skills were not associated with their demographic factors; however, children's broad math and general numeracy knowledge at the end of kindergarten were associated with their race (white vs. non-white). Specifically, children who were identified as White had significantly better broad math (M = 1.69, SD = .66) and general numeracy skills (M = 3.45, SD = 1.18) than their peers of other races and ethnicities (M_{math} = 1.23, SD = .88; $M_{numeracy}$ = 2.39, SD = 1.48), $t_{math}(63) = 2.20$, $p <$ .05, d = .59; $t_{numeracy}(63) = 3.04$, $p < .01$, d = .79; however, children's race was not predictive of their broad math or general numeracy knowledge after controlling for their age and WM, so it was not included in the final models. Data and study measure materials are available at osf.io/ekpux/.

Results

Relations Among Variables

Descriptive statistics and correlations among key variables are presented in Table 2. Significant positive zero-order correlations were found among all continuous variables, rs = .40 to .92. These significant positive relations between target variables held after controlling for age, prs = .30 to .91.

Next, we considered relations between repeating patterning, broad math, and numeracy knowledge to inform future regression models and to provide evidence of validity for our teacher-friendly repeating patterning measure. Repeating patterning at the end of preschool moderately correlated with broad math and numeracy knowledge at the end of kindergarten, even after controlling for age, $prs(62)$ = .47 and .49, $ps < .01$ (see Table 2). Further, children who could count to 100 at the end of kindergarten had higher repeating patterning knowledge at the end of pre-K (M = .32, SD = 1.24) than children who could not (M = -1.25, SD = 1.13), $t(63)$ = 4.50, $p < .01$, d = 1.32; however, children who mastered the successor principle at the end of kindergarten did not differ significantly in their repeating patterning knowledge at the end of pre-K (M = .10, SD = 1.25) from children who had not (M = -.30, SD = 1.55), $t(63)$ = 1.15, $p < .26$, d = .28. Overall, repeating patterning knowledge was moderately related to future math knowledge, including numeracy and rote counting knowledge, but not our measure of successor principle knowledge. Correlations between our repeating patterning assessment and similar constructs (e.g., math and numeracy) provided evidence of convergent validity of the measure.

Next, consider relations between WM and target variables (see Table 2). Repeating patterning knowledge was strongly related to the verbal and visual-spatial WM composite after controlling for age, $pr(62)$ = .69, $p < .01$, providing additional evidence of convergent validity for our repeating patterning measure. Similarly, broad math knowledge and general numeracy knowledge were moderately correlated with the WM composite after controlling for age, $pr_{math}(62)$ = .42, $p < .01$; $pr_{numeracy}(62)$ = .43, $p < .01$. Children who could count to 100 at the end of kindergarten had higher verbal and visual-spatial WM at the end of pre-K (M = .20, SD = .87) than children who could not (M = -.53, SD = .82), $t(63)$ = 2.91, $p < .01$, d = .86. However, children who mastered the successor principle by the end of kindergarten did not differ in verbal and visual-spatial WM at the end of pre-K (M = .12, SD = .88) from children who had not (M = -.13, SD = .95), $t(63)$ = 1.08, $p < .29$, d = .27. Overall, repeating patterning and math measures both related to verbal and visual-spatial WM, indicating the importance of controlling for both types of WM in assessing the relations between repeating patterning and math.

Predictors of Math Knowledge

In order to determine the predictive relations between repeating patterning knowledge at the end of pre-K and later math knowledge, linear and logistic regression analyses were performed with the four math knowledge measures at the end of kindergarten as the dependent variable in each model. The first regression block included age at the end of kindergarten and WM at the end of pre-K to control for general cognitive ability. Then, repeating patterning at the end of pre-K was entered into the second regression block to test our hypothesis that repeating patterning would be a unique predictor of end-of-kindergarten math knowledge on each outcome. Lastly, we added broad math knowledge at the end of pre-K in the third block to examine whether repeating patterning remained a unique predictor of math knowledge at the end-of-kindergarten after controlling for previous math knowledge on the same measure, which is a very stringent test given that previous performance on a measure is a very strong predictor of future performance on that measure. Results are presented in Tables 3 and 4 and discussed below.

For broad math knowledge, repeating patterning was a unique predictor over and above age and verbal and visual-spatial WM, $\beta = .33$, $p < .04$ (see Table 3) and explained an additional 5% of the variance in children's broad math knowledge when added to the model including age and WM. Repeating patterning additionally remained a marginal unique predictor of broad math knowledge when controlling for previous broad math knowledge, $\beta = .26$, $p < .08$. In predicting general numeracy knowledge, repeating patterning was a unique predictor over and above age and WM, $\beta = .37$, $p < .05$, explaining an additional 6% of the variance in children's general numeracy knowledge when added to the model. Repeating patterning remained a unique predictor of general numeracy knowledge when also controlling for previous broad math knowledge, $\beta = .30$, $p < .05$.

Results were similar for predicting children's success in counting to 100. The odds ratio for repeating patterning indicates that when holding age and WM constant, children with higher repeating patterning scores at the end of pre-K were 4.27 times more likely to successfully count to 100 at the end of kindergarten, $\chi^2_{\text{Wald}}(1, N = 65) = 6.87$, $p < .01$ (see Table 4). Additionally, repeating patterning knowledge accounted for an additional 20% of the variance in children's success in counting to 100, $\chi^2(1, N = 65) = 10.60$, $p < .01$ once added to the model. Further, with the inclusion of repeating patterning, our model was able to correctly classify children's count to 100 mastery 80% of the time. Repeating patterning remained a reliable and unique predictor of children's success in counting to 100 even when previous broad math knowledge was included in the model, $\chi^2_{\text{Wald}}(1, N = 65) = 7.08$, $p < .01$. In contrast, repeating patterning was not a significant predictor of mastery of the successor principle when controlling for age and the WM composite, $\chi^2_{\text{Wald}}(1, N = 65) = .28$, $p < .60$, nor when controlling for previous broad math knowledge along with age, and WM, $\chi^2_{\text{Wald}}(1, N = 65) = .18$, $p < .67$.

Discussion

Children's repeating patterning knowledge at the end of pre-K uniquely predicted children's math knowledge one year later at the end of kindergarten, for broad math, general numeracy knowledge, and the specific aspect of numeracy knowledge of counting to 100. Further, repeating patterning remained a significant predictor of kindergarten general numeracy knowledge as well as counting to 100 beyond pre-K broad math knowledge, and the relation was marginal for broad math knowledge. In addition, the contribution of repeating patterning knowledge was separable from verbal and visual-spatial WM and age. However, repeating patterning did not predict mastery of the successor principle. We discuss the implications of these findings for how repeating patterning knowledge might contribute to early mathematics development.

First, repeating patterning knowledge does not seem to simply be a proxy for other cognitive skills, particularly verbal or visual-spatial WM. WM has been proposed as a potential third variable underlying the relation between repeating patterning and math knowledge (Burgoyne et al., 2017). In particular, WM was related to math knowledge and to repeating patterning knowledge in past studies (Bull et al., 2008; Collins & Laski, 2015; Geary, 2011; Rittle-Johnson et al., 2019), and this was true in the current study as well. Verbal and visual-spatial WM should support identifying and applying relations between pattern elements to re-create the pattern using the same or new materials, with visual-spatial WM being especially important when working with visual patterns. Given the demands of repeating patterning and math tasks on WM, it is important to control for individual differences in this general cognitive skill when evaluating the predictive power of repeating patterning for later mathematics knowledge. The current study extends a previous finding that repeating patterning knowledge at the beginning of the pre-K year predicted math knowledge at the end of the school year, over and above the influence of verbal and visual-spatial WM (Rittle-Johnson et al., 2019). It indicates that repeating patterning knowledge in preschool can predict math knowledge at the end of kindergarten, after children have received formal math instruction, over and above the influence of verbal and visual-spatial WM. It could be that children's patterning knowledge helps them learn from mathematics instruction, although this requires further empirical support. We acknowledge that accounting for WM in our models may have reduced the size of our effect in relation to past work that did not consider these skills in their model (Rittle-Johnson et al., 2017).

Second, repeating patterning knowledge at the end of preK was predictive of children's future numeracy knowledge at the end of kindergarten. The current study supports our conjecture that a Big Idea for early mathematics is that "Numbers follow rules just like repeating patterns follow rules. When we find a pattern, we can create and use rules that underlie our number system." It converges with past research that children's repeating patterning knowledge at the beginning of the final year of preschool was predictive of their numeracy knowledge at the end of preschool (Rittle-Johnson et al., 2019), and that repeating patterning knowledge at the end of preschool was predictive of knowledge of two numeracy topics at the end of first grade (i.e., their knowledge of the mappings between symbolic numerals, their number names, and their magnitudes, as well as their calculation knowledge; Rittle-Johnson et al., 2017). Only the first study also controlled for verbal and visual-spatial WM. Because we also controlled for WM, we may have thus found lower effect sizes than in previous work (Rittle-Johnson et al., 2017). The current study was also the first to consider later specific pattern-intensive numeracy skills and found that repeating patterning knowledge was predictive of children's ability to count to 100, but not their successor principle knowledge.

These findings contribute to a much-needed theory for how early repeating patterning knowledge supports future math knowledge. They provide clues to potential pathways through which repeating patterning knowledge supports early numeracy development (see Figure 1). We specifically argue that repeating patterning knowledge may help children learn the numeration system, likely by helping children better detect the patterns in numbers, and thus extending their counting range. A core pattern in the base ten numeration system is that the one's digits repeat in each decade (and the ten's place in each decade increases by 1), leading to a predictable count word sequence (beyond twenty in English). Interestingly, children who can independently reach 20-99 when counting aloud can often reproduce the pattern of the one's digits in decades beyond their counting range after being given the first few count words of an unfamiliar decade (e.g., a child who can only reach 30 when counting independently can count to 49 when provided with

41, 42, 43; Siegler & Robinson, 1982). A small percentage of these children can even continue counting into the next decade without additional assistance. This may be because children in this stage of counting no longer view the count string as a single disconnected string, but rather increasingly understand the patterned relationships between adjacent count words, increasing the likelihood of being able to reproduce different parts of the count string in isolation of the entire sequence (Fuson et al., 1982). Noticing and using these repeating patterns likely also greatly reduces what children need to memorize. Other research indicates that preschool children are also noticing the structure of multidigit verbal and written numerals even though they have not linked them to specific quantities (Mix et al., 2017). It may be that children's repeating patterning knowledge helps them notice the underlying base-10 structure of multidigit verbal and written numerals.

We also proposed that repeating patterning knowledge might help children learn another foundational aspect of numeracy–the successor principle–or knowledge that the cardinality for a given count word is the cardinality of the previous count word plus one. Researchers have suggested that children may acquire this concept by detecting a pattern in the relation between the order of the count words and their set size (Carey, 2004; Cheung et al., 2017; Gelman & Gallistel, 1978; Sarnecka & Carey, 2008). However, we did not find evidence that repeating patterning knowledge was predictive of later successor principle knowledge. Learning this concept may require direct instruction, or exposure to conceptually aligned experiences, as opposed to indirect acquisition through exposure to repeating patterns (Baroody et al., 2019). Alternatively, a ceiling effect on our successor principle knowledge measure could have limited our ability to detect a relation to repeating patterning. Further, our study may have been underpowered to detect an effect after controlling for working memory and analyzing it as a categorical variable.

Future research should expand upon our proposed conceptual model to continue to explore how the development of numeracy knowledge might be supported by repeating patterning knowledge. This model is likely one of several potential theoretical models, and thus research using more sophisticated designs is needed to rule out possible alternatives and to refine and expand it. For example, patterning knowledge is concurrently and longitudinally related to verbal and non-symbolic calculation knowledge in preschool (Rittle-Johnson et al., 2017; Zippert et al., 2019), and Arabic numeral calculation knowledge in early primary school (Fyfe et al., 2017; MacKay & De Smedt, 2019). This may arise in part because noticing the pattern that the order of the addends does not impact the result (i.e., the commutative property) greatly reduces the number of calculation facts to learn. Similarly, noticing that adding one results in the next number in the count sequence simplifies calculation (Baroody et al., 2019). Further, repeating patterning knowledge is predictive of symbolic number knowledge, including symbolic magnitude comparison knowledge in preschool (Zippert et al., 2019) and the mapping of Arabic Digits and verbal number names to quantities in first grade (Rittle-Johnson et al., 2017). This may be in part because symbolic number knowledge requires understanding the rule that numbers appearing later in the count sequence are also larger in magnitude. More generally, additional research is needed to explore if and how repeating patterning knowledge supports understanding of whole numbers and whether it extends to supporting understanding of rational numbers. Given the underlying structure of our number system (Siegler & Lortie-Forgues, 2014), noticing patterns should support aspects of numerical development that are facilitated by helping us to create rules to describe patterns, and using those rules when working with numbers.

We also might expect the repeating pattern-numeracy link to be bidirectional. Numeracy knowledge might support performance on repeating pattern items by facilitating children's awareness of the number of items in a pattern unit, and the number of times the unit repeats. This knowledge allows for accurate pattern duplication and unit identification. Further, understanding that a number name or symbol can represent a specific set of objects (e.g., cardinality and mapping numerals to quantities via symbolic mapping) might allow children to apply abstract labels to describe repeating pattern items (describing a blue-blue-red-red block pattern as A-A-B-B or 1-1-2-2). Abstract labeling in particular has been shown to help preschool children detect the underlying structure of patterns and reproduce them with different materials (e.g., a green-green-yellow-yellow block pattern; Flynn et al., 2020; Fyfe et al., 2015). Preliminary empirical evidence supports this link. Specifically, non-symbolic quantity knowledge and verbal and object counting knowledge in preschool predict repeating patterning knowledge in first grade (Rittle-Johnson et al., 2017). However, more longitudinal and experimental work is needed to better understand the bidirectionality of the relations between repeating pattern and numeracy knowledge.

Implications for Education

Rather than removing repeating patterning from early math content standards, as was done in the Common Core State Standards (2010), the current study adds to growing evidence that we should encourage high-quality instruction on repeating patterning. Such instruction would address the Common Core Mathematical practice of "look for and make use of structure" especially for numbers. Improving, rather than reducing attention to repeating patterns in early education also makes sense because repeating patterning is popular among preschool children, their teachers, and their parents (Ginsburg et al., 2003; Rittle-Johnson et al., 2015; Zippert et al., 2020; Zippert & Rittle-Johnson, 2018).

However, there are concerns that teachers often do not attend to looking for and making use of structure in repeating patterns (Economopoulos, 1998). Moving beyond simply copying repeating patterns to having children extend patterns by a full pattern unit and abstracting patterns using new materials should help (Clements & Sarama, 2014). Talking about patterns, including asking children to explain their thinking (Rittle-Johnson et al., 2008) and referring to patterns using abstract labels (e.g., such as “this is an ABB pattern”; Fyfe et al., 2015), could help children learn about and use patterns. Emphasizing structure in repeating patterns could also help children to notice patterns in our verbal and written number systems.

Finally, our repeating patterning measure showed good evidence of reliability and validity. Specifically, the measure had strong internal consistency and showed both convergent (e.g., correlating with similar constructs such as math and numeracy knowledge and working memory), and construct validity (e.g., produced comparable item difficulties in relation to previously proposed construct maps with preschoolers; Rittle-Johnson et al., 2015). The quality of measurement instruments is particularly important as both researchers and teachers need access to valid and reliable tools to assess student learning (Purpura & Lonigan, 2015). Thus, the current findings allow us to recommend our teacher-friendly repeating patterning measure for use both in research and instructional contexts.

Conclusion

Repeating patterning knowledge at the end of preschool predicted children’s broad math and numeracy knowledge, including their ability to count to 100, at the end of Kindergarten. It remained a unique predictor of general numeracy knowledge and count to 100 after controlling for end-of-pre-K broad math knowledge. This predictive value was separable from that of verbal and visual-spatial working-memory. Theories of math development and early math instruction standards should thus give greater attention to the role of children's repeating patterning knowledge.

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Table 1

Descriptive Statistics for Items on Teacher-based Repeating Patterning Assessment at End of Pre-K

Item number, type, and pattern unit	Proportion correct (SD)	Item-total correlation	Item difficulty (SE)
9. Abstract AB	.74 (.44)	.52	-.65 (.29)
1. What's Next AB	.71 (.45)	.65	-.51 (.29)
6. Extend AB	.70 (.46)	.78	-.44 (.28)
3. Missing AB	.68 (.46)	.73	-.37 (.28)
5. Missing ABB	.59 (.49)	.43	.09 (.28)
7. Extend AABB	.58 (.49)	.83	.15 (.28)
10. Abstract ABBB	.56 (.50)	.62	.22 (.28)
2. What's Next ABC	.49 (.50)	.66	.53 (.27)
4. Missing ABC	.48 (.50)	.51	.59 (.27)
8. Extend ABC	.47 (.49)	.64	.65 (.27)

Note. Items are listed in order of observed difficulty, and item number indicates order in which item was given. Negative item difficulty values indicate easier items.

Table 2

Descriptive Statistics, Correlations of End-of-Pre-K WM and Patterning Knowledge and End of Kindergarten Math and Numeracy

Variables	Raw Score	Correlations						
	M(SD)	1	2	3	4	5	6	7
1. Age at end of kindergarten	6.14(.29)	---	.48*	.40*	.48*	.44*	.45*	.40*
2. Verbal WM	1.60(1.51)		---	.69*	.92*	.70*	.55*	.59*
3. Visual-spatial WM	2.83(2.05)		.62*	---	.92*	.69*	.46*	.41*
4. WM composite[a]	.02(.91)		.89*	.91*	---	.76*	.55*	.54*
5. Patterning	-.06(1.39)		.63*	.62*	.69*	---	.57*	.58*
6. Math	1.41(.83)		.42*	.34*	.42*	.47*	---	.87*
7. Numeracy	2.80(1.46)		.49*	.30*	.43*	.49*	.84*	---

Notes. Values above the diagonal are raw correlations ($df = 63$). Values below the diagonal are partial correlations after controlling for age (in months) at end of kindergarten ($df = 62$). Working Memory (WM) measures and patterning knowledge were assessed at the end of pre-K. [a]WM composite represents averaged standardized scores on the verbal and visual-spatial WM tasks. $*p < .05$.

Table 3

Linear Regression Predicting End-of-Kindergarten Broad Math and General Numeracy Skills

Model Variables	*B*(*SE*)	β	*t*	R^2
Broad Math				
Step 1[a]				.34
Age	.68(.33)	.24	2.05*	.04
WM	.40(.11)	.43	3.69**	.14
Step 2[b]				.39
Age	.59(.33)	.21	1.81†	.03
WM	.18(.14)	.20	1.27	.02
Patterning	.20(.09)	.33	2.13*	.05
Step 3[c]				.46
Age	.33(.32)	.12	1.02	.01
WM	.02(.15)	.02	.15	.00
Patterning	.16(.09)	.26	1.79†	.03
Broad Math (end of pre-K)	.35(.12)	.40	2.89**	.08
General Numeracy				
Step 1[a]				.32
Age	.89(.60)	.18	1.48	.02
WM	.73(.19)	.45	3.79**	.16
Step 2[b]				.37
Age	.71(.59)	.14	1.21	.02
WM	.31(.26)	.20	1.22	.02
Patterning	.39(.17)	.37	2.34*	.06
Step 3[c]				.45
Age	.23(.58)	.05	.40	.00
WM	.02(.26)	.01	.08	.00
Patterning	.32(.16)	.30	2.01*	.04
Broad Math (end of pre-K)	.63(.21)	.41	2.95**	.08

Note. Age represents children's age in months at the end of Kindergarten. Working memory (WM) represents a composite of standardized scores on the visual-spatial and verbal WM tasks. WM and patterning knowledge were assessed at the end of pre-K. R^2 values for each predictor variable represent squared semipartial correlations. [a]df = (2, 62). [b]df= (3, 61). [c]df= (4, 60). †p< .10. *p< .05. **p< .01

Table 4

Logistic Regressions Predicting Mastery of Specific Numeracy Skills at the End of Kindergarten

Model Variables	*B*(*SE*)	Odds Ratio	*Wald's* χ^2	*Nagelkerke* R^2
Count to 100 Mastery				
Step 1[a]				.19
Age	-.37(1.23)	.69	.09	.02
WM	1.16(.45)	3.20	6.56*	.17
Step 2[b]				.39
Age	-2.05(1.63)	.13	1.58	.02
WM	-.19(.67)	.83	.08	.17
Patterning	1.45(.55)	4.27	6.87**	.20
Step 3[c]				.39
Age	-1.64(1.73)	.19	.90	.02
WM	.03(.73)	1.03	.00	.17
Patterning	1.58(.59)	4.86	7.08**	.20
Broad Math (end of pre-K)	-.53(.73)	.59	.53	.00
Successor Principle Mastery				
Step 1[a]				.03
Age	-.21(1.01)	.81	.05	.00
WM	.34(.33)	1.41	1.11	.03
Step 2[b]				.03
Age	-.30(1.02)	.74	.08	.00
WM	.18(.45)	1.20	.16	.03
Patterning	.15(.29)	1.16	.28	.00
Step 3[c]				.04
Age	-.49(1.08)	.62	.20	.00
WM	.08(.49)	1.08	.03	.03
Patterning	.13(.29)	1.13	.18	.00
Broad Math (end of pre-K)	.24(.40)	1.27	.36	.01

Note. Age represents children's age in months at the end of kindergarten. Working memory (WM) represents a composite of standardized scores on the visual-spatial and verbal WM tasks. WM and patterning knowledge were assessed at the end of pre-K. *Nagelkerke* R^2 values for each predictor variable represent the difference between the model R^2 for each step when each variable is included and excluded from the model.

[a]$df = (2, 62)$. [b]$df = (3, 61)$. [c]$df = (4, 60)$.

* $p < .05$. **$p < .01$.

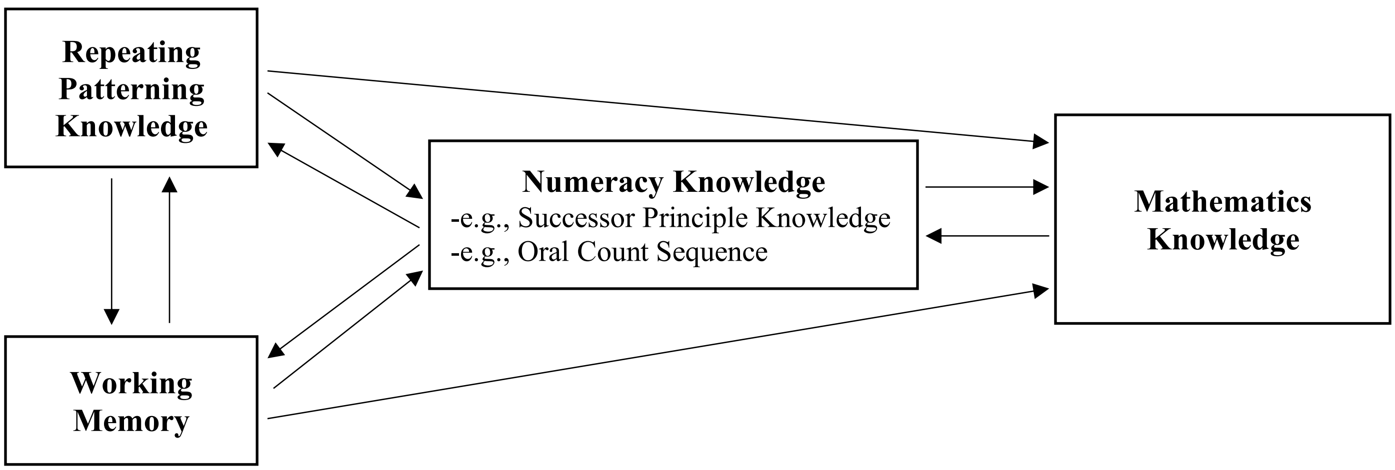
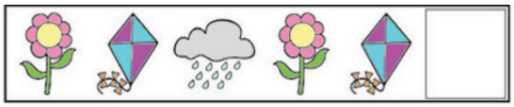


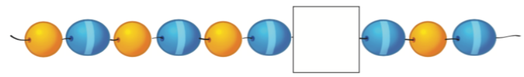
Figure 1. Conceptual model of hypothesized pathways through which repeating pattern knowledge and working memory at the end of preK support early numeracy and mathematics knowledge in kindergarten.

What Comes Next Pattern ABC



"What comes next in the pattern? Use one of these." [Experimenter gestures to picture cutout response options below.]

Missing Item Pattern AB



"Find the missing bead [experimenter gestures to picture cutout response options below] to complete the pattern [experimenter gestures across pattern]."

Extend Pattern ABC

"Can you complete the pattern?" [Experimenter gestures to circles on the right of the pattern.]

Figure 2. Sample items from the teacher-based patterning assessment. From "Not Just IQ: Patterning Predicts Preschoolers' Math Knowledge Beyond Fluid Reasoning," by E. Zippert, C. Clayback, & B. Rittle-Johnson, 2019, *Journal of Cognition and Development, 20,* p. 8. Copyright 2019 by Taylor & Francis Group, LLC. Reprinted with permission.

Supplemental Material

Table S1

Participants' Broad Math and General Numeracy Skills by their Race, Gender and Financial Assistance

Demographic	Broad Math			General Numeracy			Count to 100					Successor Principle				
	n	*M(SD)*	t^a	*n*	*M(SD)*	t^a	*n*	%	*n*	%	χ^{2b}	*n*	%	*n*	%	χ^{2b}
Race																
White	25	1.69(.66)	2.20*	25	3.45(1.18)	3.04*	18	72.0	7	28.0	.25	9	36.0	16	64.0	.51
Non-White	40	1.23(.88)		40	2.39(1.48)		31	77.5	9	22.5		18	45.0	22	55.0	
Gender																
Female	33	1.49(.81)	.79	33	2.78(1.35)	-.12	25	75.8	8	24.2	.01	22	66.7	11	33.3	1.86
Male	32	1.32(.85)		32	2.82(1.59)		24	75.0	8	25.0		16	50.0	16	50.0	
Financial Assistance																
None	29	1.62(.65)	1.91	29	3.11(1.30)	1.56	22	75.9	7	24.1	.01	18	62.1	11	37.9	.28
Some or full	36	1.23(.93)		36	2.55(1.55)		27	75.0	9	25.0		20	52.6	16	62.1	

Notes. $n = 65$.
[a] $df = 63$. [b] $df = (1)$.
* $p < .05$