Patterning Knowledge is Foundational for Mathematics Achievement

Bethany Rittle-Johnson

Emily Fyfe

Erica Zippert

EARLI 2017, Tampere, Finland

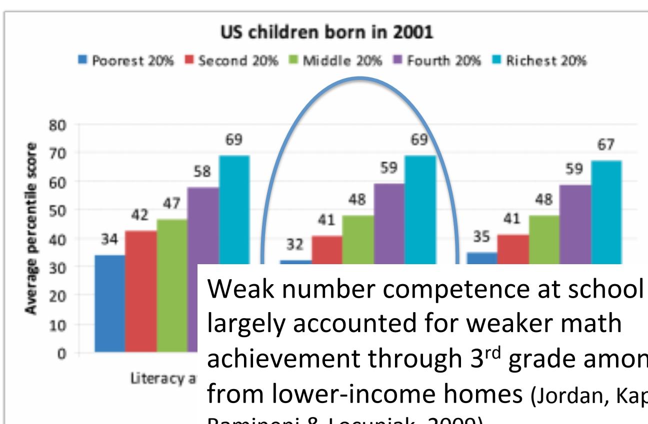
Prior Knowledge & Mathematics Achievement

Prior knowledge of math at school entry predicts later math achievement across primary and secondary school (Duncan et al., 2007; Jordan et al., 2009; Watts et al., 2014).

Duncan et al., 2007

Independent variable	Math
School-entry measure	
Reading	.10*** (.02)
Math	.42*** (.04)
Attention skills	.11*** (.02)
Externalizing problems	.01 (.01)
Internalizing problems[a]	—
Social skills	−.01 (.01)

Prior Knowledge Varies by Family Income

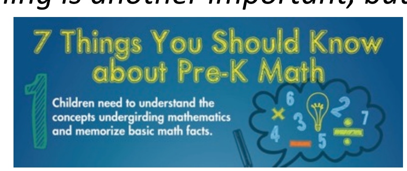


Weak number competence at school entry largely accounted for weaker math achievement through 3rd grade among children from lower-income homes (Jordan, Kaplan, Ramineni & Locuniak, 2009).

Waldfogel and Washbrook (2008) based on 10,000 children born in the United States in 2001

More than Amount of Knowledge: Types of Prior Math Knowledge

- Need to understand *types* of prior knowledge that are particularly important
  - Numeracy knowledge receives the most attention.
  - Proficiency in mathematics requires developing knowledge of multiple topics and their interrelations (National Research Council, 2009).
  - *Patterning is another important, but understudied, topic*



***Pattern**: a predictable sequence*

E.g., alternating sequence of shapes or sounds

E.g., functional relationships between two variables

Why patterning knowledge?

- Young children spontaneously engage in patterning activities (Ginsburg, Inoue, & Seo, 1999; Ginsburg, Lin, Ness, & Seo, 2003).
- Parents and preschool teachers report engaging children in patterning activities many times a week (Rittle-Johnson, Fyfe, Loehr & Miller, 2014)
- Patterning is a core skill for mathematical thinking (Charles, 2005; Sarama & Clements, 2004; Steen, 1988).
- Knowledge of patterns is included as a central algebraic topic in consensus documents in mathematics education (NCTM, 2000; NAEYC, 2014).
- Instruction on repeating patterns in preschool supported better numeracy and patterning knowledge in kindergarten (Papic et al., 2011)

NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

naeyc Accredited

Why *not* pattern knowledge?

- The (U.S.) National Mathematics Advisory Panel (2008) concluded: "In the Major Topics of School Algebra set forth in this report, **patterns are not a topic of major importance**. The prominence given to patterns in PreK–8 is not supported by comparative analyses of curricula or mathematical considerations" (p. 59).
  - Justification: Only one of the six highest performing countries on an international assessment emphasized patterns in the early grades (Schmidt & Houang, 2007).
  - Paucity of evidence available at the time of the report.
  - U.S. Common Core State Standards followed this recommendation, dropping patterning as a content standard.

Goal Today

- Briefly review development of early patterning knowledge
- Present evidence for unique importance of early patterning knowledge for mathematics achievement
  - Study 1: Concurrent relations in preschool
  - Study 2: Longitudinal evidence that prior patterning knowledge is a unique predictor of mathematics achievement at age 12

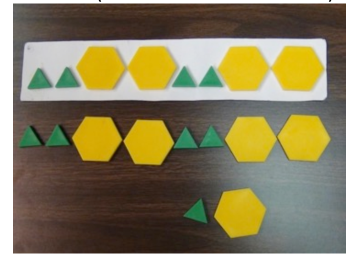
Development of Pattern Knowledge

- Age 3: Begins to develop. Children notice and fill in simple alternating AB patterns (e.g., black and white striped shirt).
- Ages 4-7: Expanding variety of patterning skills with variety of pattern units.
  - Focus on repeating patterns: a pattern unit that repeats over and over (e.g., ABBABB)

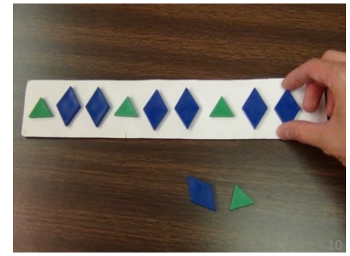
(Clements & Sarama, 2009)

Easier Pattern Skills

(Clements & Sarama 2009)



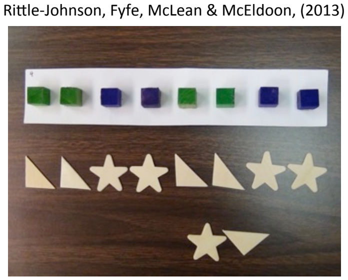
Duplicating
- Making an exact copy of a model pattern



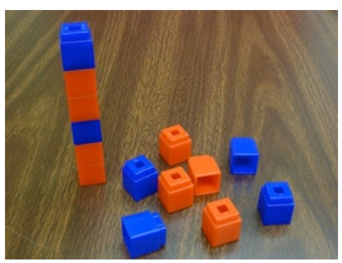
Extending
- Continuing a model pattern
  - Not just what item comes next - require extending *by one pattern unit*

Advanced Pattern Skills

(Clements & Sarama 2009; Rittle-Johnson, Fyfe, Loehr & Miller, 2015; Rittle-Johnson, Fyfe, McLean & McEldoon, (2013)



Abstracting
- Recreating a model pattern using different materials
  - E.g., "Please make the same kind of pattern here, using these shapes"



Pattern Unit Recognition
- Identify the unit of repeat in reference to a model pattern
  - E.g. ""What is the smallest tower you could make and still keep the same pattern as this?"

Study 1: Concurrent Relations in Preschool

- 77 American preschoolers, ages 4.0 to 5.2 months (mean = 4.5 yrs)
  - Sex: 37 boys and 40 girls
  - 53% Ethnic minorities
  - 35% from low-income homes
- Design
  - Individually assessed range of skills at beginning of school year
    - Math knowledge
    - Patterning knowledge (2 measures)
    - Cognitive controls

Rittle-Johnson & Zippert, under review

ies INSTITUTE OF EDUCATION SCIENCES

Math Knowledge Assessment

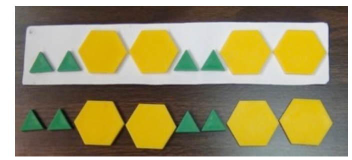
Research-Based Early Mathematics Assessment (REMA) – Short Form (Weiland, Wolfe, Hurwitz, Clements, Sarama & Yoshikawa 2012).

Math Topic	Sample Item
Non-symbolic Quantity	Shown two cards, with 4 dots and 3 dots: 'Which one has more?'
Counting	Shown 4 objects in a line: "'I'm going to show you some food boxes. Please tell me how many I have."
Symbolic Mapping	Match the numerals 1-5 to the appropriate number of grapes.
Shape	Select all triangles from a collection of 24 shapes; some are prototypic shapes and some are not.

Research-Based Patterning Assessment

Assessed pattern knowledge using 3D shapes

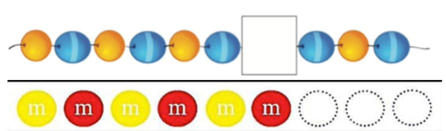
1. Four tasks
    1. Duplicating – make exact copy
    2. Extending – identify what comes next
    3. Abstracting – recreate pattern using new materials
    4. Pattern Unit Recognition – identify unit of repeat
2. Pattern Units: ABB, AAB, AABB



(Clements & Sarama, 2009; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013; Rittle-Johnson, Fyfe, Loehr & Miller, 2015)

New Teacher-Based Patterning Assessment

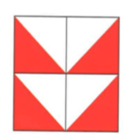
- Assessed pattern knowledge using worksheets with paper-cut outs
- Based on materials available for teachers on the internet
- Four tasks:
    - What comes next?
    - Filling-in missing item
    - Extending
    - Matching

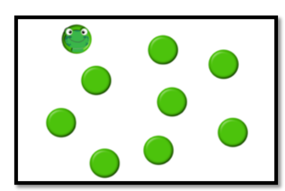


- Internal consistency: Alpha = .83
- Relation to research-based patterning: *r*(76) = .59
- Created composite Patterning measure

Cognitive Controls

- Spatial skills composite: spatial visualization, visual-spatial working memory & form perception
- Verbal working memory (WM): backward digit span
- Verbal ability: vocabulary





Results: Predicting Concurrent Math Knowledge

Variable	*B*	β	t
Pattern composite	.37(.11)**	.35	3.25
Spatial composite	.26(.12)*	.23	2.22
Verbal WM	.16(.07)*	.27	2.42
Verbal Ability	.01(.01)	.09	0.99
Age	.00(.28)	.00	-0.01

Notes. Standard errors are in parentheses. $*p < .05$. $**p < .01$.

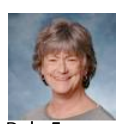
Bonus: Similar results when predicting math knowledge at end of school year

Study 2: Longitudinal Evidence

- Research question: Are patterning skills in preschool and first-grade unique predictors of middle-grades mathematics achievement?

Study 2 Method

- 513 students
  - Originally recruited in preschool
  - All from low income homes; predominantly black (79%)
  - Focus on knowledge and skills at age 5 (end of Pre-Kindergarten year) and age 7 (end of First grade)
- Follow-up children in middle school
  - Average age = 12.1 year
    - 83% finishing Grade 6
    - 17% retained a grade level, so finishing Grade 5



Dale Farran

HEISING - SIMONS FOUNDATION

ies INSTITUTE OF EDUCATION SCIENCES

PRI Middle School Follow Up Project

PEABODY research institute

Study 2 Measures

- Early predictors from pre-k and first-grade
  - **Comprehensive Research-Based Early Mathematics Assessment** (REMA; Clements, Sarama, & Liu, 2008)
    - Broken into 6 math subscales, including ***patterning*** (similar to our research-based patterning measure)
  - **4 general and cognitive skills**, including verbal & reading skills, ratings of self-regulation
- 4 Math outcomes measured in middle-grades
  - KeyMath 3 Diagnostic Assessment subtests:
    - Numeration
    - Algebra
    - Geometry
  - Quantitative concepts subtest from WJ-III

Results: Correlations

	Correlation with Patterning Knowledge	
Math Outcome	End of pre-k M Age = 5.0	End of first grade M Age = 7.0
Numeration	.51	.39
Algebra	.47	.35
Geometry	.49	.42
Quant. Concepts	.42	.33
Math Composite	.53	.42

Similar across outcomes, so used composite measure

Results: Regression Models for Math Composite

Measure	End of pre-k M Age = 5.0	End of first grade M Age = 7.0
Math Predictors		
Patterning	**.23*** **	**.09** **
Nonsymbolic		
Counting		
Symbolic Mapping		
Shape		
Calculation		
Reading		
Narrative Recall		
Work-Related Skills		
Self-regulation		

Results: Regression Models for Math Composite

Measure	End of pre-k M Age = 5.0	End of first grade M Age = 7.0
Math Predictors		
Patterning	**.23*** **	**.09** **
Nonsymbolic	.21***	.04
Counting	.00	.00
Symbolic Mapping	.11	.19***
Shape	-.02	.03
Calculation	--	.27***
Reading	.10*	.14***
Narrative Recall	.16***	.06
Work-Related Skills	.04	.24***
Self-regulation	.03	-.04

Study 2 Summary

- Patterning knowledge at the end of preschool (age 5.0) and end of first grade (age 7.0) was a consistent, unique predictor of mathematics achievement in 6th grade.
  - True for 5th grade achievement as well


CHILD DEVELOPMENT
Explore this journal >

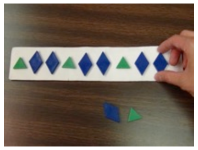
Empirical Article

Early Math Trajectories: Low-Income Children's Mathematics Knowledge From Ages 4 to 11

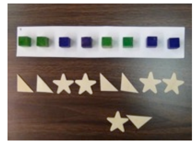
Bethany Rittle-Johnson, Emily R. Fyfe, Kerry G. Hofer, Dale C. Farran

First published: 6 December 2016 Full publication history

DOI: 10.1111/cdev.12662 View/save citation



Conclusion



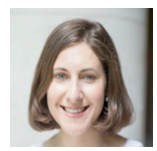
- Early patterning knowledge is a unique predictor of math knowledge, both concurrently and 5-7 years later, on a variety of math outcomes and with children from diverse backgrounds.
- Patterning knowledge should receives more attention in the research literature and policy documents.

Patterning & Math

- Deducing underlying rules core to patterning and all of math
  - E.g., Successor principle for symbol-quantity mappings (e.g., the next number name means adding one)….”
- Empirically linked to
  - *Symbolic magnitude knowledge* (e.g., Which is more? 5 or 9)
  - *Calculation knowledge* (How much is 8 plus 3?)
    - (Papic et al., 2011; Rittle-Johnson et al., 2016)
- Overall, theories of mathematics development need to integrate the role of early pattern knowledge on future mathematics learning.

Educational Implications

- We can improve children’s patterning knowledge (e.g., Papic et al., 2011)
  - Young children are paying attention to structure in the world – build on this!



Acknowledgements

Children’s Learning Lab

Peabody Research Institute

Dale Farran, PI & Kerry Hofer, co-PI

Kayla Polk, Carol Bilbrey and Dana True

Funding Sources

IES grant R305A160132 to Rittle-Johnson

IES grant R305A140126 and R305K050157 to Farran

Heising-Simons Foundation #2013-26 to Farran

ies INSTITUTE OF EDUCATION SCIENCES

HEISING - SIMONS FOUNDATION

SUPPLEMENTAL SLIDES

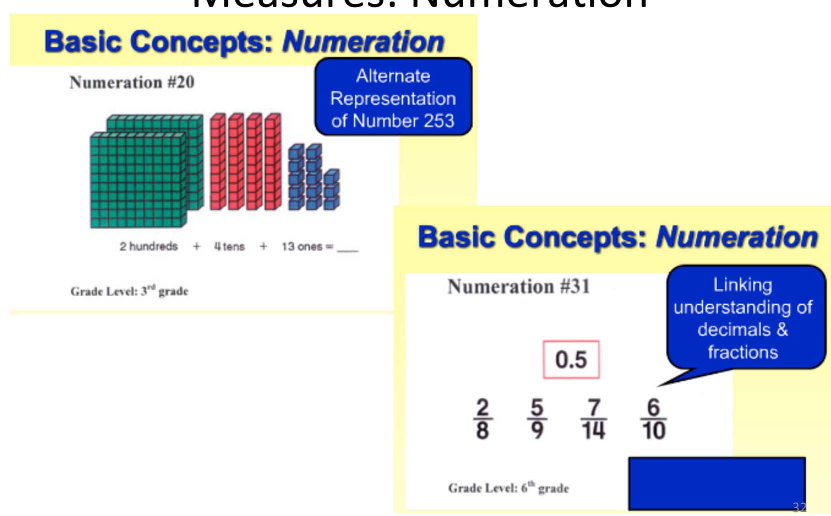
Study 2 Early Math Subscales and Sample Item

Math Topic	Sample Item
Patterning	Duplicate an AABB pattern
Non-symbolic Quantity	Shown two cards, with 4 dots and 3 dots: 'Which one has more?'
Counting	Shown 5 objects in a line: "'I bought these cans of food. Count these cans to tell me how many there are.
Symbolic Mapping	Match the numerals 1-5 to the appropriate number of grapes.
Calculation	"Here are 6 pennies. Three more are hidden under the cloth. How many are there in all?"
Shape	Select all triangles from a collection of 24 shapes; some are prototypic shapes and some are not.

Study 2 General Cognitive Skills

- *Narrative recall* skills
  - Used Renfrew Bus Story or Woodcock Johnson III Story recall.
  - Measures vocabulary, verbal IQ and working memory capacity.
- *Reading* skills
  - Used WJ Letter-Word Identification
- *Worked-related skills*
  - Used teacher rating from Cooper-Farran
- *Self-regulation skills*
  - Used teacher rating on Instrumental Competence Scale for Young Children-Short Form.

Study 2: Key Math Outcome Measures: Numeration



Key Math Algebra