16 is One More Than 15: The Role of the Successor Principle in Building Mathematics Knowledge

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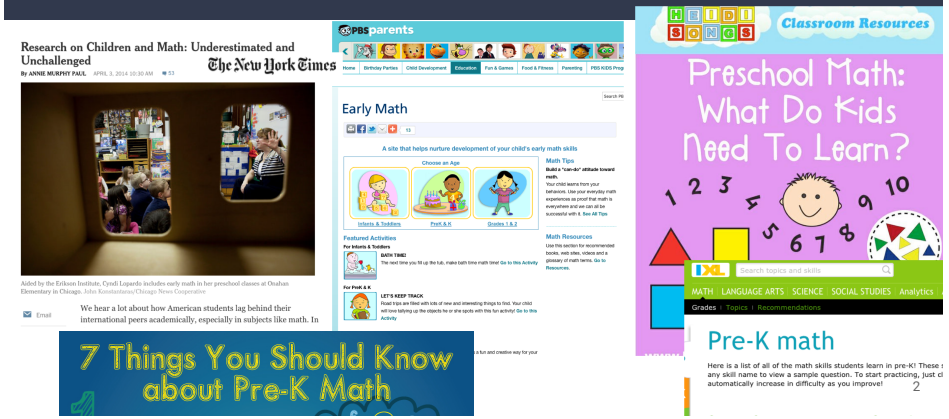
Focus on Early Math Knowledge



Individual Differences in Early Math Knowledge Predict Later Math Achievement

Math knowledge before or shortly after school entry predicts later mathematics and reading achievement across primary and secondary school (Duncan et al., 2007; Jordan et al., 2009; Watts et al., 2014).

- Focus on early number knowledge today

Developing Number Knowledge: The Successor Principle

Foundational aspects of number knowledge:

Cardinality principle – The last word in a count indicates the set size (cardinality)

Successor principle - Cardinality for each count word is cardinality of previous count word plus one.

- Successor principle considered a key conceptual insight about counting, integers and arithmetic (Gelman & Gallistel, 1978; Sarnecka & Carey, 2008).
- Unit task by Sarnecka & Carey (2008) "I'm putting FOUR frogs in here. <hide> Now watch <add one more>. Now is it FIVE or SIX?"
- Children understand the successor principle much later than cardinality principle (Cheung, Rubenson & Barner, 2017).

Current Study Motivation

- Little is known about individual differences in understanding the successor principle
  - Across a wide range of numbers
  - Both forward and backward (adding vs. subtracting one)
  - In children from diverse backgrounds
  - Nor how these individual differences relate to math knowledge beyond counting
  - Current study addresses these gaps.
- Little is known about effective methods for improving children's successor principle knowledge

Goals

1. Characterize successor principle knowledge in two age groups for children from diverse backgrounds and how it relates to their number and general math knowledge.
   1. Study 1: Near the end of Kindergarten (ages 5-6)
   2. Study 2: In final year of preschool (ages 4-5)
2. Create and evaluate a successor principle training for preschool children.

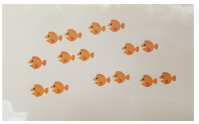
Study 1 Method

- **Participants: 65 Kindergarten students (M age = 6.14 years, SD= .29)**
  - 39% white, 46% Black, 14% bilingual, 49% male
  - 56% had received some to full financial assistance to attend preschool (through public pre-K programs)
- **Procedure: Near end of Kindergarten, assessed:**
  - Successor Principle knowledge
  - Math knowledge, including general numeracy knowledge and ability to count to 100
  - General cognitive skills
    - Visual-spatial working memory: eCorsi Block Tapping task
    - Verbal ability: Receptive vocabulary

Successor Principle Measure

Successor Principle: Fish Pond Task

- Assess children's knowledge of the successor principle using numbers ranging from 15 to 116 with 10 items. Based on Cheung et al. (2017)
  - N = 15, 20, 34, 46, 51, 62, 73, 95, 107, 116



"Fifteen fish are swimming under the lilypad. Now watch… another fish swims in! Now are there 16 or 17 fish?"

Math Knowledge

Research-Based Early Math Assessment (REMA)

Brief version (Weiland et al., 2012)

- 19 items: Broad math knowledge
  - 13 items assessed numeracy knowledge and 6 items assessed shape or patterning knowledge.

We need to match these labels to the right group of grapes.

Please match the numbers to the grapes.

I'm going to ask you which number is *smaller*. Which is smaller: 27 or 32?

Oral Count to 100

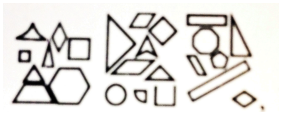
- Assess children's knowledge of the oral count sequence to 100

"How high can you count? Start at 1 and tell me… "

When stop: "What comes next? Can you go higher?"

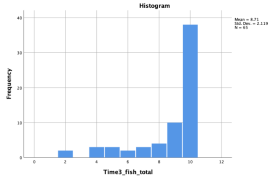
- Because 63% of students were at ceiling, scored as count to 100 or not.

Place chips on top of all the shapes that are *triangles*. Place chips only on the *triangles*.



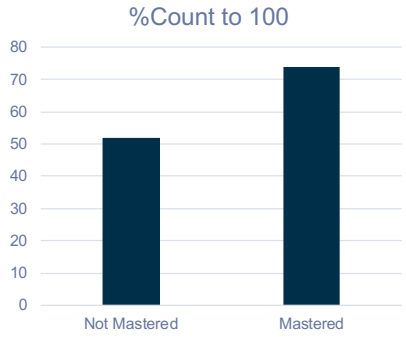
Study 1 Results: Successor Principle Knowledge at End of Kindergarten

- Accuracy was high on all items, ranging from .80 - .94.
- Average accuracy was .87.
  - 59% of children were successful on all 10 items.
  - Classified children as mastering successor principle or not.



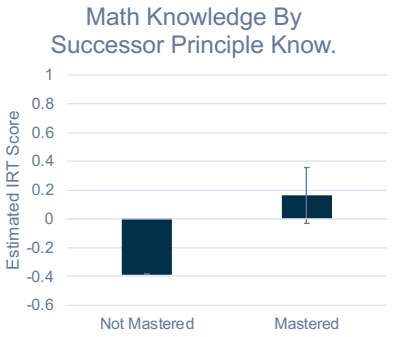
Starting Number	Proportion correct (SD)
15	.94 (.24)
20	.85 (.36)
34	.88 (.33)
46	.88 (.33)
51	.94 (.24)
62	.80 (.40)
73	.85 (.36)
95	.83 (.38)
107	.92 (.27)
116	.83 (.38)

Study 1 Results: Relation to Count Knowledge



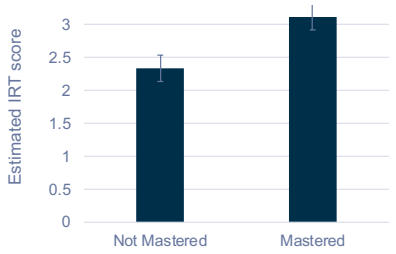
- Children who had mastered the successor principle were also more likely to be able to count to 100.

Study 1 Results: Relations to Math Knowledge at end of Kindergarten



F (1, 61) = 4.39, p = .04

Successor principle accuracy was strongly related to concurrent general numeracy (*r (59)* = .45) and to general math knowledge (r (59) = .29), controlling for age and general cognitive skills.



F (1, 61) = 5.49, p = .02

Study 1 Summary

- By the end of Kindergarten, many children from diverse backgrounds had mastered the successor principle.
- Mastering the successor principle by the end of Kindergarten was related to concurrent math and numeracy knowledge.

Study 2 Method

Participants:

- 212 children in last year of preschool (M age = 4.7 years, SD=.37)
- Recruited from 12 preschools (5 public, 7 private)
- 53% white, 29% Black, 10% bilingual, 56% male
- 35% received some to full financial assistance to attend preschool

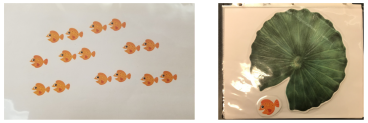
Measures:

- Successor Principle knowledge
- Math knowledge, including
  - General math and numeracy knowledge (using REMA brief)
  - Ability to count to 50 ("How high can you count? Start at 1 and tell me… "). Highest correct oral count word

Study 2 Successor Principle Measure

Successor Principle: Revised Fish Pond Task

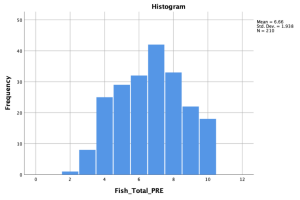
- 10 items with numbers ranging from 2 to 34. Modified from Cheung et al. (2017)
  - Seven +1 items, with N = 6, 3, 7, 15, 12, 34, 20
  - Three -1 items, with N = 2, 4, 5



"Fifteen fish are swimming under the lilypad. Now watch… another fish swims in! Now are there 16 or 17 fish?"

Study 2 Results: Successor Principle Knowledge Pre-K

- Performance was above chance for every number but 12,
- Average accuracy of 67%, with normal distribution of scores.



Starting Number & Direction	Proportion correct (SD)
6 Add	.70 (.46)
3 Add	.74 (.44)
7 Add	.59 (.49)
15 Add	.64 (.48)
12 Add	.38 (.49)
34 Add	.61 (.49)
20 Add	.61 (.49)
2 Subtract	.89 (.32)
4 Subtract	.83 (.37)
5 Subtract	.67 (.47)

Study 2 Results: Relation to Math knowledge in Pre-K

- Successor principle knowledge was strongly related to concurrent:
  - General math knowledge (*r* (207) =.48)*
  - General numeracy knowledge (*r* (207) = .43)*
  - Highest oral count word (*r* (205) = .50)*

*controlling for age

- Summary: Children are linking forward and backward count sequence to adding one and subtracting one. Substantial individual differences are related to children's general numeracy and broad math knowledge.

Part 2: Improving Children's Successor Principle Knowledge

- Research question***: How can we improve children's Successor Principle knowledge?***
  - Study 2 children randomly assigned to receive 5 training sessions that included activities focused on the successor principle (plus other content) or to receive no training (only regular classroom activities).

Our Successor Principle Training

Session	
1	Add 1 (set sizes 1-5)
2	Add 1 (set sizes 1-5)
3	Subtract 1 (set sizes 1-5)
4	Add 1 (set sizes 1-10)
5	Add and subtract 1 (set sizes 1-10)

Training based on a research-based early math curriculum

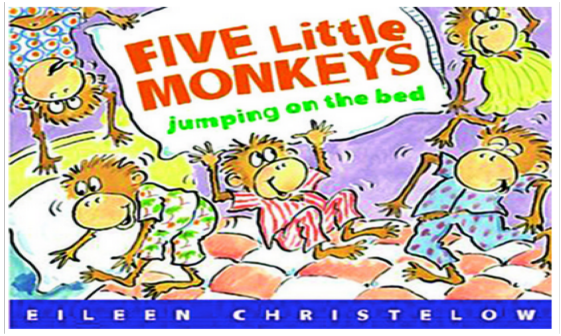
Study 2 Intervention: Hiding Bug Game

From Session 1

Study 2 Intervention: Cube stairs

From *Building Blocks Pre-K Math Curriculum*

Study 2 Intervention: Subtracting 1



Practice predicting how many monkeys are jumping on the bed each time one falls off
Finger play provides visual support.

Training Result

- No improvement pretest to posttest in successor principle knowledge scores, regardless of condition (e.g., M = 67% to 68% correct pretest to posttest)
  - Improvement for adding one to small set sizes, but similar improvement in control condition receiving no training from us.
  - If exclude children with very low number knowledge at pretest, still no effect of condition.
- Summary: Need to identify effective ways to improve successor principle knowledge.

Part 2: Improving Children's Successor Principle Knowledge

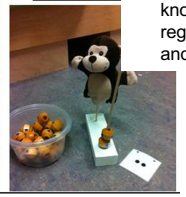
- One previous attempt: Spaepen, Gunderson, Gibson, Goldin-Meadow & Levine (2018)
  - 3-5 year old children
  - 4 one-on-one training sessions with set sizes 1-6

Spaepen et al (2018) Training Study

- Experimental condition focused on adding one more to increase set size from 1 to 6, with explicit instruction that the next number is one more than the previous.
  - "She had one wug and we gave her one more, so she would have two wugs. Now she has two wugs."
- Control condition practiced counting sets 1-6, with feedback
  - No condition difference for successor principle knowledge (up to set size 8) from pretest to posttest.
  - Cardinal-principle-knowers improved regardless of condition and 3-4-knowers did not.

Experimental materials

Control mate



Discussion

- Successor Principle Knowledge
  - Many children master it even for numbers above 99 by the end of Kindergarten.
  - Substantial individual differences in final year of preschool. Children are linking forward and backward count sequence to adding one and subtracting one.
- Individual differences in successor principle knowledge are related to children's general numeracy and broad math knowledge.
  - Supports the theory that successor principle knowledge is core to developing math knowledge (Sarnecka & Carey, 2008; Cheung, et al., 2017).

Future Directions

- Need longitudinal evidence.
- Need to develop effective instructional methods for improving successor principle knowledge. Our training on the successor principle was not successful (Study 2), nor was a past effort (Spaepen et al., 2018).
  - Mastery of cardinality principle may be necessary.
  - Educationally important. Also necessary to test for causal relations between successor principle knowledge and broader math knowledge.

Acknowledgements

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Count Sequence Knowledge: What's Next/Before Task

- 6 items taken from Spaepen, et al (2018)
- Given a number and asked what comes right after (N = 6, 3 and 7).
- Then given a number and asked what comes right before (N = 2, 4 and 5).
- Scored as # of items correct out of 6

***subsample of children (*n*= 56) received this additional measure**