The Role of Store Brand Spillover in a Retailer's Category Management Strategy

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A Store Brand Spillover Example with Two Categories

In this section, we consider an alternative setting with two retail categories to provide additional justification for the retailer's objective function we study in the main body of the paper (equation 2). In this alternative setting, the category we examine in the main body of the paper, hereafter referred as category A, represents the focal category. Category B is the second category. For notational parsimony, we assume that category B includes only one product – a store brand product.

We capture the store brand's cross-category spillover as follows. Let λ_A and λ_B denote the number of consumers who shop in categories A and B, respectively. Following the notation we use in the main body of paper, z_s and $\lambda_A z_s$ denote the SB's market share and the number of customers who purchase the SB in category A, respectively. Let $\delta \in [0,1]$ denote the spillover parameter, which captures the notion that a portion of SB shoppers in category A decide to visit category B. Mathematically, $\delta\lambda_A z_s$ captures the additional traffic the SB offered in category A generates for category B. That is, increasing the SB market share in category A creates additional demand for category B. The retailer's total profit (i.e., the sum of profits from categories A and B) can be written as

$$\Gamma_R^{\mathcal{A}} = \underbrace{\lambda_A \sum_{i\in\mathcal{A}} (p_i - w_i)z_i - K\,|\mathcal{A}|}_{\text{profit from category } A} + \underbrace{(\delta\lambda_A z_s + \lambda_B)\,(p_x - w_x)z_x - K}_{\text{profit from category } B}, \tag{28}$$

where p_x, w_x, and z_x denote the SB's retail price, wholesale price, and market share in category B, respectively. Because we assume that category B has one product, the fixed cost for category B equals K. Moreover, we assume that $w_x = 0$ to be consistent with the assumption we make in the main body of the paper that $w_s = 0$.

Let $\theta_B \sim U[0,1]$ denote consumers' quality preference parameter in category B. Furthermore, let q_x denote the SB product's quality in category B. Then $z_x = Pr\left(\theta_B q_x - p_x \geq 0\right) = 1 - \frac{p_x}{q_x}$. Accordingly, category B's category manager solves $\max_{p_x\geq 0} \left(\delta\lambda_A z_s + \lambda_B\right) p_x \left(1 - \frac{p_x}{q_x}\right) - K$. Hence, the optimal retail price for the SB product in category B equals $p_x^* = \frac{q_x}{2}$. Plugging this optimal retail price into (28) and rearranging terms yield

$$\Gamma_R^{\mathcal{A}} = \lambda_A \left(\sum_{i \in \mathcal{A}} (p_i - w_i) z_i - K\,|\mathcal{A}| + \delta \frac{q_x}{4} z_s \right) + \lambda_B \frac{q_x}{4} - K. \tag{29}$$

Recall from Equation (2) that the retailer's objective function in the main body of the paper is

$$\Pi_R^{\mathcal{A}} = \sum_{i \in \mathcal{A}} (p_i - w_i) z_i - K\,|\mathcal{A}| + \gamma z_s. \tag{30}$$

Setting $\gamma = \delta \frac{q_x}{4}$ reveals that (29) can be written as

$$\Gamma_R^{\mathcal{A}} = \lambda_A \Pi_R^{\mathcal{A}} + \lambda_B \frac{q_x}{4} - K. \tag{31}$$

That is, (29), which we obtain from a two-category setting with SB spillover, is an affine function of the retailer's objective function we consider in the main body of the paper. Consequently, both settings lead to the same equilibrium outcomes and insights. In the main body of the paper, we analyze $\Pi_R^{\mathcal{A}}$ (rather than $\Gamma_R^{\mathcal{A}}$ or another setting with multiple categories) for expositional clarity.

B Robustness Tests

In this section, we test the robustness of our managerial insights by relaxing some of our modeling assumptions.

B.1 Production Costs and Quality Differentials

Our model described in Section 3 relies on two simplifying assumptions regarding production costs and quality differentials. First, we assume that each product has zero production cost. Second, we assume equal quality differentials, $q_1 - q_2 = q_2 - q_s = \alpha$. In this section, we relax these two assumptions to demonstrate the robustness of our managerial insights.

We start our analysis with general quality expressions (i.e., q_1, q_2, and q_s). Furthermore, we let $c_1(q_1)$, $c_2(q_2)$, and $c_s(q_s)$ denote the production costs of the high-quality NB, the low-quality NB, and the SB, respectively. Hereafter, we write c_i instead of $c_i(q_i)$ for notational parsimony. Accordingly, manufacturer i's profit function for a given assortment $\mathcal{A}$ is $\pi_i^{\mathcal{A}} = (w_i - c_i) z_i$ for $i = 1, 2$. The retailer's objective function for a given assortment $\mathcal{A}$ is $\Pi_R^{\mathcal{A}} = \sum_{i \in \mathcal{A}} (p_i - w_i) z_i - K\,|\mathcal{A}| + \gamma z_s$, where $w_s = c_s$ because the retailer sources the SB at the production cost.

We characterize the equilibrium wholesale and retail prices based on the sequence of events described in Section 3. The equilibrium wholesale price of the high-quality NB is

$$w_1^* = \begin{cases} \frac{c_1+q_1}{2} & \text{if} \quad \mathcal{A}^* = \{1\}, \\ \frac{q_1[2(c_1+q_1-q_2)+c_2]}{4q_1-q_2} & \text{if} \quad \mathcal{A}^* = \{1,2\}, \\ \frac{c_1+c_s+q_1-q_s-\gamma}{2} & \text{if} \quad \mathcal{A}^* = \{1,s\}, \\ \frac{(q_1-q_s)(2c_1+c_2)+(q_1-q_2)[2(q_1-q_s)+c_s-\gamma]}{4q_1-q_2-3q_s} & \text{if} \quad \mathcal{A}^* = \{1,2,s\}. \end{cases}$$

Similarly, the equilibrium wholesale price of the low-quality NB is

$$w_2^* = \begin{cases} \frac{c_2+q_2}{2} & \text{if} \quad \mathcal{A}^* = \{2\}, \\ \frac{q_2(c_1+q_1-q_2)+2c_2q_1}{4q_1-q_2} & \text{if} \quad \mathcal{A}^* = \{1,2\}, \\ \frac{c_2+c_s+q_2-q_s-\gamma}{2} & \text{if} \quad \mathcal{A}^* = \{2,s\}, \\ \frac{c_1(q_2-q_s)+2c_2(q_1-q_s)+(q_1-q_2)[q_2-q_s+2(c_s-\gamma)]}{4q_1-q_2-3q_s} & \text{if} \quad \mathcal{A}^* = \{1,2,s\}. \end{cases}$$

Based on these wholesale prices, $p_1^* = \frac{q_1+w_1^*}{2}$ if the high-quality NB is in the assortment. Similarly, $p_2^* = \frac{q_2+w_2^*}{2}$ if the low-quality NB is in the assortment. Lastly, $p_s^* = \frac{c_s+q_s-\gamma}{2}$ if the SB is in the assortment. These analyses reveal that increases in production costs lead to increases in the wholesale and retail prices. Based on these wholesale and retail prices, the retailer selects the assortment that maximizes its payoff. The retailer's equilibrium payoff is

$$\Pi_R^* = \begin{cases} 0 & \text{if} \quad \mathcal{A}^* = \emptyset, \\ \frac{\left(q_1-w_1^*\right)^2}{4q_1} - K & \text{if} \quad \mathcal{A}^* = \{1\}, \\ \frac{\left(q_2-w_2^*\right)^2}{4q_2} - K & \text{if} \quad \mathcal{A}^* = \{2\}, \\ \frac{(q_s+\gamma-c_s)^2}{4q_s} - K & \text{if} \quad \mathcal{A}^* = \{s\}, \\ \frac{q_1-2w_1^*}{4} + \frac{q_2{w_1^*}^2-2q_2w_1^*w_2^*+q_1{w_2^*}^2}{4(q_1-q_2)q_2} - 2K & \text{if} \quad \mathcal{A}^* = \{1,2\}, \\ \frac{q_1-2w_1^*}{4} + \frac{q_1(c_s-\gamma)^2+q_s\left({w_1^*}^2+2w_1^*(\gamma-c_s)\right)}{4(q_1-q_s)q_s} - 2K & \text{if} \quad \mathcal{A}^* = \{1,s\}, \\ \frac{q_2-2w_2^*}{4} + \frac{q_2(c_s-\gamma)^2+q_s\left({w_2^*}^2+2w_2^*(\gamma-c_s)\right)}{4(q_2-q_s)q_s} - 2K & \text{if} \quad \mathcal{A}^* = \{2,s\}, \\ \frac{(q_1-w_1^*)(q_1-q_2-w_1^*+w_2^*)}{4(q_1-q_2)} + \frac{(-c_s+q_s+\gamma)(-c_sq_2+q_sw_2^*+q_2\gamma)}{4q_s(q_2-q_s)} & \\ \quad + \frac{(q_2-w_2^*)\left(c_s(q_1-q_2)+q_s(w_2^*-w_1^*)+q_2(w_1^*+\gamma)-q_1(w_2^*+\gamma)\right)}{4(q_1-q_2)(q_2-q_s)} - 3K & \text{if} \quad \mathcal{A}^* = \{1,2,s\}. \end{cases} \tag{32}$$

Although (32) shows the retailer's optimal payoff in closed form, an analytical characterization of the optimal assortment is difficult because Π_R^* is a function of eight model parameters – q_1, q_2, q_s, c_1, c_2, c_s, γ, and K. As such, we numerically examine the role of unequal quality differentials ($q_1-q_2 \neq q_2-q_s$) and positive production costs ($c_i > 0$). In our numerical study, we set $(q_1, q_2, q_s) = (1+\kappa\alpha, 1+\alpha, 1)$ and $c_i(q_i) = \beta q_i^2$ for $i \in \{1,2,s\}$, where $\kappa > 1$ and $\beta \geq 0$. Our original model is a special case of this formulation in which $\kappa = 2$ and $\beta = 0$. When $\kappa > 2$, the high-quality NB has a larger quality advantage over the other products, whereas $\kappa \in (1,2)$ implies a smaller quality advantage. Our formulation is equivalent to assuming general quality levels for the two NBs because there is a one-to-one correspondence between the (q_1, q_2) pair and the (α, κ) pair.

We design our numerical experiment as follows: α ranges from 0 to 2 in increments of 0.1, γ ranges from 0 to 1 in increments of 0.01, and K ranges from 0 to 0.4 in increments of 0.01. These values are identical to the ones we used in our numerical analysis in Section 5.1. Additionally, we set $\kappa \in \{1.5, 1, 2.5\}$ and $\beta \in \{0, 0.01, 0.1\}$. We characterize the equilibrium outcomes for 782,649 scenarios obtained from the unique combinations of 21 values of α, 101 values of γ, 41 values of K, 3 values of κ, and 3 values of β.

Our first managerial insight is that overlooking SB spillover can result in suboptimal assortment and pricing decisions, leading to financial losses for the retailer. In our main model (i.e., $\kappa = 2$ and $\beta = 0$), we find that the retailer incurs the largest losses when it fails to adjust its assortment to take SB spillover into account, whereas its losses are relatively small when it carries the right assortment but fails to adjust its prices. Table 1 shows the robustness of this finding with respect to alternative quality and cost parameters. Specifically, for each (κ, β) combination in our numerical analysis, the retailer's average loss is about 0.08 in instances in which it carries the optimal assortment but fails to adjust its prices. However, its average loss is more than 0.26 in instances in which it carries a suboptimal assortment.

Table 1. The Retailer's Average Loss Due to Overlooking SB Spillover

κ	β	Average loss when the retailer carries the optimal assortment but sets suboptimal prices	Average loss when the retailer carries a suboptimal assortment
1.5	0	0.0857	0.2805
1.5	0.01	0.0851	0.2824
1.5	0.1	0.0818	0.2793
2	0	0.0860	0.2796
2	0.01	0.0854	0.2822
2	0.1	0.0817	0.2739
2.5	0	0.0868	0.2770
2.5	0.01	0.0864	0.2808
2.5	0.1	0.0821	0.2695

Our second managerial insight is that taking SB spillover into account decreases the retailer's category profit when the degree of SB spillover is high. However, a low degree of SB spillover may enable the retailer to simultaneously increase its category profit and SB market share. In Table 2, we present an example suggesting that our insights continue to hold for unequal quality differentials and positive production costs. In Table 2, we set $(\kappa, \beta, \alpha, K) = (2.5, 0.1, 1, 0.01)$ and report the equilibrium outcomes for three γ values. When $\gamma = 0$, the equilibrium assortment is

$\mathcal{A}^* = \{1,2,s\}$, and the retailer's category profit is $\pi_R^* = 0.2748$. When $\gamma = 0.1$, $\mathcal{A}^* = \{1,2,s\}$ and $\pi_R^* = 0.2768$. Put differently, when $\gamma = 0.1$, SB spillover enables the retailer to simultaneously increase its category profit (i.e., $\Delta\pi_R = 0.2768 - 0.2748 = 0.002$) and SB market share. In contrast, when $\gamma = 0.9$, $\mathcal{A}^* = \{s\}$, which leads to a negative category profit, $\pi_R^* = -0.01$. Indeed, a high γ decreases the retailer's category profit (i.e., $\Delta\pi_R = -0.01 - 0.2748 = -0.2848$).

Table 2. An Example on the Impact of SB Spillover on the Retailer's Category Profit

κ	β	α	K	γ	$\mathcal{A}^*$	$p_1^* - w_1^*$	$p_2^* - w_2^*$	$p_s^* - w_s^*$	z_1^*	z_2^*	z_s^*	π_R^*	$\Delta\pi_R$
2.5	0.1	1	0.01	0	$\{1,2,s\}$	0.9292	0.7208	0.4500	0.1389	0.1319	0.1792	0.2748	0.0000
2.5	0.1	1	0.01	0.1	$\{1,2,s\}$	0.9375	0.7375	0.4000	0.1333	0.1042	0.2625	0.2768	0.0020
2.5	0.1	1	0.01	0.9	$\{s\}$			0.0000			0.9000	-0.0100	-0.2848

Our third managerial insight is that SB spillover is never beneficial for the low-quality NB, but may increase the high-quality NB's profit. Our third insight continues to hold for unequal quality differentials and positive production costs. For example, when $(\kappa, \beta, \alpha, \gamma, K) = (2.5, 0.01, 2, 0, 0.17)$, the equilibrium assortment is $\mathcal{A}^* = \{1,2\}$, and the high-quality NB's profit is $\pi_1^* = 0.4191$. If γ increases from 0 to 0.42, the retailer switches its assortment from $\{1,2\}$ to $\{1,s\}$ by replacing the low-quality NB with its SB. When $\gamma = 0.42$, $\pi_1^* = 0.4473$, which is higher than its profit when $\gamma = 0$. This example illustrates that the high-quality NB may benefit from SB spillover. On the contrary, we do not find any parameter combinations in which SB spillover increases the low-quality NB's profit.

Based on these numerical results, we conclude that our managerial insights are not driven by the equal quality differentials (i.e., $q_1 - q_2 = q_2 - q_s = \alpha$) and/or the zero production costs (i.e., $c_1 = c_2 = c_s = 0$) assumptions.

B.2 Number of National Brand Manufacturers

Our model described in Section 3 focuses on a category with two NB manufacturers and one SB. In this section, we extend our analyses to models with three and four NB manufacturers. For consistency with our original formulation, we assume zero production costs and set $(q_1, q_2, q_3, q_s) = (1+3\alpha, 1+2\alpha, 1+\alpha, 1)$ in the model with three NBs. Similarly, we set $(q_1, q_2, q_3, q_4, q_s) = (1+4\alpha, 1+3\alpha, 1+2\alpha, 1+\alpha, 1)$ in the model with four NBs. The demand structure we present in Section 4.1 extends to larger assortments. For example, when $\mathcal{A} = \{1,2,3,4,s\}$, $z_1 = 1 - \frac{p_1 - p_2}{q_1 - q_2} = 1 - \frac{p_1 - p_2}{\alpha}$, $z_i = \frac{p_{i-1} - p_i}{q_{i-1} - q_i} - \frac{p_i - p_{i+1}}{q_i - q_{i+1}} = \frac{p_{i-1} + p_{i+1} - 2p_i}{\alpha}$ for $i = 2,3,4$, and $z_s = \frac{p_4 - p_s}{q_4 - q_s} - \frac{p_s}{q_s} = \frac{p_4 - (1+\alpha)p_s}{\alpha}$.

We characterize the equilibrium wholesale and retail prices for each assortment based on the sequence of events described in Section 3. In the last stage of the game, the retailer sets $p_s = \frac{q_s - \gamma}{2}$

if the SB is in the assortment. Similarly, for given wholesale price, w_i, the retailer sets $p_i = \frac{q_i+w_i}{2}$ if NB i is in the assortment. In the second stage, NBs that are in the assortment jointly optimize their wholesale prices. For example, when $\mathcal{A} = \{1, 2, 3, 4, s\}$, the equilibrium wholesale prices are $(w_1, w_2, w_3, w_4) = \left(\frac{56\alpha-\gamma}{97}, \frac{15\alpha-2\gamma}{97}, \frac{4\alpha-7\gamma}{97}, \frac{\alpha-26\gamma}{97}\right)$. Consistent with our original model, the wholesale prices increase in the degree of product differentiation, α, and decrease in the degree of SB spillover, γ. In the first stage of the game, we use the wholesale and retail prices from the last two stages of the game to calculate the retailer's payoff for each assortment. We then find the assortment that maximizes the retailer's payoff. Although the number of feasible assortments increases in the number of NBs in the category, the two-step approach described in Section 4.2 continues to be useful in identifying suboptimal assortments. For example, when the category has four NBs, the retailer prefers $\{1\}$ over $\{2\}$, $\{3\}$ and $\{4\}$ due to its high quality. Consequently, as in our original model, identifying the best assortment of size one requires a comparison between $\{1\}$ and $\{s\}$.

Similar to the numerical analyses performed in Section 5.1, we characterize the equilibrium outcomes for 86,961 scenarios obtained from the unique combinations of 21 values of α, 101 values of γ, and 41 values of K. Table 3 validates the robustness of our first managerial insight that overlooking SB spillover leads to the largest losses when the retailer fails to adjust its assortment, whereas the retailer's losses are relatively small when it carries the right assortment but fails to adjust its prices. For example, when there are four NBs in the category, the retailer's average loss is 0.0835 when it carries the right assortment but sets suboptimal prices, while its average loss is 0.2002 when it does not carry the optimal assortment.

Table 3. The Retailer's Average Loss Due to Overlooking SB Spillover

Number of NBs	Average loss when the retailer carries the optimal assortment but sets suboptimal prices	Average loss when the retailer carries a suboptimal assortment
2	0.0859	0.2796
3	0.0818	0.2276
4	0.0835	0.2002

Our insights regarding the retailer's category profit continue to hold when there are three or four NBs in the category. For example, when there are four NBs with $(\alpha, \gamma, K) = (0, 0.75, 0.25)$, if the retailer overlooks SB spillover, it carries $\{1, 2\}$ and its category profit is 0.4766. However, when the retailer takes SB spillover into account, it carries $\{s\}$ (i.e., it replaces two high-quality NBs with the SB) and its category profit is -0.1406. Thus, when γ is high, taking SB spillover into account decreases the retailer's category profit. Indeed, the retailer's category profit may be negative when the degree of SB spillover is high. In contrast, a low degree of SB spillover may enable the retailer to simultaneously increase its category profit and SB market share. For example, when there are four NBs with $\mathcal{A}^*(\alpha, \gamma, K) = \{1, 2, 3, s\}$, $z_s^*(\alpha, \gamma, K) = \frac{1}{194} + \frac{71\gamma}{194\alpha} + \frac{\gamma}{2}$, which increases in γ, and $\Delta\pi_R(\alpha, \gamma, K) = \gamma\left(\frac{112}{9409} + \frac{4173\gamma}{18818\alpha} - \frac{\gamma}{4}\right)$, which is positive when γ is small.

Last, we examine whether our insights regarding the impact of SB spillover on the NBs continue to hold when the category has three or four NBs. We find that the lowest quality NB never benefits from SB spillover. However, other NBs, which have medium or high quality, may benefit from SB spillover when the retailer removes their lower-quality competitors from the assortment to take SB spillover into account. Table 4 illustrates this with a numerical example in which there are three NBs with $\alpha = 0.6$ and $K = 0$. When $\gamma = 0$, the retailer carries $\mathcal{A}^*(\alpha, \gamma, K) = \{1, 2, 3, s\}$, and all three NBs have positive profits. SB spillover is never beneficial for the low-quality NB manufacturer. That is, $\Delta\pi_3(\alpha, \gamma, K) = \pi_3^*(\alpha, \gamma, K) - \pi_3^*(\alpha, 0, K) < 0$ for all $\gamma > 0$. In contrast, the high- and medium-quality NBs may benefit from SB spillover. When $\gamma = 0.1$, the retailer removes the lowest-quality NB from its assortment and carries $\mathcal{A}^*(\alpha, \gamma, K) = \{1, 2, s\}$. This assortment change increases the high- and medium-quality NBs' profits. When $\gamma = 0.13$, SB spillover hurts the medium-quality NB ($\Delta\pi_2(\alpha, \gamma, K) = -0.0004$), but benefits the high-quality NB ($\Delta\pi_1(\alpha, \gamma, K) = 0.0138$). Finally, when $\gamma = 0.75$, SB spillover harms all three NBs. The same insights also hold when the category has four NBs.

Table 4. An Example on the Impact of SB Spillover on the NB Manufacturers

				High-quality NB		Medium-quality NB		Low-quality NB	
α	γ	K	$\mathcal{A}^*(\alpha, \gamma, K)$	π_1^*	$\Delta\pi_1$	π_2^*	$\Delta\pi_2$	π_3^*	$\Delta\pi_3$
0.6	0	0	$\{1, 2, 3, s\}$	0.0999	0.0000	0.0142	0.0000	0.0009	0.0000
0.6	0.1	0	$\{1, 2, s\}$	0.1021	0.0022	0.0156	0.0014	0.0000	-0.0009
0.6	0.13	0	$\{1, 2, s\}$	0.1003	0.0005	0.0138	-0.0004	0.0000	-0.0009
0.6	0.75	0	$\{1, s\}$	0.0766	-0.0233	0.0000	-0.0142	0.0000	-0.0009

Based on these findings, we conclude that our managerial insights regarding the retailer's losses associated with overlooking SB spillover and the impact of SB spillover on the retailer's category profit continue to hold when the category has three or four NBs. Moreover, we find that the lowest quality NB that is in direct competition with the SB never benefits from SB spillover. However, medium- and high-quality NBs may benefit from the removal of their lower-quality competitors from the assortment.

B.3 Sequence of Events

Our model described in Section 3 is a three-stage game in which the retailer moves first by selecting its assortment, $\mathcal{A}$. The NB manufacturers that are in the assortment move next by setting their wholesale prices, w_i. Finally, the retailer sets the retail prices p_i for all $i \in \mathcal{A}$. In this section, we analyze an alternative sequence of events to demonstrate the robustness of our managerial insights with respect to the timing of the assortment and pricing decisions. In the alternative sequence, the NB manufacturers move first by setting their wholesale prices. The retailer moves next by selecting its assortment and setting the retail prices for the products in its assortment. All other assumptions (e.g., $(q_1, q_2, q_s) = (1+2\alpha, 1+\alpha, 1)$ and zero production costs) are identical to the ones in our original model for consistency.

The retail price expressions we derived in Section 4.1 continue to hold because the last stage of the new sequence is identical to the last stage of our original sequence. That is, $p_1 = \frac{q_1+w_1}{2}$ when the high-quality NB is in the assortment. Similarly, $p_2 = \frac{q_2+w_2}{2}$ when the low-quality NB is in the assortment. Lastly, $p_s = \frac{q_s-\gamma}{2}$ when the SB is in the assortment. Based on these retail prices, the retailer's payoff is

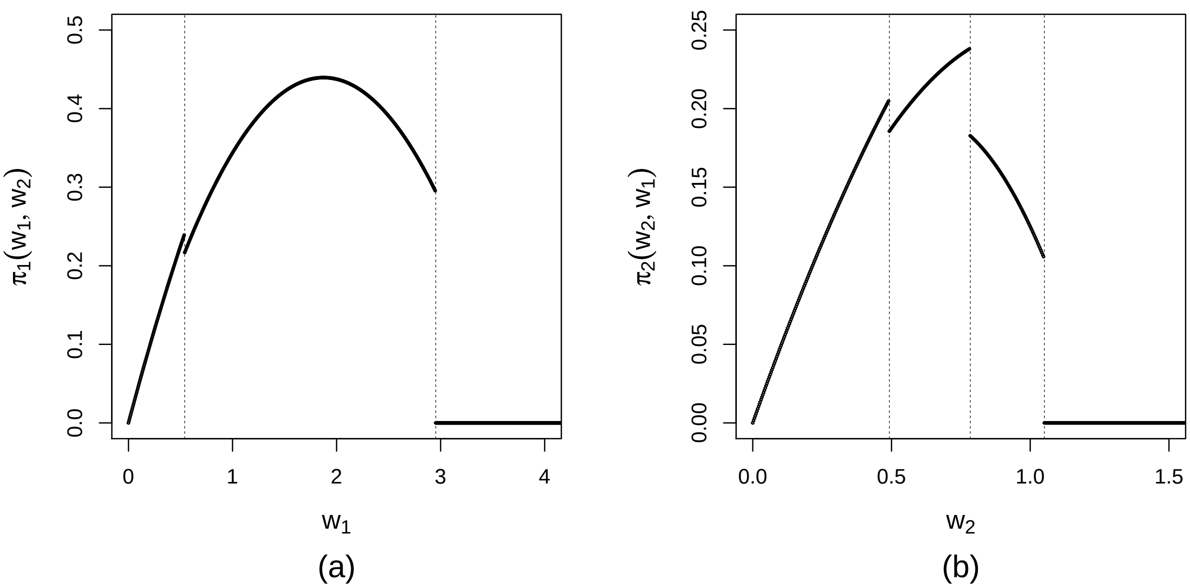
$$\Pi_R(w_1, w_2) = \begin{cases} 0 & \text{if} \quad \mathcal{A} = \emptyset, \\ \frac{(1+2\alpha-w_1)^2}{4(1+2\alpha)} - K & \text{if} \quad \mathcal{A} = \{1\}, \\ \frac{(1+\alpha-w_2)^2}{4(1+\alpha)} - K & \text{if} \quad \mathcal{A} = \{2\}, \\ \frac{(1+\gamma)^2}{4} - K & \text{if} \quad \mathcal{A} = \{s\}, \\ \frac{1}{4} + \frac{\alpha}{2} + \frac{(w_1-w_2)^2}{4\alpha} - \frac{w_1}{2} + \frac{w_2^2}{4(1+\alpha)} - 2K & \text{if} \quad \mathcal{A} = \{1,2\}, \\ \frac{\gamma^2+2\alpha\left(2\alpha+\gamma^2+1\right)+w_1^2+2w_1(\gamma-2\alpha)}{8\alpha} - 2K & \text{if} \quad \mathcal{A} = \{1,s\}, \\ \frac{(\alpha+1)\gamma^2+\alpha+(w_2-\alpha)^2+2\gamma w_2}{4\alpha} - 2K & \text{if} \quad \mathcal{A} = \{2,s\}, \\ \frac{\gamma^2+\alpha\left(2\alpha+\gamma^2+1\right)+w_1^2-2w_1(\alpha+w_2)+2w_2^2+2\gamma w_2}{4\alpha} - 3K & \text{if} \quad \mathcal{A} = \{1,2,s\}, \end{cases} \tag{33}$$

Let $\tilde{\mathcal{A}}(w_1, w_2)$ denote the assortment that maximizes the retailer's payoff, (33). Then, the high- and low-quality NB manufacturer's profits are respectively

$$\pi_1(w_1|w_2) = \begin{cases} \frac{w_1(2\alpha-w_1+1)}{4\alpha+2} & \text{if } \tilde{\mathcal{A}} = \{1\}, \\ \frac{w_1(\alpha-w_1+w_2)}{2\alpha} & \text{if } \tilde{\mathcal{A}} = \{1,2\}, \\ \frac{w_1(2\alpha-\gamma-w_1)}{4\alpha} & \text{if } \tilde{\mathcal{A}} = \{1,s\}, \\ \frac{w_1(\alpha-w_1+w_2)}{2\alpha} & \text{if } \tilde{\mathcal{A}} = \{1,2,s\}, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } \pi_2(w_2|w_1) = \begin{cases} \frac{w_2(1+\alpha-w_2)}{2(1+\alpha)} & \text{if } \tilde{\mathcal{A}} = \{2\}, \\ \frac{w_2((1+\alpha)w_1-2\alpha w_2+w_2)}{2\alpha(1+\alpha)} & \text{if } \tilde{\mathcal{A}} = \{1,2\}, \\ \frac{w_2(\alpha-\gamma-w_2)}{2\alpha} & \text{if } \tilde{\mathcal{A}} = \{2,s\}, \\ \frac{w_2(w_1-2w_2+\gamma)}{2\alpha} & \text{if } \tilde{\mathcal{A}} = \{1,2,s\}. \\ 0 & \text{otherwise.} \end{cases}$$

The first two stages of the game are analytically intractable for two reasons. First, the retailer's optimal assortment, $\tilde{\mathcal{A}}(w_1, w_2)$, depends on five parameters, w_1, w_2, α, γ, and K. Consequently, the boundaries at which the retailer switches from one assortment to another are difficult to characterize. Second, and more importantly, as shown in Figure 5, the NB manufacturers' profit functions have jump discontinuities due to the retailer's assortment switches. As a result, the equilibrium wholesale prices may be in one of the boundary points. Thus, having derived each player's objective function, we numerically examine whether our managerial insights continue to hold under the alternative sequence of events.

Figure 5. The NB Manufacturer's Profit Functions



Notes. $(\alpha, \gamma, K) = (2, 0.25, 0.04)$ in both graphs. Figure 5(a) shows π_1 as a function of w_1 when the low-quality NB sets $w_2 = \frac{q_2}{2} = 1.5$. In this figure, the retailer's optimal assortment, $\tilde{\mathcal{A}}(w_1, 1.5)$, is $\{1\}$ when $w_1 \in [0, 0.5401)$, $\{1, s\}$ when $w_1 \in [0.5401, 2.9530)$, and $\{s\}$ when $w_1 \geq 2.9530$. Figure 5(b) shows π_2 as a function of w_2 when the high-quality NB sets $w_1 = \frac{q_1}{2} = 2.5$. In this figure, the retailer's optimal assortment, $\tilde{\mathcal{A}}(2.5, w_2)$, is $\{2\}$ when $w_2 \in [0, 0.4925)$, $\{2, s\}$ when $w_2 \in [0.4925, 0.7838)$, $\{1, 2, s\}$ when $w_2 \in [0.7838, 1.0511)$, and $\{1, s\}$ when $w_2 \geq 1.0511$. The vertical dashed lines represent the assortment switching points.

Let $\tilde{w}_1(w_2) \equiv \arg\max_{w_1} \pi_1(w_1|w_2)$ and $\tilde{w}_2(w_1) \equiv \arg\max_{w_2} \pi_2(w_2|w_1)$ denote the high- and low-quality NB manufacturers' best response functions, respectively. For a given (α, γ, K), we derive $\tilde{w}_1(w_2)$ for $w_2 \in [0, q_2]$ and $\tilde{w}_2(w_1)$ for $w_1 \in [0, q_1]$. An equilibrium exists when there exists a (w_1^*, w_2^*) pair such that $\tilde{w}_1(w_2^*) = w_1^*$ and $\tilde{w}_2(w_1^*) = w_2^*$. An equilibrium does not exist for some parameter combinations due to the discontinuities in the NB manufacturers' profit functions. For example, we cannot find an equilibrium in 271 out of 86,961 unique (α, γ, K) combinations we examined in

Section 5.1. Nevertheless, our three managerial insights continue to hold under the new sequence of events. First, using the $86,961 - 271 = 86,690$ scenarios in which there is an equilibrium, we find that the retailer's average loss is 0.0346 in scenarios in which the retailer carries the right assortment but sets suboptimal prices due to overlooking SB spillover. The retailer's average loss is 0.2804 in instances in which it carries a suboptimal assortment due to overlooking SB spillover. These findings show the robustness of our first managerial insight that the retailer incurs the largest losses when it fails to adjust its assortment to take SB spillover into account, whereas its losses are relatively small when it carries the right assortment but fails to adjust its prices.

Second, there are cases in which SB spillover enables the retailer to simultaneously increase its category profit and SB market share. For example, when $(\alpha, \gamma, K) = (1, 0, 0)$, the retailer carries $\mathcal{A}^* = \{1, 2, s\}$, and its equilibrium profit and SB market share are 0.5153 and 0.0714, respectively. When $(\alpha, \gamma, K) = (1, 0.1, 0)$, the retailer continues to carry $\mathcal{A}^* = \{1, 2, s\}$, but its equilibrium profit and SB market share are higher (0.5187 and 0.1571, respectively). Nonetheless, there are also cases in which SB spillover leads to a decrease in the retailer's category profit. For example, $\pi_R^* = 0.1257$ when $(\alpha, \gamma, K) = (0.5, 0, 0.25)$, and $\pi_R^* = -0.16$ when $(\alpha, \gamma, K) = (0.5, 0.8, 0.25)$. In this example, a high γ not only decreases the retailer's category profit but also forces the retailer to incur a loss in the focal category.

Last, we find that SB spillover may increase the high-quality NB's profit. For example, when $(\alpha, \gamma, K) = (3, 0, 0.003)$, $\mathcal{A}^* = \{1, 2\}$, and the high-quality manufacturer's profit is $\pi_1^* = 0.4714$. If γ increases from 0 to 0.1, the retailer switches its assortment from $\{1, 2\}$ to $\{1, 2, s\}$ by introducing its SB. When $\gamma = 0.1$, the high-quality NB's equilibrium profit is $\pi_1^* = 0.4817$, which is higher than its profit when $\gamma = 0$. This example illustrates that the high-quality NB manufacturer may benefit from SB spillover. Conversely, we do not find any parameter combinations in which SB spillover increases the low-quality NB manufacturer's profit.

Based on these numerical results, we conclude that whether the NB manufacturers set their wholesale prices before or after the retailer selects its assortment does not change our managerial insights.