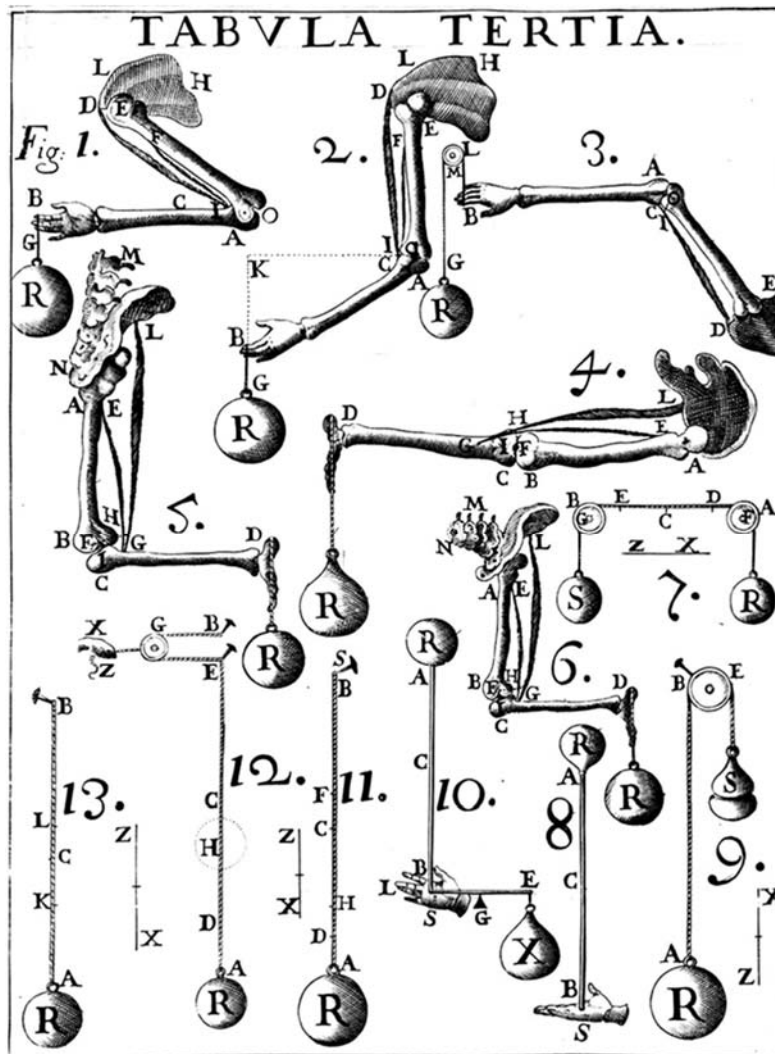


# Introductory Physics for the Life Sciences I

## PHYS 1501L

### Laboratory Manual

### Fall 2019



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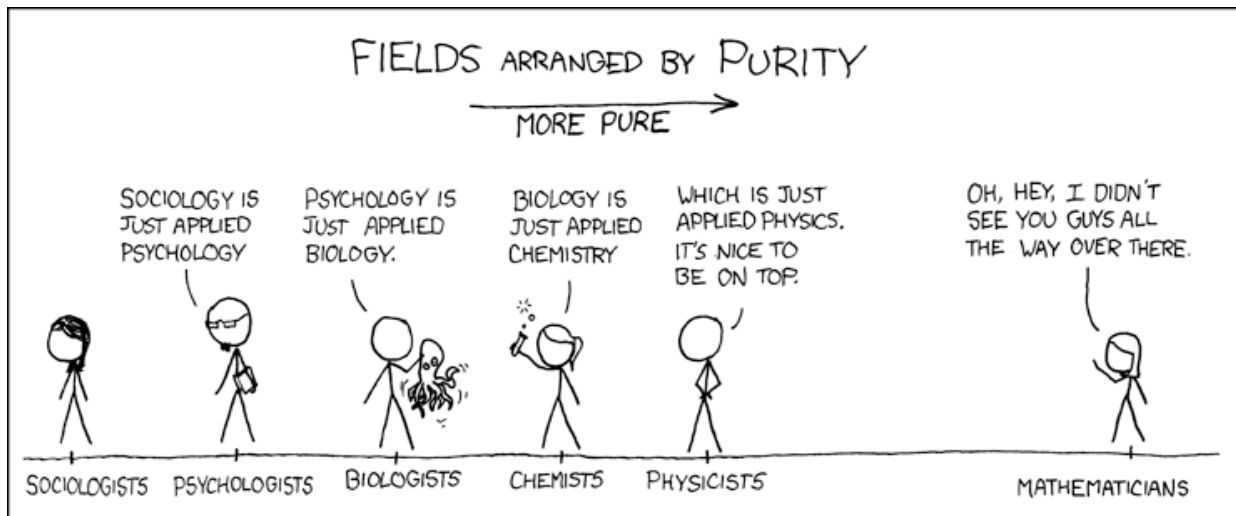
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## Acknowledgment

While this manual is (in at least the legal sense) my own work, I am very much indebted to my predecessors and colleagues, particularly Ken Shriver, Richard Helms, and Sherry Thompson. I am also grateful for much valuable feedback from my Teaching Assistants. About the only parts of the manual which are completely my own are the many mistakes. As you discover these errors, please point them out to your TA.

Oh, and I am also indebted to Randall Munroe, Wikipedia, and all who contribute to the Creative Commons. I plan to join them as soon as the lawyers let me.

Forrest T. Charnock



xkcd.com

**Cover Illustration:** Plate from *De Motu Animalium, Pars Prima* by Giovanni Alfonso Borelli, 1680. This was the first published work on biomechanics.

# Introductory Physics for the Life Sciences I

## PHYS 1501L

(Prior to Fall 2015, this lab was known as PHYS 114A.)

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## Useful Physical Constants\*

Universal gravitational constant:	$G = 6.67408(31) \times 10^{-11} \frac{m^3}{kg \ s^2}$
Speed of light (exact)	$c = 2.99792458 \times 10^8 \frac{m}{s}$
Avogadro's Constant (exact) <sup>†</sup>	$N_A = 6.02214076 \times 10^{23} \text{ mol}^{-1}$
Boltzmann's Constant (exact) <sup>†</sup>	$k = 1.380649 \times 10^{-23} \text{ J/K}$
Universal gas constant	$R = 8.3144598(48) \frac{J}{mol \ K}$
Acceleration due to gravity at Vanderbilt <sup>‡</sup> :	$g = 9.7943(32) \frac{m}{s^2}$
Standard atmospheric pressure	$1 \text{ atm} \equiv 1.01325 \times 10^5 \text{ Pa}$
Absolute zero	$0 \text{ K} = -273.15^\circ\text{C}$

---

\* Values of fundamental constants are from NIST ([physics.nist.gov/cuu](http://physics.nist.gov/cuu)). The number in parentheses is the standard uncertainty of the final digits of the main number. For example:  $6.67408 \pm 0.00031 = 6.67408(31)$

<sup>†</sup> As of May 20, 2019, this exact value is adopted by the General Conference on Weights and Measures (GCWM)

<sup>‡</sup> Dr. Medford Webster, Vanderbilt University

## Introduction

### The Sermon

The speed of light is  $2.99792458 \times 10^8$  m/s. This is not science.

The Wikipedia entry on Newton's 2<sup>nd</sup> law of motion is not science.

Nor is the periodic table of the elements.

Science is not a collection of facts. (Not even true facts!) Rather, science is a **process** for figuring out what is really going on. *What is the underlying principle here? How does this relate to some other observation?* If you are not involved in such a process, you are not doing science. A brilliant, dedicated, A+ student memorizing a list of equations is not doing science. A baby dropping peas on the floor to see what happens: now *that's science!!* (Does oatmeal fall too? Let's find out!!)

This is a science lab. I expect you to do some science in it.

*"Yeah, yeah, Dr. Charnock, I've heard this sermon before."*

Perhaps so, but I have seen too many brilliant and dedicated students who have learned to succeed in their other science classes by learning **lots of stuff**. So, they come into physics planning to memorize every equation they encounter and are completely overwhelmed. **You cannot succeed in physics by learning lots of stuff**. There are simply too many physics problems in the world; you cannot learn them all.

Instead, **you should learn as little as possible!**\* More than any other science, physics is about fundamental principles, and those few principles<sup>†</sup> must be the focus of your attention. Identify and learn those fundamental principles and how to use them. Then you can **derive** whatever solution that you need. And that **process** of derivation is the **process** of science.

*"OK, thanks for the advice for the class, but this is a lab!"*

It's still about fundamental principles. Look, each week you will come to lab and do *lots of stuff*. By following the instructions and copying (. . . oh, I mean *sharing* . . .) a few answers from your lab partners, you can blunder through each lab just fine. The problem is that the following week you will have a quiz, and you will not remember everything you did in that lab the week before.

When you are doing each lab, consciously relate your experiments to the underlying principles.

*How did I measure this? Which principle am I applying? Why are we doing this?*

On the subsequent quiz, instead of having to **remember what you did**, you can apply the principles to **figure out what you did**. Trust me. It really is easier this way.

---

\* . . . but not less.

†  $F = ma$ , conservation of energy and momentum, oscillations and waves, trigonometry. You will learn a few more in the second semester.

## GOALS AND OBJECTIVES

Physics is about the real world, not some idealized Platonic world that only exists in your head.\* The purpose of this lab is to relate the theories and equations you are learning in the classroom to reality. Hopefully, we'll convince you that all that physics stuff actually does work. Of course, reality can be messy, and along the way you will learn to deal with experimental uncertainty, loose cables, bad sensors, sticky wheels, temperamental software, temperamental lab partners, your own awful handwriting, and the typos in this lab book.

Welcome to experimental physics!



xkcd.com

## CORRELATION WITH LECTURE

Most of the topics covered in the lab will also be covered in your lecture, although not necessarily in the same sequence or at the same time during the semester. Given the scheduling (and re-scheduling) of the different lecture sections (some are MWF and some are TR), and the different lab sections (the first lab is Monday at 1 PM, the last is Thursday at 4 PM), *perfect correlation of lecture and lab topics is not possible*. The TA will provide a brief overview of the physics concept being explored in the lab during the first part of each lab section.

Occasionally, to improve the correlation with the lecture, the order of the labs may be changed from the sequence in this lab book. If so, you will be informed by your TA. **Check your email regularly.**

\* That's the Mathematics Dept. Walk over to Bldg. 3.

## PREPARATION

Prior to coming to lab, you should read over each experiment. Furthermore, for each laboratory, you must complete a *pre-lab* activity printed at the beginning of each lab in this manual. The pre-lab should be completed before the lab and turned in at the **beginning** of the lab. See the course syllabus for more details. In some labs, you may also be required to complete experimental predictions and enter them in your lab manual before you come to lab. Your TA will discuss this with you when necessary. Bring the following to each lab:

- Your complete lab manual secured in a 3-ring binder, including your previous graded labs.
- Your completed pre-lab.
- A scientific calculator. Graphing calculators are nice but not necessary. For some calculations, you may find a spreadsheet (such as Excel), Matlab, or some other computer based tools more appropriate. You are welcomed and encouraged to use such tools, but you still need a calculator.
- A pen, pencil and an eraser.

Often, the pre-lab includes online media for you to watch. Direct URL links are printed in the text, but clickable links may be found here:

<https://my.vanderbilt.edu/physicslabs/videos/>



xkcd.com

## PROCEDURE IN THE LABORATORY

In the laboratory, you will need to be efficient in the use of your time. We encourage a free exchange of ideas between group members and among students in the section, and we expect you to share both in taking data and in operating the computer, but **you should do your own work (using your own words) in answering questions in the lab manual and on the review questions handed out in lab.**

## HONOR CODE

The Vanderbilt Honor Code applies to all work done in this course. Violations of the Honor Code include, but are not limited to:

- Copying another student's answers on a pre-lab, lab questions, review questions, or quiz;
- Submitting data as your own when you were not involved in the acquisition of that data; and
- Copying data or answers from a prior term's lab (even from your own, in the event that you are repeating the course).

## GRADING

Your lab reports will be graded each week and returned to you the following week. Grades (including lab and quiz grades) will be posted on Brightspace.

- ***Mistakes happen!*** Check that the scores on Brightspace are correct. If you don't do this, *no one will.*
- Retain your lab reports so that any such errors can be verified and corrected.
- Details of grading may be found on the online syllabus.

## MAKING UP MISSED LABS

For details, I refer you to the syllabus (see below), but the main points are . . .

- **All** labs must be completed.
- If you know ahead of time that you will miss a lab, you must email both Dr. Charnock and your TA no later than the Friday before you will miss the lab.

forrest.t.charnock@vanderbilt.edu

In that email, include

- Your lab (1601L, 1602L, 1501L, or 1502L)
- Section number
- TA name
- A brief explanation of why you are missing lab.
- If arranging a make-up ahead of time is not possible, email us as soon as possible.
  - If you are abducted by aliens, whip out your phone and compose an email describing your predicament while the tractor beam is lifting you into the air. Make sure to hit SEND before the iris door closes or the message won't go out.\* Update us on your situation as soon as you are returned to Devil's Tower.
- You must be pro-active in making up labs.
  - Do NOT passively wait for someone to tell you what to do.
  - If you do not receive a reply from Dr. Charnock within 24 hrs, ***email him again.*** Repeat as necessary.†

---

\* Aliens rarely share their Wi-Fi passwords.

† Luke 18:1-5



**SYLLABUS:** available online

<https://my.vanderbilt.edu/physicslabs/documents/>

## How to Count Significant Figures\*

For all *measured* quantities (excepting counted quantities<sup>†</sup>), there will always be an associated uncertainty. For example,

$$\text{height of Mt. Everest}^{\ddagger} = 8844.43 \text{ m} \pm 0.21 \text{ m}$$

Understanding the uncertainty is crucial to understanding the quantity. However, it is usually not necessary to provide a precise uncertainty range as shown above. The simplest way to represent uncertainty is the method significant figures. Here, the  $\pm$  is dropped and the uncertainty is implied by the figures that are shown. An individual digit is usually considered significant if its uncertainty is less than  $\pm 5$ . In the case of Mt. Everest, the uncertainty is greater than 0.05 m; thus making the "3" uncertain. Rounding to the nearest 0.1 meter, we can write

$$\text{height of Mt. Everest} = 8844.4 \text{ m.}$$

This quantity has five significant figures. Notice that a digit does not need to be precisely known to be significant. Maybe the *actual* height is 8844.2 m. Maybe it is 8844.6 m. But the Chinese Academy of Sciences is confident that it is NOT 8844.7 m. Hence, that final "4" is worth recording.

In general, the rules for interpreting a value written this way are . . .

- All non-zero digits are significant
- All zeros written between non-zero digits are significant
- All zeros right of the decimal AND right of the number are significant
- Unless otherwise indicated, all other zeros are implied to be mere place-holders and are not significant.

Consider the following examples. The significant digits are underlined

1023

102300

102300.00

001023.450

0.0010230

---

\* Even if you think you understand significant figures, read this anyway. Some of what you think you know may be wrong.

<sup>†</sup> For example: "There are **exactly** 12 eggs in that carton."

<sup>‡</sup> 2005, Chinese Academy of Sciences, [https://en.wikipedia.org/wiki/Mount\\_Everest](https://en.wikipedia.org/wiki/Mount_Everest)

Occasionally, a zero that appears to be a mere place-holder is actually significant. For example, the length of a road may be measured as  $15000 \text{ m} \pm 25 \text{ m}$ . The second zero is significant. There are two common ways to write this.

- Use scientific notation (preferable):  $1.500 \times 10^4 \text{ m}$
- Use a bar to indicate the least significant figure:  $150\overline{00} \text{ m}$  or  $150\underline{00} \text{ m}$

### Addition and Subtraction

If several quantities are added or subtracted, the result will be limited by the number with the largest uncertain decimal position. Consider the sum below:

$$\begin{array}{r} 123.4500 \\ 12.20 \\ 0.00023 \\ \hline 135.65023 \\ 135.65 \end{array}$$

This sum is limited by 12.20; the result should be rounded down to the nearest hundredth. Again, consider another example:

$$\begin{array}{r} 3210\underline{00} \\ 12.30 \\ -333 \\ \hline 320679.3 \\ 320680 \end{array}$$

In  $3210\underline{00}$ , the last zero is not significant. The final answer is rounded up to the ten's position.

### Multiplication and Division

When multiplying or dividing quantities, the quantity with the fewest significant figures will determine the number of significant figures in the answer.

$$\frac{123.45 \times 0.0555}{22.22} = 0.30834721 = 0.308$$

0.0555 has the fewest significant figures with three. Thus, the answer must have three significant figures.

To ensure that round off errors do not accumulate, *keep at least one digit more* than is warranted by significant figures during intermediate calculations. Do the final round off at the end.

### *A Number Like 5.5 is exactly in the middle? Do I round up or down?*

For a fraction of 0.5, I always round up\* (for example,  $5.5 \rightarrow 6$ ), but others have different opinions†. Counting significant figures is literally an order-of-magnitude approximation, so rounding up or down does not really matter that much.

### *How Can This Break Down?*

Remember, counting significant figures is NOT a perfect way of accounting for uncertainty. It is only a first approximation that is easy to implement and is often sufficient.

For transcendental functions (sines, cosines, exponentials, *etc.*) these rules simply don't apply. When doing calculations with these, I usually keep one extra digit to avoid throwing away resolution.

However, even with simple math, naively applying the above rules can cause you to needlessly lose resolution.

Suppose you are given two measurements  $10m$  and  $9s$ . You are asked to calculate the speed.

With  $10m$  I will assume an uncertainty of about 0.5 out of 10 or about 5%.‡

With  $9s$  you have *almost* the same uncertainty (0.5 out of 9), but technically we only have one significant digit instead of two.

If I naively apply the rules . . .

$$\frac{10m}{9s} = 1.1111 \frac{m}{s} = 1 \frac{m}{s}$$

. . . my answer has an uncertainty of 0.5 out of 1!!! 50%!!!

This is what I call the **odometer problem**: When you move from numbers that are close to rolling over to the next digit (0.009, 8, 87, 9752953, *etc.*) to numbers that have just barely rolled over (0.001, 1.4, 105, 120258473, *etc.*), the estimate of the uncertainty changes by a factor of 10.§ Here, we really need to keep a second digit in the answer.

$$\frac{10m}{9s} = 1.1111 \frac{m}{s} = 1.1 \frac{m}{s}$$

Notice: In the problem above, if the numbers are flipped, the odometer problem goes away:

$$\frac{9m}{10s} = 0.9000 \frac{m}{s} = 0.9 \frac{m}{s}$$

---

\* . . ., and for good mathematical reasons, mind you. But still, it does not really matter that much.

† Google it, if you want to waste an hour of your life.

‡ Of course, I don't *really* know what the uncertainty is. It could be much larger, but bear with me anyway.

§ . . . and, vice-versa.

*Oh Great! I thought this was supposed to be easy.*

Well . . ., it is! But, you still have to use your head!

- Apply the rules.
- Look out for the “odometer problem”.
- If warranted, keep an extra digit.
- Simple!

Remember: counting significant figures is literally an order-of-magnitude approximation. So, don't get too uptight about it. If you need something better than an order-of-magnitude approximation, see Lab 1 and Appendix C.

What you should *never* do is willy-nilly copy down every digit from your calculator. If you are in the habit of doing that, STOP IT. You are wasting your time and lying to yourself. If you ever claim that your cart was traveling at 1.35967494 m/s, expect your TA to *slap you down*. **That is just wrong!**

**What makes a result *scientific* is honesty, not precision.**

To say that the altitude of Mt. Everest is about 9000 m is perfectly true.

To say that the altitude of Mt. Everest is 8844.4324 m is a lie.

Forrest T. Charnock  
Director of the Undergraduate Laboratories  
Vanderbilt Physics

## The Greek Alphabet

The 26 letters of the Standard English alphabet do not supply enough variables for our algebraic needs. So, the sciences have adopted the Greek alphabet as well. You will have to learn it eventually, so go ahead and learn it now, particularly the lower case letters.

Alpha	A	$\alpha$
Beta	B	$\beta$
Gamma	$\Gamma$	$\gamma$
Delta	$\Delta$	$\delta$
Epsilon	E	$\epsilon$
Zeta	Z	$\zeta$
Eta	H	$\eta$
Theta	$\Theta$	$\theta$
Kappa	K	$\kappa$
Lambda	$\Lambda$	$\lambda$
Mu	M	$\mu$
Nu	N	$\nu$
Xi	$\Xi$	$\xi$
Omicron	O	$\omicron$
Pi	$\Pi$	$\pi$
Rho	P	$\rho$
Sigma	$\Sigma$	$\sigma$
Tau	T	$\tau$
Upsilon	Y	$\upsilon$
Phi	$\Phi$	$\phi$ or $\varphi$
Chi	X	$\chi$
Psi	$\Psi$	$\psi$
Omega	$\Omega$	$\omega$

**Important:**  $\omega$  is NOT  $w$ ! Note how the Greek  $\omega$  is curvy, the Latin  $w$  is pointy. Please do not call  $\omega$  'double-u'; it is 'omega'.

And  $\rho$  is a Greek *rho*, NOT a Latin *p*!

Name \_\_\_\_\_ Section \_\_\_\_\_ Date \_\_\_\_\_

**Pre-Lab Preparation Sheet for Lab 0:  
Problem Solving**

(Due at the Beginning of Lab)

**Directions:**

Read the essay *How to Count Significant Figures*, then read over the following lab. Answer the following questions.

1. Applying the rules of significant figures, calculate the following

$$123.4 + 120 + 4.822 - 21 =$$

$$\frac{185.643 \times 0.0034}{3022} =$$

$$(523400 \times 0.0032) + 253 =$$

# GUIDE TO CONVERTING TO METRIC

**TEMPERATURE**

- 60°C EARTH'S HOTTEST
- 45°C DUBAI HEAT WAVE
- 40°C SOUTHERN US HEAT WAVE
- 35°C NORTHERN US HEAT WAVE
- 30°C BEACH WEATHER
- 25°C WARM ROOM
- 20°C ROOM TEMPERATURE
- 10°C JACKET WEATHER
- 0°C SNOW!
- 5°C COLD DAY (BOSTON)
- 10°C COLD DAY (MOSCOW)
- 20°C FUCKFUCKFUCKCOLD
- 30°C FUUUUUUUUUUCK!
- 40°C SPT GOES "CUNK"



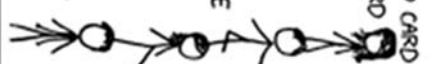
**SPEED**

kph	m/s	ACTIVITY
5	1.5	WALKING
13	3.5	JOGGING
25	7	SPRINTING
35	10	FASTEST HUMAN
45	13	HOUSECAT
55	15	RABBIT
75	20	RAPTOR
100	25	SLOW HIGHWAY
110	30	INTERSTATE (65 MPH)
120	35	SPEED YOU ACTUALLY GO WHEN IT SAYS "65"
140	40	RAPTOR ON HOVERBOARD

THE KEY TO CONVERTING TO METRIC IS ESTABLISHING NEW REFERENCE POINTS. WHEN YOU HEAR "26°C," INSTEAD OF THINKING "THAT'S 79°F" YOU SHOULD THINK, "THAT'S WARMER THAN A HOUSE BUT COOL FOR SWIMMING." HERE ARE SOME HELPFUL TRIBLES OF REFERENCE POINTS:

**LENGTH**

- 1cm WIDTH OF MICROSD CARD
- 3cm LENGTH OF SD CARD
- 12cm CD DIAMETER
- 14cm PENIS
- 15cm BIC PEN
- 80cm DOORWAY WIDTH
- 1m LIGHTSABER BLADE
- 170cm SUMMER GAU
- 200cm DARTH VADER
- 2.5m CEILING
- 5m CAR-LENGTH
- 16m<sup>1/2</sup>m HUMAN TOWER OF SERENITY CREW



**VOLUME**

- 3mL BLOOD IN A FIELD MOUSE
- 5mL TEASPOON
- 30mL NASAL PASSAGES
- 40mL SHOT GLASS
- 350mL SODA CAN
- 500mL WATER BOTTLE
- 3L TWO-LITER BOTTLE
- 5L BLOOD IN HUMAN MALE
- 30L MILK CRATE
- 55L SUMMER GAU
- 65L DENNIS KUCINICH
- 75L RON PAUL
- 200L FRIDGE

SO, WHEN IT'S BLOCKED, THE MUCUS IN YOUR NOSE COULD ABOUT FILL A SHOT GLASS.


RELATED: I'VE INVENTED THE WORST MIXED DRINK EVER.

55 + 65 + 75 < 200



**MASS**

- 3g PEANUT M&M
- 100g CELL PHONE
- 500g BOTTLED WATER
- 1kg ULTRAPORTABLE LAPTOP
- 2kg LIGHT-MEDIUM LAPTOP
- 3kg HEAVY LAPTOP
- 5kg LCD MONITOR
- 15kg CRT MONITOR
- 4kg CAT
- 4.1kg CAT (WITH CAPTION)
- 60kg LADY
- 70kg DUDE
- 150kg SHAG
- 200kg YOUR MOM
- 220kg YOUR MOM (INCL. CHEAP JEWELRY)
- 223kg YOUR MOM (ALSO INCL. MAKEUP)





Name \_\_\_\_\_ Date \_\_\_\_\_ Partners \_\_\_\_\_  
 TA \_\_\_\_\_ Section \_\_\_\_\_

## Lab 0: Estimation and Problem Solving

*Measure what can be measured, and make measurable what cannot be measured.*

*--Galileo Galilei*

**Objective:** To learn how to approach a problem by asking the right questions.

**Equipment:** meter stick, 1 kg mass, ruler, caliper, your brain

### DISCUSSION

Before you can really *know anything*, you have to *measure something*, be it distance, time, acidity, or social status. However, measurements cannot be exact. Rather, all measurements have some uncertainty associated with them.\* Ideally, all measurements should consist of two numbers: the **value** of the measured quantity  $x$  and its **uncertainty**†  $\Delta x$ . The uncertainty reflects the reliability of the measurement. The range of measurement uncertainties varies widely. Some quantities, such as the mass of the electron  $m_e = (9.1093897 \pm 0.0000054) \times 10^{-31}$  kg, are known to better than one part per million. Other quantities are only loosely bounded: *There are 100 to 400 billion stars in the Milky Way.*

Note that **we are not talking about “human error”!** We are not talking about *mistakes!* Rather, uncertainty is inherent in the instruments and methods that we use **even when perfectly applied.** The goddess Athena cannot not read a digital scale any better than you.

---

\* The only exceptions are counted quantities. “There are *exactly* 12 eggs in that carton.”

† Sometimes this is called the *error* of the measurement, but *uncertainty* is the better term. *Error* implies a variance from the correct value:

$$error = x_{measured} - x_{correct}$$

But, of course, we don’t know what the correct value is. If we did, we would not need to make the measurement in the first place. In fact, we cannot know the error in principle! But we can measure the uncertainty.

## RECORDING UNCERTAINTY

In general, uncertainties are usually quoted with no more precision than the measured result; and the last significant figure of a result should match that of the uncertainty. For example, a measurement of the acceleration due to gravity on the surface of the Earth might be given as

$$g = 9.7 \pm 1.2 \text{ m/s}^2 \quad \text{or} \quad g = 9.9 \pm 0.5 \text{ m/s}^2$$

But you should **never** write

$$g = 9.7 \pm 1.25 \text{ m/s}^2 \quad \text{or} \quad g = 9.92 \pm 0.5 \text{ m/s}^2.$$

In the last two cases, the precision of the result and uncertainty do not match.

Uncertainty is an inherently fuzzy thing to measure; so, it makes little sense to present the uncertainty of your measurement with extraordinary precision. It would be silly to say that I am  $(1.823643 \pm 0.124992)\text{m}$  tall. Therefore, the stated uncertainty will usually have only one significant digit. For example

$$23.5 \pm 0.4 \quad \text{or} \quad 13600 \pm 700$$

However, if the uncertainty is between 1.0 and 2.9 (or 10 and 29, or 0.0010 and 0.0029, *etc.*) it **may** be better to have two significant digits. For example,

$$124.5 \pm 1.2$$

There is a big difference between saying  $\pm 1$  and  $\pm 1.4$ . There is not a big difference between  $\pm 7$  and  $\pm 7.4$ . (This is related to the odometer problem. See the above essay *How to Count Significant Figures*.)

## VOCABULARY

Here we define some useful terms (with examples) and discuss how uncertainties are reported in the lab.

**Absolute uncertainty:** This is the magnitude of the uncertainty assigned to a measured physical quantity. It has the same units as the measured quantity.

Example 1. Again, consider the example above:

$$L = (1.56 \pm 0.03) \text{ cm}$$

Here, the uncertainty is given in units of length: 0.03 cm. When the uncertainty has the same dimension as the measurement, this is an *absolute uncertainty*.

**Relative uncertainty:** This is the ratio of the absolute uncertainty and the value of the measured quantity. It has no units, that is, it is dimensionless. It is also called the *fractional uncertainty* or, when appropriate, the *percent uncertainty*.

Example 2. In the example above the *fractional uncertainty* is

$$\frac{\Delta V}{V} = \frac{0.03 \text{ cm}}{1.56 \text{ cm}} = 0.019$$

The *percent uncertainty* would be 1.9%.

#### Exercise A

1. Measure the height of one member of your group.
2. Estimate the uncertainty of this measurement
3. Now, have another group measure the height of the same student.

Name	height	uncertainty

Since there are always measurement uncertainties, your numbers are probably different.\* But these two measurements may still be consistent with each other.

4. Are these different measurements consistent with each other?
5. What do you mean when you say they are or are not consistent?

---

\* If the numbers are the same, you could probably add another digit of precision to your measurement.

## Exercise B

Suppose we were to lie all of the students head to foot on the ground.

6. What would be the length and corresponding uncertainty of this trail of bodies?
  
  
  
  
  
  
  
  
  
  
7. When adding measurements together, is it better to think in terms of absolute or relative uncertainty?
  
  
  
  
  
  
  
  
  
  
8. How many students, lying head to foot would be required to encircle a football field? Estimate your uncertainty of this number?

## Exercise C

9. How many sheets of paper could you fit inside this room?
  
  
  
  
  
  
  
  
  
  
10. What is the dominant source of error in this estimate?

Name \_\_\_\_\_ Section \_\_\_\_\_ Date \_\_\_\_\_

**Pre-Lab Preparation Sheet for Lab 1:  
Measurement, Uncertainty, and Uncertainty Propagation**

(Due at the Beginning of Lab)

Read over the lab, then answer the following questions.

Consider two quantities:

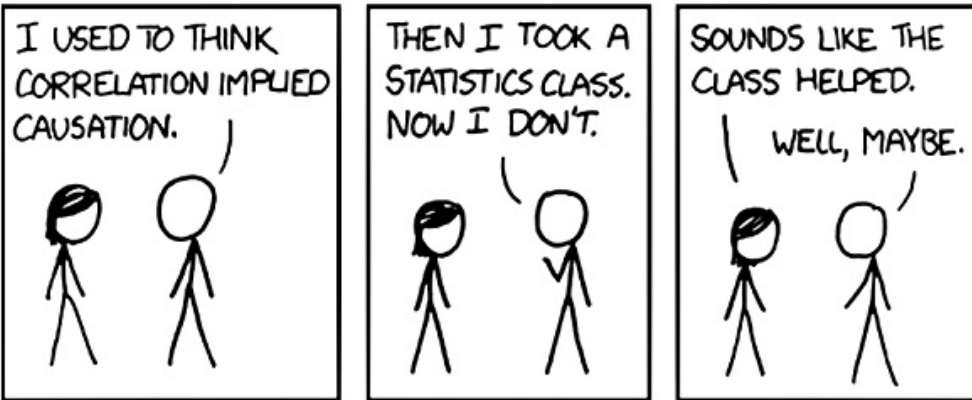
$$A = (10 \text{ m} \pm 1 \text{ m})$$

$$B = (100 \text{ m} \pm 5 \text{ m})$$

1. What are the relative uncertainties of A and B?

2. What is the uncertainty of  $(A + B)$ ?

3. What is the uncertainty of  $(A - B)$ ?



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## Lab 1: Measurement, Uncertainty, and Uncertainty Propagation

*“The first principle is that you must not fool yourself – and you are the easiest person to fool.”*

*--Richard Feynman*

**Objective:** To understand how to report both a measurement and its uncertainty.

Learn how to propagate uncertainties through calculations

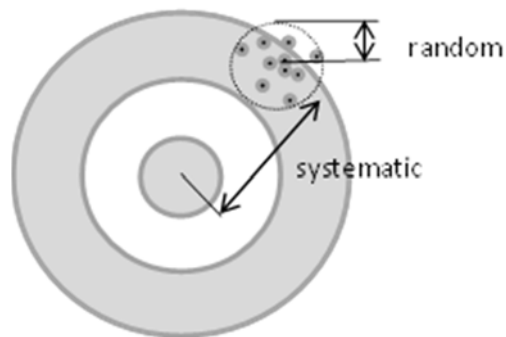
Define *mean*, *standard deviation*, and *standard deviation of the mean*.

**Equipment:** meter stick, 1 kg mass, ruler, caliper, short wooden plank, graduated cylinder

### Types of uncertainties

*Random uncertainties* occur when the results of repeated measurements vary due to truly random processes. For example, random uncertainties may arise from small fluctuations in experimental conditions or due to variations in the stability of measurement equipment. These uncertainties can be estimated by repeating the measurement many times.

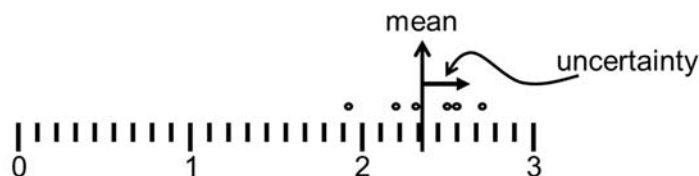
A *systematic uncertainty* occurs when all of the individual measurements of a quantity are biased by the same amount. These uncertainties can arise from the calibration of instruments or by experimental conditions. For example, slow reflexes while operating a stopwatch would systematically yield longer measurements than the true time duration.



*Mistakes* can be made in any experiment, either in making the measurements or in calculating the results. However, by definition, mistakes can also be avoided. Such blunders and major systematic errors can only be avoided by a thoughtful and careful approach to the experiment.

### Estimating uncertainty

*Repeated observation:* Suppose you make repeated measurements of something: say with a stopwatch you time the fall of a ball. Due to random variations, each measurement will be a little different. From the spread of the measurements, you can calculate the uncertainty of your results.



Shortly, we will describe the formal procedure to do this calculation. (Oddly enough, truly random uncertainties are the easiest to deal with.)

*By eye or reason:* Sometimes, repeated measurements are not relevant to the problem. Suppose you measure the length of something with a meter stick. Meter sticks are typically ruled to the mm; however, we can often read them more precisely than that.



Consider the figure above. Measuring from the left side of each mark and considering the position uncertainties of both ends of the bar, I can confidently say that the bar is  $(1.56 \pm 0.03)$  cm. Perhaps your younger eyes could read it with more confidence, but when in doubt it is better to overestimate uncertainty.

Could I do a better job by measuring several times? Not always. Sometimes with repeated measurements, it still comes down to “*Looks like  $(1.56 \pm 0.03)$  cm to me.*” But that’s ok. Your reasoned judgment is sufficient. **Science is defined by rigorous honesty, not rigorous precision!**



### Reducing random uncertainty by repeated observation

By taking a large number of individual measurements, we can use statistics to reduce the random uncertainty of a quantity. For instance, suppose we want to determine the mass of a standard U.S. penny. We measure the mass of a single penny many times using a balance. The results of 17 measurements on the same penny are summarized in Table 1.

Table 1. Data recorded measuring the mass of a US penny.

	mass (g)	deviation (g)		mass (g)	deviation (g)
1	2.43	-0.088	10	2.46	-0.058
2	2.49	-0.028	11	2.52	0.002
3	2.49	-0.028	12	2.4	-0.118
4	2.58	0.062	13	2.58	0.062
5	2.52	0.002	14	2.61	0.092
6	2.55	0.032	15	2.49	-0.028
7	2.52	0.002	16	2.52	0.002
8	2.64	0.122	17	2.46	-0.058
9	2.55	0.032			

The **mean** value  $\bar{m}$  (that is, the average) of the measurements is defined to be

$$\bar{m} = \frac{1}{N} \sum_{i=1}^N m_i = \frac{1}{17} (m_1 + m_2 + \dots + m_{17}) = 2.518\text{g} \quad (1)$$

The **deviation**  $d_i$  of the  $i$ th measurement  $m_i$  from the mean value  $\bar{m}$  is defined to be

$$d_i = m_i - \bar{m} \quad (2)$$

Fig. 1 shows a histogram plot of the data on the mass of a US penny. Also on the graph is a plot of the smooth bell curve (that is a *normal distribution*) that represents what the distribution of measured values would look like if we took many, many measurements. The result of a large set of repeated measurements (when subject only to random uncertainties) will always approach a normal distribution which is symmetrical about  $\bar{m}$ .

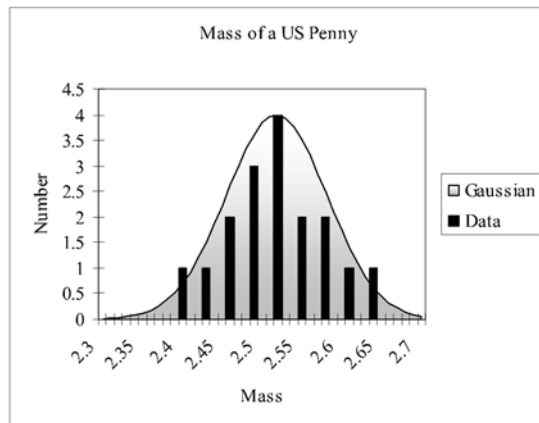


Figure 1. The Gaussian or normal distribution for the mass of a penny  $N=17$ ,  $\bar{m}=2.518$  g,  $\Delta m=0.063$  g.

*OK, now I have all of these measurements. How accurate is any one of these measurements?*

For this, we now define the **standard deviation**  $\Delta m$  as

$$\Delta m = \sqrt{\frac{\sum_{i=1}^N (m_i - \bar{m})^2}{(N-1)}} = \sqrt{\frac{\sum_{i=1}^N (m_i - \bar{m})^2}{16}} = 0.063 \text{ g} \quad (3)$$

For normal distributions, 68% of the time the result of an individual measurement would be within  $\pm \Delta m$  of the mean value  $\bar{m}$ . Thus,  $\Delta m$  is the experimental uncertainty for an *individual* measurement of  $m$ .

*The mean  $\bar{m}$  should have less uncertainty than any individual measurement. What is **that** uncertainty?*

The uncertainty of the final average is called the **standard deviation of the mean**. It is given by

$$\Delta M = \frac{\Delta m}{\sqrt{N}} \quad (4)$$

With a set of  $N=17$  measurements, our result is

$$\begin{aligned} \text{mass of a penny} &= \bar{m} \pm \Delta M = \bar{m} \pm \frac{\Delta m}{\sqrt{N}} \\ &= 2.518 \text{ g} \pm \frac{0.063 \text{ g}}{\sqrt{17}} \\ &= (2.518 \pm 0.015) \text{ g} \end{aligned} \quad (5)$$

Thus, if our experiment is only subject to random uncertainties in the individual measurements, we can improve the precision of that measurement by doing it repeatedly and finding the average.

Note, however, that the precision improves only as  $\frac{1}{\sqrt{N}}$ . To reduce the uncertainty by a factor of

10, we have to make 100 times as many measurements. We also have to be careful in trying to get better results by letting  $N \rightarrow \infty$ , because the overall accuracy of our measurements will eventually be limited by systematic errors, which *do not cancel out* like random errors do.

**Exercise 1:**

- a. With a caliper, measure the width and thickness of the plank. Make at least five measurements of each dimension and enter the result into Table 2.
- b. With a ruler, measure the length of the wooden plank on your table as precisely as possible. Estimate the uncertainty, and enter the result below.
- c. For both the width and thickness, calculate your final result (the *mean*) and the uncertainty (the *standard deviation of the mean*). Enter the final results below the table.

Table 2

width	deviation	thickness	deviation

width = \_\_\_\_\_

thickness = \_\_\_\_\_

length = \_\_\_\_\_

## Propagation of uncertainties

Usually, to obtain a final result, we have to measure a variety of quantities (*say, length and time*) and mathematically combine them to obtain a final result (*speed*). How the uncertainties in individual quantities combine to produce the uncertainty in the final result is called the *propagation of uncertainty*.

Here we summarize a number of common cases. For the most part these should take care of what you need to know about how to combine uncertainties.\*

### *Uncertainties in sums and differences:*

If several quantities  $x_1, x_2, x_3$  are measured with absolute uncertainties  $\Delta x_1, \Delta x_2, \Delta x_3$  then the absolute uncertainty in  $Q$  (where  $Q = x_1 \pm x_2 \pm x_3$ ) is

$$\Delta Q = |\Delta x_1| + |\Delta x_2| + |\Delta x_3| \quad (6)$$

In other words, *for sums and differences, add the **absolute** uncertainties.*

### *Uncertainties in products and quotients:*

Several quantities  $x, y, z$  (with uncertainties  $\Delta x, \Delta y, \Delta z$ .) combine to form  $Q$ , where

$$Q = \frac{x y}{z}$$

(or any other combination of multiplication and division). Then the fractional uncertainty in  $Q$  will be

$$\frac{\Delta Q}{|Q|} = \left| \frac{\Delta x}{x} \right| + \left| \frac{\Delta y}{y} \right| + \left| \frac{\Delta z}{z} \right| \quad (7)$$

In other words, *for products and quotients, add the **fractional** uncertainties.*

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\* These expressions for the propagation of uncertainty are an **upper limit** to the resulting uncertainty. In this case, you could actually do better. See Appendix C for details.

**Exercise 2:**

- d. Calculate the total volume of the block and the associated uncertainty. Show your math below.

**Exercise 3:**

You can also find the volume of an object by measuring the volume of water displaced by the object when it is submerged.

- e. Measure the volume of water in the graduated cylinder and the associated uncertainty.
- f. Submerge the block (holding it under the surface with a pen or pencil), and find the resulting volume and the associated uncertainty.
- g. Calculate the volume of the plank and the associated uncertainty. Show your math below.
- h. Is this answer consistent with your result from Exercise 2? Explain.

**Exercise 4:**

- a. Calculate the total surface area of the block and the associated uncertainty. Show your math below.





Name \_\_\_\_\_ Section \_\_\_\_\_ Date \_\_\_\_\_

## Pre-Lab Preparation Sheet for Lab 2: Position, Velocity, and Acceleration in one-dimensional motion

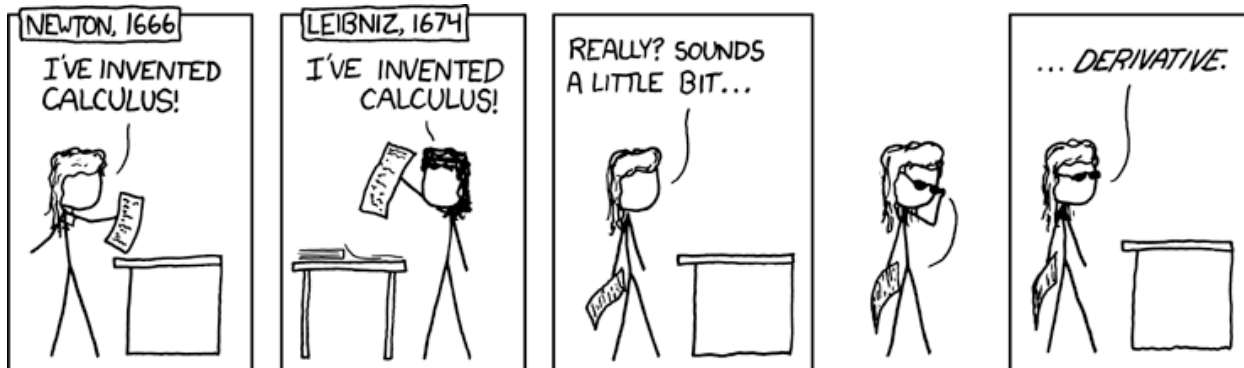
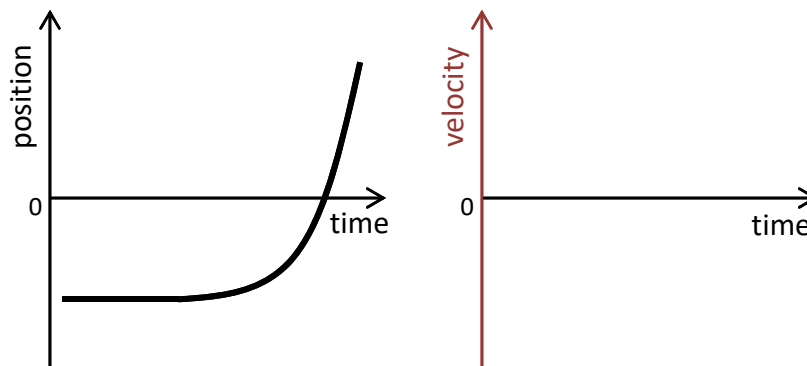
(Due at the Beginning of Lab)

Watch the video *Introduction to Optimization and Curve Fitting* found at the following site:

[my.vanderbilt.edu/physicslabs/videos/](http://my.vanderbilt.edu/physicslabs/videos/)

Read over the lab and then answer the following questions

- Given the following position curve, sketch the corresponding velocity curve.



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2. Imagine kicking a box across the floor: it suddenly starts moving then slides for a short distance before coming to a stop. Make a sketch of the position and velocity curves for such motion.

