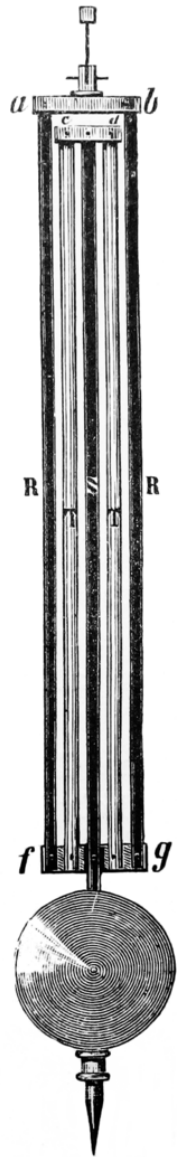


**General Physics Laboratory I**  
**PHYS 1601L**

**Laboratory Manual**  
**Spring 2020**



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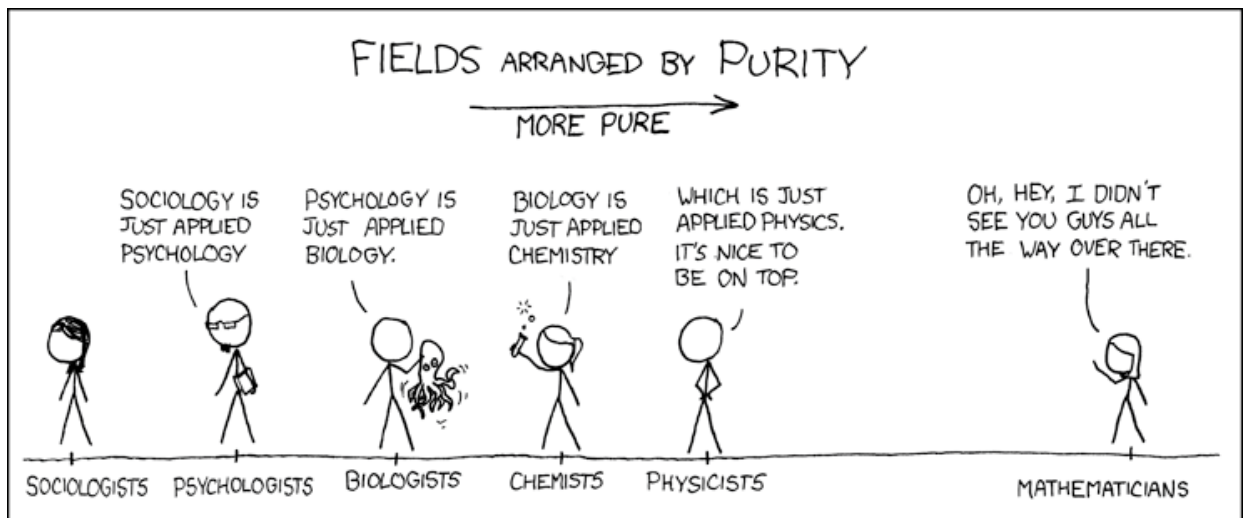
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## Acknowledgment

While this manual is (in at least the legal sense) my own work, I am very much indebted to my predecessors and colleagues, particularly Ken Shriver, Richard Helms, and Sherry Thompson. I am also grateful for much valuable feedback from my Teaching Assistants. About the only parts of the manual which are completely my own are the many mistakes. As you discover these errors, please point them out to your TA.

Oh, and I am also indebted to Randall Munroe, Wikipedia, and all who contribute to the Creative Commons. I plan to join them as soon as the lawyers let me.

Forrest T. Charnock



xkcd.com

**Cover Illustration:** Diagram of a temperature compensating gridiron pendulum, invented by John Harrison, 1726. By combining metal rods of with different coefficients of thermal expansion, the length of the pendulum is kept constant. If you have never heard of John Harrison, you should. Google him.

# General Physics Laboratory I

## PHYS 1601L

(Prior to Fall 2015, this lab was known as PHYS 116A.)

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## Useful Physical Constants\*

Universal gravitational constant:	$G = 6.67408(31) \times 10^{-11} \frac{m^3}{kg \ s^2}$
Speed of light (exact by definition)	$c = 2.99792458 \times 10^8 \frac{m}{s}$
Avogadro's Constant (exact) <sup>†</sup>	$N_A = 6.02214076 \times 10^{23} \text{ mol}^{-1}$
Boltzmann's Constant (exact) <sup>†</sup>	$k = 1.380649 \times 10^{-23} \text{ J/K}$
Universal gas constant	$R = 8.3144598(48) \frac{J}{mol \ K}$
Acceleration due to gravity at Vanderbilt <sup>‡</sup> :	$g = 9.7943(32) \frac{m}{s^2}$
Standard atmospheric pressure	$1 \text{ atm} \equiv 1.01325 \times 10^5 \text{ Pa}$
Absolute zero	$0 \text{ K} = -273.15^\circ\text{C}$

---

\* Values of fundamental constants are from NIST ([physics.nist.gov/cuu](http://physics.nist.gov/cuu)). The number in parentheses is the standard uncertainty of the final digits of the main number. For example:  $6.67408 \pm 0.00031 = 6.67408(31)$

<sup>†</sup> As of May 20, 2019, this exact value is adopted by the General Conference on Weights and Measures (GCWM)

<sup>‡</sup> Dr. Medford Webster, Vanderbilt University

## Introduction

### The Sermon

The speed of light is  $2.99792458 \times 10^8$  m/s. This is not science.

The Wikipedia entry on Newton's 2<sup>nd</sup> law of motion is not science.

Nor is the periodic table of the elements.

Science is not a collection of facts. (Not even true facts!) Rather, science is a **process** for figuring out what is really going on. *What is the underlying principle here? How does this relate to some other observation?* If you are not involved in such a process, you are not doing science. A brilliant, dedicated, A+ student memorizing a list of equations is not doing science. A baby dropping peas on the floor to see what happens: now *that's science!!* (Does oatmeal fall too? Let's find out!!)

This is a science lab. I expect you to do some science in it.

*"Yeah, yeah, Dr. Charnock, I've heard this sermon before."*

Perhaps so, but I have seen too many brilliant and dedicated students who have learned to succeed in their other science classes by learning **lots of stuff**. So, they come into physics planning to memorize every equation they encounter and are completely overwhelmed. **You cannot succeed in physics by learning lots of stuff**. There are simply too many physics problems in the world; you cannot learn them all.

Instead, **you should learn as little as possible!**\* More than any other science, physics is about fundamental principles, and those few principles<sup>†</sup> must be the focus of your attention. Identify and learn those fundamental principles and how to use them. Then you can **derive** whatever solution that you need. And that **process** of derivation is the **process** of science.

*"OK, thanks for the advice for the class, but this is a lab!"*

It's still about fundamental principles. Look, each week you will come to lab and do *lots of stuff*. By following the instructions and copying (. . . oh, I mean *sharing* . . .) a few answers from your lab partners, you can blunder through each lab just fine. The problem is that the following week you will have a quiz, and you will not remember everything you did in that lab the week before.

When you are doing each lab, consciously relate your experiments to the underlying principles.

*How did I measure this? Which principle am I applying? Why are we doing this?*

On the subsequent quiz, instead of having to **remember what you did**, you can apply the principles to **figure out what you did**. Trust me. It really is easier this way.

---

\* . . . but not less.

†  $F = ma$ , conservation of energy and momentum, oscillations and waves, trigonometry. You will learn a few more in the second semester.

## GOALS AND OBJECTIVES

Physics is about the real world, not some idealized Platonic world that only exists in your head.\* The purpose of this lab is to relate the theories and equations you are learning in the classroom to reality. Hopefully, we'll convince you that all that physics stuff actually does work. Of course, reality can be messy, and along the way you will learn to deal with experimental uncertainty, loose cables, bad sensors, sticky wheels, temperamental software, temperamental lab partners, your own awful handwriting, and the typos in this lab book.

Welcome to experimental physics!



xkcd.com

## CORRELATION WITH LECTURE

Most of the topics covered in the lab will also be covered in your lecture, although not necessarily in the same sequence or at the same time during the semester. Given the scheduling (and re-scheduling) of the different lecture sections (some are MWF and some are TR), and the different lab sections (the first lab is Monday at 1 PM, the last is Thursday at 4 PM), *perfect correlation of lecture and lab topics is not possible*. The TA will provide a brief overview of the physics concept being explored in the lab during the first part of each lab section.

Occasionally, to improve the correlation with the lecture, the order of the labs may be changed from the sequence in this lab book. If so, you will be informed by your TA. **Check your email regularly.**

\* That's the Mathematics Dept. Walk over to Bldg. 3.

## PREPARATION

Prior to coming to lab, you should read over each experiment. Furthermore, for each laboratory, you must complete a *pre-lab* activity printed at the beginning of each lab in this manual. The pre-lab should be completed before the lab and turned in at the **beginning** of the lab. See the course syllabus for more details. In some labs, you may also be required to complete experimental predictions and enter them in your lab manual before you come to lab. Your TA will discuss this with you when necessary. Bring the following to each lab:

- Your complete lab manual secured in a 3-ring binder, including your previous graded labs.
- Your completed pre-lab.
- A scientific calculator. Graphing calculators are nice but not necessary. For some calculations, you may find a spreadsheet (such as Excel), Matlab, or some other computer based tools more appropriate. You are welcomed and encouraged to use such tools, but you still need a calculator.
- A pen, pencil and an eraser.

Often, the pre-lab includes online media for you to watch. Direct URL links are printed in the text, but clickable links may be found here:

<https://my.vanderbilt.edu/physicslabs/videos/>



xkcd.com

## PROCEDURE IN THE LABORATORY

In the laboratory, you will need to be efficient in the use of your time. We encourage a free exchange of ideas between group members and among students in the section, and we expect you to share both in taking data and in operating the computer, but **you should do your own work (using your own words) in answering questions in the lab manual and on the review questions handed out in lab.**

## HONOR CODE

The Vanderbilt Honor Code applies to all work done in this course. Violations of the Honor Code include, but are not limited to:

- Copying another student's answers on a pre-lab, lab questions, review questions, or quiz;
- Submitting data as your own when you were not involved in the acquisition of that data; and
- Copying data or answers from a prior term's lab (even from your own, in the event that you are repeating the course).

## GRADING

Your lab reports will be graded each week and returned to you the following week. Grades (including lab and quiz grades) will be posted on Brightspace.

- ***Mistakes happen!*** Check that the scores on Brightspace are correct. If you don't do this, *no one will.*
- Retain your lab reports so that any such errors can be verified and corrected.
- Details of grading may be found on the online syllabus.

## MAKING UP MISSED LABS

For details, I refer you to the syllabus (see below), but the main points are . . .

- **All** labs must be completed.
- If you know ahead of time that you will miss a lab, you must email both Dr. Charnock and your TA no later than the Friday before you will miss the lab.

forrest.t.charnock@vanderbilt.edu

In that email, include

- Your lab (1601L, 1602L, 1501L, or 1502L)
- Section number
- TA name
- A brief explanation of why you are missing lab.
- If arranging a make-up ahead of time is not possible, email us as soon as possible.
  - If you are abducted by aliens, whip out your phone and compose an email describing your predicament while the tractor beam is lifting you into the air. Make sure to hit SEND before the iris door closes or the message won't go out.\* Update us on your situation as soon as you are returned to Devil's Tower.
- You must be pro-active in making up labs.
  - Do NOT passively wait for someone to tell you what to do.
  - If you do not receive a reply from Dr. Charnock within 24 hrs, ***email him again.*** Repeat as necessary.†

---

\* Aliens rarely share their Wi-Fi passwords.

† Luke 18:1-5



**SYLLABUS:** available online

<https://my.vanderbilt.edu/physicslabs/documents/>

## How to Count Significant Figures\*

For all *measured* quantities (excepting counted quantities<sup>†</sup>), there will always be an associated uncertainty. For example,

$$\text{height of Mt. Everest}^{\ddagger} = 8844.43 \text{ m} \pm 0.21 \text{ m}$$

Understanding the uncertainty is crucial to understanding the quantity. However, it is usually not necessary to provide a precise uncertainty range as shown above. The simplest way to represent uncertainty is the method significant figures. Here, the  $\pm$  is dropped and the uncertainty is implied by the figures that are shown. An individual digit is usually considered significant if its uncertainty is less than  $\pm 5$ . In the case of Mt. Everest, the uncertainty is greater than 0.05 m; thus making the "3" uncertain. Rounding to the nearest 0.1 meter, we can write

$$\text{height of Mt. Everest} = 8844.4 \text{ m.}$$

This quantity has five significant figures. Notice that a digit does not need to be precisely known to be significant. Maybe the *actual* height is 8844.2 m. Maybe it is 8844.6 m. But the Chinese Academy of Sciences is confident that it is NOT 8844.7 m. Hence, that final "4" is worth recording.

In general, the rules for interpreting a value written this way are . . .

- All non-zero digits are significant
- All zeros written between non-zero digits are significant
- All zeros right of the decimal AND right of the number are significant
- Unless otherwise indicated, all other zeros are implied to be mere place-holders and are not significant.

Consider the following examples. The significant digits are underlined

1023

102300

102300.00

001023.450

0.0010230

---

\* Even if you think you understand significant figures, read this anyway. Some of what you think you know may be wrong.

<sup>†</sup> For example: "There are **exactly** 12 eggs in that carton."

<sup>‡</sup> 2005, Chinese Academy of Sciences, [https://en.wikipedia.org/wiki/Mount\\_Everest](https://en.wikipedia.org/wiki/Mount_Everest)

Occasionally, a zero that appears to be a mere place-holder is actually significant. For example, the length of a road may be measured as  $15000 \text{ m} \pm 25 \text{ m}$ . The second zero is significant. There are two common ways to write this.

- Use scientific notation (preferable):  $1.500 \times 10^4 \text{ m}$
- Use a bar to indicate the least significant figure:  $150\overline{0}0 \text{ m}$  or  $150\underline{0}0 \text{ m}$

### Addition and Subtraction

If several quantities are added or subtracted, the result will be limited by the number with the largest uncertain decimal position. Consider the sum below:

$$\begin{array}{r} 123.4500 \\ 12.20 \\ 0.00023 \\ \hline 135.65023 \\ 135.65 \end{array}$$

This sum is limited by 12.20; the result should be rounded down to the nearest hundredth. Again, consider another example:

$$\begin{array}{r} 3210\underline{0}0 \\ 12.30 \\ -333 \\ \hline 320679.3 \\ 320680 \end{array}$$

In  $3210\underline{0}0$ , the last zero is not significant. The final answer is rounded up to the ten's position.

### Multiplication and Division

When multiplying or dividing quantities, the quantity with the fewest significant figures will determine the number of significant figures in the answer.

$$\frac{123.45 \times 0.0555}{22.22} = 0.30834721 = 0.308$$

0.0555 has the fewest significant figures with three. Thus, the answer must have three significant figures.

To ensure that round off errors do not accumulate, **keep at least one digit more** than is warranted by significant figures during intermediate calculations. Do the final round off at the end.

### ***How Do I Round a Number Like 5.5? Do I round up or down?***

For a fraction of 0.5, I always round up\* (for example,  $5.5 \rightarrow 6$ ), but others have different opinions†. Counting significant figures is literally an order-of-magnitude approximation, so rounding up or down does not really matter that much.

### ***How Can This Break Down?***

Remember, counting significant figures is NOT a perfect way of accounting for uncertainty. It is only a first approximation that is easy to implement and is often sufficient.

For transcendental functions (sines, cosines, exponentials, *etc.*) these rules simply don't apply. When doing calculations with these, I usually keep one extra digit to avoid throwing away resolution.

However, even with simple math, naively applying the above rules can cause you to needlessly lose resolution.

Suppose you are given two measurements  $10m$  and  $9s$ . You are asked to calculate the speed.

With  $10m$  I will assume an uncertainty of about 0.5 out of 10 or about 5%.‡

With  $9s$  you have *almost* the same uncertainty (0.5 out of 9), but technically we only have one significant digit instead of two.

If I naively apply the rules . . .

$$\frac{10m}{9s} = 1.1111 \frac{m}{s} = 1 \frac{m}{s}$$

. . . my answer has an uncertainty of 0.5 out of 1!!! 50%!!!

This is what I call the **odometer problem**: When you move from numbers that are close to rolling over to the next digit (0.009, 8, 87, 9752953, *etc.*) to numbers that have just barely rolled over (0.001, 1.4, 105, 120258473, *etc.*), the estimate of the uncertainty changes by a factor of 10.§ Here, we really need to keep a second digit in the answer.

$$\frac{10m}{9s} = 1.1111 \frac{m}{s} = 1.1 \frac{m}{s}$$

Notice: In the problem above, if the numbers are flipped, the odometer problem goes away:

$$\frac{9m}{10s} = 0.9000 \frac{m}{s} = 0.9 \frac{m}{s}$$

---

\* . . . and for good mathematical reasons, mind you. But still, it does not really matter that much.

† Google it, if you want to waste an hour of your life.

‡ Of course, I don't *really* know what the uncertainty is. It could be much larger, but bear with me anyway.

§ . . . and, vice-versa.

*Oh Great! I thought this was supposed to be easy.*

Well . . ., it is! But, you still have to use your head!

- Apply the rules.
- Look out for the “odometer problem”.
- If warranted, keep an extra digit.
- Simple!

Remember: counting significant figures is literally an order-of-magnitude approximation. So, don't get too uptight about it. If you need something better than an order-of-magnitude approximation, see Lab 1 and Appendix C.

What you should *never* do is willy-nilly copy down every digit from your calculator. If you are in the habit of doing that, STOP IT. You are wasting your time and lying to yourself. If you ever claim that your cart was traveling at 1.35967494 m/s, expect me to *slap you down*. **That is just wrong!**

**What makes a result *scientific* is honesty, not precision.**

To say that the altitude of Mt. Everest is about 9000 m is perfectly true.

To say that the altitude of Mt. Everest is 8844.4324 m is a lie.

Forrest T. Charnock  
Director of the Undergraduate Laboratories  
Vanderbilt Physics

### The Greek Alphabet

The 26 letters of the Standard English alphabet do not supply enough variables for our algebraic needs. So, the sciences have adopted the Greek alphabet as well. You will have to learn it eventually, so go ahead and learn it now, particularly the lower case letters.

Alpha	A	$\alpha$
Beta	B	$\beta$
Gamma	$\Gamma$	$\gamma$
Delta	$\Delta$	$\delta$
Epsilon	E	$\epsilon$
Zeta	Z	$\zeta$
Eta	H	$\eta$
Theta	$\Theta$	$\theta$
Kappa	K	$\kappa$
Lambda	$\Lambda$	$\lambda$
Mu	M	$\mu$
Nu	N	$\nu$
Xi	$\Xi$	$\xi$
Omicron	O	$\omicron$
Pi	$\Pi$	$\pi$
Rho	P	$\rho$
Sigma	$\Sigma$	$\sigma$
Tau	T	$\tau$
Upsilon	Y	$\upsilon$
Phi	$\Phi$	$\phi$ or $\varphi$
Chi	X	$\chi$
Psi	$\Psi$	$\psi$
Omega	$\Omega$	$\omega$

**Important:**  $\omega$  is NOT  $w$ ! Note how the Greek  $\omega$  is curvy, the Latin  $w$  is pointy. Please do not call  $\omega$  'double-u'; it is 'omega'.

And  $\rho$  is a Greek *rho*, NOT a Latin *p*!

Name \_\_\_\_\_ Section \_\_\_\_\_ Date \_\_\_\_\_

**Pre-Lab Preparation Sheet for Lab 0:  
Problem Solving**

(Due at the Beginning of Lab)

**Directions:**

Read the essay *How to Count Significant Figures*, then read over the following lab. Answer the following questions.

1. Applying the rules of significant figures, calculate the following

$$123.4 + 120 + 4.822 - 21 =$$


$$\frac{185.643 \times 0.0034}{3022} =$$

$$(523400 \times 0.0032) + 253 =$$

# GUIDE TO CONVERTING TO METRIC

**TEMPERATURE**

- 60°C EARTH'S HOTTEST
- 45°C DUBAI HEAT WAVE
- 40°C SOUTHERN US HEAT WAVE
- 35°C NORTHERN US HEAT WAVE
- 30°C BEACH WEATHER
- 25°C WARM ROOM
- 20°C ROOM TEMPERATURE
- 10°C JACKET WEATHER
- 0°C SNOW!
- 5°C COLD DAY (BOSTON)
- 10°C COLD DAY (MOSCOW)
- 20°C FUCKFUCKFUCKCOLD
- 30°C FUUUUUUUUUUCK!
- 40°C SPT GOES "CUNK"



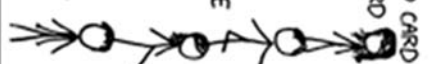
**SPEED**

kph	m/s	ACTIVITY
5	1.5	WALKING
13	3.5	JOGGING
25	7	SPRINTING
35	10	FASTEST HUMAN
45	13	HOUSECAT
55	15	RABBIT
75	20	RAPTOR
100	25	SLOW HIGHWAY
110	30	INTERSTATE (65 MPH)
120	35	SPEED YOU ACTUALLY GO WHEN IT SAYS "65"
140	40	RAPTOR ON HOVERBOARD

THE KEY TO CONVERTING TO METRIC IS ESTABLISHING NEW REFERENCE POINTS. WHEN YOU HEAR "26°C," INSTEAD OF THINKING "THAT'S 79°F" YOU SHOULD THINK, "THAT'S WARMER THAN A HOUSE BUT COOL FOR SWIMMING." HERE ARE SOME HELPFUL TRIBLES OF REFERENCE POINTS:

**LENGTH**

- 1cm WIDTH OF MICROSD CARD
- 3cm LENGTH OF SD CARD
- 12cm CD DIAMETER
- 14cm PENIS
- 15cm BIC PEN
- 80cm DOORWAY WIDTH
- 1m LIGHTSABER BLADE
- 170cm SUMMER GAU
- 200cm DARTH VADER
- 2.5m CEILING
- 5m CAR-LENGTH
- 16m<sup>1/2</sup> HUMAN TOWER OF SERENITY CREW



**VOLUME**

- 3mL BLOOD IN A FIELD MOUSE
- 5mL TEASPOON
- 30mL NASAL PASSAGES
- 40mL SHOT GLASS
- 350mL SODA CAN
- 500mL WATER BOTTLE
- 3L TWO-LITER BOTTLE
- 5L BLOOD IN HUMAN MALE
- 30L MILK CRATE
- 55L SUMMER GAU
- 65L DENNIS KUCINICH
- 75L RON PAUL
- 200L FRIDGE

SO, WHEN IT'S BLOCKED, THE MUCUS IN YOUR NOSE COULD ABOUT FILL A SHOT GLASS.


RELATED: I'VE INVENTED THE WORST MIXED DRINK EVER.

55 + 65 + 75 < 200



**MASS**

- 3g PEANUT M&M
- 100g CELL PHONE
- 500g BOTTLED WATER
- 1kg ULTRAPORTABLE LAPTOP
- 2kg LIGHT-MEDIUM LAPTOP
- 3kg HEAVY LAPTOP
- 5kg LCD MONITOR
- 15kg CRT MONITOR
- 4kg CAT
- 4.1kg CAT (WITH CAPTION)
- 60kg LADY
- 70kg DUDE
- 150kg SHAG
- 200kg YOUR MOM
- 220kg YOUR MOM (INCL. CHEAP JEWELRY)
- 223kg YOUR MOM (ALSO INCL. MAKEUP)





Name \_\_\_\_\_ Date \_\_\_\_\_ Partners \_\_\_\_\_  
 TA \_\_\_\_\_ Section \_\_\_\_\_

## Lab 0: Estimation and Problem Solving

*Measure what can be measured, and make measurable what cannot be measured.*

*--Galileo Galilei*

**Objective:** To learn how to approach a problem by asking the right questions.

**Equipment:** meter stick, 1 kg mass, ruler, caliper, your brain

### DISCUSSION

Before you can really *know anything*, you have to *measure something*, be it distance, time, acidity, or social status. However, measurements cannot be exact. Rather, all measurements have some uncertainty associated with them.\* Ideally, all measurements should consist of two numbers: the *value* of the measured quantity  $x$  and its *uncertainty*†  $\Delta x$ . The uncertainty reflects the reliability of the measurement. The range of measurement uncertainties varies widely. Some quantities, such as the mass of the electron  $m_e = (9.1093897 \pm 0.0000054) \times 10^{-31}$  kg, are known to better than one part per million. Other quantities are only loosely bounded: *There are 100 to 400 billion stars in the Milky Way.*

Note that **we are not talking about “human error”!** We are not talking about *mistakes!* Rather, uncertainty is inherent in the instruments and methods that we use **even when perfectly applied.** The goddess Athena cannot not read a digital scale any better than you.

---

\* The only exceptions are counted quantities. “There are *exactly* 12 eggs in that carton.”

† Sometimes this is called the *error* of the measurement, but *uncertainty* is the better term. *Error* implies a variance from the correct value:

$$error = x_{measured} - x_{correct}$$

But, of course, we don’t know what the correct value is. If we did, we would not need to make the measurement in the first place. In fact, we cannot know the error in principle! But we can measure the uncertainty.

## RECORDING UNCERTAINTY

In general, uncertainties are usually quoted with no more precision than the measured result; and the last significant figure of a result should match that of the uncertainty. For example, a measurement of the acceleration due to gravity on the surface of the Earth might be given as

$$g = 9.7 \pm 1.2 \text{ m/s}^2 \quad \text{or} \quad g = 9.9 \pm 0.5 \text{ m/s}^2$$

But you should **never** write

$$g = 9.7 \pm 1.25 \text{ m/s}^2 \quad \text{or} \quad g = 9.92 \pm 0.5 \text{ m/s}^2.$$

In the last two cases, the precision of the result and uncertainty do not match.

Uncertainty is an inherently fuzzy thing to measure; so, it makes little sense to present the uncertainty of your measurement with extraordinary precision. It would be silly to say that I am  $(1.823643 \pm 0.124992)\text{m}$  tall. Therefore, the stated uncertainty will usually have only one significant digit. For example

$$23.5 \pm 0.4 \quad \text{or} \quad 13600 \pm 700$$

However, if the uncertainty is between 1.0 and 2.9 (or 10 and 29, or 0.0010 and 0.0029, *etc.*) it **may** be better to have two significant digits. For example,

$$124.5 \pm 1.2$$

There is a big difference between saying  $\pm 1$  and  $\pm 1.4$ . There is not a big difference between  $\pm 7$  and  $\pm 7.4$ . (This is related to the odometer problem. See the above essay *How to Count Significant Figures*.)

## VOCABULARY

Here we define some useful terms (with examples) and discuss how uncertainties are reported in the lab.

**Absolute uncertainty:** This is the magnitude of the uncertainty assigned to a measured physical quantity. It has the same units as the measured quantity.

Example 1. Again, consider the example above:

$$L = (1.56 \pm 0.03) \text{ cm}$$

Here, the uncertainty is given in units of length: 0.03 cm. When the uncertainty has the same dimension as the measurement, this is an *absolute uncertainty*.

**Relative uncertainty:** This is the ratio of the absolute uncertainty and the value of the measured quantity. It has no units, that is, it is dimensionless. It is also called the *fractional uncertainty* or, when appropriate, the *percent uncertainty*.

Example 2. In the example above the *fractional uncertainty* is

$$\frac{\Delta V}{V} = \frac{0.03 \text{ cm}}{1.56 \text{ cm}} = 0.019$$

The *percent uncertainty* would be 1.9%.

#### Exercise A

1. Measure the height of one member of your group.
2. Estimate the uncertainty of this measurement
3. Now, have another group measure the height of the same student.

Name	height	uncertainty

Since there are always measurement uncertainties, your numbers are probably different.\* But these two measurements may still be consistent with each other.

4. Are these different measurements consistent with each other?
5. What do you mean when you say they are or are not consistent?

---

\* If the numbers are the same, you could probably add another digit of precision to your measurement.

## Exercise B

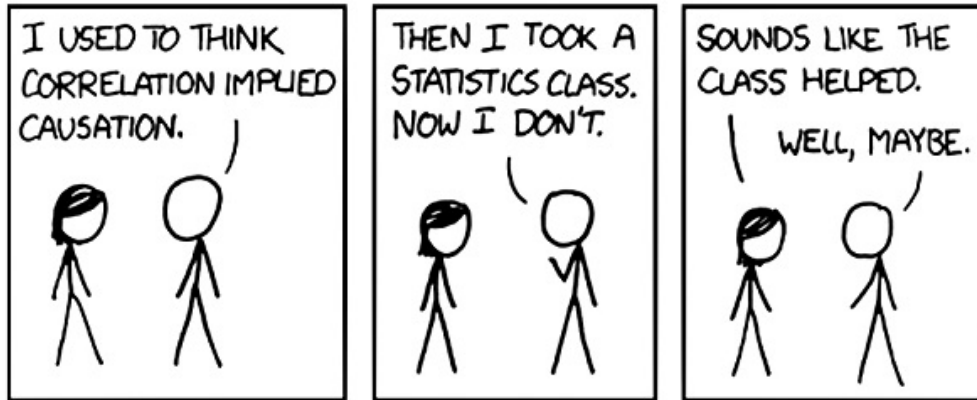
Suppose we were to lie all of the students head to foot on the ground.

6. What would be the length and corresponding uncertainty of this trail of bodies?
  
  
  
  
  
  
  
  
  
  
7. When adding measurements together, is it better to think in terms of absolute or relative uncertainty?
  
  
  
  
  
  
  
  
  
  
8. How many students, lying head to foot would be required to encircle a field in front of the library? Estimate your uncertainty of this number?

## Exercise C

9. How many sheets of paper could you fit inside this room?
  
  
  
  
  
  
  
  
  
  
10. What is the dominant source of error in this estimate?





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## Lab 1: Measurement, Uncertainty, and Uncertainty Propagation

*“The first principle is that you must not fool yourself – and you are the easiest person to fool.”*

*--Richard Feynman*

**Objective:** To understand how to report both a measurement and its uncertainty.

Learn how to propagate uncertainties through calculations

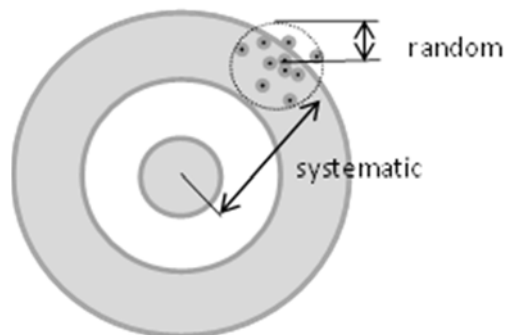
Define *mean*, *standard deviation*, and *standard deviation of the mean*.

**Equipment:** meter stick, 1 kg mass, ruler, caliper, short wooden plank, graduated cylinder

### Types of uncertainties

*Random uncertainties* occur when the results of repeated measurements vary due to truly random processes. For example, random uncertainties may arise from small fluctuations in experimental conditions or due to variations in the stability of measurement equipment. These uncertainties can be estimated by repeating the measurement many times.

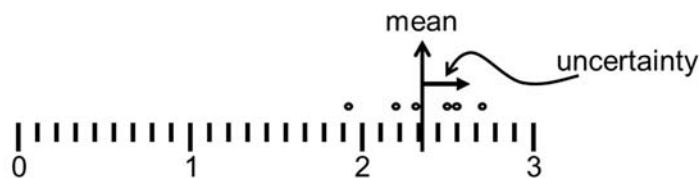
A *systematic uncertainty* occurs when all of the individual measurements of a quantity are biased by the same amount. These uncertainties can arise from the calibration of instruments or by experimental conditions. For example, slow reflexes while operating a stopwatch would systematically yield longer measurements than the true time duration.



*Mistakes* can be made in any experiment, either in making the measurements or in calculating the results. However, by definition, mistakes can also be avoided. Such blunders and major systematic errors can only be avoided by a thoughtful and careful approach to the experiment.

### Estimating uncertainty

*Repeated observation:* Suppose you make repeated measurements of something: say with a stopwatch you time the fall of a ball. Due to random variations, each measurement will be a little different. From the spread of the measurements, you can calculate the uncertainty of your results.



Shortly, we will describe the formal procedure to do this calculation. (Oddly enough, truly random uncertainties are the easiest to deal with.)

*By eye or reason:* Sometimes, repeated measurements are not relevant to the problem. Suppose you measure the length of something with a meter stick. Meter sticks are typically ruled to the mm; however, we can often read them more precisely than that.



Consider the figure above. Measuring from the left side of each mark and considering the position uncertainties of both ends of the bar, I can confidently say that the bar is  $(1.56 \pm 0.03)$  cm. Perhaps your younger eyes could read it with more confidence, but when in doubt it is better to overestimate uncertainty.

Could I do a better job by measuring several times? Not always. Sometimes with repeated measurements, it still comes down to “*Looks like  $(1.56 \pm 0.03)$  cm to me.*” But that’s ok. Your reasoned judgment is sufficient. **Science is defined by rigorous honesty, not rigorous precision!**



### Reducing random uncertainty by repeated observation

By taking a large number of individual measurements, we can use statistics to reduce the random uncertainty of a quantity. For instance, suppose we want to determine the mass of a standard U.S. penny. We measure the mass of a single penny many times using a balance. The results of 17 measurements on the same penny are summarized in Table 1.

Table 1. Data recorded measuring the mass of a US penny.

	mass (g)	deviation (g)		mass (g)	deviation (g)
1	2.43	-0.088	10	2.46	-0.058
2	2.49	-0.028	11	2.52	0.002
3	2.49	-0.028	12	2.4	-0.118
4	2.58	0.062	13	2.58	0.062
5	2.52	0.002	14	2.61	0.092
6	2.55	0.032	15	2.49	-0.028
7	2.52	0.002	16	2.52	0.002
8	2.64	0.122	17	2.46	-0.058
9	2.55	0.032			

The **mean** value  $\bar{m}$  (that is, the average) of the measurements is defined to be

$$\bar{m} = \frac{1}{N} \sum_{i=1}^N m_i = \frac{1}{17} (m_1 + m_2 + \dots + m_{17}) = 2.518\text{g} \quad (1)$$

The **deviation**  $d_i$  of the  $i$ th measurement  $m_i$  from the mean value  $\bar{m}$  is defined to be

$$d_i = m_i - \bar{m} \quad (2)$$

Fig. 1 shows a histogram plot of the data on the mass of a US penny. Also on the graph is a plot of the smooth bell curve (that is a *normal distribution*) that represents what the distribution of measured values would look like if we took many, many measurements. The result of a large set of repeated measurements (when subject only to random uncertainties) will always approach a normal distribution which is symmetrical about  $\bar{m}$ .

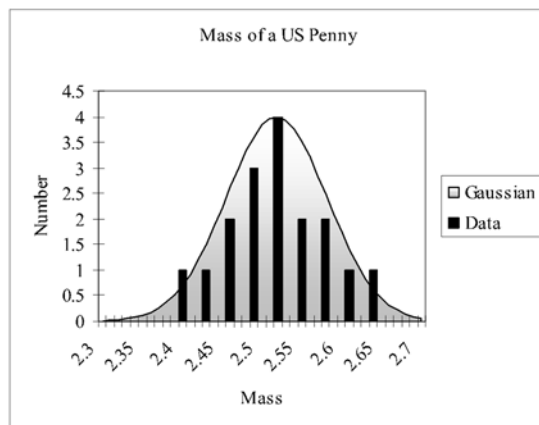


Figure 1. The Gaussian or normal distribution for the mass of a penny  $N=17$ ,  $\bar{m}=2.518$  g,  $\Delta m=0.063$  g.

*OK, now I have all of these measurements. How accurate is any one of these measurements?*

For this, we now define the **standard deviation**  $\Delta m$  as

$$\Delta m = \sqrt{\frac{\sum_{i=1}^N (m_i - \bar{m})^2}{(N-1)}} = \sqrt{\frac{\sum_{i=1}^N (m_i - \bar{m})^2}{16}} = 0.063 \text{ g} \quad (3)$$

For normal distributions, 68% of the time the result of an individual measurement would be within  $\pm \Delta m$  of the mean value  $\bar{m}$ . Thus,  $\Delta m$  is the experimental uncertainty for an *individual* measurement of  $m$ .

*The mean  $\bar{m}$  should have less uncertainty than any individual measurement. What is **that** uncertainty?*

The uncertainty of the final average is called the **standard deviation of the mean**. It is given by

$$\Delta M = \frac{\Delta m}{\sqrt{N}} \quad (4)$$

With a set of  $N=17$  measurements, our result is

$$\begin{aligned} \text{mass of a penny} &= \bar{m} \pm \Delta M = \bar{m} \pm \frac{\Delta m}{\sqrt{N}} \\ &= 2.518 \text{ g} \pm \frac{0.063 \text{ g}}{\sqrt{17}} \\ &= (2.518 \pm 0.015) \text{ g} \end{aligned} \quad (5)$$

Thus, if our experiment is only subject to random uncertainties in the individual measurements, we can improve the precision of that measurement by doing it repeatedly and finding the average.

Note, however, that the precision improves only as  $\frac{1}{\sqrt{N}}$ . To reduce the uncertainty by a factor of

10, we have to make 100 times as many measurements. We also have to be careful in trying to get better results by letting  $N \rightarrow \infty$ , because the overall accuracy of our measurements will eventually be limited by systematic errors, which *do not cancel out* like random errors do.

**Exercise 1:**

- a. With a caliper, measure the width and thickness of the plank. Make at least five measurements of each dimension and enter the result into Table 2.
- b. With a ruler, measure the length of the wooden plank on your table as precisely as possible. Estimate the uncertainty, and enter the result below.
- c. For both the width and thickness, calculate your final result (the *mean*) and the uncertainty (the *standard deviation of the mean*). Enter the final results below the table.

Table 2

width	deviation	thickness	deviation

width = \_\_\_\_\_

thickness = \_\_\_\_\_

length = \_\_\_\_\_

## Propagation of uncertainties

Usually, to obtain a final result, we have to measure a variety of quantities (*say, length and time*) and mathematically combine them to obtain a final result (*speed*). How the uncertainties in individual quantities combine to produce the uncertainty in the final result is called the *propagation of uncertainty*.

Here we summarize a number of common cases. For the most part these should take care of what you need to know about how to combine uncertainties.\*

### *Uncertainties in sums and differences:*

If several quantities  $x_1, x_2, x_3$  are measured with absolute uncertainties  $\Delta x_1, \Delta x_2, \Delta x_3$  then the absolute uncertainty in  $Q$  (where  $Q = x_1 \pm x_2 \pm x_3$ ) is

$$\Delta Q = |\Delta x_1| + |\Delta x_2| + |\Delta x_3| \quad (6)$$

In other words, *for sums and differences, add the **absolute** uncertainties.*

### *Uncertainties in products and quotients:*

Several quantities  $x, y, z$  (with uncertainties  $\Delta x, \Delta y, \Delta z$ .) combine to form  $Q$ , where

$$Q = \frac{x y}{z}$$

(or any other combination of multiplication and division). Then the fractional uncertainty in  $Q$  will be

$$\frac{\Delta Q}{|Q|} = \left| \frac{\Delta x}{x} \right| + \left| \frac{\Delta y}{y} \right| + \left| \frac{\Delta z}{z} \right| \quad (7)$$

In other words, *for products and quotients, add the **fractional** uncertainties.*

---

\* These expressions for the propagation of uncertainty are an **upper limit** to the resulting uncertainty. In this case, you could actually do better. See Appendix C for details.

**Exercise 2:**

- d. Calculate the total volume of the block and the associated uncertainty. Show your math below.

**Exercise 3:**

You can also find the volume of an object by measuring the volume of water displaced by the object when it is submerged.

- e. Measure the volume of water in the graduated cylinder and the associated uncertainty.
- f. Submerge the block (holding it under the surface with a pen or pencil), and find the resulting volume and the associated uncertainty.
- g. Calculate the volume of the plank and the associated uncertainty. Show your math below.
- h. Is this answer consistent with your result from Exercise 2? Explain.

**Exercise 4:**

- a. Calculate the total surface area of the block and the associated uncertainty. Show your math below.





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## Pre-Lab Preparation Sheet for Lab 2: Position, Velocity, and Acceleration in one-dimensional motion

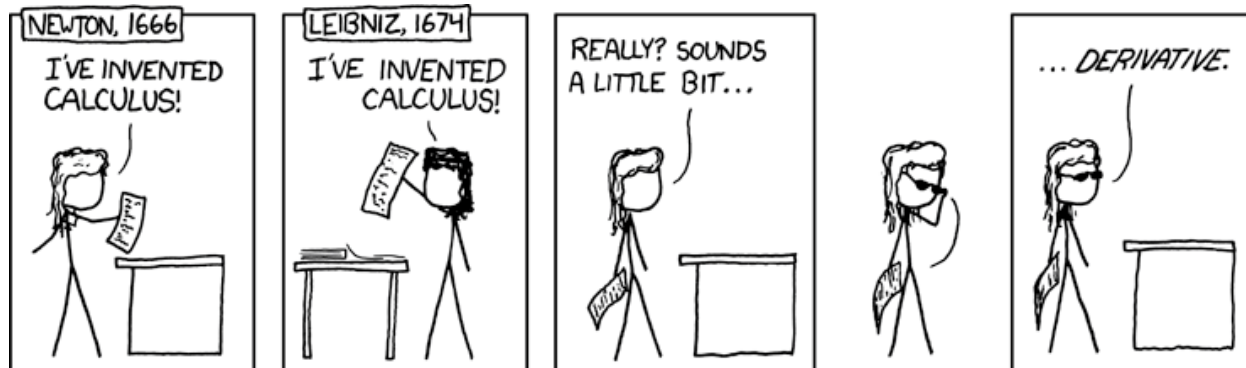
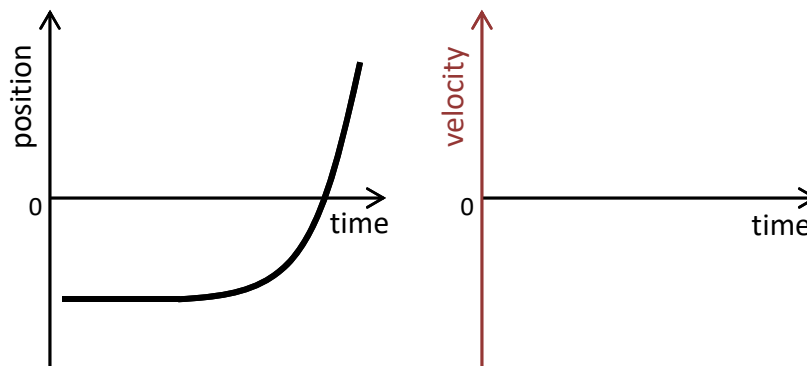
(Due at the Beginning of Lab)

Watch the video *Introduction to Optimization and Curve Fitting* found at the following site:

[my.vanderbilt.edu/physicslabs/videos/](http://my.vanderbilt.edu/physicslabs/videos/)

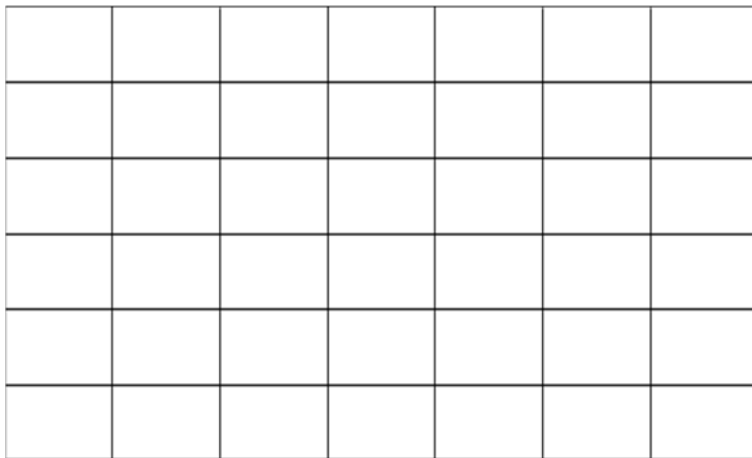
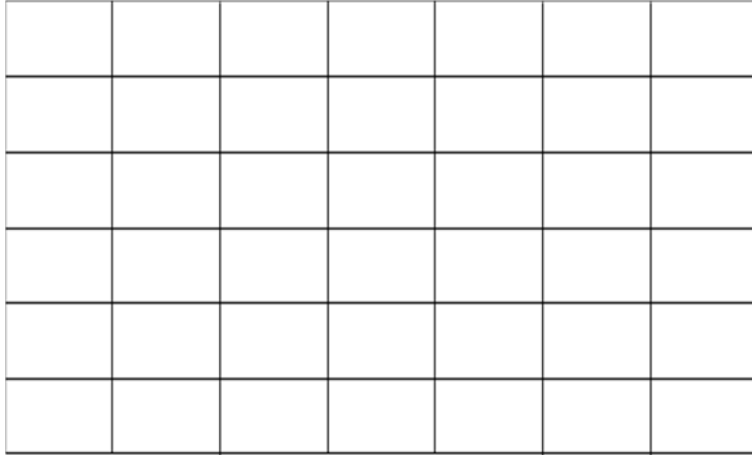
Read over the lab and then answer the following questions

- Given the following position curve, sketch the corresponding velocity curve.



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2. Imagine kicking a box across the floor: it suddenly starts moving then slides for a short distance before coming to a stop. Make a sketch of the position and velocity curves for such motion.



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## Lab 2: Position, Velocity, and Acceleration in One-Dimensional Motion

*"God does not care about our mathematical difficulties. He integrates empirically."  
 --Albert Einstein*

### Objectives:

- To understand graphical descriptions of the motion of an object.
- To understand the mathematical and graphical relationships among position, velocity and acceleration

### Equipment:

- 2.2-meter track w/ adjustable feet and end stop
- A block to raise one end of the cart
- Motion sensor
- Torpedo level
- PASCO dynamics cart and friction cart

### DISCUSSION

Velocity is the rate of change or time derivative of position.

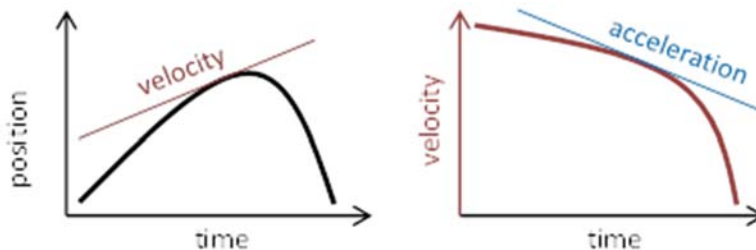
$$\vec{v} = \frac{d\vec{x}}{dt} \quad (1)$$

On a Cartesian plot of position vs. time, the slope of the curve at any point will be the instantaneous velocity.

Likewise, acceleration is the rate of change or time derivative of velocity (the 2<sup>nd</sup> derivative of position).

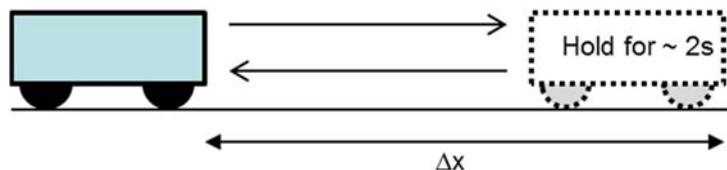
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2} \quad (2)$$

On a Cartesian plot of velocity vs. time, the slope of the curve at any point will be the instantaneous acceleration.

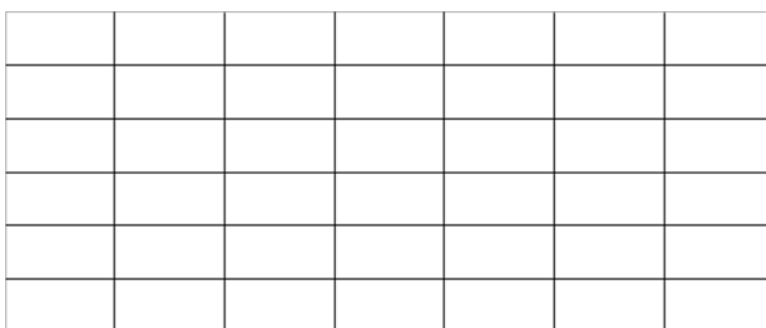





Thus, the shape of any one curve (position, velocity, or acceleration) can determine the shape of the other two.

### Exercise 1: Back and Forth



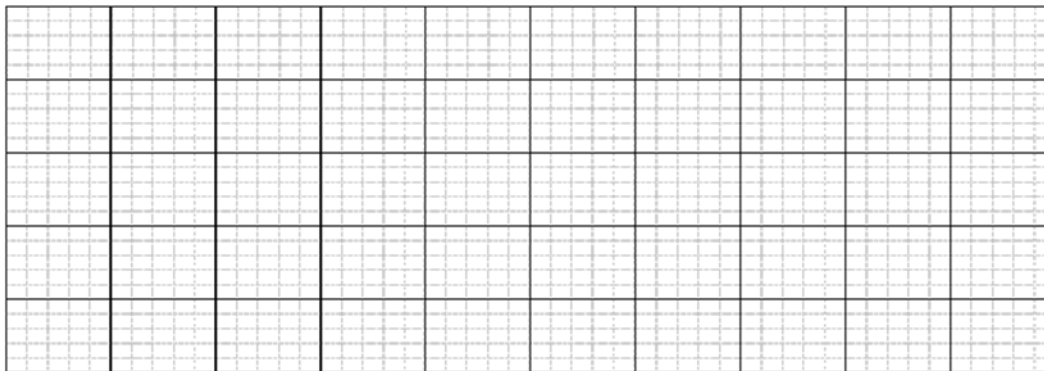
- a. Place the friction cart on the track. (That is the one with the friction pad on the bottom. Without letting go of the cart, quickly push it toward the detector by about a foot, then stop it for 1 or 2 seconds. Then quickly but smoothly return the cart to the starting point. Note the distance it travels, and sketch the position vs. time curve for the block on the plot below.




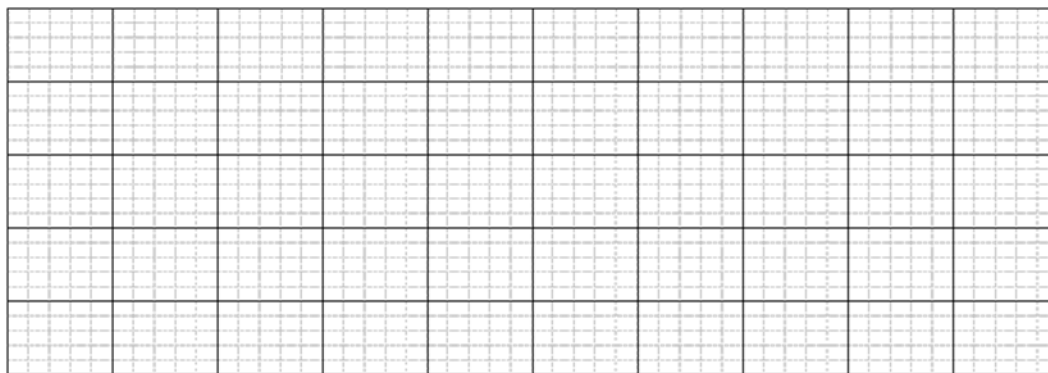
- b. Now, open the Labfile directory found on your computer's desktop. Navigate to A Labs/Lab2 and select the program Position. The PASCO Capstone program should open and present you with a blank position vs. time graph.
- c. Click the **Record**  button (lower left side of the screen), and repeat the experiment above. Click **Stop** to cease recording data. Note how the PASCO plot compares to yours. **Note:** The cart may bounce or stutter in its motion. If you don't get a smooth curve, delete the data\* and repeat the run with more Zen†.
- d. By clicking the scaling icon  (top left corner of the Graph window) you can better fill the screen with the newly acquired data.
- e. Select the slope icon . A solid black line will appear on the screen. By dragging this line to points along the plot, you can measure the slope of the curve at those points. Using this tool, find the steepest part of the curve (that is, the largest velocity). Then, sketch the velocity curve for the block in the graph below. Add appropriate numbers to the x and y axes.


\* To delete data: click the icon on the bottom of the screen.

† “*This time, let go your conscious self and act on instinct.*” Obi-Wan Kenobi



- f. How does the *shape* of the position curve determine the *sign* of the velocity curve?
- g. Now, let's see how well you drew it! Double-click on the new plot icon  (middle top of the screen) and select **Velocity (m/s)** for the y-axis. Note the shape and position of the curve and see how well it matches your sketch. Also note how it aligns with the position curve.
- h. Use the slope tool to find the changing slope along the velocity curve. With this information, sketch the acceleration curve for the block. Again, appropriately mark the axes.




- i. Let's see what PASCO says about the acceleration. Again, create a new plot  and select **Acceleration ( $m/s^2$ )** for the y-axis. Compare it to your acceleration curve and PASCO's velocity curve.
- j. How does the *shape* of the position curve determine the *sign* of the acceleration curve?
  
- k. Print out the three PASCO plots. Annotate these plots to show the times when the push began, when the push ended, when it was slowing, and when it stopped.

### Exercise 2: Skidding to a Stop

Delete your previous runs. (Bottom bar, Experiment, Delete ALL Data Runs). With a left click of the mouse, you can remove the slope tools.

- a. Move the cart to end of the track opposite the detector.
- b. Start recording data, then give the cart a quick, firm push so that it slides a few feet before coming to rest. Then, stop the data acquisition.
 


By clicking the scaling icon , you can better fill the screen with the newly acquired data. You can also adjust the scale by clicking and dragging along the x or y axis, or zooming with the scroll wheel.
- c. Again, if the plotted data is not reasonably smooth, delete the data and repeat the experiment with more Zen.
- d. Print out the curves and annotate on the graphs with the times when the push began, when the push ended, and when the cart was sliding on its own.



- i. Similarly, apply a numerical fit to the position data. Record the results below. Are the results consistent with the velocity and acceleration curves? (That is, do the uncertainties overlap?)



**Exercise 3: Up and Down**

- a. Place a block under one of the track stands to form a ramp. The detector must be on the raised end.
- b. Place a low friction cart on the track and give it a push so that it rolls a few feet up the incline and then rolls back. After a few practice runs, run the detector and acquire motion data.
- c. With a click and drag of the mouse, highlight that section of the data where the cart is freely rolling along the track. Then use the scaling tool  to zoom-in on that section of the data.
- d. Print out these plots and annotate the graphs with the following information.
  - When and where does the velocity of the cart go to zero?
  - When is the velocity positive, and when is it negative?
  - What is the acceleration when the velocity is zero?
  - Does the acceleration of the cart go to zero? If so, where?
- e. Find the average acceleration going up the slope and the average acceleration going down the slope. Record the results below.
  
- f. How does the acceleration up the slope compare with the acceleration down the slope? What might account for the difference?



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**Pre-Lab Preparation Sheet for Lab 3:**  
**Force, Mass, and Acceleration**  
(Due at the Beginning of Lab)

**Directions:**

Read over the lab and then answer the following questions.

Consider the experimental configuration shown in Figure 1.

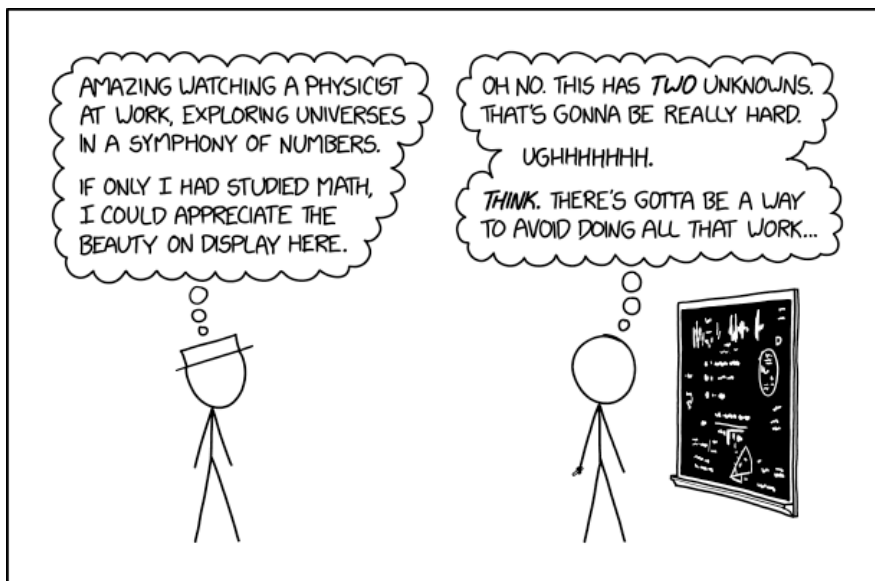
1. Draw and label the free body diagram.

2. Starting from Newton's 2<sup>nd</sup> law, show that the acceleration of the cart is given by:

$$a = \frac{mg}{m + M} .$$

3. In the limit\* where  $M \ll m$ , what is the acceleration?

4. In the limit where  $M \gg m$ , what is the acceleration? (Hint: the answer is *not* zero.)



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\* Notation:  $\gg$  means *much greater than*;  $\ll$  means *much less than*.

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### Lab 3: Force, Mass, and Acceleration

*“Well, the Force is what gives a Jedi his power . . . It surrounds us and penetrates us. It binds the galaxy together.”*

*-- Obi-Wan Kenobi on Newton's 2<sup>nd</sup> Law*

#### Equipment

Motion sensor	Force sensor
Low-friction cart	2.2 meter track
Torpedo level	Low –friction pulley
Foam crash pad	0.02, 0.05, 0.10, and 0.20 kg hooked masses

**Note:** The acceleration due to gravity varies with location. Here at Vanderbilt, this acceleration has been measured as

$$g = (9.7943 \pm 0.0032) \frac{m}{s^2}$$

Use this value throughout the semester.

#### Introduction

Newton's 2<sup>nd</sup> Law is the most important concept you will learn in this class:

$$\vec{F}_{net} = m\vec{a}$$

If you know the **mass** and **net force** on an object, you know the **acceleration** of the object.

If you know the **acceleration** of an object and its **initial velocity** and its **initial position**, you know the complete **trajectory** of the object.

Consider the problem illustrated below: a frictionless wheeled cart is pulled by string attached to a falling mass.

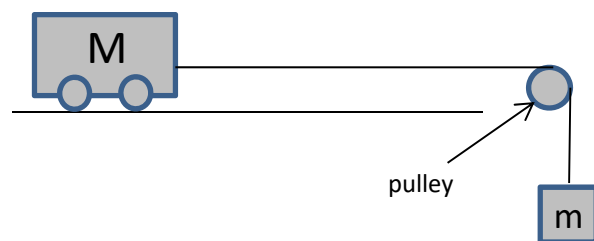


Fig. 1

It can be shown that the acceleration  $a$  is given by

$$a = \frac{mg}{M + m} \quad (1)$$

Of course, that is the theory. In the real world, things can get messy. Welcome to experimental physics.

### Exercise 1: Data Acquisition

1. Label the individual forces on the diagram above.
2. Mass the combined cart and force sensor with the electronic scale

$$M = \underline{\hspace{10cm}}$$

**Note:** Before each measurement with the scale, you should *tare* the scale. Empty the scale, then press the Z or TARE button found on the panel. This resets the zero point of the scale.

You will be using a set of hooked weights for your falling masses. These have nominal values of 0.020 kg, 0.050 kg, 0.100 kg, 0.200 kg.

3. Measure their precise masses with the digital scale and record the results and associated uncertainties on Table 1 below.

You will use the PASCO Force Sensor to measure the tension in the string. Note the sign convention for the direction of the force on the hook.

Assemble the cart, force sensor, string, and falling mass. The falling mass will be one of four hooked weights: 0.020 kg, 0.050 kg, 0.100 kg, 0.200 kg. **Verify that a crash pad is positioned underneath the falling mass.**

4. Run the Capstone program Velocity&Force.

**Note:** Before each measurement with the force sensor, you must *tare* the force sensor. Remove any force from the hook, then press the TARE

button found on the side of the sensor. This will ensure that when the force is zero, the device returns zero.

5. Holding cart stationary, measure the static tension  $T_{static}$  on the string and the corresponding uncertainty. Record the result in Table 1 below.

**Note:** Do not assume that every digit which Capstone reports is significant.

6. **Dedicate one of your members to catching the cart before it crashes into the pulley.** Please, do not turn this into a projectile motion lab!
7. Start recording, then release the cart. Record the dynamic tension on the string  $T_{dynamic}$ , the acceleration  $a$ , and the associated uncertainties. Justify your determination of each of these uncertainties below.

Table 1: Measured values

$m$	$T_{static}$	$T_{dynamic}$	$a$

8. Why are  $T_{dynamic}$  and  $T_{static}$  different?

**Exercise 2: Data Analysis**

While you have measured the acceleration above, it can also be calculated from Equ. 1, or from the measured tension of the string and the cart mass.

9. Fill in the table below. Include the associated uncertainties

Table 2: Calculated accelerations

<i>a from Equ. (1)</i>	<i>a from T/M</i>

10. Are the measured values of  $a$  (from Table 1) consistent with the accelerations calculated from Equ. (1) (from Table 2)? Explain.



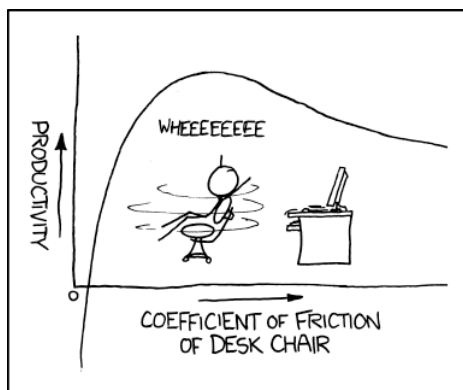


13. Thus far, we have been blithely ignoring friction of the cart. Given that friction is present, how would that effect the acceleration of the cart?

14. Would the addition of a constant friction force resolve any discrepancy between the observed acceleration and  $\frac{mg}{M + m}$ ? Draw the free body diagram and apply the 2<sup>nd</sup> Law.



3. Show that basic logic requires that  $\mu_s \geq \mu_k$ .



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## Lab 4: Static and Kinetic Friction

*Q: How does a physicist milk a cow?*

*A: First, let's assume a spherical cow . . . .*

*--old physics joke (Physicists find this hilarious. \*)*

### Equipment

2.2 meter track	Force sensor
Motion sensor	Mass set
Wooden block	Masonite board

### Introduction

Friction is incredibly complicated. A full science discipline (tribology) is devoted to it. Its microscopic origins remains a subject of high level research. Nevertheless, in many cases, a very simple model (a so-called *spherical cow*) describes the behavior of friction with surprising accuracy:

If we have two solid materials in contact, we can define two types of friction:

**Static friction: the force that keeps two surfaces from slipping.** The force of static friction is parallel to the surface. Its magnitude is whatever is required to prevent slippage. Its maximum magnitude is the coefficient of static friction  $\mu_s$  times the normal force  $n$ .

$$0 \leq F_s \leq \mu_s n \quad (1)$$

**Kinetic friction: a constant force between two slipping surfaces.** The force is parallel to the surface, and its direction is opposite the motion. The magnitude is

$$F_k = \mu_k n \quad (2)$$

This model is purely empirical; there is no underlying explanation or principle. The coefficients ( $\mu_s$  and  $\mu_k$ ) must be measured on a case by case basis. But this naive model happens to work well enough to be extremely useful. Note that this model makes several assumptions:

- Friction is binary: static or kinetic, nothing else.
- Kinetic friction is independent of both contact area and velocity.

---

\* Sad, I know.

- While  $\mu_s \geq \mu_k$ , the coefficients are otherwise unrelated.

Table 1 provides a short list of examples.

Table 1: Friction coefficients

<i>Material 1</i>	<i>Material 2</i>	$\mu_s$	$\mu_k$
glass	glass	0.9 – 1.0	0.4
glass	nickel	0.78	0.56
rubber	concrete	1.0	0.8
Teflon™	Teflon™	0.04	0.04

### Experiments

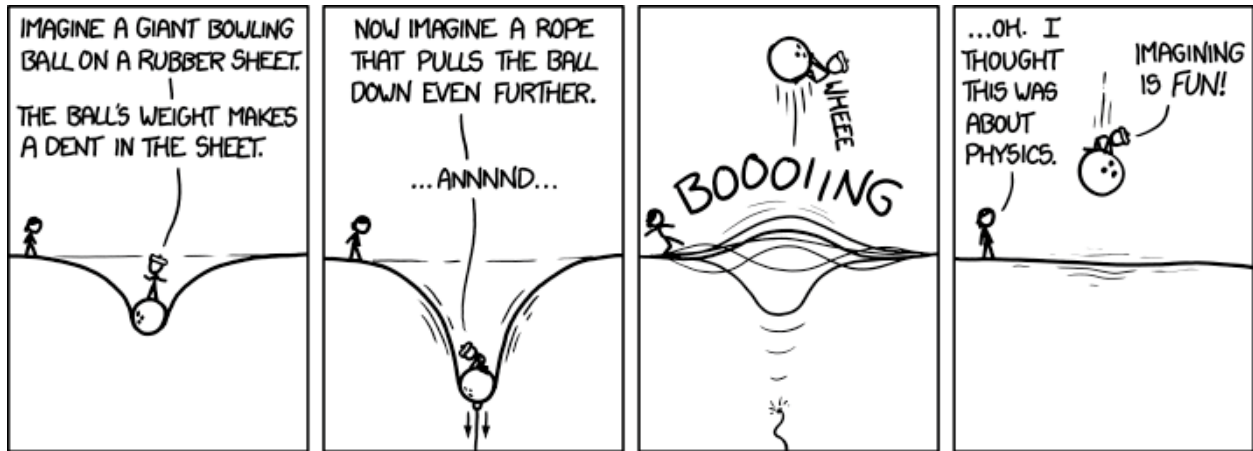
1. Along with your lab partners, develop a procedure to measure the coefficient of static friction and corresponding uncertainty for the wooden block and the Masonite\* board. Clearly describe your procedure and present your results.
2. Along with your lab partners, develop a procedure to measure the coefficient of kinetic friction and corresponding uncertainty for the wooden block and the Masonite board.
3. **Experimentally** verify your answers to the following questions:
  - a. Is  $f_k$  independent of the velocity?
  - b. Is  $f_k$  independent of the surface area?

Clearly describe your procedure and present your results.

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\* A type of glorified cardboard.





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## Lab 5: Scaling and the Properties of Elastic Materials

*To see a world in a grain of sand  
 And a heaven in a wild flower,  
 Hold infinity in the palm of your hand,  
 And eternity in an hour.*

--William Blake

### GOALS AND OBJECTIVES

- To develop a general appreciation of the effects of scale on the properties of materials.
- To determine the domain of applicability of Hooke's Law for particular objects.
- To recognize the difference between material elastic properties and object elastic properties.
- To predict the elastic properties of an object of arbitrary scale.

### EQUIPMENT

2-meter measuring stick	Clamp, mount and 1.5-m long rod
Rubber strips and bands	Clips
SONAR reflector	Hanging masses
Motion sensor	Force sensor
Right-angle clamp and rod for mounting the force sensor	
Safety goggles	

### Galilean Scaling

In his final book, *Discourses and Mathematical Demonstrations Relating to Two New Sciences*<sup>\*</sup>, Galileo described the mathematics of scaling. Consider two dogs, Bruno and Pip. Bruno is exactly  $3\times$  the size (in every direction: height, depth, and width) as Pip, but is otherwise identical. Clearly, Bruno will be heavier. Since mass is proportional to **volume**, Bruno's weight will be  $3^3$  or  $27\times$  the weight of Pip.

Of course, Bruno will also be stronger, but how much stronger? The strength of a bone is proportional to the **area** of the bone's cross section. Thus he will only be  $3^2$  or  $9\times$  stronger than Pip. *Pound for pound, Pip is much stronger than Bruno!* This is the problem of *scaling*: an apparently simple change in size can

---

<sup>\*</sup> A remarkably boring title, but considering what he went through after publishing *The Starry Messenger*, that can be forgiven.

lead to surprising changes in an object's other properties.



An illustration from Galileo's book.

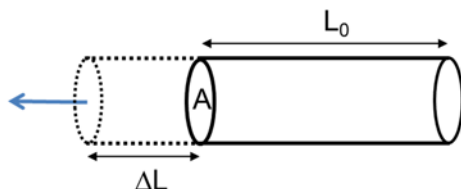
To make up for this scaling problem, the bones of large animals must be proportionally much thicker than the bones of small animals. King Kong could never exist. He would collapse under his own weight. Likewise, ants are famously strong for their size, able to lift objects many times heavier than themselves. This is no great mystery but a straightforward consequence of scaling.

### Rubber Bands

Different systems scale in different ways. In this lab, you will study the scaling properties of rubber cords. (And, you will learn a few other things about rubber as well.)

For a wide range of elastic materials, the relationship between force and deformation is well described by Hooke's Law:

$$F_s = -k\Delta L \quad (3)$$



However, this is not a *Law* as much as *Spherical Cow*: an approximation that is acceptable over a limited range of deformation.\* Within this limited range, one can characterize an object's elastic properties by its spring constant  $k$ .

### Exercise 1: Inventory

Use the calipers to carefully measure the thickness of each rubber band. Take care not to compress the bands as you measure them. You want the relaxed dimensions.

---

\* Stretch it too much and Murphy's Law takes over.

	Diameter (m)	cross-sectional area (m <sup>2</sup> )
Small	_____	_____
Large	_____	_____

**Exercise 2: Scaling the Spring Constant with length**

Use the large cord for the following.

1. Place the two hose clamps on the cord separated by 1.00 m.
2. Hang one clamp from the force sensor. Hang the sonar reflector from the other clamp.
3. Start the program Spring Constant. Then, gently pull the cord ~2 cm.

Stop acquiring data before releasing the cord.

4. Repeat two more times.
5. Find the average spring constant.

measured  $k$  (1.00 m) = \_\_\_\_\_

6. Given your pre-lab prediction for how the spring constant should scale with length, estimate the spring constants for 0.50 m and 0.25 m cords.
7. Now, measure these spring constants as you did for the 1.00 m cord.

	estimated	measured	% diff
$k$ (0.50 m)	_____	_____	_____
$k$ (0.25 m)	_____	_____	_____

Strictly speaking, a spring constant only conveys information about a particular spring or cord. A parameter that was independent of length would be more generally applicable.

8. Rewrite the spring constant as a function of the length of the cord. (*I'm looking for an expression with numbers.*)

$$k(L) =$$

### Exercise 3: Scaling in all dimensions

Now, suppose you had a double cord (two cords side by side)

9. Estimate the spring constant for two parallel cords 0.50 m long. Then measure it.

estimate

measured

% diff

$k_{\text{double}}(0.50 \text{ m})$

---

Now, consider changing the diameter of a single cord.

10. How will the spring constant scale with the diameter? For example, will it scale proportionally? Or with the square? Or with the cube?

11. Write an expression for the spring constant as a function of the diameter of the cord,  $k(D)$ . (*I'm looking for a number here.*)

12. Write an expression for the spring constant of a cord with arbitrary diameter and length.

13. Estimate the spring constants for the small cord with lengths of 0.50 m and 0.25 m.

	estimate	measured	% diff
$k_{\text{small}} (0.50 \text{ m})$	_____		
$k_{\text{small}} (0.25 \text{ m})$	_____		

**Exercise 4: Wild extrapolation**

14. How should the spring constant of a cord scale with uniform changes in its over-all size (length, and diameter)?

**Exercise 5: Professor, I Shrank My Lab Partner!**

Imagine your lab partner is playing with a paddle ball.\* Out of nowhere, Rick Moranis<sup>†</sup> zaps her with his shrink ray making her and the toy 100 times smaller (in all three dimensions). Setting aside her psychological trauma, how does this change of scale affect her game-play? Discuss this with your partners, and then your TA. (Focus on toy; don't worry about the girl's strength.)



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\* For those of you unfamiliar with toys from the mid-20<sup>th</sup> century, it consists of a rubber ball attached to a paddle by a thin rubber band. You then whack the ball repeatedly with the paddle, sort of like playing Ping-Pong with yourself. In the pre-iPhone era, this was regarded as fun.

<sup>†</sup> You know, the nerdy father from *Honey, I Shrank the Kids*. (Yeah, I'm old.)





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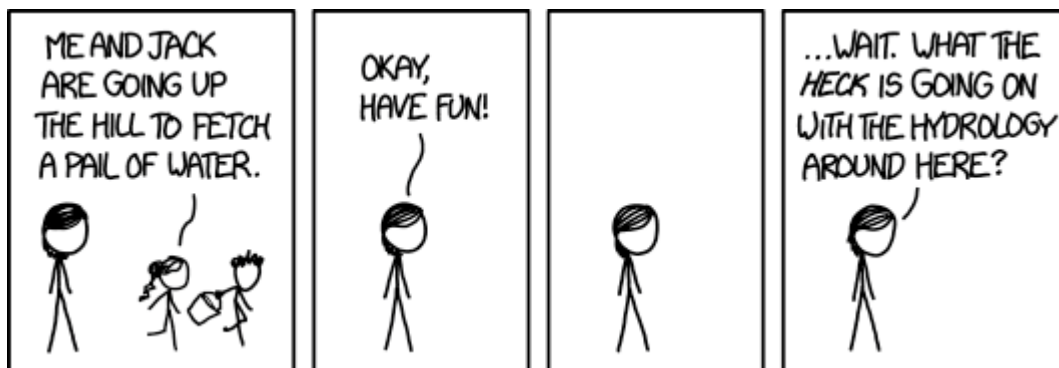
**Pre-Lab Preparation Sheet for Lab 6:****Energy, Work, and Power**

(Due at the Beginning of Lab)

1. Power is defined as the time derivative of work.

$$P \equiv \frac{dW}{dt}$$

Given this and the definition of Work, derive an expression for Power as a function of velocity. Show your algebra.



xkcd.com

2. A constant friction force  $f$  is applied to a moving block of mass  $m$  causing it to slow down.

a. Write an expression for the Work of friction as a function of time:  $W_f(t)$  .

b. Write an expression for the Power provided by friction as a function of time.

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## Lab 6: Energy, Work, and Power

*“Capital isn’t this pile of money sitting somewhere, it’s an accounting construct.”*

*--Bethany McLean (The same can be said of energy.)*

### Equipment

2.2 meter track	Force sensor
Motion sensor	Friction cart
Frictionless cart	
0.1 kg hooked mass	

### Introduction

Energy is like money. An energy problem is essentially an accounting problem

- A system starts with some initial energy.
- Energy can then be added to the system. (That is, a force can do positive work on the system.)
- Energy can be removed from a system. (That is, a force can do negative work on the system.)
- Energy can be transformed from one form into another. (Such as kinetic into potential energy.)

Your job in this lab is to do the accounting: verify that your assets, earnings, and losses add up. Let’s say that the total energy of a system is its mechanical energy  $ME$ .\*

$$\begin{aligned} ME &\equiv \textit{kinetic energy} + \textit{potential energy} \\ &= K + U \end{aligned} \quad (1)$$

Then, the accounting equation is simply

$$ME_f = ME_i + (\textit{positive work on the system}) - (\textit{negative work on the system}) \quad (2)$$

Work is the change in energy on an object due to an applied force. For a tiny step  $d\vec{s}$ ,

$$\begin{aligned} dW &= \vec{F} \cdot d\vec{s} \\ &= F_x dx + F_y dy + F_z dz \end{aligned} \quad (3)$$

---

\* We could also include the thermal energy, chemical energy, electro-magnetic energy, magical energy, or monolith energy, but let’s keep it simple for now.

Integrating over the path of motion, we have

$$W = \int_{\vec{s}_i}^{\vec{s}_f} \vec{F} \cdot d\vec{s} \quad (4)$$

$$= \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

The sign of the work is determined by relative directions of the applied force and the motion.

**Note:** the sign conventions of your several measuring devices have no respect for each other or Equation (4). Do not naively accept what the computer is telling you. Instead, figure out the sign of the work yourself.

### Exercise 1

Note the design of the friction cart. Here, the normal force on the friction pad is from a spring force and *not* the weight of the cart.

Combine the friction cart and force sensor.

1. Level the track.
2. Measure the friction force on the cart:

$$f = \underline{\hspace{10cm}}$$

3. Level the track and tare the force meter.
4. Select the Velocity and Force tab on the Capstone program.
5. Gently pull the cart about a meter away from the motion detector. Slowly vary the applied force so that the force vs. position curve is a little wavy. Then, gently bring the cart to a stop

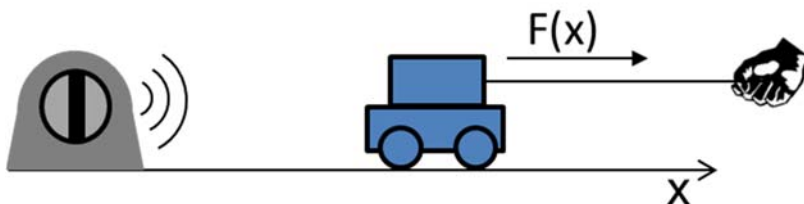



Fig. 1

6. Using the integration tool , measure the work of your hand pulling the cart.

$$W_h = \underline{\hspace{10cm}}$$

7. Calculate the work of friction on the cart. (Show your calculation.)

$$W_f = \underline{\hspace{10cm}}$$



8. Calculate the percent difference between the work of friction and the work of your hand.

9. Now, define the *final* condition as the point of maximum velocity. Verify Equation (2) for this point by calculating the percent difference between the left and right hand sides of the equation.

Power is the time derivative of work:

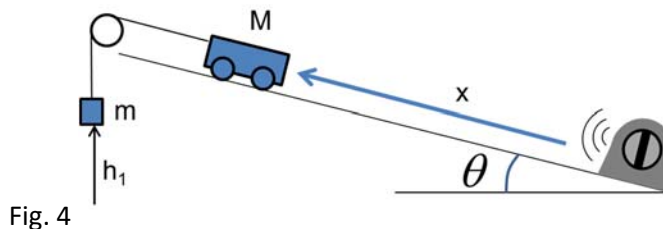
$$\begin{aligned} P &= \frac{dW}{dt} \\ &= \frac{\vec{F} \cdot d\vec{x}}{dt} = \vec{F} \cdot \vec{v} \end{aligned} \quad (5)$$

Using the calculator function in Capstone, calculate the power supplied by your hand.

- a. Select the Calculator icon  from the left side toolbar.
- b. Enter the expression for power you found in the pre-lab. Use the right mouse button to select you parameters (Force, etc.) from the data.
- c. Create a new plot  and plot the resulting power vs. position curve.
- d. Print out the resulting velocity, force and power plots. Annotate these plots to indicate
  - The beginning and end of the motion.
  - The maximum velocity.
  - The maximum force of your hand.
  - The maximum power of your hand.

**Exercise 2**

1. Tilt the ramp to about  $15^\circ$ . Measure the actual angle as accurately as possible.
2. Using the frictionless cart and a 0.100 kg hanging mass, assemble the system shown below.  
**NOTE:** For this apparatus, accurately measuring the angle of the ramp and the friction is relatively difficult. However, measuring acceleration is relatively easy.



3. Write an algebraic expression describing the initial energy of the system in terms of  $M$ ,  $m$ ,  $x$ , and  $\theta$ .
4. Write an algebraic expression describing the total mechanical energy of the system in terms of  $M$ ,  $m$ ,  $x$ ,  $\theta$ , and  $v$ .
5. Open the program *Work & Energy.cap*. You will find that several parameters have been created using the Calculator function in Capstone. However, there are several mistakes. Correct these mistakes so that Capstone accurately displays the Kinetic ( $K$ ), Potential ( $U$ ), and Mechanical Energies ( $ME$ ) of the system.
6. While acquiring data, release the cart from rest and let it roll down the ramp. Print out the resulting plot. Are the results consistent with a nearly frictionless system? Explain.

7. Replace the frictionless cart with the frictional cart and repeat the experiment. Print out the resulting plot.
8. What aspect of this plot relates to the power consumed by friction?
9. From this plot, find the maximum power consumed by friction. Explain your procedure.
10. From this plot, find the force of friction. Explain your procedure.



11. Does this match the friction you measured earlier?



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**Pre-Lab Preparation Sheet for Lab 7:  
Momentum**

(Due at the Beginning of Lab)

**Directions:**

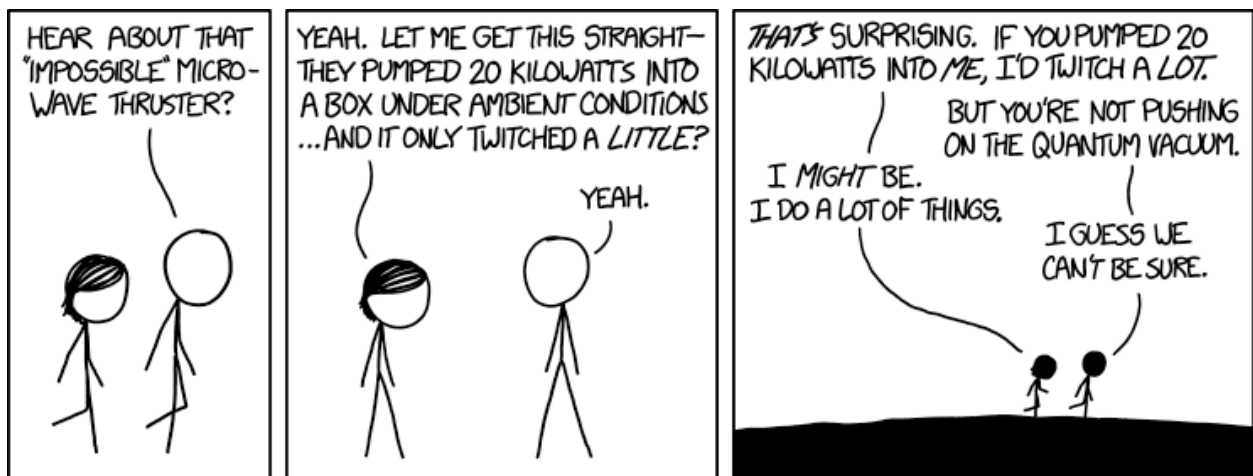
Read over the following lab, then answer the following questions.

1. Wile E. Coyote steps off a cliff edge. He reasons that
  - a. his vertical momentum is zero,
  - b. momentum is conserved,  
therefore . . .
  - c. he cannot fall.

Correct his reasoning. How is momentum conserved in this case?

2. Define *closed system*.

3. Algebraically show that if two carts of identical mass collide and stick, kinetic energy cannot be conserved.



xkcd.com (If you don't get this, Google "resonant cavity thruster".)

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## Lab 7: Momentum

*When he was within six miles of the place,  
There Number Four stared him straight in the face.  
He turned to his fireman, said Jim you'd better jump,  
For there're two locomotives that are going to bump.*

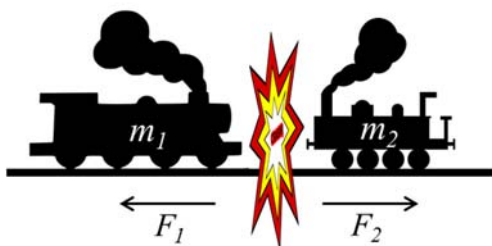
*--The Ballad of Casey Jones, variant by  
Eddie Newton and T. Lawrence Seibert*

**Objective:** To observe conservation of momentum and conservation of kinetic energy (or not).

**Equipment:** 2m track, magnetic cart, plunger cart, two motion sensors, 2 ( $\frac{1}{2}$  kg) cart masses

### The 3<sup>rd</sup> Law and Momentum

Two locomotives of masses  $m_1$  and  $m_2$  collide. During the collision, each locomotive exerts an equal but opposite force on the other.  $\vec{F}_1$  is the force on  $m_1$  from  $m_2$ .  $\vec{F}_2$  is the force on  $m_2$  from  $m_1$ .



Newton's 3<sup>rd</sup> Law: For every action there is an equal and opposite reaction.

$$\begin{aligned}
 \vec{F}_1 &= -\vec{F}_2 \\
 m_1 \vec{a}_1 &= -m_2 \vec{a}_2 \\
 m_1 \vec{a}_1 + m_2 \vec{a}_2 &= 0 \\
 m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} &= 0 \\
 \frac{d}{dt}(m_1 \vec{v}_1 + m_2 \vec{v}_2) &= 0
 \end{aligned} \tag{6}$$

Hence, the quantity  $(m_1 \vec{v}_1 + m_2 \vec{v}_2)$  does not change with time. If we define *momentum* to be\*

$$\vec{p} \equiv m\vec{v} \quad , \tag{7}$$

we have the following principle:

**The total momentum of a closed system is constant.**

$$\vec{p}_1(t) + \vec{p}_2(t) = \vec{p} \tag{8}$$

### Important things to note:

- The *total momentum* of a closed system is always conserved. No exceptions. If the momentum of your system is changing, your system is not closed; something else is meddling with it.
- Momentum is a vector. The direction matters.
- There is no such thing as *kinetic momentum* or *potential momentum* or *internal momentum*. There is only *momentum*.<sup>†</sup>

### Types of collisions

We can define three basic types of collisions:

- **Totally elastic collision:** Two or more objects collide. Afterward, the total kinetic energy is conserved.

$$\begin{aligned}
 K_i &= K_f \\
 \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 + \dots &= \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 + \dots
 \end{aligned}$$

- **Totally inelastic collision:** Two or more objects collide and stick together after the collision. This implies that kinetic energy is not conserved.

$$K_i \neq K_f$$

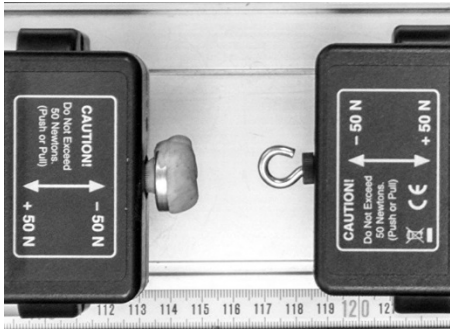
\* Because representing momentum with  $\vec{p}$  makes perfect sense.

† Angular momentum (which you will study later) never transforms into momentum, and momentum never transforms into angular momentum.

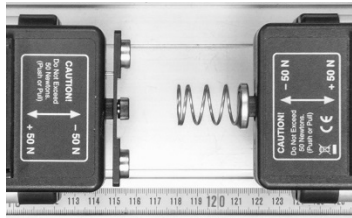
- **Partially elastic collision:** The objects do not stick together after the collision, but kinetic energy is not conserved.

$$K_i \neq K_f$$

### Exercise 1: fully inelastic collisions

1. Attach the hook to one force sensor. Attach the clay to the other.
  2. Level the track.
  3. Run the program Collisions.cap. Note that this program defaults to measuring position and force as functions of time, but you are welcome to change these axes as you see fit.
  4. Print out the Collisions plot. Then annotate the plot noting the duration of the collision, and the change of velocity of each cart.
- 
5. Conceive and execute a procedure to verify conservation of momentum. Briefly describe your procedure and present your results and your percent uncertainties. Are your measurements of the initial and final momenta consistent with conservation of momentum?

### Exercise 2: Elastic Collisions

6. Replace the clay and hook with a spring and plate.
  7. Print out the Collisions plot. Then annotate the plot noting the duration of the collision, and the change of velocity of each cart.
- 
8. Conceive and execute a procedure to verify conservation of momentum and conservation of kinetic energy. Briefly describe your procedure and present your results and your deviations.
    - a. Are your measurements of the initial and final momenta consistent with conservation of momentum?
    - b. Are your measurements of the initial and final kinetic energies consistent with conservation of kinetic energy?

### Discussion

9. The position sensors work by assuming that the speed of sound is 344 m/s and measuring the time for the echo to bounce off the target. Doubtless, this speed is off by a bit. How would the uncertainty in the speed of sound affect your determination of the conservation of momentum or kinetic energy? Explain.

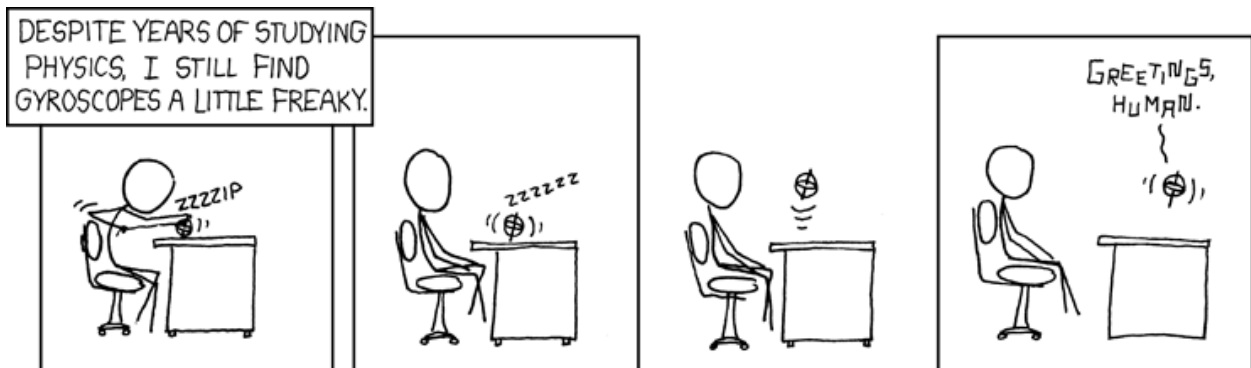






3. A car is traveling North. In which direction are the wheels turning?

4. How is the  $z$ -axis of a Right Handed coordinate system related to the  $z$ -axis of a Left Handed coordinate system?



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## Lab 8: Torque and Rotational Inertia

*“You make the rocking world go round”*  
 --B. May *et al.*, *Jazz*, 2 (1978).

### Objectives

- To relate an objects rotational inertia to its geometry
- To understand Newton’s laws in terms of rotational motion
- To observe the conservation of energy in terms of rotational kinetic energy

### Equipment

- PASCO Rotational Motion Sensor with aluminum holder
- Calipers
- thread
- electronic balance
- Flat aluminum disk
- 10 g mass
- Table clamp, 2 rods, and right angle clamp
- Gyroscope, gimbal, and motor

### DISCUSSION:

Angular velocity  $\vec{\omega}$  and angular momentum  $\vec{L} = I\vec{\omega}$  are vectors. The direction of these vectors is given by the right hand rule. The equations describing Newton’s 2<sup>nd</sup> law for rotational motion are fully analogous to the equations for linear motion. A net torque on an object will change its angular velocity  $\vec{\omega}$  :

$$\vec{\tau} = I \frac{d\vec{\omega}}{dt} = I\vec{\alpha} \quad (1)$$

where  $I$  is the rotational inertia\*, and the angular acceleration  $\alpha = \frac{d\vec{\omega}}{dt}$ . We may also write the 2<sup>nd</sup> Law to relate the net torque to change angular momentum  $\vec{L}$ .

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad (2)$$

---

\* Also known as the moment of inertia

Thus, the direction of the applied torque is always in the same direction as the resulting angular acceleration and the change in the angular momentum.

If multiple objects are attached together, the total rotational inertia is the sum of the individual moments of inertia.

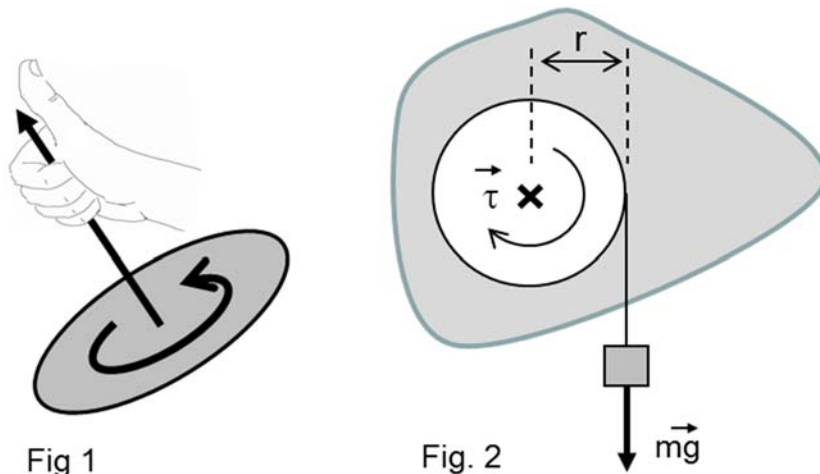


Fig 1

Fig. 2

Consider Fig. 2. An arbitrary object (in this case, the lump sketched above is attached to a pulley). A torque is applied to the object by a hanging weight from the pulley. The net torque on the system is

$$\begin{aligned}\vec{\tau} &= I\vec{\alpha} \\ \vec{r} \times (m\vec{g}) &= (I_{\text{object}} + I_{\text{weight}})\vec{\alpha} \\ rmg &= (I_{\text{object}} + mr^2)\alpha\end{aligned}\quad (3)$$

Note that *the hanging weight must be included in the sum* even though it is moving in a straight line!

There is another complication. Since OSHA does not permit us to perform undergraduate laboratories in frictionless vacuums, we should also include the torque of friction in the calculation as well.

$$rmg + \tau_f = (I_{\text{object}} + mr^2)\alpha\quad (4)$$

### Exercise 1: Rotational inertia of the flat disk

From theory, we know that the rotational inertia of a flat disk is

$$I_{\text{disk}} = \frac{1}{2}MR^2\quad (5)$$

Measure the radius and mass of the disk and calculate its rotational inertia.

$$I_{\text{disk}} = \underline{\hspace{10em}}$$

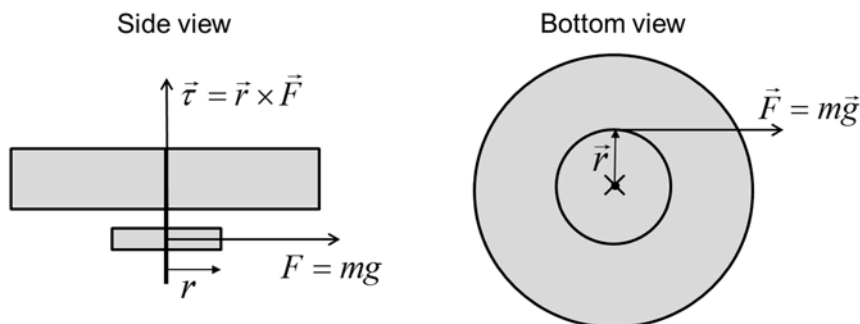


Fig. 3

### Exercise 2: Rotational inertia of the PASCO rotational motion sensor

First, we will measure the rotational inertia of the motion device.

- Open the file Rotational Motion.cap
- Measure the radius of the largest pulley on the PASCO rotational motion device.
- Wind the string attached to the 10g mass around that pulley until the mass is suspended about 1m above the floor, but make sure the string falls freely from the pulley before the mass hits the floor.
- Start data acquisition and let the mass drop.
- On the angular velocity plot, you should notice two distinct slopes. These correspond to the angular accelerations of the device before and after the weight fell away. Using the slope tool measure these angular accelerations. Note the sign.

$$\alpha_1 = \underline{\hspace{10em}}$$

$$\alpha_2 = \underline{\hspace{10em}}$$

- Place the disk on the device and repeat the above experiment to measure the angular accelerations before and after the weight falls free.

$$\alpha_3 = \underline{\hspace{10em}}$$

$$\alpha_4 = \underline{\hspace{10em}}$$

Print out these plots. Annotate this plot to indicate the start of the fall and the release of the weight. Also include the acceleration during each segment of the plot. For a moment, let's ignore friction. Using equation (3), calculate the following moments of inertia.

The PASCO device by itself,  $I_{PASCO} =$  \_\_\_\_\_

The combination of the PASCO device and the disk,  $I_{total} =$  \_\_\_\_\_

Taking the difference, find the rotational inertia of just the disk, and the percent difference between this value and the value found in Exercise 1.

$I_{disk} =$  \_\_\_\_\_ % diff = \_\_\_\_\_

### Exercise 3: Accounting for friction

Now, let's face reality and deal with friction. As the mass is accelerating the pulley system, the physics is described by the expression using friction

$$rmg + \tau_f = (I + mr^2)\alpha_1 \quad (6)$$

After the mass falls free, this simplifies to

$$\tau_f = I\alpha_2 \quad (7)$$

With these two equations, we have two unknowns:  $I$  and  $\tau_f$ . Do the algebra (on a separate sheet) to find expressions for  $I$  and  $\tau_f$  in terms of known values. Write them below.

Now, again calculate the following moments of inertia while accounting for friction:

The PASCO device by itself,  $I_{PASCO} =$  \_\_\_\_\_

The combination of the PASCO device and the disk,  $I_{total} =$  \_\_\_\_\_

Taking the difference, find the rotational inertia of just the disk, and the percent difference between this value and the value found in Exercise 1.

$I_{disk} =$  \_\_\_\_\_ % diff = \_\_\_\_\_

#### Exercise 4: Changing the axis of rotation

The rotational inertia of a thin disk rotating about its edge is

$$I_{edge} = \frac{1}{4}MR^2 \quad (8)$$

- g. From your measurements of the mass and radius, calculate the rotational inertia for the disk when rotating on its edge.

- h. Using the same method of Exercise 3, measure the rotational inertia in this orientation.



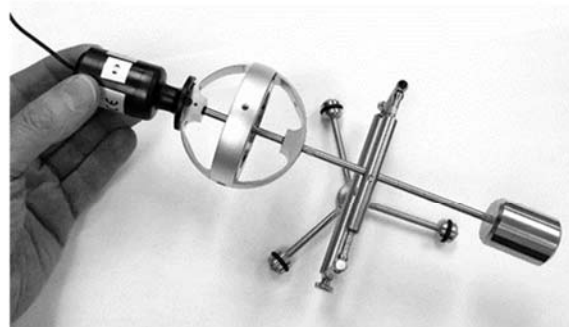
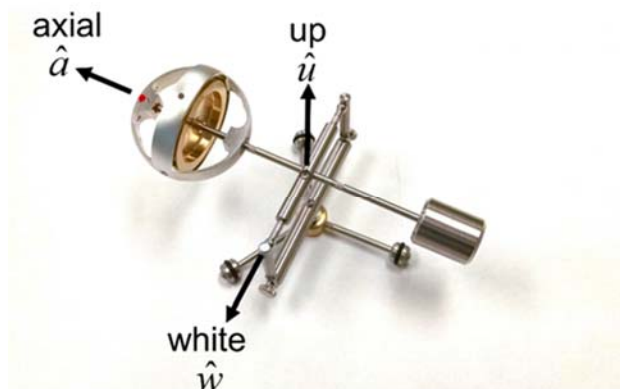


1. Suppose the mass of the string were not negligible. How would that affect the data?

### Exercise 5: Gyroscopes

In the previous exercises, you applied the torque in the same direction as the angular momentum of the system  $\vec{L}_0$ . In this exercise, we're going to explore what happens when you apply an additional torque in a different direction from the angular momentum  $\vec{L}_0$ .

Take a moment to examine the gimbaled gyroscope you will be using. The spin of the brass disk defines the angular momentum  $\vec{L}_0$  of the gyroscope along its axis, or what we will refer to as the axial direction  $\hat{a}$ . (Note the red dot on the gyroscope.) However, the gimbals permit the system to also rotate along the white direction  $\hat{w}$  (note the white dot on the gimbal), and the up direction  $\hat{u}$ .



Spinning up the gyroscope

By spinning the brass disk, we give the gyroscope its angular momentum  $\vec{L}_0$ . To change this angular momentum, we must apply a torque. From Equation (2), the angular momentum changes in the *same* direction as the applied torque.

Suppose the gyroscope was spinning in the  $\hat{a}$  direction. You then apply a force  $\vec{F}$  to the gimbal as shown on the image below. Draw and label the vectors for the initial angular momentum  $\vec{L}_0$  and the applied torque  $\vec{\tau}$  on the picture below. Describe how the gyroscope will then move.



As we speed up the disk, we apply even more torque to the system, increasing the angular momentum as well. On the picture above, draw the vectors  $\vec{\tau}_2$  and  $\vec{L}_2$  representing the direction of the torque and angular momentum that result from increasing the speed of the disk even more.

Now, let's actually speed up the disk and apply different torques to the system. Attach the motor and speed up the brass disk. You'll know that the disk is spinning fast enough when the pitch that you hear stays relatively constant. When answering the questions in this exercise, think about the right hand rule and how to apply it. If you need a refresher, consult Appendix B

- a. Make an hypothesis: If you apply a torque in the upward direction, in which direction would the angular momentum change?
  
- b. In which direction should the gyroscope move for an upward change of the angular momentum?
  
- c. At what point and in which direction should you apply a force to produce an upward

torque?

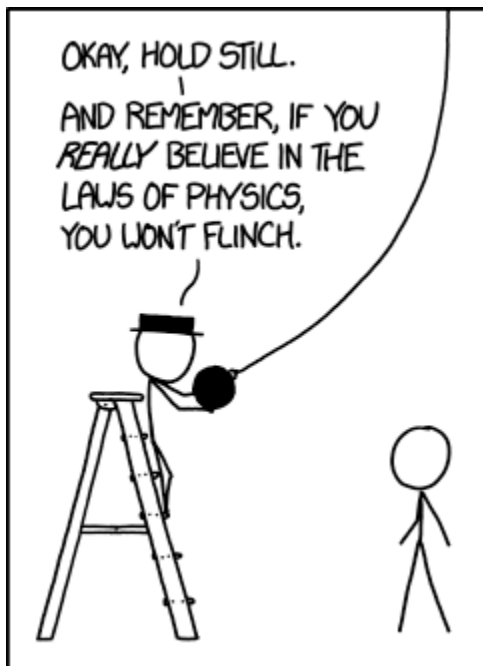
- d. Apply the upward torque and describe the effect on the system.
- e. At what point and in which direction should you apply a force to produce a downward torque?
- f. Apply the downward torque and describe the effect on the system.
- g. Let's say that we took a hanging weight and hung it beside the counterweight. In what direction would that apply a torque? In the picture below, draw the vectors for the force  $\vec{F}$ , torque  $\vec{\tau}$ , and angular momentum  $\vec{L}$  that are now acting on the system.







3. An OK mechanical watch may vary from true time by up to 1 minute a day. What is the percent uncertainty of its period?
4. The Martin Burgess Clock B (the world's most accurate pendulum clock) lost 0.625s in 100 days. What is its percent error?



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## Lab 9: Harmonic Motion

*“It appears to me that those who rely simply on the weight of authority to prove any assertion, without searching out the arguments to support it, act absurdly.”*

--Galileo Galilei

500 g hooked mass

Rotational motion sensor

### Objective

- To test principles to the limit of experimental resolution.
- To understand the relationship between simple harmonic oscillators and time-keeping.

An object is in equilibrium when there is no net force on it. We define three basic types of equilibrium. Consider a ball on a track.

If the track is flat, there is no net force on the ball. If the ball received a bump it would roll in the direction of the bump at a constant speed. This is *neutral equilibrium*.



If the ball is on the peak of a hill, again there is no net force on it. But, if it receives a bump to the left (or right) it experiences a repelling force which further accelerates the ball to the left (or right) away from its equilibrium position. This is *unstable equilibrium*.



If the ball is at the bottom of a trough, again there is no net force on it. However, if it receives a bump to the left, it experiences a restoring force to the right returning the ball to its equilibrium position. The ball will then oscillate back and forth about its equilibrium position.\* This is *stable equilibrium*.

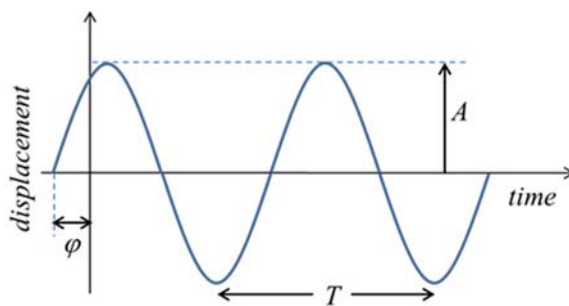


### Simple Harmonic Oscillators and Springs

The simplest type of oscillator is called a *Simple Harmonic Oscillator* (SHO)<sup>†</sup>. Here, the restoring force is linearly proportional to the displacement.

$$F \propto -\text{displacement} \quad (1)$$

It can be shown (see your textbook) that with such a force, the motion will be sinusoidal.



For a pendulum moving in an arc, we can measure the displacement with an angle  $\theta$ . Hence, the motion should be given by

$$\theta(t) = A \sin(\omega t + \varphi) \quad (2)$$

Here,

- $A$  is the amplitude of the oscillation
- $\omega$  is the angular frequency (in radians/sec)
- $\varphi$  is the phase shift

\* . . . unless there is excessive damping.

<sup>†</sup> Physicists like to keep our naming conventions as obvious as possible.



We can relate  $\omega$  to the frequency  $f$  and period  $T$  by

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (3)$$

### Clocks

All clocks are oscillators. Something in the clock (a balance wheel, a quartz crystal, or an electron in a cesium 133 atom) moves back and forth in a very regular pattern. The clock measures time by counting each oscillation.

While Galileo was still a student in Piza, he once observed a lamp swinging back and forth. Apparently uninterested in the sermon, he timed the swing of the pendulum against his pulse and discovered something quite extraordinary:

*The period of a pendulum is independent of the amplitude of its motion.*

He first wrote about this *isochronism* in 1602, but did not consider controlling a clock with a pendulum until the last year of his life 1642. 100 years later, John Harrison had built a pendulum clock accurate to within 1 second per month.\*

Why would isochronism be valuable to a clockmaker? Why not build a pendulum clock that runs with a constant amplitude?

---

\* John Harrison's pendulum is illustrated on the cover of this lab manual.

**Task 1**

Galileo determined that the frequency of a pendulum is inversely proportional to the square root of its length.

$$\left(\frac{1}{\omega}\right)^2 \propto L \quad (4)$$

Empirically verify this relationship.

1. Create a pendulum of known length.
2. Let it oscillate for many cycles with an amplitude of about 0.2 radians.
3. Fit the resulting motion with the Capstone software to determine  $\omega$ . Make note of the corresponding uncertainty in  $\omega$ .

NOTE: Capstone will often not display coefficients with a precision matching the uncertainty of the coefficient. You will have to increase the number of digits displayed by hand.

- Right click on the fit results
  - Select curvefit properties
  - Select numerical format
  - Select coefficients
  - Increase the number of significant figures
4. In Excel, plot  $\left(\frac{1}{\omega}\right)^2$  vs.  $L$ . Properly label your axes. Was Galileo right? Explain your reasoning.

**Task 2**

5. With a long pendulum, measure  $\omega$  for several different amplitudes ranging from about 0.03 to 0.5 radians. Plot the results in Excel. Include the error bars. Properly label your axes.

NOTE: An example Excel file using error bars can be found alongside the Capstone file. By editing this file, you can easily produce the require plot.

6. Does  $\omega$  vary significantly as a function of amplitude, or is  $\omega$  effectively constant given the uncertainties in your measurements? Explain.

7. How does the uncertainty in amplitude affect the uncertainty in the period?

8. With your results in mind, explain why grandfather clocks use long pendulums.





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## Lab 10: Damped Driven Harmonic Motion

*“If you want to find the secrets of the universe, think in terms of energy, frequency and vibration.”*

*--Nikola Tesla*

### Objectives

- To understand damped-driven motion
- Understand what factors change the damping constant
- Discover how the damping constant affects the frequency of oscillation

### Equipment

Force sensor, Oscillating mass and springs, vibration generator, signal generator

### Damped Harmonic Motion

With Simple Harmonic Motion, the force on a mass is given by

$$\begin{aligned}
 F(x) &= -kx \\
 ma &= -kx \\
 m \frac{d^2x}{dt^2} &= -kx
 \end{aligned}
 \tag{1}$$

The solution to this differential equation is

$$x(t) = A \sin(\omega_0 t + \phi) \tag{2}$$

where  $\omega_0 = \sqrt{\frac{k}{m}}$ .

In this idealized situation, the mass would oscillate back and forth forever. However, in the real world, oscillations fade. We can model this by adding a damping force to the motion. In particular, we will add a drag force that scales with the velocity of the mass:

$$F(x) = -kx - bv$$

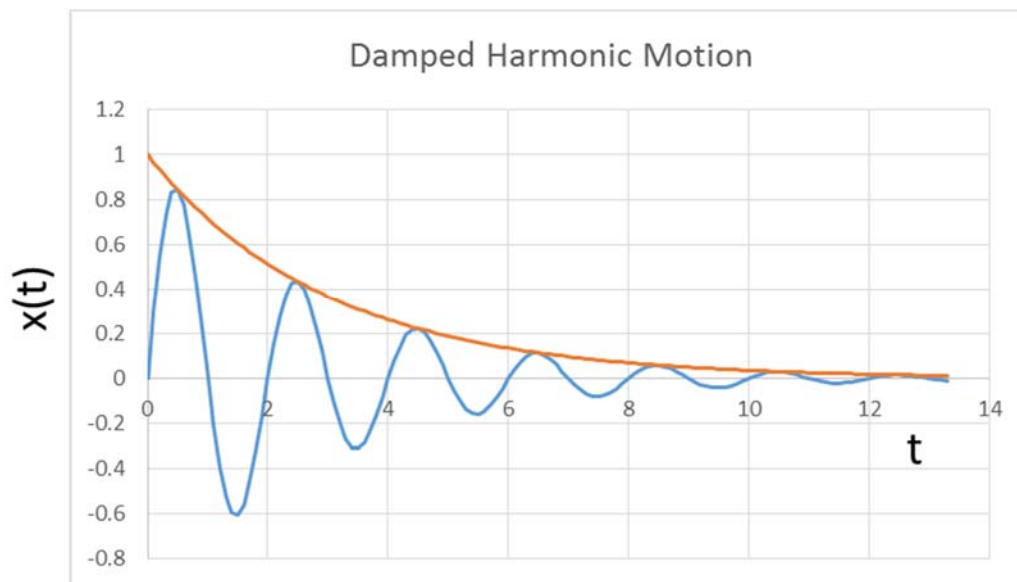
$$m \frac{d^2 x}{dt^2} = -kx - b \frac{dx}{dt} \quad (3)$$

Here,  $b$  is the damping constant and is a measure of how much drag we are adding to the system. (An oscillator suspended in water would have a smaller  $b$  than an oscillator suspended in honey.)

The solution\* to this differential equation happens to be . . .

$$x(t) = Ae^{-t/\tau} \sin(\omega_0 t + \varphi) \quad (4)$$

Here,  $A$  is the amplitude,  $\varphi$  is the phase shift,  $\tau = \frac{2m}{b}$ . We call  $\tau$  the *time constant* of the oscillator. It describes how quickly the motion fades away. After a period of time  $\tau$ , the amplitude of the motion has fallen by a factor of  $e^{-1}$  (or about 0.37).



\* This assumes that the damping force is small compared to the driving force. In the lingo, this is an “underdamped system”. For all of the ugly details, see your textbook.



**Exercise 1: Finding the natural frequency  $\omega_0$** 

For this exercise, we are going to exclude the driving force so that we can find the natural angular frequency  $\omega_0$  of the system.

1. Begin taking data on the computer. Pull the mass down by a few centimeters and then release. Because there is no driving force, this graph should look similar to what you saw in the last lab.
2. Fit the graph that you see with eq.(4) to determine  $\omega_0$ .
3. Calculate the natural frequency in Hertz (cycles per second)  $f_0$ .

$$\omega_0 = \underline{\hspace{4cm}} \text{ rad/s} \qquad f_0 = \underline{\hspace{4cm}} \text{ Hz}$$

**Exercise 2: Dampening and the Magnet Spacing**

In this exercise, we are going to see how the distance of the magnet effects the damping of the system.

4. First, place the magnet about 2 mm from mass, pull the mass down by a few cm, and release.
5. Now, move the magnet about 4 mm away and do the same thing. Observe what happens to the system with the magnet at various distances from the mass. Empirically describe the damping of the system.

## Damped Driven Harmonic Motion

If one wishes for the motion to continue despite the damping, one can add an extra driving force

$$D_{max} \cos(\omega_d t).$$

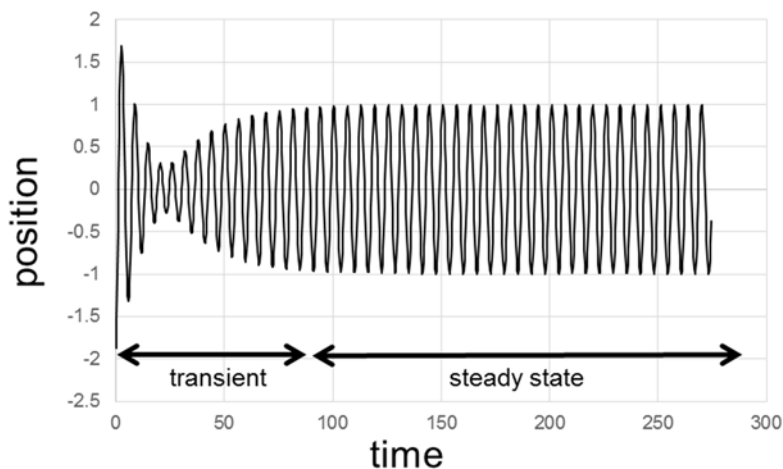
$$F(x) = -kx - b \frac{dx}{dt} + D_{max} \cos(\omega_d t) \quad (5)$$

$$m \frac{d^2 x}{dt^2} = -kx - b \frac{dx}{dt} + D_{max} \cos(\omega_d t)$$

The solution to this differential equation happens to be . . .

$$x(t) = A_{transient} e^{-t/\tau} \sin(\omega_0 t + \varphi) + A_{steady} \cos(\omega_d t) \quad (6)$$

Here  $\varphi$  is still the phase shift,  $\omega_0$  is the natural, undamped frequency of the system  $\sqrt{\frac{k}{m}}$ . The first part of the equation we shall call the transient state. This motion can be somewhat chaotic, but this state fades away with a time constant of  $\tau = \frac{2m}{b}$ . The second part is the steady state solution which is simple sinusoidal motion, and it continues for as long as the driving force is applied. The graph of a damped-driven oscillation looks something like this:



This time constant  $\tau$  is a measure of the duration of the transient segment of the oscillations. After a period of about three time constants ( $t = 3\tau$ ), the transient part of the equation becomes small enough that we can ignore it. In this lab, we are going to focus on the steady state solution. We can solve for the amplitude of the steady state with the equation:



### Exercise 4: Damped Driven Harmonic Motion

Now, we will rigorously examine the damping effect of the magnet.

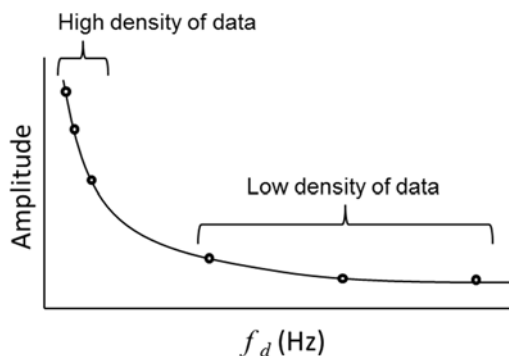
6. Position the magnet about **1 mm** away from the mass.
7. Begin taking data, then pull the mass down, release it, and allow it to oscillate. Fit your data with the equation that looks similar to Eq. (4) and find the time constant  $\tau_a$  for the decaying oscillation:

$$\tau_a = \underline{\hspace{4cm}}$$

Now, we are going to turn on the driving force and map out the response of the oscillator to different driving frequencies.

8. Open the program Damped Driven Harmonic Oscillator.cap. Turn on the oscillator.
9. Set the driving frequency  $f_d$  to the natural frequency  $f_0$ , and acquire data. Fit the steady state portion of your oscillation to a sine wave and measure the amplitude. Record the result on an Excel spreadsheet.
10. Increase the driving frequency by about 0.1 Hz and repeat the measurement.
11. Continue until the amplitude levels off.

Note: The point here is to produce a smooth plot of driving frequency vs. Amplitude. If  $A$  is changing rapidly with  $f_d$ , then you will need a high density of data points. If it changes slowly, you can use fewer data points. Adjust the frequency interval of your measurements as necessary.



12. Repeat for the driving frequencies lower than  $f_0$ .

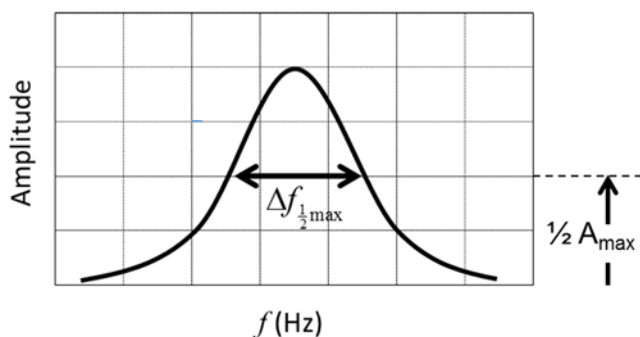
13. Using Excel, plot  $A$  vs  $f_d$ . Print out the resulting curve. At what driving frequency do you find the maximum amplitude?
14. We are now going to repeat what we just did with the magnet at **5 mm** from the mass. Find the time constant for the system with the magnet at this distance.

$$\tau_b = \underline{\hspace{2cm}}$$

15. Again, produce a plot of driving frequency vs. amplitude. How does this plot compare to the previous plot? What is the effect of increasing the damping force?

### Exercise 6: $Q$ -Factor

We will define the bandwidth of the resonance ( $\Delta f_{\frac{1}{2}\max}$ ) as the width of the frequency response at  $\frac{1}{2}$  the maximum amplitude.



The  $Q$ -factor measures the quality of the resonance. We will define it as the ratio of the natural frequency to the bandwidth of the resonance.\*

$$Q \equiv \frac{f_0}{\Delta f_{\frac{1}{2}\max}} \quad (8)$$

\* There are several definitions. This is the simplest.

A high  $Q$  value is an indication of a lower rate of energy loss and a long time constant  $\tau$ . A low  $Q$  value is an indication of a higher rate of energy loss and a shorter time constant  $\tau$ .

16. Using our data from exercise 4, calculate the  $Q$  factor for two damping cases you studied above.

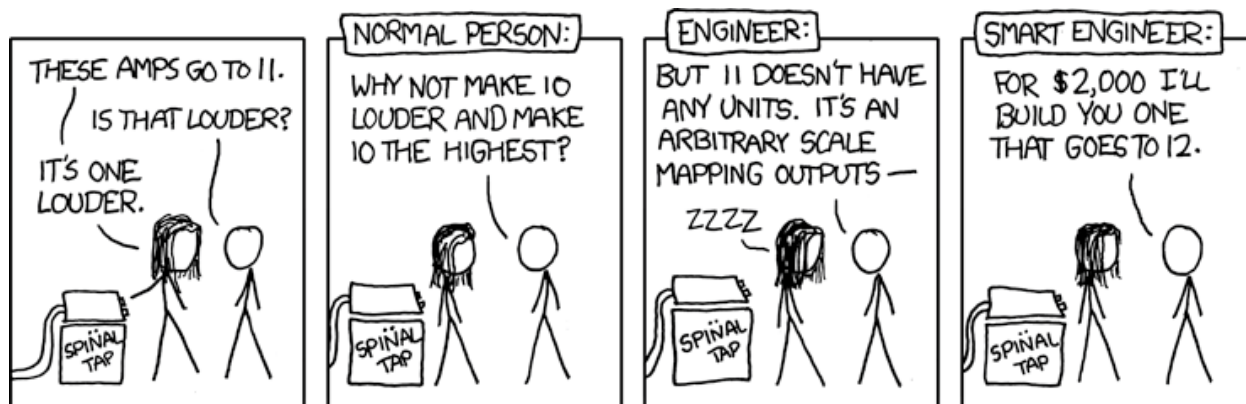
$$Q_a = \underline{\hspace{4cm}}$$

$$Q_b = \underline{\hspace{4cm}}$$

17. What relationship do you see between the damping force and the  $Q$  factor?



3. The notes A4 (440.000 Hz) and D4 (293.665 Hz) constitute a fifth: a frequency ratio of (almost) 3:2. Find three harmonic frequencies which these notes share? (Or very nearly share.)



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## Lab 12: Standing Waves and Resonance

*There is geometry in the humming of the strings; there is music in the spacing of the spheres.*

--Pythagoras

### Objectives

- Understand the relationship between harmonic oscillators and waves
- Understand how boundary conditions lead to resonance

### Equipment

- Function generator with mason string. 100 g for tension
- Oscillator
- Precision mass set
- Large slotted mass, 4 kg total
- Steel wire
- Microphone
- Spectrum Lab
- Super pulley + small pulley
- 3 table clamps
- Risers for sound board / equipment box
- Bridge
- Tuning fork

### Introduction: The harmonic oscillator vs. the unbounded wave

The equation for a simple harmonic oscillator describes the motion of a single rigid object.

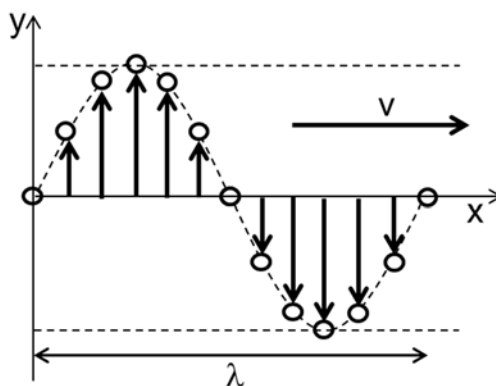


$$y(t) = A \cos(\phi - \omega t) \quad (1)$$

Here,  $\omega$  is determined by the mass and spring constant of the oscillator:

$$\omega = \sqrt{\frac{k}{m}} \quad (2)$$

A wave (for example, waves on a string) can be regarded as a collection of coupled simple harmonic oscillators, where each oscillator is a tiny segment of the string. However, each segment of the string is slightly out of phase with its neighbor. Thus, the phase shift  $\phi$  of each oscillator is a function of  $x$ .



$$\begin{aligned} y(x,t) &= A \cos(kx - \omega t) \\ &= A \cos\left(\frac{2\pi}{\lambda}x - 2\pi ft\right) \\ &= A \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi v}{\lambda}t\right) \\ &= A \cos\left(\frac{2\pi}{\lambda}[x - vt]\right) \end{aligned} \quad (3)$$

**Simulation 1:** <http://physics.bu.edu/~duffy/HTML5/wave.html>

The speed of the wave  $v$  is determined by the tension  $T$  and the linear mass density  $\mu$  of the string.

$$v = \sqrt{\frac{T}{\mu}} \quad (4)$$

The speed is related to (*but not determined by*) the wavelength  $\lambda$  and frequency  $f$ :

$$v = \lambda f \quad (5)$$

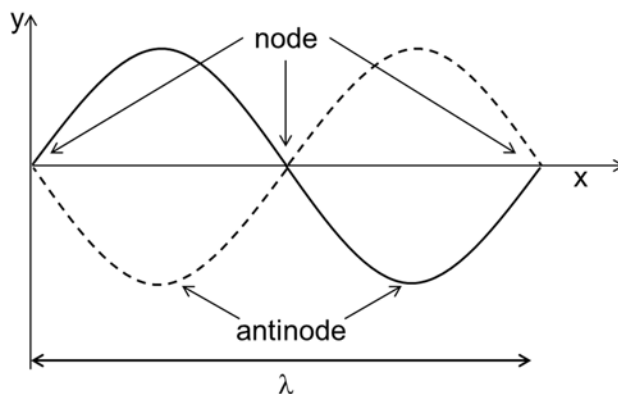
In other words, if you increase the wavelength you will lower the frequency, but the speed remains unchanged.

## Interference and standing waves

If two waves of the same frequency are simultaneously traveling in opposite directions, they will interfere with each other to create a **standing wave**.

**Simulation 2:** [http://physics.bu.edu/~duffy/HTML5/transverse\\_standing\\_wave.html](http://physics.bu.edu/~duffy/HTML5/transverse_standing_wave.html)

Note how the wave shape remains, but the wave does not appear to be going anywhere. Instead, a pattern of **nodes** (where the string does not move) and **antinodes** (where the string exhibits maximum motion) appear.



### Exercise 1: Real strings and boundaries

Equation (3) nicely describes a wave traveling along an infinitely long string which is magically suspended in space. Any frequency/wavelength may be produced on the string

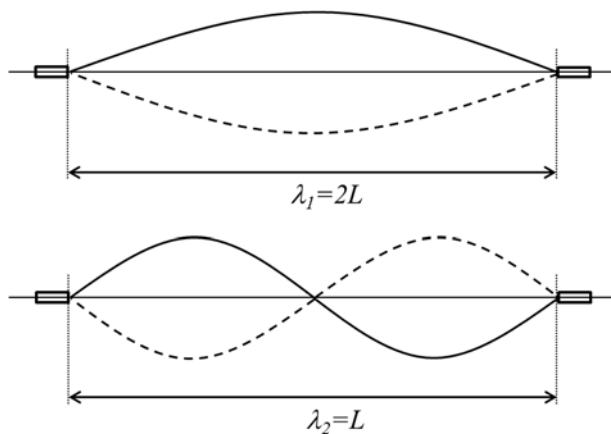
However, real strings have ends and are held in place by clamps, bridges, nuts, and whatnot. These create **boundary conditions**: points where we constrain the position of the string to a particular value. Consider a string of length  $L$  clamped at opposite ends of its length.

- Since waves reflect off the clamps, waves will be traveling in both directions on the string. This will create a standing wave.
- Since the ends cannot move, only standing waves with nodes separated by a distance  $L$  can exist on the string.

Hence, *the allowed wavelengths are quantized*\*: only those waves which match the boundary conditions can exist on the string. The first two solutions are illustrated below, but other solutions also exist. Each solution is called a **resonance**.

---

\* No, this is not quantum mechanics. But it is very analogous to the quantization of energy levels in quantum mechanical systems.



1. In the blank space below, sketch the next two solutions, and record each wavelength  $\lambda_n$  in terms of  $L$ .
2. Then, write down an expression for the wavelength  $\lambda$  for all possible solutions. The longest possible wavelength  $\lambda_1$  is called the **fundamental** or the **first harmonic**.

Half of this wavelength  $\lambda_1/2$  is the **second harmonic**.

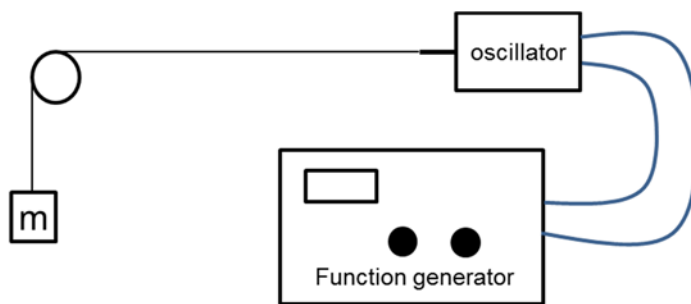
One third of the fundamental wavelength  $\lambda_1/3$  is called the **third harmonic**.

*Et cetera.*\*

3. Label your sketches with the corresponding harmonic.

---

\* For some systems (such as certain organ pipes) not all harmonics are allowed.

**Exercise 2: Driven waves on a string**

4. Measure the linear mass density (mass / length) of the of the mason's twine.

$$\mu = \underline{\hspace{10cm}}$$

5. Apply 100 g to tension the string. Calculate the velocity of waves on the string.

$$v = \underline{\hspace{10cm}}$$

6. Turn on the oscillator and adjust the frequency of the function generator to about 5 Hz. Then, steadily increase the frequency until the string resonates with a maximum amplitude in its fundamental mode. Adjust the amplitude knob as needed.

$$f_1 = \underline{\hspace{10cm}}$$

7. Describe the boundary conditions for the wave on this string. How is the string constrained at each end. (Note: this string is not clamped at one end.)

8. Noting the positions of the nodes, measure the wavelength of the fundamental  $\lambda_1$ .

9. Knowing the frequency and wavelength, calculate the speed of waves on the string. Then, calculate the percent difference between this value and the wave-speed calculated earlier.

$v =$  \_\_\_\_\_

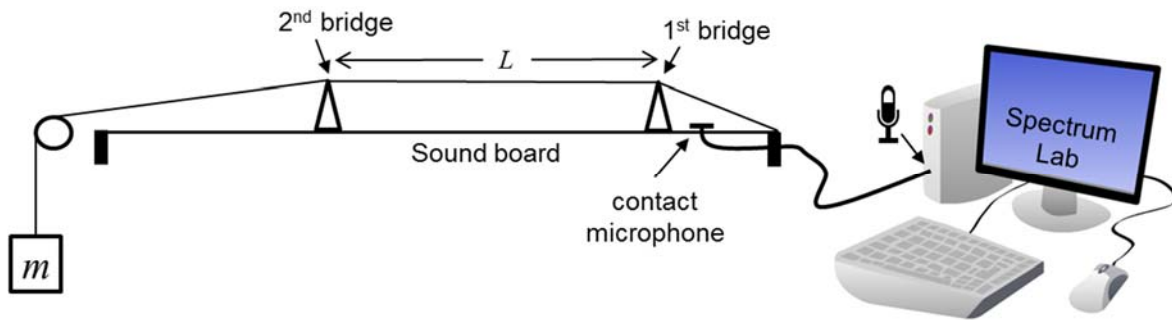
% diff = \_\_\_\_\_

10. Estimate the frequencies of the next three harmonics and enter the values in the table below. Then with the function generator, experimentally measure the frequency of each harmonic.

	Estimated frequency	Measured frequency
$f_1$	XXXXXXXXXXXXXXXXXXXXXXXXXX	
$f_2$		
$f_3$		
$f_4$		

**Exercise 3: The Tennessee music box.\***

In the previous exercise, you actively drove the string at particular frequencies to see which frequencies would resonate. In this exercise, you will randomly dump energy into the wire (*i.e.*, pluck the wire) and see what frequencies naturally appear.



11. Study the steel wires and how tension is applied to each. Describe the boundary conditions of their vibrating segments (that is the segment between the 1<sup>st</sup> and 2<sup>nd</sup> bridges).

---

\* Minus the frets.

Tension the fine wire with 2 kg and the heavier wire with 3 kg. Set the length  $L$  of a fine wire to 60 cm and one of a heavy wire to 60 cm.

A contact microphone detects vibrations of the sound board. Verify that it is plugged into the mic input of the computer.

Run the program Spectrum Lab (found on the desktop).

This program performs a *Fourier transform* to separate the sound into its component frequencies. Your cochlea (the snail-shell like structure behind your eardrum) performs the same task for your brain.



12. Pluck the fine wire and measure the frequencies of the first five harmonics. (You can use the mouse to identify the frequency components on the screen.) Fill in the table below and verify the higher harmonics are indeed multiples of the fundamental.

$n$	1	2	3	4	5
$f_n$					
$f_n / f_1$					

13. Reduce the tension of the fine wire by removing 1kg. Measure the resulting fundamental frequency. Verify that this frequency is consistent with equations (4) and (5). Show your work.

14. From frequency measurements, calculate the mass density  $\mu$  of the fine and heavy strings.

$$\mu_{\text{fine}} = \underline{\hspace{10cm}}$$



$$\mu_{\text{heavy}} = \underline{\hspace{10cm}}$$

### Consonance

Consonance (or a pleasing harmony) is perceived when two different notes share many of the same frequencies in their harmonic sequences.

Tension the fine wire with 2 kg. Set the vibrating lengths on the fine wire to 60 cm and 40 cm.

15. Measure the fundamental frequencies, and the frequency ratio.

$$f_{60} = \underline{\hspace{10cm}} \qquad f_{40} = \underline{\hspace{10cm}}$$

$$\text{frequency ratio } \underline{\hspace{10cm}}$$

16. Which harmonics do these notes share?

17. Adjust the length of the shorter string to find two different consonances. The strings should make a pleasing sound; if you are tone deaf, ask for help from a more musically inclined student.. Calculate the frequency ratio for each of these new consonances.

String lengths:

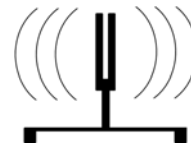
Frequency ratio:





**Exercise 6: Harmonic patterns**

22. Select a tuning fork within your vocal range. Gently strike the fork with the rubber mallet and place the bottom of the fork against the sound board.
23. With the computer, note the pattern of harmonics displayed on the screen.
24. Replace the contact mic with a regular microphone.
25. From your group, select the best opera singer. Holding the microphone, with a steady clear voice sing “Ahhhhhhhhhhhh”. Try to match the tone of the tuning fork. Note the pattern of harmonics displayed on the screen.
26. Using the same pitch as before, sing “Eeeeeeeeeeeeeeeee”.
27. Briefly describe how the harmonic patterns of these three sounds differ.



28. Hiss into the microphone: “Sssssssssssss”. How does this frequency pattern compare the vowel sounds above? Try some other non-vowel sounds. Why is it only possible to sing with vowels?

## Appendix A: The Small Angle Approximation

Often, you will find yourself dealing with small angles. In such cases, the following approximations can make your calculations a lot easier.

**If an angle  $\theta$  is measured in radians**, the arc length of the corresponding circle segment  $s$  is given by

$$s = r\theta .$$

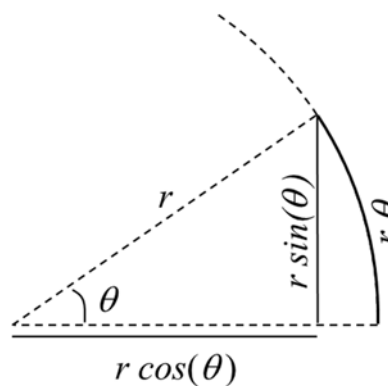
By definition

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} .$$

Inspecting the diagram, it is obvious that for small angles

$$r \sin(\theta) \approx r\theta$$

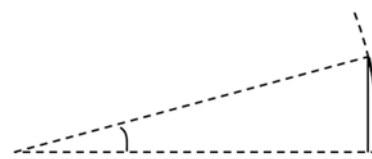
$$\boxed{\sin(\theta) \approx \theta} .$$



It is also obvious that for small angles

$$r \cos(\theta) \approx r$$

$$\boxed{\cos(\theta) \approx 1}$$



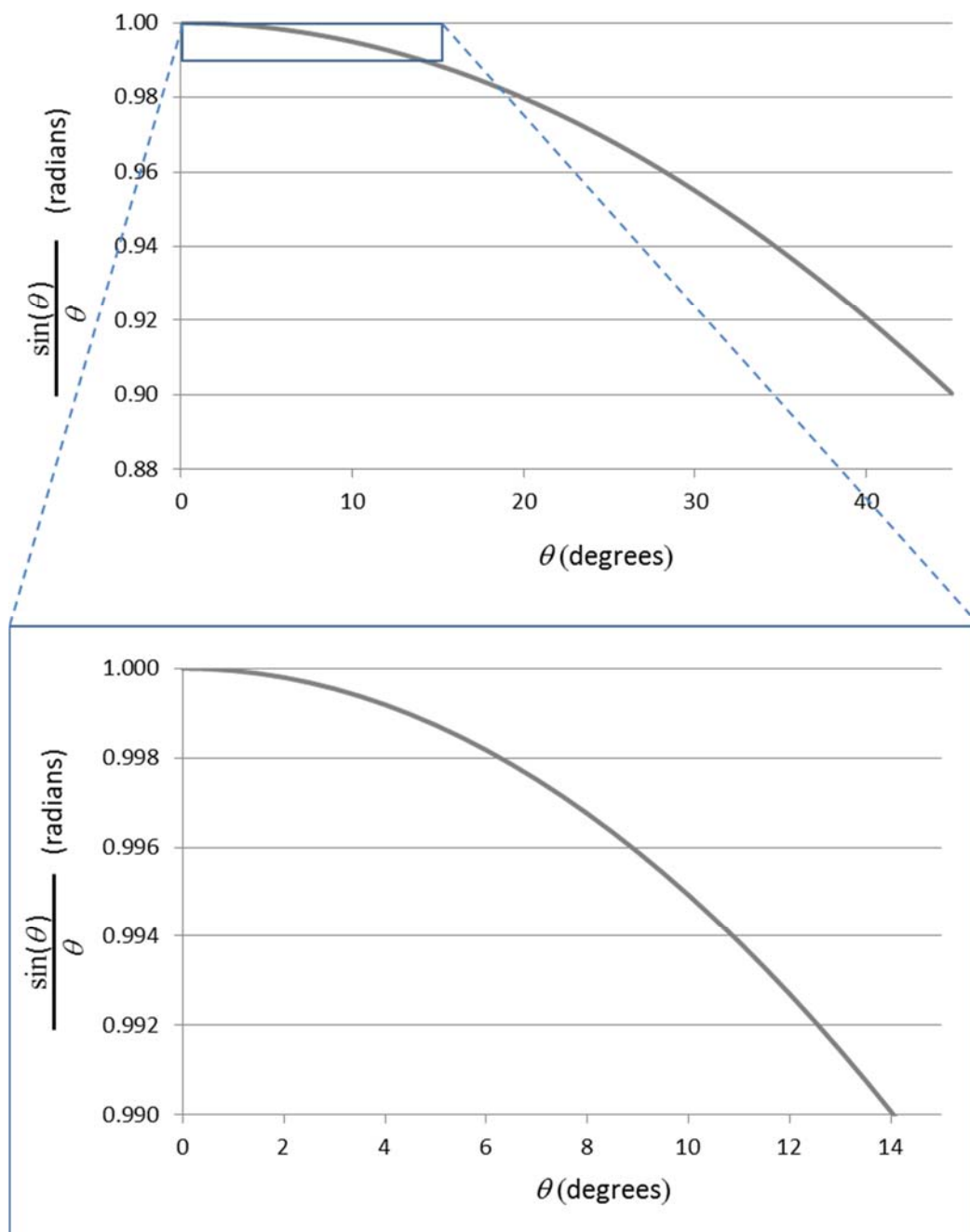
Thus,

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\boxed{\begin{aligned} \tan(\theta) &\approx \sin(\theta) \\ &\approx \theta \end{aligned}}$$

So, for what angles does this approximation work? Well, it depends on how accurate you need to be. The graph below is a useful guide. The closer  $\frac{\sin(\theta)}{\theta}$  is to 1, the better the approximation.

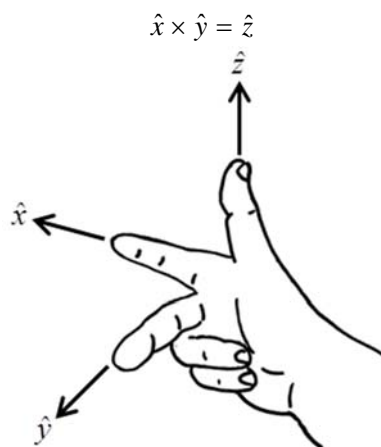
**Note:** The angles on the x-axis are in degrees. The angles on the y-axis are calculated in radians.



## Appendix B: The Right Hand Rule and Right Handed Coordinates

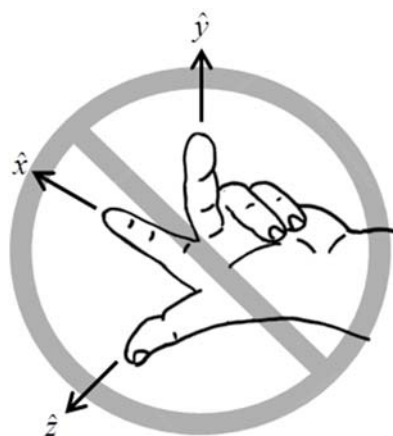
Many of the equations used in this lab and the associated class require that you use a *Right Handed Coordinate System*, particularly those equations involving cross products. Using the wrong coordinates can lead to considerable confusion and grief; so, **get in the habit of drawing your coordinate axes correctly!**

Consider the illustration of a Right Handed Coordinate axes below. The unit vectors  $\hat{x}$ ,  $\hat{y}$ , and  $\hat{z}$  correspond to the index finger, middle finger, and thumb respectively. Here



Right Handed Coordinates

Below is an example of a Left Handed Coordinate system where  $\hat{x} \times \hat{y} = -\hat{z}$ , and *that's wrong!!*



Left Handed Coordinates

## Appendix C: Advanced Propagation of Uncertainties

Suppose you had measured three different rods:

$$x = (2.34 \pm 0.03) \text{ m} \quad y = (5.43 \pm 0.05) \text{ m} \quad z = (8.31 \pm 0.02) \text{ m}$$

What would be the length of these three rods placed end to end?

In Lab 1, you were instructed to combine the uncertainties by simply adding the absolute uncertainties.

$$\begin{aligned} L &= x + y + z \\ &= (2.34 + 5.43 + 8.31) \text{ m} \pm (0.03 + 0.05 + 0.02) \text{ m} \\ &= (16.08 \pm 0.10) \text{ m} \end{aligned}$$

In fact, this is an upper limit of the uncertainty. In this particular case, we can actually do a bit better. The logic goes like this:

*If you make several independent measurements subject to random uncertainties, we expect some measurements will be a little bit high and some to be a little bit low. These variations should partially cancel out.*

I'll spare you the ugly details of the math, but it boils down to this:

**When adding several independent measurements**

$$Q = q_1 + q_2 + \dots + q_n$$

**the corresponding absolute uncertainties are added in quadrature. (That is, add them like the Pythagorean theorem.)**

$$\Delta Q = \sqrt{\Delta q_1^2 + \Delta q_2^2 + \dots + \Delta q_n^2}$$

Thus, in the example above we have

$$\begin{aligned} L &= x + y + z \\ &= (2.34 + 5.43 + 8.31) \text{ m} \pm \left( \sqrt{0.03^2 + 0.05^2 + 0.02^2} \right) \text{ m} \\ &= (16.08 \pm 0.06) \text{ m} \end{aligned}$$



Note that for this to apply, we must be adding independent measurements. Consider for a moment a different problem:

*You are given a rod of length  $r = (1.54 \pm 0.04)\text{m}$ , What would be the length of 3 similar rods placed end to end?*

Here, **we only have a measurement for a single rod**; so, there is no longer a partial canceling out of uncertainties. In this case, the best we can do is to simply add the uncertainties:

$$\begin{aligned} 3r &= (1.54 + 1.54 + 1.54)\text{m} \pm (0.04 + 0.04 + 0.04)\text{m} \\ &= 3(1.54 \pm 0.04)\text{m} \\ &= (4.62 \pm 0.12)\text{m} \end{aligned}$$

What about when multiplying (or dividing) measurements? The method is similar:

**When multiplying (or dividing) several independent measurements**

$$Q = q_1 \times q_2 \times \dots \times q_n$$

**the corresponding relative uncertainties are added in quadrature.**

$$\frac{\Delta Q}{Q} = \sqrt{\left(\frac{\Delta q_1}{q_1}\right)^2 + \left(\frac{\Delta q_2}{q_2}\right)^2 + \dots + \left(\frac{\Delta q_n}{q_n}\right)^2}$$

So, if I were to take the measurements in the first example and multiply them together (say, to find a volume), the result would be

$$\begin{aligned} V &= x \times y \times z \\ &= (2.34 \times 5.43 \times 8.31) \text{ m}^3 \pm \left( \sqrt{\left(\frac{0.03}{2.34}\right)^2 + \left(\frac{0.05}{5.43}\right)^2 + \left(\frac{0.02}{8.31}\right)^2} \right) 100\% \\ &= 16.08 \text{ m}^3 \pm (0.016)100\% \\ &= 16.08 \text{ m}^3 \pm 1.6\% \end{aligned}$$

But, suppose I had a measured only one rod and imagined constructing a cube 3 rods long by 2 rods tall and 1 rod deep. What would be the volume of this structure?

Now with only one actual measurement

$$r = (1.54 \pm 0.04)\text{m}$$

the relative uncertainties are simply added:

$$\begin{aligned} 3r \times 2r \times r &= (6 \times 1.54^3)\text{m}^3 \pm \left( \frac{3 \times 0.04}{3 \times 1.54} + \frac{2 \times 0.04}{2 \times 1.54} + \frac{0.04}{1.54} \right) \times 100\% \\ &= 21.91 \text{ m}^3 \pm (0.08) \times 100\% \\ &= 21.91 \text{ m}^3 \pm 8\% \end{aligned}$$

**Moral of the story:**

**Independent measurements: add uncertainties in quadrature.**

**Non-independent measurements: simply add the uncertainties.**