

## THE DETECTION OF COHERENCE IN MOVING RANDOM-DOT PATTERNS<sup>1</sup>

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**Abstract**—If a random-dot pattern is presented in two successive displays in which the second is spatially displaced in relation to the first, then under certain conditions observers are able to accurately discriminate the direction of their apparent motion. The accuracy of detecting this coherent relationship between the two stimuli was found to be a rapidly decreasing function of their separation in space and time and an increasing function of the number of elements in the pattern. The visual system seems to utilize a process similar to cross-correlation to detect coherent, position-invariant patterns of stimulation.

Visual stimulation is changing stimulation. The world moves and the observer moves. In the psychophysical laboratory the sequential transformations are sometimes rapid and unsystematic so that stimuli interfere with each other visually, but in natural environments the changing stimulation is typically blended into a coherent visual percept. In general, the visual system reliably represents both the transformations and the invariance of the changing stimulation. Indeed, these two aspects are complementary: The invariant- or constant aspects correspond to stable objects in the environment with which the organism must interact, while the transformations of these objects correspond to events and changes to which the organism must adapt. The transformations may even be said to define the objects that remain invariant under the transformations. Apparently, the visual system is an efficient detector of coherence<sup>3</sup> in the spatio-temporal pattern of stimulation.

Rather little is known, however, about how this coherence-detection is accomplished. Several relevant research paradigms have developed more or less in-

dependently, but have not yielded a unified picture of the spatio-temporal interactions in vision. One finds examples of both interference (masking) and facilitation (integration) in the perceptual interactions between successive stimuli, depending on whether the successive spatial patterns are uncorrelated or correlated (e.g. Eriksen, 1966; Eriksen and Collins, 1967; Lappin and Bell, 1972). When a single form is stroboscopically presented in two different spatial positions, the appearance of motion suggests a visual integration of the two stimuli, while the detection of the change suggests a differentiation of the spatial and temporal positions of the two stimuli. Whether a single visual process accounts for the spectrum of spatio-temporal interactions that occur in these various experimental paradigms is as yet unclear.

Considering the specific phenomenon of stroboscopic motion, at least two different experimental procedures have been employed, and these procedures may reflect different visual processes. The conventional procedure has utilized a very simple pattern that is sequentially presented in two different spatial and temporal positions. Since the stimulus pattern is simple and distinct and since its successive spatial and temporal positions are usually easily discriminated by the observer, subjective reports of the compellingness of the motion experience have traditionally been used instead of discrimination accuracy as a dependent measure for the phenomenon. A different procedure for studying the perception of motion utilizes a random-dot pattern that is sufficiently complex to preclude detecting its spatial displacement merely by remembering the position of a distinct contour (see Julesz, 1971). This stimulus arrangement is sufficient to provide a compelling impression of motion and to permit accurate discriminations of the direction of displacement (Julesz, 1971; Bell and Lappin, 1973). The question of whether these two experimental procedures for producing stroboscopic motion reflect different underlying visual processes might be answered by examining their functional dependencies on spatial and temporal parameters.

Briefly summarizing the effects of spatial and temporal displacements between two successive stimuli in the conventional stroboscopic motion procedure, the following relationships have commonly been

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<sup>3</sup> We use the term "coherence" in the same sense it is used in statistical communications theory—to refer to dependence between the detailed values of separate signals. This concept includes but is more general than the special use of "coherence" in optics associated with lasers and holographs. For the case of visual stimulation originating from objects moving in three-dimensional space, the coherence is between patterns separated in time and space. One means for detecting the coherence between a pair of signals is by cross-correlation. This linear statistical operation is applicable to the specific stimulus arrangements employed in this experiment, as will be discussed in greater detail below.

reported (see Graham, 1965; Kahneman, 1967; Kahneman and Wolman, 1970; Kolers, 1972): (a) Exposure duration of the first stimulus and the interstimulus interval (ISI) interact to affect the subjective quality of apparent motion. Specifically, for exposure durations less than about 100 msec, the subjective quality of apparent motion depends primarily upon the stimulus onset asynchrony (SOA), usually reaching a maximum at an SOA in the neighborhood of 100–150 msec and gradually decreasing for SOAs less or greater than this value. (b) For a given exposure duration, the spatial separation required to produce optimal apparent motion increases with ISI. This relationship is often known as Korte's third law of apparent motion. (c) Good apparent motion may often be reported by most observers over a wide range of ISIs, up to as much as 500 msec for almost any value of exposure duration and spatial separation.

In contrast, we have found that the discrimination of the direction of displacement of a random-dot pattern is a rapidly decreasing function of the spatial separation between the two successive presentations (Bell and Lappin, 1973). We might also expect that performance would decrease rapidly with ISI, insofar as the discrimination of merely a difference between successive random patterns was found to fall to a chance level at ISIs of only about 30 msec (Lappin and Bell, 1972). Similarly, research on temporal integration has typically found that most stimuli cease their visual interactions at ISIs of about 100 msec, and the same temporal parameters might be expected to govern the detection of coherent relations between sequential random-dot patterns.

A primary objective of this study was to determine the effects of spatial and temporal displacements on the detection of coherence in a dynamic random-dot pattern. Observers were asked to identify the direction of displacement of the second of a pair of corresponding patterns. In the first experiment, the stimuli were varied independently with respect to the distance of spatial displacement, ISI, and duration of the first pattern. In a second experiment aimed at testing a cross-correlational model for the underlying visual process, the stimulus patterns were varied in size both in terms of the number of component elements and also in terms of the spatial separation between elements.

## EXPERIMENT 1

### Method

**Stimuli.** The stimuli were pairs of sequentially presented random-dot patterns, one of which is illustrated in Fig. 1. A small digital computer (PDP-8/1) generated and controlled each pattern on a CRT display scope (Tektronix 604) equipped with a rapidly decaying phosphor (P-15). Each pattern contained 288 dots randomly positioned in the cells of a  $24 \times 24$  matrix, with the constraint that each row and each column contained 12 dots. The dots were sequentially plotted with an interval of  $45 \mu\text{sec}$  between successive points; the entire display was continuously re-plotted every 12.9 msec until the desired exposure duration was reached. (The first dots in the display were thus likely to be plotted once more than the last dots in the sequence.) The displays were binocularly viewed in a dimly lit room at a distance of about 127 cm. The visual angle subtended

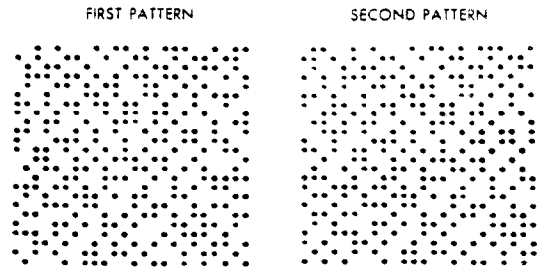


Fig. 1. An example of the random-dot patterns used in this experiment. In this example, the second pattern has been displaced to the right by three columns. (The black/white relation is reversed in this photographic reproduction.)

by the pattern was approx  $1.5^\circ$  in the horizontal and vertical directions.

A stimulus presentation consisted of a pair of random-dot patterns separated by an ISI of 0, 25, 50 or 100 msec. The exposure duration of the first pattern in each pair was either 50, 100, 200 or 400 msec and the duration of the second was constant at 100 msec. The two successive patterns in each pair were identical except that all of the dots in the first pattern were displaced to the right or left in the second pattern by 1, 2, 3 or 4 columns. The outside boundaries of the two patterns were in the same position on the face of the CRT, so that when the pattern was displaced two columns to the right, for example, the two columns on the far right edge of the first pattern were removed and reinserted on the left edge of the second pattern.

**Subjects.** Four students in psychology at Vanderbilt served as paid volunteer observers.

**Procedure.** Each of the four observers served in eight experimental sessions. No practice or training beyond a brief verbal description was necessary; performance was essentially constant for various observers and various amounts of practice. Within each session the duration of the first pattern was held constant while the ISI, distance of displacement, and direction of the displacement were randomly varied across trials. The ordering of the exposure durations was counterbalanced within and between observers. Each session consisted of 400 trials. On each trial the observer was required to identify the direction of displacement—right or left. The trial was initiated by the observer depressing two telegraph keys; the first pattern appeared 500 msec later; the observer then responded by releasing the right or left telegraph key to indicate the direction of displacement; and this was followed by an auditory signal if the response was correct. Each session required about 25 min.

### Results

The main results are shown in Figure 2. As may be seen, the accuracy of discrimination decreased rapidly as a function of both ISI and spatial displacement. The effect of exposure duration is not shown because there was no discernible effect of that variable and no interaction with the other two variables. The average percentages of correct discriminations were 75.0, 76.3, 76.6 and 76.2 for the 50, 100, 200 and 400 msec durations, respectively.

The effects of these variables were statistically tested by a five-way repeated-measures analysis of variance—with spatial displacement, ISI, exposure duration, and direction of displacement as fixed factors and observers as a random variable. The effects

of spatial displacement, ISI, and their interaction were all highly significant ( $P < 0.001$ ). The main effect of exposure duration ( $F < 1.0$ ) and its interactions with spatial displacement ( $F[9/27] = 1.06$ ) and ISI ( $F < 1.0$ ) were all insignificant. The main effect for direction of displacement was insignificant ( $F < 1.0$ ), although there was a marginally significant interaction between direction, ISI, and exposure duration ( $F[9/27] = 2.40$ ,  $P < 0.05$ ) for which we have no interpretation. There were no other significant effects.

One of the basic questions about these results concerns the functional interaction between the spatial and temporal displacements of the two patterns. In contrast with Korte's third law of apparent motion, there is no indication that when the ISI is increased motion can be more accurately discriminated at greater spatial separations. Increases in either spatial or temporal displacement rapidly produce decrements in discriminating the direction of displacement,

<sup>4</sup> The independence condition requires that the rank order of the dependent responses to the various levels of each independent variable remain invariant under all levels of the other independent variable. Since there are a total of six possible pair-wise comparisons among the four values in each row and each column of the  $4 \times 4$  matrix, a total of 48 comparisons are involved in the test of independence for each  $4 \times 4$  matrix. It may be seen in Fig. 2 that there is one violation of independence in the overall displacement-by-ISI data matrix—involving a reversal in the order of the 25 and 50 msec. ISI conditions under the largest amount of displacement. This violation appears likely to be due to the contributions of random error, however. Similar tests may be made on the  $4 \times 4$  matrices obtained for each value of exposure duration (averaged across observers) and for each observer (averaged across durations). (The 16 matrices for each combination of duration and observer tended to have enough random fluctuation to make the tests on them more ambiguous.) In the four matrices obtained under each level of exposure duration, there were 3, 2, 2 and 6 violations of independence (out of 48 comparisons in each matrix) for the shortest to longest durations, respectively. For the four observers there were 2, 3, 6 and 7 violations of independence. As would be expected from Fig. 2, all of these reversals in rank order occurred where performance was near asymptote, usually near the chance level. Unfortunately, we have no adequate model for the contribution of random error, and are thus unable to distinguish real violations of independence from the chance perturbations of random error.

Assuming for the moment that the independence condition is suitably satisfied in these data, then we may proceed to the more stringent test of the double cancellation condition. One test of double cancellation may be made on each  $3 \times 3$  matrix of data for the combinations of the two independent variables. Since 16 different  $3 \times 3$  matrices can be obtained from one  $4 \times 4$  matrix, there are 16 tests of the double cancellation condition to be made on each replication of the  $4 \times 4$  displacement-by-ISI experiment. First, for the overall matrix shown in Fig. 2, there was one violation of double cancellation in nine cases (out of 16) in which both antecedent conditions were satisfied. In the four matrices for the shortest to longest exposure durations, respectively, there were 0, 1, 0 and 4 violations of double cancellation. In the four matrices for the individual observers, there were 4 violations of double cancellation for one of the observers and none for the other three. The greatest violations of double cancellation occurred in the same matrices with the greatest violations of independence.

although there is some indication that the effect of spatial displacement is reduced at longer ISIs. The functional significance of the statistically reliable interaction shown in Fig. 2 is uncertain, however. Much of the apparent interaction is due to the imposition of upper and lower boundaries on the percentage of correct responses, but this particular dependent measure is not likely to be linearly related to the amount of information obtained by the underlying perceptual process. There might exist some monotonic transform of the percentage of correct responses that would render the curves on Fig. 2 parallel, and thus provide a representation as an additive combination of two functionally independent variables.

The possibility that spatial and temporal displacements do not interact and combine by an additive process was assessed by means of conjoint-measurement (see Krantz and Tversky, 1971). The theory of conjoint-measurement specifies necessary and sufficient conditions for the existence of an additive composition of the two variables based simply upon the ordinal relations among the responses to each combination of the independent variables. The data of interest, then, are contained in a  $4 \times 4$  table, giving the percentage of correct responses for each combination of the four spatial displacements with the four ISIs. This basic experiment was replicated for each of the four observers and for each of the four exposure durations. Since the latter variable had a negligible and insignificant effect, the data for the four durations can be treated as independent replications of the basic displacements-by-ISI design. There are two necessary conditions that must be satisfied if an additive representation exists for the composition of two variables: *independence* and *double cancellation*. There was a small number of violations of both of these conditions.<sup>4</sup> Although there is some uncertainty about the extent to which the violations of both double cancellation and independence can be attributed to the unsystematic perturbations of random error, there is some doubt that the composition of spatial displacement and ISI can be properly represented as an additive combination of two functionally independent variables. The violations of double cancellation generally indicated that the effects of ISI

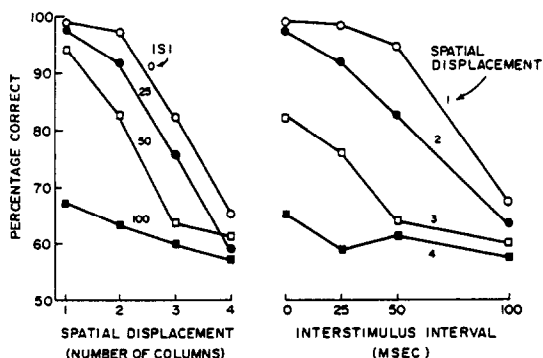


Fig. 2. The accuracy of discrimination of the direction of displacement as a function of the spatial displacement and interstimulus interval between two successive patterns. The two graphs contain the same data, replotted simply to provide a different view of the effect of each independent variable.

were reduced at larger values of displacement, or equivalently, that the effects of displacement were reduced at larger values of ISI. The explanation (post hoc) may be that at longer ISIs the observers were sometimes able to identify the direction of displacement by remembering a few features of the first pattern—e.g. clusters of dots or empty spaces—and then comparing their positions in the two displays. One observer did report doing this on some trials.

### Discussion

The principal result of this experiment is that identification of the direction of displacement in two successive presentations of a random-dot pattern was a decreasing function of both the spatial and temporal separation between the two presentations. Apparently, the visual system becomes less able to match the two successive patterns as they are made more discrepant in either space or time—it seems to be a problem of detecting the relationship between the two patterns rather than of perceiving their apparent motion as in the conventional stroboscopic paradigm. To gain some perspective on these results it is useful to consider several alternative hypotheses about spatio-temporal interactions of vision.

First, these results seem inconsistent with a process in which the visual system integrates the temporal distribution of stimulation at a particular retinal position, as has been proposed to account for many cases of visual masking (e.g. Eriksen, 1966). If the two successive patterns were simply superimposed and combined without regard for their spatial displacement, then the additive combination would be only a random collection of blacks, whites and grays, with no clue to the direction of displacement.

Second, the perception of motion in this experimental set-up is evidently based upon a process that is different from the process responsible for the subjective impression of motion of a single form in the conventional stroboscopic motion paradigm. The temporal characteristics of these two cases of motion-perception are quite different: (a) Whereas a subjective impression of motion can be obtained over a wide range of spatial and temporal separations (see Kolers, 1972), accurate discriminations of the direction of motion were obtained in this experiment over only a small range of spatial and temporal separations. (b) In most stroboscopic motion demonstrations the exposure duration of the first stimulus interacts strongly with the ISI (Kahneman and Wolman, 1970), but in the present experiment variation of exposure duration in the range of 50–400 msec produced neither main effect nor interaction with ISI. (c) Finally, the effects of ISI and spatial separation in the present experiment are inconsistent with Korte's third law of apparent motion—by which the decrement in apparent motion produced by increasing spatial separation can be counteracted by corresponding increases in temporal separation (see Graham, 1965; Kolers, 1972).

Third, the present results seem not to be based upon a discrimination between the remembered position of the first pattern and the perceived position of the second pattern, as has been proposed by Kinchla and Allan (1969). The reason this seems unlikely is that discrimination accuracy was found to decrease with the spatial separation of the two patterns, whereas spatial discrimination should increase with the distance of displacement.<sup>5</sup> Moreover, the rapid decrease in performance with the ISIs in the range of 0–100 msec seems to reflect a more rapid decrease than is indicated in most studies of memory.

<sup>5</sup> As the patterns were displaced in one direction a corresponding number of columns at that edge of the first display were removed and replaced at the opposite edge in the second display. Thus, one might wonder whether the decrement in performance associated with increasing distances of displacement might be due to the subjects perceiving the motion of this small number of columns in the opposite direction. This was almost certainly not the case, however, for the following reasons: (a) No subject ever reported noticing that these columns at the edge of the first display matched those at the opposite edge of the second display. (b) Even if such opposite motion had been detected, it would have correctly informed our subjects about the actual direction of displacement. (c) Virtually the same effects of distance of displacement have been obtained in subsequent experiments using rotated circular displays in polar coordinates, where all of the radii (corresponding to columns in the present experiment) were displaced in the same direction, with none being removed and reinserted in another position. (d) The results of experiment 2 show that the detectability of this opposite motion would have been negligibly small even for the four-column displacements. Nevertheless, the decrements in performance associated with greater displacements may be accentuated to a small degree by the fact that greater displacements were correlated with a reduced number of corresponding elements in the two displays (if the reinserted columns are perceptually unrelated to any elements in the first display). Experiment 2 shows, however, that performance decreases with distance of displacement even when this reduced correspondence is taken into account—see Fig. 3.

### EXPERIMENT 2

The results of Experiment 1 suggest that identifying the direction of displacement of a random-dot pattern is essentially a problem in detecting the coherent relationship between the two presentations of the pattern. Insofar as "coherence" is commonly measured in optics and in communications theory by a cross-correlation of the signals or patterns in question (cf. Born and Wolf, 1970; Goldman, 1953; Rosenfeld, 1969; Weiner, 1949), we may inquire further whether this mathematical operation might also have some descriptive validity for how the visual system detects relationships in complex moving patterns. One of the basic assumptions in using cross-correlation to describe visual processes is that the underlying neural system functions as a linear system, in which separate component elements contribute additively to the detectability of the coherent relationship between the pair of patterns. In general, the operation of a system is said to be linear if it satisfies the principle of superposition: If  $f$  and  $g$  are two different input patterns, and if  $L(f)$  is the response of the system  $L$  to the input  $F$ , then  $L$  is a linear operation if  $L(f + g) = L(f) + L(g)$  for all pairs of inputs  $f$  and  $g$ . Thus, in the present case we might consider each of the component dots as a separate input; the question is whether these contribute additively to the

detection of the global coherence of the pattern. If cross-correlation is a valid description of visual process, then the detectability of the coherent relation between the two successive patterns should increase as some linear function of the number of elements in the pattern. The purpose of experiment 2 was to test this prediction.

Specifically, we can define the cross-correlation between a pair of random-dot patterns as follows. Let  $f(i, j)$  be a pattern of dots in rectilinear coordinates, where  $i = \{1, 2, \dots, R\}$  is the vertical or rows index and  $j = \{1, 2, \dots, C\}$  is the horizontal or columns index and let the numerical value of  $f(i, j)$  at any given point  $(i, j)$  be equal to +1 if the position contains a dot of -1 if the position is empty. The cross-correlation function for a pair of such patterns  $f_1(i, j)$  and  $f_2(i, j)$  is then given by

$$f_1 \times f_2(a, b) = \frac{\psi}{RC} \sum_{i=1}^R \sum_{j=1}^C f_1(i, j) f_2(i + a, j + b) \quad (1)$$

where  $a$  and  $b$  are integers corresponding to displacements in the horizontal and vertical directions respectively, and  $0 \leq \psi \leq 1$  is a scalar that decreases with the spatial and temporal separation between the two patterns as indicated by experiment 1. The cross-correlation function  $f_1 \times f_2(a, b)$  thus represents a new pattern of values in rectilinear coordinates like those of the original pattern  $f(i, j)$ . When the values of  $a$  and  $b$  correspond to the physical displacement of the second pattern, then the cross-correlation  $f_1 \times f_2(a, b)$  would have an expected value of  $\mu\psi$ , where  $\mu$  is the signal/noise ratio of coherent to random components, and for all other values of  $a$  and  $b$  the cross correlation would have an expected value of 0 (assuming that the patterns were generated by a random process in which the probability of a dot occurring in any given position was independent of the occurrence of dots in all other positions).

Of course the obtained value of the cross-correlation at any given displacement  $(a, b)$  will in general not equal only 0 or  $\mu\psi$ , but will be variable, due to the contributions of underlying random processes. The visual task of determining the direction of motion of the pattern can thus be represented as a signal detection problem in which the observer must decide whether the cross-correlation at any given displacement  $(a, b)$  is the result of a corresponding physical motion or whether it is due to the random correlation between the two patterns. Accordingly, the detectability of the coherent relation can be measured by the signal/noise ratio,

$$d' = \mu\psi/\sigma \quad (2)$$

where  $\mu\psi$  is the difference between the expected value of the cross-correlation  $f_1 \times f_2$  a displacement corresponding to the physical motion of the pattern and the expected value of  $f_1 \times f_2$  at any other displacement, and where  $\sigma$  is the standard deviation of  $f_1 \times f_2$  at displacements not corresponding to the motion of the pattern. The variance  $\sigma^2$  may be considered as the sum of the variance from two separate sources: One source is associated with the binomial distribution of the proportion of the total positions in which

the two patterns  $f_1(i, j)$  and  $f_2(i + a, j + b)$  have the same value, which has a variance

$$\sigma_b^2 = \frac{pq}{n} = \frac{(0.5)^2}{RC} = \frac{0.25}{RC}$$

since in these experiments, the probability of a correspondence in any two positions selected at random is approx 1/2 and since the total number of elements in each pattern is  $n = RC$ , where  $R$  and  $C$  are the number of rows and columns respectively. The second source of variance may be associated with the ability of the visual system to detect a match or non-match of the two patterns, which we may designate as  $\sigma_v^2$  for each element of the pattern and  $\sigma_v^2/RC$  for the variance of the mean over all  $RC$  of the pattern elements. We can now substitute into equation (2) as follows:

$$\begin{aligned} d' &= E(f_1 \times f_2)/\sigma \\ &= \mu\psi/\sqrt{(0.25/RC + \sigma_v^2/RC)} \\ &= (\sqrt{RC}\mu\psi)/\sqrt{(0.25 + \sigma_v^2)} \end{aligned} \quad (3)$$

If the detection of a coherent displacement of these random-dot patterns is governed by an underlying cross-correlation process, then we can predict that the detection of the displacement as measured by  $d'$  will be directly proportional to the square root of the size of the pattern,  $\sqrt{RC}$ . The critical assumption is that all of the elements of the display are processed independently, contributing equally to the total variance of the cross-correlation.

To test this prediction we varied both the number of rows and number of columns of the patterns. In order to distinguish the effects of the number of elements from those of the retinal area subtended by the pattern, we also varied the spacing between adjacent dot positions. In a previous experiment (Bell and Lappin, 1973), we had found that the effect of the magnitude of the spatial displacement was determined by the number of rows or columns of displacement, independent of the retinal area subtended by the pattern. In that previous experiment the patterns were composed of black and white squares that also varied in size with the area of the whole pattern, but in the present experiment the size of the component dots was constant and only the spacing between dots varied with the area of the whole pattern.

#### Method

*Stimuli.* The stimulus patterns were similar to those of experiment 1, with the following exceptions. The number of rows and columns of the patterns were independently manipulated; the alternative values were 8, 16 or 32 for both variables. A dot appeared in half of the total positions in each column. The temporal interval between successive dots was inversely proportional to the total number of dots in the pattern, so that the total time required to display the complete pattern and the brightness of each spot were constant for all pattern sizes. The duration of both the first and second display of each pattern was constant at approx 300 msec. (For the  $8 \times 32$  and  $32 \times 8$  patterns the cycle time for each refreshment of the total pattern was 18.5 instead of 14.5 msec and the total display duration was 388 msec instead of the approximate 300-msec duration for the other patterns. The results of both experiment 1 and experiment 2 indicate, however, that this had

no discernible effect on performance.) The spacing between dots was varied by a factor of 2, with approx 1.4 and 2.8 mm center-to-center spacing between dots; the smaller value corresponded to that used in experiment 1. With the smaller space between dots, the 32 × 32 patterns subtended approx 2° in the horizontal and vertical dimensions, and the larger spacing produced a pattern of about 4° for the 32 × 32 pattern. The second pattern of each pair was displaced either two or three columns to the right or left. Since the outside boundaries were the same for the two presentations, two or three columns at one edge of the first display disappeared and were replaced by the same number of new randomly generated columns at the other edge.

**Subjects.** The subjects were four male Vanderbilt undergraduates who served as paid volunteers. None had been a subject in experiment 1.

**Procedure.** Each subject served for five sessions of 360 trials. All four independent variables—rows, columns, displacement, and spacing—were randomly varied from trial to trial, resulting in an average of 50 trials for each subject under each of the 36 experimental conditions. Other aspects of the procedure were the same as in experiment 1.

**Results**

Table 1 gives the percentage of correct identifications of the direction of displacement in each experimental condition. As may be seen, accuracy increases as a function of both the number of rows and number of columns in the pattern and decreases with the distance of the displacement; performance is similar for both the small and large spacing between dots, but tends to be slightly higher for the smaller spacing (an average of 77 vs 74% correct).

These effects are corroborated by analysis of variance, with subjects as the random variable. The main effects for the numbers of rows and columns and for the distance of displacement are all highly significant ( $P < 0.001$ ). The main effect of spacing is much smaller, but is statistically significant ( $0.01 < P < 0.05$ ). The only significant interaction is between the spacing and number of columns ( $0.01 < P < 0.05$ ), indicating a somewhat smaller increase in accuracy as a function of the number of columns for the larger spacing. We have no explanation for this effect; there is no indication of a corresponding rows × spacing interaction ( $F < 1.0$ ).

Figure 3 provides a comparison of these results with the prediction given by equation (3) that was derived from the cross-correlation model. In order to

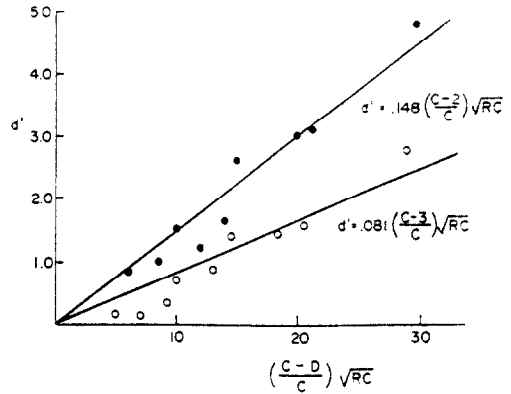


Fig. 3. The accuracy of discrimination of the direction of displacement as a function of the number of elements in the pattern. ( $R$  is the number of rows,  $C$  is the number of columns, and  $D$  is the distance of displacement in columns.)

take account of the reduced coherence produced by the displacement, the parameter  $\mu$  in equation (3) must be proportional to the fraction of columns that are the same in the two patterns. Since the outside boundaries of the two patterns are the same, the displaced columns disappear and are replaced at the opposite edge by an equal number of new columns. In the present experiment the ratio,  $\mu$ , of coherent to the total elements in the two patterns is equal to  $(C-D)/C$ , where  $C$  is the number of columns and  $D$  is the distance of displacement in columns. The detectability of the coherent relationship between the patterns is also assumed to decrease with the distance of displacement, as indicated by the results of experiment 1. Thus, for each distance of displacement the  $d'$  measure of discrimination accuracy should increase in direction proportion to  $(\sqrt{RC})(C-D)/C$ . The theoretical predictions of Fig. 3 are thus a pair of straight lines passing through the origin—one line for each displacement. The percentages of correct discrimination for each of the 18 combinations of  $R$ ,  $C$  and  $D$  were transformed to  $d'$  by assuming that the responses were generated from two overlapping normal distributions of equal variance with no response bias for responding “right” or “left”. (If the performance of each individual subject were characterized by the hypothesized straight line through the origin, then the  $d'$  transformation of the total percentages of correct responses over all of the subjects and values of spacing between dots will also generate a straight line through the origin.) The resultant data are plotted in Fig. 3, and as may be seen these data are roughly described by two straight lines whose slopes depend on the distance of displacement. The two lines plotted in Fig. 3 minimize the squared error in  $d'$ , and account for 93 and 88% of the variance in  $d'$  for the displacement of 2 and 3 columns respectively. Though there are hints of small but systematic deviations from the predictions, the cross-correlational model provides a reasonable first approximation for the effects of the size of the patterns—accounting for more than 90% of the variance in the 18 data points with only two free parameters.

Table 1. Percentage of correct discriminations of direction of displacement in each experimental condition

Spacing	Rows	Number of Columns Displaced						
		2			3			
		Columns	8	16	32	Columns	8	16
Small	8	63	83	97	54	53	80	
	16	68	83	95	50	64	83	
	32	73	96	99	61	82	98	
Large	8	68	72	84	53	60	72	
	16	69	76	92	55	69	74	
	32	72	90	99	66	74	86	

### Discussion

The finding that the accuracy of discriminating the direction of displacement is an increasing function of the size of the pattern clearly indicates that the underlying process is a global operation on the whole pattern rather than a local operation applied to a limited subset of elements of the pattern. These results are inconsistent with the hypothesis that the perceived motion of a few component elements is used to infer the motion of the complete pattern. The results are consistent with the statistical properties of a cross-correlation of the two successive patterns.

As in experiment 1, performance was found to decrease with the distance of displacement. The measure of this distance appears to be based mainly on the number of elements of the pattern rather than on the angular distance across the retina—a relative rather than absolute measure. With the closer spacing, displacements of two and three columns corresponded to absolute retinal displacements of approx 7.5' and 11.25' and produced averages of 84.1 and 69.4% correct discrimination; whereas doubling the space between elements resulted in absolute displacements of approx 15' and 22.5' but the discrimination accuracy dropped to only 80.4 and 67.5% correct for the relative displacements of two and three columns respectively. There was no interaction between spacing and distance of displacement ( $F < 1.0$ ).

The finding that the relative distance of displacement was more important than the absolute distance conflicts with a recent study by Braddick (1974) that reaches the opposite conclusion: that "the limit on the displacement was its absolute size (maximum about 15') rather than the number of elements' widths." The most reasonable explanation for this discrepancy is that when Braddick increased the absolute distance of displacement he also decreased the number of component elements, holding the outside dimensions of the pattern constant. Thus, for a given relative displacement, an increase in absolute displacement was correlated with a decrement in performance for a purely statistical reason. Decreasing the number of elements by a factor of four, as in Braddick's experiment, would be expected to divide the signal/noise ratio in half for detecting the coherently moved pattern, which in turn might be counteracted by cutting the relative displacement in half, so that performance might be more highly correlated with the absolute than with the relative displacement.

### GENERAL DISCUSSION

The results of this study reflect two complementary phenomena—the detection of motion and the detection of coherent patterns invariant under displacement in retinal position.

This investigation as well as that of Braddick (1974), demonstrates that the discrimination of the direction of displacement of these random-dot patterns is governed by processes different from those responsible for the perception of motion of simple patterns in the conventional stroboscopic motion paradigm. The discrepancy with the better-known stroboscopic motion phenomena concerns the effects of the temporal parameters and the interaction (or lack of it in the present case) between the spatial and temporal

separations of the two successive patterns. The effects of spatial separation and of the number of elements in the patterns are also inconsistent with the hypothesis that the visual system "remembers" the positions of individual elements of the first pattern and then discriminates these from the positions of elements in the second pattern. Instead, the performance in these experiments seems to depend upon a global process for matching the two complete patterns.

In addition to the detection of motion, an essential aspect of visual perception is the detection of the coherent patterns of stimulation that remain invariant under displacement on the retina. The world moves and the observer moves, but the observer's perception of environmental objects must to some extent remain constant and invariant. The importance of this visual achievement has often been emphasized by James Gibson (e.g. 1966). The present study suggests the mechanism by which this perceptual invariance may be accomplished—by a cross-correlation of the sequentially occurring spatial patterns. Reichardt (1961), has explicitly proposed that movement perception in the fly's visual system is based upon an auto-correlation of the input optical patterns. Uttal (1973) has also recently proposed that the detection of stationary patterns embedded in random noise is based upon an auto-correlation of the input optical patterns. Insofar as cross-correlation is a linear, position-invariant operation for detecting periodicity, it is thus similar in several respects to a Fourier transform. The present results are therefore compatible with contemporary enthusiasm for Fourier analyses for describing the visual processing of spatial patterns (e.g. Campbell, 1974; Cornsweet, 1970; Harmon and Julesz, 1973; Ratliff, Knight, Dodge and Hartline, 1974). Similarly, the present investigation is also compatible with analogies drawn between neural processing and holography (e.g. Julesz, 1971; Westlake, 1970).

Although the neural basis for these behavioral phenomena is as yet uncertain, they do suggest two properties that seem likely to be associated with the underlying neural organization. First, the ability to visually fuse two successive patterns that are spatially displaced in relation to each other suggests an extensive interaction between neighboring elements of the visual system, which presumably must depend upon a densely interconnected neural network. Second, the finding that detection of the coherent relation is roughly proportional to the square root of the number of pattern elements indicates that the stimulated neural units are stochastically independent and function in parallel; this seems in turn to suggest a linear homogeneous network of functionally similar units. Identifying the neural basis for these phenomena remains, however, an intriguing problem for future research.

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