**VANDERBILT UNIVERSITY**
**CIVIL AND ENVIRONMENTAL ENGINEERING DEPARTMENT**

# Course: CE 6200-01 – Continuum Mechanics

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| **Semester:** | Fall 2017 |
| **Instructor:** | Ravindra Duddu<br>Office – 288 Jacobs Hall<br>Telephone – (615) 343 4891<br>E-mail – ravindra.duddu@vanderbilt.edu<br>Office hours are flexible, so please send me an email. |
| **Class Hours:** | M, W (2:10 to 3:25pm) |
| **Venue:** | FGH 200 |
| **Textbook:** | Introduction to the Continuum Mechanics, by M. W. Lai, D. Rubin, E. Krempl, Elsevier, Ed. 4th, 2010.<br><br>(The above book is available as an e-book online through the Knovel system via Vanderbilt Library) |

## Scope of the Course

**Course Overview:** Continuum Mechanics provides a mathematical framework to describe the kinematics and mechanical behavior of materials (solid and fluid) as a continuous medium, rather than as discrete particles. The goal of this course is to provide an introduction to the interdisciplinary field of continnum mechanics that permeates all disciplines of engineering, applied mathematics, and engineering sciences. Given the vastness of this field, we will mainly restrict our attention to linearly elastic solids and kinematics with an emphasis on finite strain (or large deformation); whereas, nonlinear elasticity will be covered in lesser detail. Thus, this course lays the theoretical foundations for the design and analysis of structures using the finite element method. As a part of the course, the students will be required to: (1) complete textbook readings prior to each class and provide comments on Brightspace; (2) submit weekly homework assignments that may involve developing computer programs in Matlab (or Python) and/or performing simulation studies in Abaqus software; (3) deliver a 25 minute presentation/discussion on a journal article of their choice in the area of computational continuum mechanics.

**Course Objectives:** After completing this course, you should be able to:

1. Read and write the language of continuum mechanics, specifically, using the tensorial, indicial and matrix notations
2. Prove and derive tensor identities related to tensor algebra, calculus and mechanics

3. Explain the physical interpretation of various stress, deformation and strain measures in three-dimensions described using tensor operations
4. Describe failure theories based on stress or strain invariants
5. Derive and explain the fundamental laws of mass and momentum balance using Lagrangian and Eulerian descriptions
6. Establish the constitutive relations for isotropic and anisotropic linearly elastic solid and isotropic nonlinearly elastic solids.
7. Write computer programs by using indicial and matrix operations in Matlab and solve simpler nonlinear elasticity problems.

## Schedule of Topics

We will follow a module based approach instead of the content based approach, because the former approach is generally considered to more effective in terms of student learning and retention. The course will be broken down into three modules as given below:

*Module I. Tensor Calculus and Notation*

- I.1 The continuous media assumption
- I.2 Scalars, vectors and tensors
- I.3 Indicial, tensor and matrix notation
- I.4 Scalar, vector and tensor products
- I.5 Change of orthonormal basis
- I.6 Principal values and principal directions
- I.7 Tensor calculus and cylindrical coordinates

*Module II. Isotropic and Anisotropic Linearly Elastic Solid*

- II.1 Traction and Cauchy stress tensor
- II.2 Invariants of stress tensor, volumetric and deviatoric stress
- II.3 Equations of motion: principle of linear momentum
- II.4 Stress-based material failure theories
- II.5 Motion and deformation, current and reference configurations
- II.6 Strain measures, linearized displacement gradients
- II.7 Linear isotropic elasticity
- II.8 Infinitesimal theory of isotropic elasticity
- II.9 Anisotropic elasticity: material symmetry and material constants

*Module III. Isotropic Nonlinearly Elastic Solid*

- III.1 Geometric measures of deformation
- III.2 Finite strain measures and polar decomposition
- III.3 Change of area and volumes due to deformation
- III.4 Piola-Kirchhoff stress tensors and work conjugates
- III.5 Balance of mass, momentum and energy
- III.6 Nonlinear isotropic elasticity

## Grading Policy

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| Homework | 30% |
| Midterm I | 15% |
| Midterm II | 15% |
| Final Exam | 20% |
| Oral Presentation | 10% |
| Reading Assignments | 10% |

### Homework

Homework will be assigned from the textbook problem sets and the students are required to turn them in neat handwriting. Alternatively, homeworks may be typed in MS Word or Latex. Submission of the homework is typically 1-2 weeks after it has been assigned. MATLAB programming may be a part of some of the assignments and students are required to email the working codes. All the assignments and the due dates will be posted on Brightspace (also see tentative course schedule).

### Midterms and Final Exam

The exams are usually closed book, however, upto two-page formula sheets are permitted. Calculators and Laptops may be allowed for performing matrix and vector calculations in Matlab; however, you are not permitted to refer to the online notes or search the web for course material during the examination.

### Oral Presentation

The student will choose a journal article of their preference in the broad area of computational continuum mechanics and will make an 25 minute oral presentation using power point slides (or similar software) including the Q&A session. The student should adequately demonstrate the ability to understand and explain the principles of continuum mechanics employed in the paper and derive or interpret the key results in the paper. The oral presentation will be scheduled for a day at the end of the semester.

### Reading Assignment

Typically, the reading assignment will comprise of 5-6 sections of a chapter from the main course text book "Introduction to the Continuum Mechanics, by M. W. Lai, D. Rubin, E. Krempl, Elsevier, 2010." Every student is REQUIRED to complete the assigned readings before class and post their feedback either through perusall or discussion boards through Brightspace. Ideally, the feedback would indicate the muddy concepts encountered during your reading or any extending questions not discussed in the chapters. Your feedback will be due on/before 11 am on the day of class. Each class will begin with a 5 minute review of the discussion board comments.

## Other Reference Books:


1. Gurtin, M.E., "An Introduction to Continuum Mechanics," Academic Press.
2. Malvern, L. E., "Introduction to the Mechanics of a Continuous Medium," Prentice-Hall, 1969.
3. Gonzalez, O. and Stuart, A.M., "A First Course in Continuum Mechanics," Cambridge University Press.
4. Timoshenko. S.P. and Goodier, J.N., "Theory of Elasticity," McGraw-Hill Book Co.
5. Pook, L.P., "Linear Elastic Fracture Mechanics for Engineers: Theory and Applications," Computational Mechanics Inc.
6. Love, A.E.H., "A Treatise on the Mathematical Theory of Elasticity," Dover
7. Sokolnikoff, I.S., "Mathematical Theory of Elasticity," McGraw-Hill
8. Boresi, A.P. and Chong, K.P., " Elasticity in Engineering Mechanics," Elsevier
9. Filonenko-Borodich, M., "Theory of Elasticity," Foreign Language Publishing House, Moscow
10. Fung, Y.C., "Foundations of Elasticity, " Prentice-Hall
11. Mall, A.K. and Singh, S., "Deformation of Elastic Solids," Prentice-Hall
12. Mushkelisvilli, N.I., "Some Basic Problems of the Mathematical Theory of Elasticity," Noordhoff
13. Scada, "Elasticity - Theory and Applications," Pergamon Press
14. Wang, C. , "Applied Elasticity," McGraw-Hill

## CE 6200 – Tentative Course Schedule

| Class | Date | Topic | Req. Reading | Assignment due |
|---|---|---|---|---|
| 1 | 8/23 | I.1 The continuous media assumption<br>I.2 Scalars, vectors and tensors | Pages_1- 7_from Mase et al. | |
| 2 | 8/28 | I.3 Indicial, tensor and matrix notation | Chapter 2– Tensors | |
| 3 | 8/30 | I.4 Scalar, vector and tensor products | | HW #1 |
| 4 | 9/4 | I.5 Change of orthonormal basis (rotation of axis) | | |
| 5 | 9/6 | I.6 Principal values and principal directions | | HW #2 |
| 6 | 9/11 | I.7 Tensor calculus | | |
| 7 | 9/13 | | Chapter 4 – Stress | HW #3 |
| 8 | 9/18 | II.1 Traction and stress tensor, normal and shear stress | | |
| 9 | 9/20 | II.2 & II.3 Invariants of stress tensor, Equations of motion: principle of linear momentum | | HW #4 |
| 10 | 9/25 | II.4 Stress-based failure theories | Failure Theories | |
| 11 | 9/27 | II.5 Motion and deformation, current and reference configurations | Chapter 3 – Kinematics | HW #5 |
| 10/2 | | Midterm #1 | Module I, Module II.1 – II.3 | |
| 12 | 10/4 | II.6 Strain measures, linearized displacement gradients | Chapter 3 – Kinematics | |
| 13 | 10/6 | II.7 Linear isotropic elasticity | Chapter 5 – Elastic Solid | |
| 14 | 10/16 | | | HW# 6 |
| 15 | 10/18 | II.8 Infinitesimal theory of isotropic elasticity | | |
| 16 | 10/20 | | | |
| 17 | 10/27 | II.9 Anisotropic elasticity, material symmetry and material constants | | HW# 7 |
| 18 | 10/30 | | | |
| 19 | 11/1 | III.1 Geometric measures of deformation | Chapter 3 – Kinematics | |
| 20 | 11/3 | III.2 Finite strain measures and polar decomposition | | |
| 21 | 11/6 | | | HW# 8 |
| 11/8 | | Midterm# 2 | Module II.4 –II.9 | |
| 22 | 11/13 | III.3 Change of area and volumes due to deformation | Chapter 3 – Kinematics | HW# 9 |
| 23 | 11/16 | III.4 Piola–Kirchhoff stress tensors and work conjugates | Chapter 4 – Stress | |
| 11/18 – 11/26 | | Thanksgiving Holiday Break | No class | |
| 24 | 11/29 | III.5 Balance of mass and momentum | Chapter 4 – Stress | HW# 10 |
| 25 | 12/4 | III.6 Nonlinear isotropic elasticity | Chapter 5 – Elastic Solid | |
| 26 | 12/6 | Oral Presentation | | |
| 12/8 | | Final Exam | Modules I – III | |

*** Note that HW assignments will be posted on OAK. http://www.vanderbilt.edu/oak/**