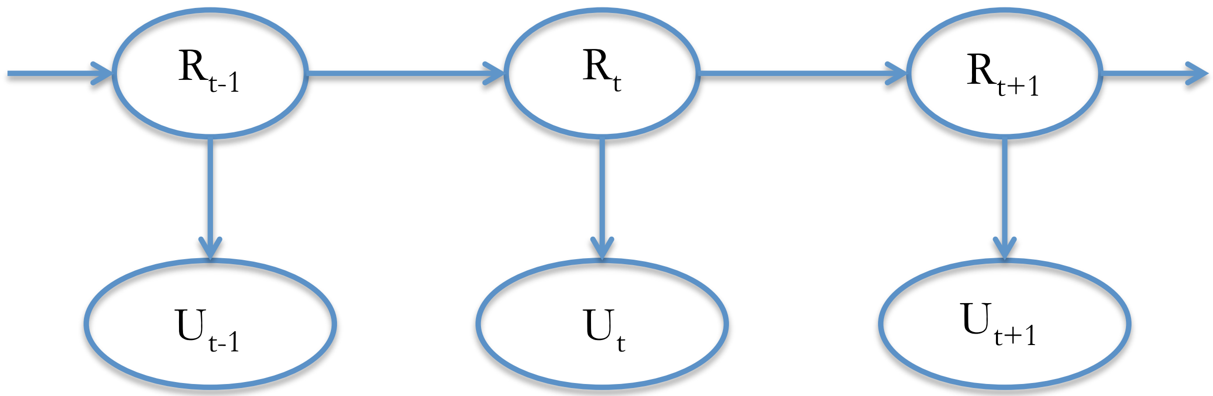
Consider the following Bayesian Network structure and conditional distributions

$P(R_0 = t) = 0.5$

R_{t-1}	$P(R_t \mid R_{t-1})$
$R_{t-1}=t$	$P(R_t=t \mid R_{t-1}=t)=0.7$
$R_{t-1}=f$	$P(R_t=t \mid R_{t-1}=f)=0.3$



R_t	$P(U_t \mid R_t)$
$R_t=t$	$P(U_t=t \mid R_t=t)=0.9$
$R_t=f$	$P(U_t=t \mid R_t=f)=0.2$

1) Compute $P(U_0=f \mid R_0=t)$
2) Compute $P(R_1=t)$
3) Compute $P(R_1=t \mid U_1 = t)$
4) Compute $P(R_1=t \mid U_1 = f)$
5) Compute $P(R_2=t \mid U_1 = t)$
6) Compute $P(R_2=t \mid U_1 = f)$
7) Compute $P(R_2=t \mid U_1 = t, U_2 = t)$
8) Compute $P(R_2=t \mid U_1 = t, U_2 = f)$

Use the FORWARD update procedure. Show your work

Did you reference the textbook while solving these?

If so, did you feel you needed to do so, or did you do so just to check your work after you finished?

Answer here Name: ______________________________

1) Compute $P(U_0=f|R_0=t) = 1 - P(U_0=t|R_0=t) = 1- 0.9 = 0.1$

2) Compute $P(R_1=t) = P(R_1=t|R_0=t)\ P(R_0=f) + P(R_1=t|R_0=f)P(R_0=f) = 0.7*0.5 + 0.3*0.5 = 0.5$

3) Compute $P(R_1=t \mid U_1 = t) = \alpha P(U1=t \mid R1=t)P(R1=t) = \alpha 0.9*0.5 = \alpha 0.45$, where $\alpha = 1/P(U1=t)$

   a) Compute $P(R_1=f \mid U_1 = t) = \alpha P(U1=t \mid R1=f)P(R1=f) = \alpha 0.2*0.5 = \alpha 0.1$, where $\alpha = 1/P(U1=t)$

   Scale to sum to 1.0 : $\alpha(0.45 + 0.1) = 1.0 \rightarrow \alpha = 1.0 /0.55 = 1.82$
   $P(R_1=t \mid U_1 = t) = 1.82 * 0.45 = 0.818$;
   $P(R_1=f \mid U_1 = t) = 1.82 * 0.1 = 0.182$

4) Compute $P(R_1=t \mid U_1 = f) = \alpha P(U1=f \mid R1=t)P(R1=t) = \alpha 0.1*0.5 = \alpha 0.05$, where $\alpha = 1/P(U1=f)$

   a) Compute $P(R_1=f \mid U_1 = f) = \alpha P(U1=f \mid R1=f)P(R1=f) = \alpha 0.8*0.5 = \alpha 0.4$, where $\alpha = 1/P(U1=f)$

   Scale to sum to 1.0 : $\alpha(0.05 + 0.4) = 1.0 \rightarrow \alpha = 1.0 /0.45 = 2.22$
   $P(R_1=t \mid U_1 = f) = 2.22 * 0.05 = 0.11$;
   $P(R_1=f \mid U_1 = f) = 2.22 * 0.4 = 0.89$

5) One formulation that stems from definition of Bayesian Network (not the FORWARD formulation illustration)

Compute $P(R_2=t \mid U_1 = t) = \alpha\, (P(U_1=t, R_2=t, R_1=t)+P(U_1=t, R_2=t, R_1=f))$

$= \alpha\, (P(U_1=t, R_2=t \mid R_1=t)P(R_1=t)+P(U_1=t, R_2=t \mid R_1=f)P(R_1=f))$

$= \alpha\, (P(U_1=t \mid R_1=t)P(R_2=t \mid R_1=t)P(R_1=t)$

$+P(U_1=t \mid R_1=f)\, P(R_2=t \mid R_1=f)P(R_1=f))$

$= \alpha\, (0.9*0.7*0.5 + 0.2*0.3*0.5) = \alpha*0.345$

Compute $P(R_2=f \mid U_1 = t) = \alpha\, (P(U_1=t, R_2=f, R_1=t)+P(U_1=t, R_2=f, R_1=f))$

$= \alpha\, (P(U_1=t, R_2=f \mid R_1=t)P(R_1=t)+P(U_1=t, R_2=f \mid R_1=f)P(R_1=f))$

$= \alpha\, (P(U_1=t \mid R_1=t)P(R_2=f \mid R_1=t)P(R_1=t)$

$+P(U_1=t \mid R_1=f)\, P(R_2=f \mid R_1=f)P(R_1=f))$

$= \alpha\, (0.9*0.3*0.5 + 0.2*0.7*0.5) = \alpha*0.205$

Scale to sum to 1: $\alpha(0.345 + 0.205) = 1.0 \rightarrow \alpha = 1.0 / 0.55 = 1.82$

$P(R_2=t \mid U_1 = t) = 1.82 * 0.345 = 0.627$

$P(R_2=f \mid U_1 = t) = 1.82 * 0.205 = 0.373$

5) Alternate formulation consistent with the FORWARD algorithm (top of page 573)

Compute $P(R_2{=}t \mid U_1 = t) = \alpha\,(P(U_1{=}t, R_2{=}t, R_1{=}t)+P(U_1{=}t, R_2{=}t, R_1{=}f))$

$= \alpha\,(P(R_1{=}t, R_2{=}t \mid U_1{=}t)P(U_1{=}t)+P(R_1{=}f, R_2{=}t \mid U_1{=}t)P(U_1{=}t))$

$= \alpha\,(P(R_1{=}t \mid U_1{=}t)P(R_2{=}t \mid R_1{=}t)P(U_1{=}t)$

$+P(R_1{=}f \mid U_1{=}t)\,P(R_2{=}t \mid R_1{=}f)P(U_1{=}t))$, where $\alpha = 1/P(U_1{=}t)$

$= P(R_1{=}t \mid U_1{=}t)P(R_2{=}t \mid R_1{=}t) + P(R1{=}f \mid U1{=}t)\,P(R_2{=}t \mid R_1{=}f)$

$= 0.818 * 0.7 + 0.182 * 0.3 = 0.627$

Compute $P(R_2{=}f \mid U_1 = t)$ ~~$= \alpha\,(P(U1{=}t, R2{=}f, R1{=}t)+P(U1{=}t, R2{=}f, R1{=}f))$~~

~~$= \alpha\,(P(U1{=}t, R2{=}f \mid R1{=}t)P(R1{=}t)+P(U1{=}t, R2{=}f \mid R1{=}f)P(R1{=}f))$~~

~~$= \alpha\,(P(U1{=}t \mid R1{=}t)P(R2{=}f \mid R1{=}t)P(R1{=}t)$~~

~~$+P(U1{=}t \mid R1{=}f)\,P(R2{=}f \mid R1{=}f)P(R1{=}f))\ \alpha = 1/P(U1{=}t)$~~

$= P(R_1{=}t \mid U_1{=}t)P(R_2{=}f \mid R_1{=}t) + P(R_1{=}f \mid U_1{=}t)\,P(R_2{=}f \mid R_1{=}f)$

$= 0.818 * 0.3 + 0.182 * 0.7 = 0.373$

6) Compute $P(R_2{=}t \mid U_1 = f)$ – on your own

7) Compute $P(R_2{=}t \mid U_1 = t, U_2 = t)$ – next slide (and top of page 573)

8) Compute $P(R_2{=}t \mid U_1 = t, U_2 = f)$ – on your own

7) Alternate formulation consistent with the FORWARD algorithm illustration (pp. 572-573)

Compute $P(R_2{=}t \mid U_1 = t, U_2 = t)$

$= \alpha\ (P(U2{=}t \mid R2{=}t, U1{=}t)P(R2{=}t \mid U1{=}t))$

$= \alpha\ (P(U2{=}t \mid R2{=}t)P(R2{=}t \mid U1{=}t))$ eq 15.4 p. 572 sensor Markov assumption

$= \alpha(0.9*0.627)$

$= \alpha(0.564)$

$= 1.565 * 0.564 = 0.883$

Compute $P(R_2{=}f \mid U_1 = t, U_2 = t)$

$= \alpha\ (P(U2{=}t \mid R2{=}f, U1{=}t)P(R2{=}f \mid U1{=}t))$

$= \alpha\ (P(U2{=}t \mid R2{=}f)P(R2{=}f \mid U1{=}t))$ eq 15.4 p. 572 sensor Markov assumption

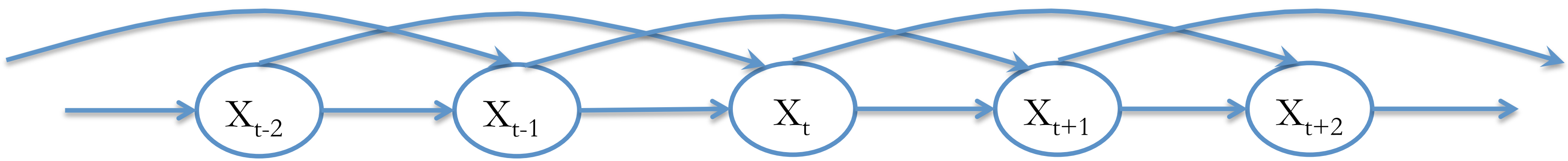
$= \alpha(0.2*0.373)$

$= \alpha(0.075)$

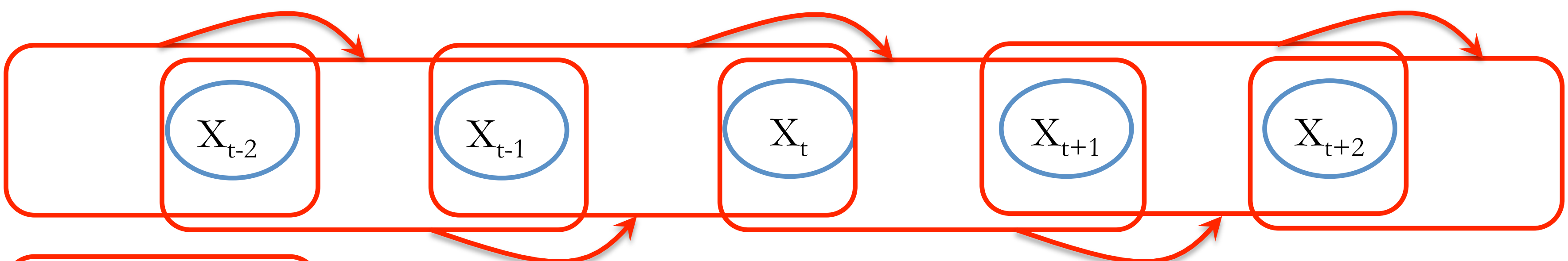
$= 1.565 * 0.075 = 0.117$

Scale to sum to 1: $\alpha(0.564+0.075) = 1 \Rightarrow \alpha = 1/0.639 = 1.565$

Show that any second-order Markov process can be rewritten as a first-order Markov process with an augmented set of state variables. Can this always be done *parsimoniously*, i.e., without increasing the number of parameters needed to specify the transition model? (problem 15.1 of text)



Each X_i is a set of the same state variables , but at different times (p. 567)



New states, each with suitably labeled state variables (X_{i-1} union X_i) = $X_{i-1,i}$

Parsimonious? Yes – same number of conditioning state variables in each $P(X_t|X_{t-1},X_{t-2})$ and $P(X_{t,t-1}|X_{t-1,t-2})$, with t-1 variable values shared