Addendum to *Agenda Constrained Legislator Ideal Points and the Spatial Model*

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In this addendum we demonstrate how to reinterpret Lemma 1 of Clinton and Meirowitz (2001) in the space of unconstrained problems for a fixed d, L, T. Denote this space as Φ. An element of this space (called a problem) is a pair $(\mathbf{h}, f)$, where $\mathbf{h}$ is a dataset of roll call votes and f is a likelihood function $f : \mathbf{H} \times X^{L=2T} \to \mathbb{R}^1$. For two problems ϑ and ξ, we define the distance between the problems as $dist(\vartheta,\xi) := \sup_{Ad(\vartheta,\xi)} \|(\mathbf{a}^u, \mathbf{x}^u, \mathbf{q}^u)(\vartheta) - (\mathbf{a}^u, \mathbf{x}^u, \mathbf{q}^u)(\xi)\|$, where $(\mathbf{a}^u, \mathbf{x}^u, \mathbf{q}^u)(\vartheta)$ is a solution to an unconstrained problem ϑ and $Ad(\vartheta,\xi)$ is the set of pairs of extrema to the unconstrained problems (ϑ,ξ). We need to introduce the complexity of taking the *sup* over $Ad(\vartheta,\xi)$ because there is no guarantee that the problems elicit unique extrema. This distance is not a metric on the space Φ because there exist multiple distinct problems which induce the same set of extrema. Thus, there are distinct ϑ,ξ for which $dist(\vartheta,\xi) = 0$. To solve this problem we can consider a different space Φ' which contains one element of each equivalence class of solutions. On this space the operator $dist(\vartheta,\xi)$ is a metric. By Ψ we denote the topology on Φ' induced by this metric. By $\digamma$ we denote the sigma algebra generated by Ψ. Let μ be an arbitrary measure $\mu : \digamma \to \mathbb{R}^1_+$ satisfying the condition: $\mu(A) = 0$ if there is no set $B \subset A$ with $B \in \Psi$. So that the measure assigns measure 0 to any set with empty interior. Then the reinterpretation of Lemma 1 becomes.

Lemma 2: Fix d, L, T. Let $\mathbf{A}$ be the subset of Φ' for which the constraint does not bind, then $\mu(\mathbf{A}) = 0$.

Proof: We first construct the extrema correspondence $\varkappa : \Phi' \rightarrow\rightarrow X^{L+2T}$ that identifies extrema in X^{L+2T} with problems in Φ'. By Lemma 1, the subset A of X^{L+2T} for which the constraint does not bind has *Lebesgue* measure 0. This means that for any $a \in A$, any neighborhood of a contains a point which is not in A. It is sufficient given the condition imposed on the measure μ to show that for any $\vartheta \in \mathbf{A}$, every set in Ψ containing ϑ contains a point $\xi \in \Phi' \backslash \mathbf{A}$. So for arbitrary $\vartheta \in \mathbf{A}$ we now construct such a point. Pick $\vartheta \in \mathbf{A}$. This implies that $\varkappa(\vartheta) \subset A$. Now pick any point $x \in \varkappa(\vartheta)$. By above we know that for any arbitrarily small neighborhood (in X^{L+2T}) of x, there exists a point y in the neighborhood that is not in A. By the definition of $dist(\cdot,\cdot)$ this means that there is a problem ξ for which $y \in \varkappa(\xi)$ but ξ is the same distance from ϑ as x is from y. Thus, the fact that every point in A is arbitrarily close to points that are not in A implies that every point in $\mathbf{A}$ is arbitrarily close to points that are not in $\mathbf{A}$. Thus, the result is established.$\blacksquare$