

Ordinal sums in bounded commutative integral residuated lattices

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The ordinal sum construction has played a major role in the study of residuated structures, in particular of BL-algebras: bounded commutative integral residuated lattices (or BCIRLs) that are representable and satisfy the divisibility condition. Indeed, Aglianò and Montagna in a paper of 2003 show that every subdirectly irreducible BL-algebra can be decomposed into an ordinal sum of sum-irreducible components (that is, Wajsberg hoops), and they use this result for the study of the lattice of subvarieties of BL-algebras.

We will show that if divisibility is dropped altogether, we cannot obtain the same kind of results, even though subdirectly irreducible MTL-algebras (representable BCIRLs) can be seen as ordinal sums of sum-irreducible components. We will then explore generalizations of the ordinal sum construction.

Furthermore, we shall notice that many relevant varieties of BCIRLs are not closed by the operation of ordinal sum. Starting with a focus on varieties that instead have this property, we will describe characteristics of projective algebras in varieties of BCIRLs.