On commutative (pseudo-) BCK-algebras

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Pseudo-BCK-algebras (or biresiduation algebras) are the $\{\backslash, /, 1\}$-subreducts of integral residuated lattices, and pseudo-ŁBCK-algebras (or cone algebras) are the $\{\backslash, /, 1\}$-subreducts of integral GMV-algebras. We call a pseudo-BCK-algebra commutative if it satisfies the equation $(x/y)\backslash x \approx y/(x\backslash y)$. Pseudo-ŁBCK-algebras may be characterized as commutative pseudo-BCK-algebras satisfying the prelinearity law.

First, we characterize ŁBCK-algebras by means of "forbidden subalgebras" and then we find and axiomatize the covers of certain varieties in the lattice of subvarieties of commutative BCK-algebras.

Second, we focus on commutative pseudo-BCK-algebras that have "enough idempotents"; we call an element a idempotent if $a\backslash(a\backslash x) = a\backslash x$ for all x. We show that such algebras are in fact pseudo-ŁBCK-algebras and that the presence of enough idempotents allows us to construct the left adjoint to the forgetful functor from bounded GMV-algebras to pseudo-ŁBCK-algebras.

Finally, we prove a version of Cantor-Bernstein theorem for commutative pseudo-BCK-algebras.