THE STRUCTURE OF BOOLEAN COMMUTATIVE IDEMPOTENT RESIDUATED LATTICES

PETER JIPSEN

A residuated lattice is *Boolean* if its lattice reduct is the reduct of a Boolean algebra. For example the residuated lattice reducts of relation algebras are Boolean residuated lattices. In case the Boolean algebra is complete and atomic, the structure of the monoid operation is determined by a ternary relation R on the set of atoms. We show that if the residuated lattice is idempotent then R is determined by two binary relations P, Q and if it is commutative then P and Q coincide and are characterized by being a preorder forest with singleton roots.

We use this result to construct all finite Boolean commutative idempotent residuated lattices and determine when they are subdirectly irreducible. Furthermore, we consider to what extent the description of the Boolean case can be extended to cover distributive commutative idempotent residuated lattices.