Lattice-ordered groups and their subreducts

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Lattice-ordered groups (ℓ-groups) have inverse-free reducts that are distributive as lattices and multiplication distributes over both meet and join (totally distributive ℓ-monoids). It is known that in the abelian case, the inverse-free subreducts satisfy more equations than the above and they are actually not finitely based. We prove that in the general case of all ℓ-groups, the inverse-free subreducts are exactly the totally distributive ℓ-monoids. Also, for the intermediate case of subreducts of semilinear ℓ-groups (subdirect products of chains) we show that special equations hold. We further provide an equational axiomatization for the variety of all semilinear totally distributive ℓ-monoids. A proof theory for ℓ-groups exists but it is complicated, relying on hypersequents. On the other hand a tame proof-theoretic calculus exists for totally-ordered ℓ -monoids. We provide a syntactic transformation from an ℓ-group identity to an inverse-free identity so that they are both equi-valid in ℓ-groups; so the existing calculus for totally distributive ℓ-monoids can be used for deciding ℓ-group equations. The translation can be seen as an application of the density rule in proof theory.