NILPOTENCE, SEMILINEARITY AND THE HAMILTONIAN PROPERTY

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Nilpotent lattice-ordered groups of class $c \in \mathbb{N}^+$ are those lattice-ordered groups (ℓ-groups, for short) whose group reduct is nilpotent of class c, and form a variety that can be defined relative to the variety of ℓ-groups by a semigroup equation ([4, 6]). Hamiltonian ℓ-groups are those for which every convex subalgebra is normal. The connection between these two classes goes beyond the fact that they can be seen as generalizations of the variety of Abelian ℓ-groups. For instance, it was shown in [3] that every nilpotent ℓ-group is Hamiltonian (cf. [8]), and a negative answer to the question whether the variety generated by all nilpotent ℓ-groups is the largest Hamiltonian variety was given in [1].

In this joint work with Constantine Tsinakis, we exploit well-known results about nilpotent and Hamiltonian ℓ-groups to argue for similar results about nilpotent and Hamiltonian prelinear cancellative residuated lattices. We begin by giving a positive answer to the question whether there exists a largest variety of Hamiltonian prelinear cancellative residuated lattices. Later, we extend work done in [5], and provide a categorical equivalence between nilpotent cancellative residuated lattices and nilpotent ℓ-groups endowed with a conucleus. By means of this result, nilpotent prelinear cancellative residuated lattices are proved to be semilinear and Hamiltonian. We also exploit the categorical equivalence to provide a characterization of monoid subreducts of nilpotent ℓ-groups.

Since both Hamiltonian and nilpotent cancellative prelinear residuated lattices are proved to be semilinear, this work is partly concerned with semilinear residuated lattices as well, with particular interest for integral cancellative varieties. We show, *inter alia*, that any variety of integral semilinear cancellative residuated lattices defined by semigroup equations is generated by those chains which are finitely generated as monoids. This leads to a generation result for varieties of integral nilpotent cancellative residuated lattices in terms of integrally ordered relatively free monoids, inspired by an analogue result holding for nilpotent ℓ-groups.

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