Compare & Discuss Problems

Topic 5: Solving Quadratic Equations

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Implementation Checklist

Compare	**Prepare to Compare** Students took time to understand what the problem was asking and understand both methods.	☐ Yes ☐ No
	Make Comparisons Students identified mathematical similarities and differences between the two methods.	☐ Yes ☐ No
Discuss	**Prepare to Discuss** Think: Students spent around 1 minute thinking independently about the worksheet prompts.	☐ Yes ☐ No
	Pair: Students spent around 2 minutes working in pairs or small groups discussing the worksheet prompts.	☐ Yes ☐ No
	Discuss Connections Share: A 3-6 minute whole-class conversation occurred where students discussed connections that included question asking and answering by the teacher and students. • Most students were involved in this whole class conversation. • The teacher asked follow-up questions in response to students' thinking, such as "Why do you think that's true?", "Do you agree or disagree? Why?", "Can you say more about that?", and "What did you like about their answer?".	☐ Yes ☐ No
	Identify the Big Idea The teacher showed the Big Idea page to the class to provide a clear, explicit statement of the Big Idea. Students identified the Big Idea and summarized it in their own words.	☐ Yes ☐ No

Timing	At least 8 minutes were spent in the Compare phase and at least 12 minutes were spent in the Discussion phase. Students spent more than half their time in the Discussion phase.	☐ Yes ☐ No

Compare & Discuss: Algebra 1 PD Institute
Discussion Resources

<u>Why have a mathematical discussion?</u>

- To deepen students' understanding of the mathematical content.
- To enhance student engagement and interest in mathematics.

<u>What should a teacher do to have a good mathematical discussion?</u>

- **<u>BEFORE</u>** the discussion starts:
    - Thoroughly **solve** the problems that will be discussed.
    - **Anticipate** student responses, errors, and difficulties.
    - **Plan** questions to ask, as well as problem extensions to use.
- **<u>DURING</u>** the discussion:
    - **Ask** lots of open-ended questions, using the following question stems to spark and continue conversation:
        - *Do you agree with Layla? Why?*
        - *Can you summarize what Riley said?*
        - *Can you give another example?*
        - *Can you describe that in more detail?*
        - *What do you mean by XXXX?*
        - *How did you do that?*
        - *What might be confusing about this example?*
    - **Re-voice** and **summarize** student contributions to keep the conversation going, saying things like:
        - *What I am hearing is XXXX. Is that what you mean?*
        - *Are you saying XXXX?*
        - *I am not sure I understand what you mean. Can you explain it again?*
    - **Manage** flow of the conversation, involving many voices from the class.
    - **Involve as many students** in the discussion as possible.
        - Be sure to **solicit** participation from students who do not have their hands raised, using *equity sticks*, *note cards*, *spinners*, or a *random name generator* for randomly selecting students to speak.
        - **Consider keeping track** of which students have spoken with a clipboard of the class roster, both to remember who has spoken and to ensure equitable participation.
    - **Hold** students accountable for listening to and understanding others' contributions, saying things like:
        - *Gloria thinks that XXXX. Tim, can you summarize what Gloria said, in your own words?*
    - **Provide** students credit for discussion participation as part of their grade.

Prepare to Compare & Discuss: Teacher Prep Checklist

For each Worked Example Pair, it is important you review the problem and its associated worksheets in advance before presenting it to the class. When prepping, keep the following checklist in mind:

- ✓ **Ensure you understand** each method in the WEP.
- ✓ **Read** the Big Idea message so you know where the discussion should lead by the conclusion of the exercise.
- ✓ **Review** the prompt on the *Discuss Connections* worksheet, and:
  - **Add** extension questions that will push your students to dig deeper during the discussion, OR
  - **Create** additional, supporting questions that will allow struggling students to grapple with the prompt more successfully.
- ✓ **Determine** when in the class you plan to present the WEP.
- ✓ **Make sufficient copies** of the worksheet(s) for participating students.

That's it! For each WEP, we don't anticipate more than 5-10 minutes of prep. Please remember to reach out to research staff if any questions or concerns come up during planning.

Differentiating Compare & Discuss Problems

We strongly believe, and our research supports, that Compare & Discuss problems can be an effective way to engage in mathematics for all learners. Below, you will find a general list of recommendations to keep in mind as you consider differentiating the Compare & Discuss problems to fit your students' needs.

DON'T:

- **Change** the examples such that they are a far removal from the implementation model.
- **Skip** whole chapters.
- **Change or adapt** the tests.
    o For research purposes, it is important every student takes the same test, even if content on the test was not covered in your class.
- **Eliminate** the side-by-side comparison of the solution methods.
- **Rush through/gloss over** the WEPs (don't save them for the last 5 minutes of class!).
    o If you are working with students and the Compare phase seems like it is moving quickly, that might not be a problem—it gives you more time to work on the Discuss phase, incorporating more extension questions for deeper discussion.

DO:

✓ **Plan ahead** with research staff.
✓ **Adapt** WEPs for content not covered, rather than skipping the examples altogether.
✓ **Blend** comparison types – types are not mutually exclusive (some can be both Why does it work? & Which is better?).
    o This may influence your extension questions for the Discuss phase.
✓ **Address** changes for later chapters with lower level classes (content in earlier chapters [1, 3, and 5] tends to be covered in all levels, but you may need to change/adapt content for later chapters [7, 9]).
✓ **Adapt** Which is correct?/How do they differ? WEPs for lower level classes.
    o Some students may be overwhelmed by a comparison with two different problems; others may struggle with determining which method is incorrect. Discuss with research staff ways to adapt these two comparison types for struggling students.

*Lastly, we encourage **<u>creativity</u>**! We're happy to work with you to find ways to incorporate the Compare & Discuss problems into your class as a yearlong theme (e.g. using **Holiday greeting cards, dress-up days**, etc.).*

Topic 5: Solving Quadratic Equations- Overview

Section	Table of Contents (Page #)	WEP Type	Suggested Use
5.1	7	Which is better?	Beginning of lesson
5.2	11	Which is better?	Mid-lesson
5.3	15	Why does it work?	Beginning of lesson
5.4	19	Which is better?	Beginning of lesson
5.5	23	Which is correct?	Beginning of lesson
5.6	27	Why does it work?	Mid-lesson
5.7	31	Which is better?	Review activity

Compare (8 minutes)

Prepare to Compare
- What is the problem asking?
- What is happening in the first method?
- What is happening in the second method?

Make Comparisons
- What are the similarities and differences between the two methods?
  - Which method is better?
  - Which method is correct?
  - Why do both methods work?
  - How do the problems differ?

Discuss (12minutes)

Prepare to Discuss (think, pair)
- How does this comparison help you understand this problem?
- How might you apply these methods to a similar problem?

Discuss Connections (share)
- What ideas would you like to share with the class?

Identify the Big Idea
- Can you summarize the Big Idea in your own words?

Topic 5.1: Properties of Radicals

WEP Type: Which is better?
Suggested use: Beginning of lesson

Problem: Riley and Gloria were asked to simplify $\sqrt{\frac{15}{18}}$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Riley simplify the expression in his first step? Why did Gloria write the prime factors for the square root of 18 in her first step?**
Make Comparisons	**Which method is better?** *In this case, Riley's "simplify the fraction first" way is better. Reducing the fraction first resulted in fewer steps and made rationalizing the denominator easier.*
Prepare to Discuss (Think, Pair)	**Create a problem where Riley's "simplify the fraction first" way would be easier to use than Gloria's "split up the square root first" way.**
Discuss Connections (Share)	*Riley's "simplify the fraction first" method will be easier than Gloria's "split up the square root first" when the fraction is easily reduced and the denominator is not a number that contains perfect square factors.* *For example,* $\sqrt{\frac{111}{555}}$ *or* $\sqrt{\frac{13}{26}}$.
Identify the Big Idea	**How do you know when you can use Riley's "simplify the fraction first" way on a problem?** *On problems involving radical expressions and fractions, it might be easier to simplify the fractions first before taking the square root.*

Riley and Gloria were asked to simplify $\sqrt{\frac{15}{18}}$.

Riley's "simplify the fraction first" way

I simplified the expression.

$$\sqrt{\frac{15}{18}}$$

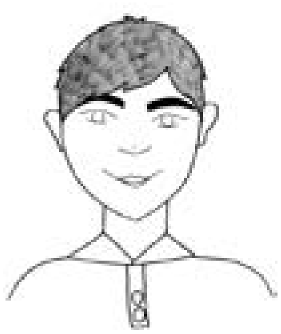
$$\sqrt{\frac{5}{6}}$$

$$\frac{\sqrt{5}}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$$

$$\frac{\sqrt{30}}{6}$$

Then I rationalized the denominator.

I simplified to get my answer.



Gloria's "split up the square root first" way

$$\sqrt{\frac{15}{18}}$$

$$\frac{\sqrt{15}}{\sqrt{2\bullet 3^2}}$$

$$\frac{\sqrt{15}}{3\sqrt{2}}$$

$$\frac{\sqrt{15}}{3\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$\frac{\sqrt{30}}{6}$$

I wrote the prime factors for the square root of 18.

Then I factored the 3 out of the radical.

I rationalized the denominator and simplified to get my answer.

How did Riley simplify the expression in his first step? Why did Gloria write the prime factors for the square root of 18 in her first step?

Which method is better, Riley's "simplify the fraction first" way or Gloria's "split up the square root first" way?

Discuss Connections

Create a problem where Riley's "simplify the fraction first" way would be easier to use than Gloria's "split up the square root first" way.

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

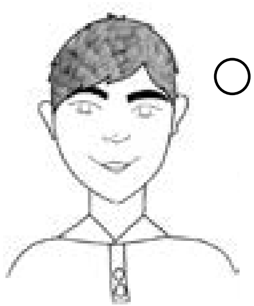
Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Riley and ... were asked ... plify $\sqrt{\frac{15}{18}}$.

Riley's "s... ... **root first"**

How do I know when I can use my "simplify the fraction first" way on a problem?



On problems involving radical expressions and fractions, it might be easier to simplify the fractions first before taking the square root.

How did Riley simplify the expr... n in his first step? W... id Gloria write the prime factors for the square root of 18 in her f... st step?

Which method is better, Riley's "simplify the fraction first" way or Gloria's "split up the square root first" way?

Topic 5.2: Properties of Radicals

WEP Type: Which is better?

Suggested use: Mid-lesson

Problem: Emma and Layla were asked to simplify $\frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}}$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Layla factor out the common factor in her first step? How did Emma simplify the expression in her second step?** **How did Emma get 2 + 10 in the numerator of her second step?** **Why does the $\sqrt{2}$ term cancel out in Layla's second step?**
Make Comparisons	**Which method do you think is better, Emma's "rationalize the denominator first" way or Layla's "simplify the expression first" way? Why?** *In this case, Layla's "simplify the expression first" way is better. Factoring out a common factor of $\sqrt{2}$ makes her calculations easier in later steps.*
Prepare to Discuss (Think, Pair)	**For the problem $\frac{\sqrt{8}+4\sqrt{2}}{\sqrt{8}}$, is it possible to use Layla's "simplify the expression first" method? Why or why not?**
Discuss Connections (Share)	*Yes, you can use Layla's "simplify the expression first" method for this problem like this:* $\frac{\sqrt{8}+4\sqrt{2}}{\sqrt{8}} \rightarrow \frac{2\sqrt{2}+4\sqrt{2}}{2\sqrt{2}} \rightarrow \frac{(2+4)\sqrt{2}}{2\sqrt{2}} \rightarrow \frac{6\sqrt{2}}{2\sqrt{2}} \rightarrow 3$
Identify the Big Idea	**How do you know if you chose a good way to solve this problem?** *Rationalizing denominators can get messy. If it is possible to simplify the expression first, it might make things easier.*

Emma and Layla were asked to simplify $\frac{\sqrt{2}+5\sqrt{2}.}{\sqrt{2}}$

Emma's "rationalize the denominator first" way

$$\frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}}$$

$$\frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

↓

$$\frac{2+10}{2}$$

↓

$$\frac{12}{2}$$

↓

$$6$$

I rationalized the denominator first.

Then I simplified the expression.

I got 12 over 2, or 6.

Layla's "simplify the expression first" way

$$\frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}}$$

$$\frac{\sqrt{2}(1+5)}{\sqrt{2}}$$

↓

$$\frac{\cancel{\sqrt{2}}(1+5)}{\cancel{\sqrt{2}}}$$

↓

$$6$$

I factored out a common factor from the numerator.

Then I simplified the expression.

I got 6.

How did Layla factor out the common factor in her first step? How did Emma simplify the expression in her second step?

Which method is better, Emma's "rationalize the denominator first" way or Layla's "simplify the expression first" way? Why?

For the problem $\frac{\sqrt{8}+4\sqrt{2}}{\sqrt{8}}$, is it possible to use Layla's "simplify the expression first" method?

Why or why not?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Emma and Lay... were asked to sim...lify $\frac{\sqrt{2}+5\sqrt{2}.}{\sqrt{2}}$

Emma's "r... the d...

...plify th... on first"

I ... th... den... first.

Then I ... sim...

I go... over 2,...

...ator.

How do I know if I chose a good way to solve this problem?

Rationalizing denominators can get messy. If it is possible to simplify the expression first, it might make things easier.

How did Layla factor out the co... ...ctor in her fi... ...How did Emma simplify the expression in her second step?

Which method is better, Emma's "rationalize the denominator first" way or Layla's "simplify the expression first" way? Why?

Topic 5.3: Solving Quadratic Equations by Graphing

WEP Type: Why does it work?
Suggested use: Beginning of lesson
Problem: Gloria and Tim were asked to draw the graph of a quadratic equation that has solutions at x = 2 and x = 4.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Gloria know to put points at (2, 0) and (4, 0)? How did Tim know to do this?** **Was it okay for Gloria to guess where to put the vertex? Why?** **Why are the x-intercepts the solutions to a quadratic equation?**
Make Comparisons	**How is the graph that Gloria drew similar or different to the graph that Tim drew?** *Gloria's and Tim's graphs both have the same solutions, meaning the same x-intercepts. The two graphs have different vertices; in addition, one is pointing up, and one is pointing down. Also, both vertices are on the line x = 3.*
Prepare to Discuss (Think, Pair)	**Can both Gloria and Tim be correct in their graphs? Why or why not?**
Discuss Connections (Share)	*Yes, both Gloria and Tim can be correct because they were only given the two solutions for a quadratic equation. When we are only given the two solutions, there could be many different graphs – any parabola that crosses the x-axis at (2, 0) and at (4, 0). But note that all quadratics with solutions at x = 2 and x = 4 have the equation* $y = a(x - 2(x - 4)$ *or* $y = a(x^2 - 6x + 8)$. *In vertex form, this would be* $y + a = a(x-3)^2$, *indicating that the vertex would be at (3, -a).*
Identify the Big Idea	**Why do both ways work?** *The solutions for a quadratic equation are the points where the graph crosses the x-axis. In this problem, we weren't told anything else about the parabola. Both graphs have the same solutions, but they are two different quadratic equations.*

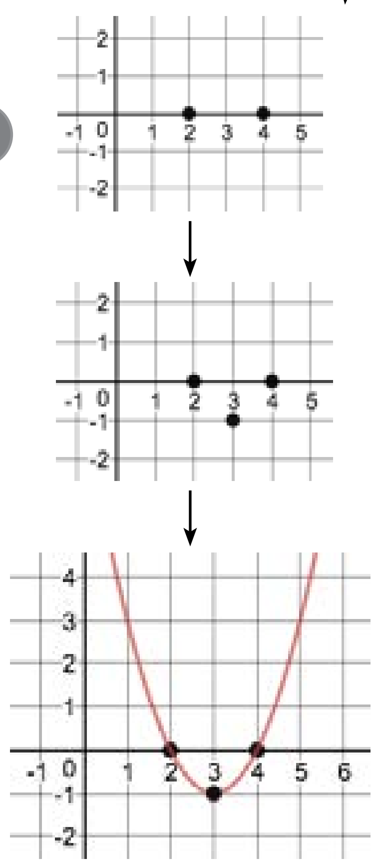
Gloria and Tim were asked to draw the graph of a quadratic equation that has solutions at x = 2 and x = 4.

Gloria's "put points at the solutions" way

I put points at (2, 0) and (4, 0).

I'm not sure where to put the vertex, so I guessed.

Then I drew the parabola.

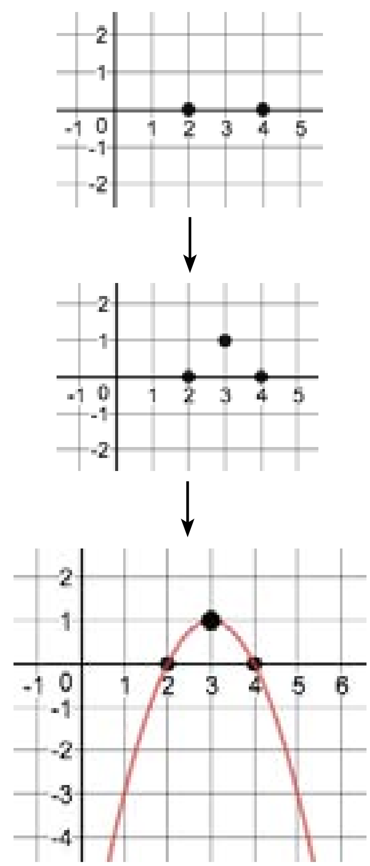


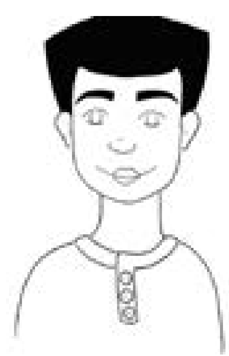
Tim's "use the x-intercepts" way

I put points at the x-intercepts.

Then I put a dot where the vertex is.

Then I drew the parabola.





How did Gloria know to put points at (2, 0) and (4, 0)? How did Tim know to do this?

How is the graph Gloria drew similar or different to the graph that Tim drew?

Discuss Connections

Can both Gloria and Tim be correct in their graphs? Why or why not?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Gloria and Tim were asked to draw the graph of a quadratic equation that has solutions at x = 2 and x = 4.

Gloria's "put points at the ...

...**se the x-int**... **pts" way**

I put points at (2, 0) and 4, 0).

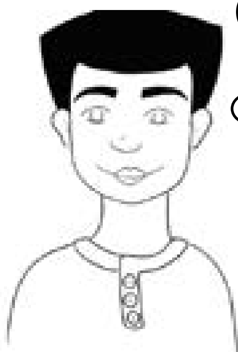
I'm not ... where to ... the verte... I gue...

The... the para...

I put points at the x-intercepts.

... I put ... ot where ... vertex is.

... the ...abola.



Why do both ways work?

The solutions for a quadratic equation are the points where the graph crosses the x-axis. In this problem, we weren't told anything else about the parabola. Both graphs have the same solutions, but they are two different quadratic equations.

How did Gloria know to put po... (2, 0) and (4, 0)... ...id Tim know to do this?

How is the graph Gloria drew similar or different to the graph that Tim drew?

Topic 5.4: Solving Quadratic Equations Using Square Roots

WEP Type: Which is better?
Suggested use: Beginning of lesson
Problem: Tim and Emma were asked to solve $(2x-2)^2 = 16$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**What does Emma mean when she says she "rewrote the equation in standard form"? What steps did she take to do this?** **Why did Tim write ± 4 in his second step?** **Why did the 4 disappear in Emma's fifth step?**
Make Comparisons	**Which method is better, Tim's "take the square root of both sides first" way, or Emma's "change to standard form first" way? Why?** *In this example, Tim's method is better. When the quadratic equation is written so that two perfect squares are on each side of the equal sign, the easiest solution method is to take the square root of both sides first.*
Prepare to Discuss (Think, Pair)	**Will one method always be better than the other? Why or why not?**
Discuss Connections (Share)	*No, because it depends on how the equation is written. If the equation were written differently, e.g. in standard form, it would not make sense to use Tim's method to factor the equation in a way that would allow you to take the square root of both sides.* *Facilitator note: You may want to write an example and non-example of equations that can use Tim's "take the square root of both sides first" method for students to discuss.*
Identify the Big Idea	**How do you know if you chose a good way to solve this problem?** *When both the left and right sides of the equation are expressions that are perfect squares, it might be easier to take the square root of both sides first.*

Tim and Emma were asked to solve $(2x - 2)^2 = 16$.

Tim's "take the square root of both sides first" way

I took the square root of both sides of the equation.

$(2x - 2)^2 = 16$

$\sqrt{(2x - 2)^2} = \sqrt{16}$

(?) $2x - 2 = \pm 4$

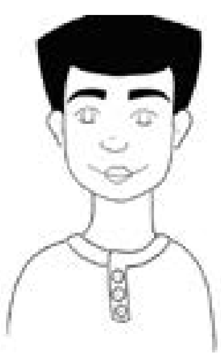
$2x = 2 \pm 4$

Then I solved for x.

$x = \dfrac{2 \pm 4}{2}$

$x = 3$ or $x = -1$

I got 3 and -1.



Emma's "change to standard form first" way

I rewrote the equation in standard form.

$(2x - 2)^2 = 16$

$(2x - 2)(2x - 2) = 16$

$4x^2 - 8x + 4 = 16$

$4x^2 - 8x - 12 = 0$

Then I factored out a common factor of 4.

(?) $4(x^2 - 2x - 3) = 0$

$x^2 - 2x - 3 = 0$

I factored the equation and solved.

$(x - 3)(x + 1) = 0$

$x = 3$ or $x = -1$

I got 3 and -1.

(?) Why did Tim write ±4 in his second step?

(⟷) Which method is better, Tim's "take the square root of both sides first" way or Emma's "change to standard form first" way? Why?

Discuss Connections

Will one method always be better than the other? Why or why not?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Tim and Emm were asked to solv $(2x - 2)^2 = 16$.

Tim's "tak ... ro ... nge t ... form first"

I took the square root of both sid equa

Then I so for x.

I rewrote the equation standard

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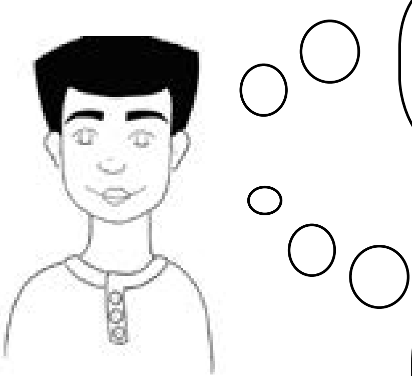


How do I know if I chose a good way to solve this problem?

When both the left and right sides of the equation are expressions that are perfect squares, it might be easier to take the square root of both sides first.

Why did Tim write ±4 in his seco ep?

Which method is better, Tim's "take the square root of both sides first" way or Emma's "change to standard form first" way? Why?

Topic 5.5: Solving Quadratic Equations Using Square Roots

WEP Type: Which is correct?

Suggested use: Beginning of lesson

Problem: Riley and Gloria were asked to solve $4(x+5)^2 = 64$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Riley and Gloria find the solutions to the equation?** **Why did Riley and Gloria both have to use the ± sign?** **Why did Gloria divide a 4 from both sides first?** **How did Riley take the square root of both sides?**
Make Comparisons	**Which method is correct, Riley's "take the square root first" way or Gloria's "divide from both sides first" way? How do you know?** *Gloria's "divide from both sides first" method is correct. Riley's "take the square root first" method is incorrect because he did not take the square root of the number outside the square binomial when he took the square root of both sides. You can prove which method is correct by plugging the answer choices back into the original equation.*
Prepare to Discuss (Think, Pair)	**Explain why Gloria's "divide from both sides first" way cannot be easily used for the problem $9(x+5)^2 = 25$, then solve using Riley's "take the square root first" way.**
Discuss Connections (Share)	*Gloria's "divide from both sides first" way cannot be easily used in this problem because 9 is not a factor of 25. If we divide by 9 on both sides, it will leave us with a fraction on the right side. Instead, we can solve using Riley's "taking the square root first" way like this:* $9(x+5)^2 = 25$ $\sqrt{9(x+5)^2} = \sqrt{25}$ $3(x+5) = \pm 5$ $3x + 15 = 5$ ⎮ $3x + 15 = -5$ $3x = -10$ ⎮ $3x = -20$ $x = -\frac{10}{3}$ ⎮ $x = -\frac{20}{3}$
Identify the Big Idea	**How can you avoid the mistake Riley made on the first problem?** *If you decide to take the square root of both sides, you should remember to take the square root everywhere you are supposed to.*

Riley and Gloria were asked to solve $4(x+5)^2 = 64$.

Riley's "take the square root first" way

$4(x+5)^2 = 64$

$\sqrt{4(x+5)^2} = \sqrt{64}$

I took the square root of each side of the equation.

$4(x+5) = \pm 8$

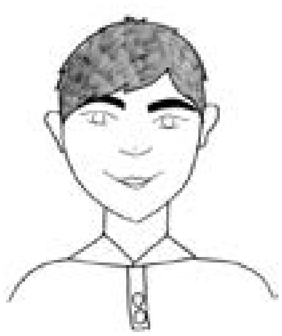
$x + 5 = \pm 2$

I simplified to solve.

$x = -5 \pm 2$

$x = -7$ or $x = -3$

I got -7 and -3.



Gloria's "divide from both sides first" way

$4(x+5)^2 = 64$

$(x+5)^2 = 16$

$\sqrt{(x+5)^2} = \sqrt{16}$

I divided both sides of the equation by 4. Then I took the square root.

$x + 5 = \pm 4$

I simplified to solve.

$x = -5 \pm 4$

$x = -9$ or $x = -1$

I got -9 and -1.

Why did Riley and Gloria both have to use the $\pm$ sign?

Which method is correct, Riley's "take the square root first" way or Gloria's "divide from both sides first" way? How do you know?

Discuss Connections

Explain why Gloria's "divide from both sides first" way cannot be easily used for the problem $9(x + 5)^2 = 25$**, then solve using Riley's "take the square root first" way.**

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Riley and Gloria were asked to solve $4(x + 5)^2 = 64$.

Riley's "tal... ro... **...ide from ... des first"**

I divided both sides of the e... 4. ... the ... root.

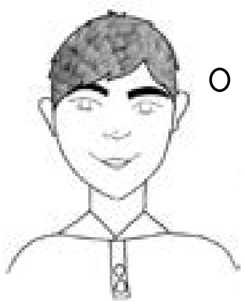
I simplifie... solve...

I got -9... -1.

I divided both sides of the ...

...lified to

...nd



How can I avoid the mistake I made on the first problem?

If I decide to take the square root of both sides, I should remember to take the square root everywhere I am supposed to.

Why did Riley and Gloria both ha... use the ± sign?

Which method is correct, Riley's "take the square root first" way or Gloria's "divide from both sides first" way? How do you know?

Topic 5.6: Solving Quadratic Equations by Completing the Square

WEP Type: Why does it work?
Suggested use: Mid-lesson
Problem: Emma and Layla were asked to solve $x^2 + 6x + 4 = 0$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why did Emma move the c term first?** **Why did Layla add 5 to both sides of the equation in her first step?**
Make Comparisons	**What is similar about Emma's "move the constant term to the other side to complete the square" method and Layla's "set the equation equal to 0 to complete the square" method, and what is different? How did they arrive at the same solutions?** *Emma and Layla both solved the equation by completing the square. The only difference in their methods is how they arranged the terms in their first steps to complete the square. Emma subtracted the constant term, while Layla added 5 to both sides to complete the square right away.*
Prepare to Discuss (Think, Pair)	**Explain how Emma and Layla's methods could be used to solve** $x^2 + 8x + 3 = 0$.
Discuss Connections (Share)	*Emma would start by subtracting 3 from each side. She would then add 16 to each side in order to create the perfect square trinomial* $x^2 + 8x + 16$ *on the left side, which would result in 13 on the right side. Layla would start by setting the right side to 0, which in this case is already done. She would add 13 to each side in order to create the perfect square trinomial* $x^2 + 8x + 16$ *on the left side, which would result in 13 on the right side. From that point on, the two methods are the same.* *Facilitator note: You may decide whether you want the students to solve using Emma and Layla's ways, or just identify that Emma would move over the 3 to the right-hand side and add 16 to both sides, and Layla would keep the 3 as is and add 13 to both sides.*
Identify the Big Idea	**Why does moving the constant term to the other side work?** *It doesn't matter how you arrange the constant terms as long as you create a perfect square trinomial. Moving the constant term to the other side of the equation helped me see how to complete the square.*

Emma and Layla were asked to solve $x^2 + 6x + 4 = 0$.

Emma's "move the constant term to the other side to complete the square" way

$$x^2 + 6x + 4 = 0$$

$$x^2 + 6x = -4$$

I moved the constant, or 4, term to the other side of the equation. Then I completed the square.

$$x^2 + 6x + 9 = -4 + 9$$

$$x^2 + 6x + 9 = 5$$

I factored.

$$(x + 3)^2 = 5$$

I took the square root of both sides and simplified to get my answers.

$$\sqrt{(x+3)^2} = \sqrt{5}$$

$$x + 3 = \pm\sqrt{5}$$

$$x = -3 \pm \sqrt{5}$$

Layla's "set the equation equal to 0 to complete the square" way

$$x^2 + 6x + 4 = 0$$

$$x^2 + 6x + 4 = 0$$

I added five to both sides of the equation.

$$x^2 + 6x + 4 + 5 = 0 + 5$$

$$x^2 + 6x + 9 = 5$$

I factored.

$$(x + 3)^2 = 5$$

I took the square root of both sides and simplified to get my answers.

$$\sqrt{(x+3)^2} = \sqrt{5}$$

$$x + 3 = \pm\sqrt{5}$$

$$x = -3 \pm \sqrt{5}$$

Why did Layla add 5 to both sides of the equation in her first step?

What is similar about Emma's "move the constant term to the other side to complete the square" method and Layla's "set the equation equal to 0 to complete the square" method, and what is different? How did they arrive at the same solutions?

Discuss Connections

Explain how Emma's "move the constant term to the other side to complete the square" method and Layla's "set the equation equal to 0 to complete the squre" method could be used to solve $x^2 + 8x + 3 = 0$.

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

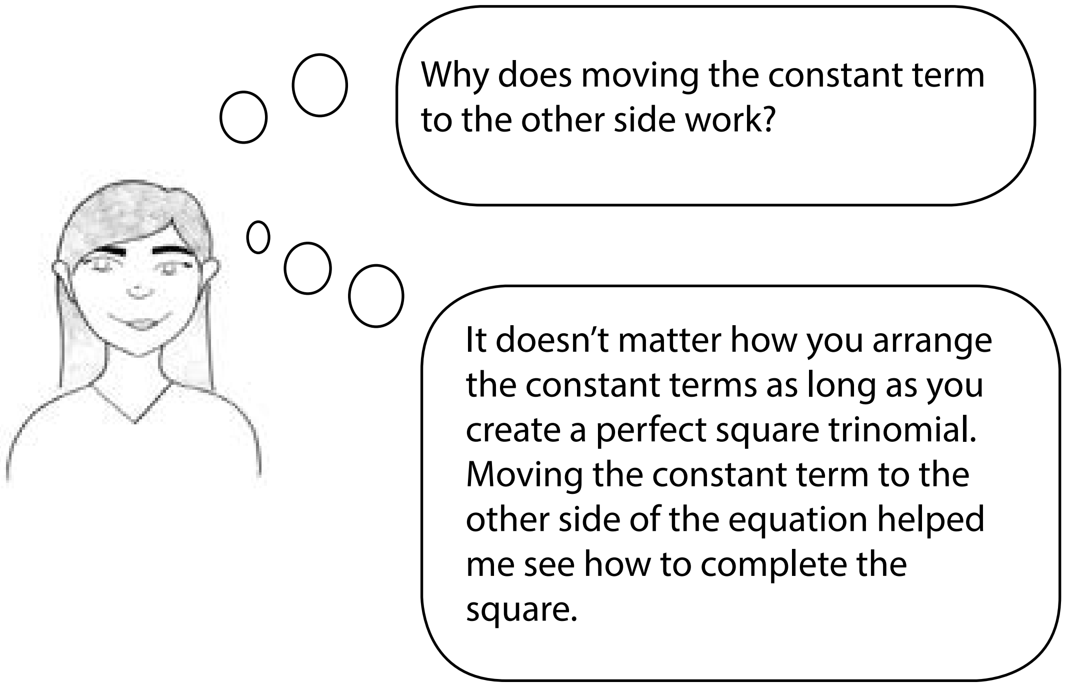
Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Emma and Layla were asked to solve $x^2 + 6x + 4 = 0$.

Emma's "move the constant ... other side ... plete the

... t the equation equal to 0 to ... lete the ... way



Why did Layla add 5 to both s... equation ... tep?

What is similar about Emma's "m... e the constant term t... e other side to complete the square" method and Layla's "set the equation equal to 0 to complete the square" method, and what is different? How did they arrive at the same solutions?

Topic 5.7: Choosing a Method to Solve Quadratic Equations

WEP Type: Which is better?
Suggested use: Review activity
Problem: Find a partner. Each of you will solve $(x+1)(x+1) = 36$, and the two methods you use must be ***different***.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Which methods did you and your partner use to solve the equation?** *Facilitator Note: Identify the methods used by asking partners to present out their work. Keep track of common methods used to guide discussion towards why that method may have been used most, and why other methods were used less frequently.*
Make Comparisons	**What are the advantages and disadvantages of each method? Which method do you think is better for solving this problem?** *Facilitator Note: Ask partners to pair-share the advantages and disadvantages to each method they used before having them share out for whole group discussion. Consider asking students to come to the board to contribute to a pros/cons T-chart for different methods used.*
Prepare to Discuss (Think, Pair)	**Create a problem where Partner A's way would work better. Then create a problem where Partner B's way would work better.**
Discuss Connections (Share)	*Facilitator Note: Answers may vary significantly at this point, and that's ok! Encourage students to work with their partners to identify problems that work better for each method they used, circulating and assisting groups as needed. Ask volunteers to share their ideas with the class. For whole group discussion, ask guided questions about how groups created the problems and what characteristics of the problems lend themselves to certain methods over others.*
Identify the Big Idea	**What do you think? Fill in your reasons why your method is a good one for this problem.** *A number of methods to solve quadratic equations exist, but if you look at the features of the problem, you can determine the most efficient way of solving. For problems like this, you want to look out for perfect squares. If there is a square binomial on one side of the equation and a perfect square constant on the other side, you can solve quickly by taking the square root of both sides.* *Facilitator Note: Encourage students to share their opinions by listing out all the methods they used and taking a class vote on which method is best. This is a suggested takeaway for this type of problem, though students may have other ideas! Ask volunteers to justify their responses for which method is best.*

Find a partner. Each of you will solve $(x+1)(x+1)=36$, **and the two methods you use must be *<u>different</u>*. Write your name in the space below, and show the work for your method.**

______________________________'s way	______________________________'s way

Which methods did you and your partner use to solve the equation?

What are the advantages and disadvantages of each method? Which method do you think is better for solving this problem?

Discuss Connections

Create a problem where Partner A's way would work better. Then create a problem where Partner B's way would work better.

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Find a partner. Each of you will solve $(x+1)(x+1)=36$**, and the two methods you use must be *different*. Write your name in the space below, and show the work for your method.**

________________________ 's way

What do you think? Fill in your reasons why your method is a good one for this problem.

Which methods did you and your partner use to solve the equation?

What are the advantages and disadvantages of each method? Which method do you think is better for solving this problem?