Compare & Discuss Problems

Topic 4: Polynomials and Factoring

 compare@gse.harvard.edu

Implementation Checklist

Compare	**Prepare to Compare** Students took time to understand what the problem was asking and understand both methods.	☐ Yes	☐ No
	Make Comparisons Students identified mathematical similarities and differences between the two methods.	☐ Yes	☐ No
Discuss	**Prepare to Discuss** Think: Students spent around 1 minute thinking independently about the worksheet prompts.	☐ Yes	☐ No
	Pair: Students spent around 2 minutes working in pairs or small groups discussing the worksheet prompts.	☐ Yes	☐ No
	Discuss Connections Share: A 3-6 minute whole-class conversation occurred where students discussed connections that included question asking and answering by the teacher and students. • Most students were involved in this whole class conversation. • The teacher asked follow-up questions in response to students' thinking, such as "Why do you think that's true?", "Do you agree or disagree? Why?", "Can you say more about that?", and "What did you like about their answer?".	☐ Yes	☐ No
	Identify the Big Idea The teacher showed the Big Idea page to the class to provide a clear, explicit statement of the Big Idea. Students identified the Big Idea and summarized it in their own words.	☐ Yes	☐ No

Timing	At least 8 minutes were spent in the Compare phase and at least 12 minutes were spent in the Discussion phase. Students spent more than half their time in the Discussion phase.	☐ Yes	☐ No

Compare & Discuss: Algebra 1 PD Institute
Discussion Resources

<u>Why have a mathematical discussion?</u>

- To deepen students' understanding of the mathematical content.
- To enhance student engagement and interest in mathematics.

<u>What should a teacher do to have a good mathematical discussion?</u>

- **<u>BEFORE</u>** the discussion starts:
  - Thoroughly **solve** the problems that will be discussed.
  - **Anticipate** student responses, errors, and difficulties.
  - **Plan** questions to ask, as well as problem extensions to use.
- **<u>DURING</u>** the discussion:
  - **Ask** lots of open-ended questions, using the following question stems to spark and continue conversation:
    - *Do you agree with Layla? Why?*
    - *Can you summarize what Riley said?*
    - *Can you give another example?*
    - *Can you describe that in more detail?*
    - *What do you mean by XXXX?*
    - *How did you do that?*
    - *What might be confusing about this example?*
  - **Re-voice** and **summarize** student contributions to keep the conversation going, saying things like:
    - *What I am hearing is XXXX. Is that what you mean?*
    - *Are you saying XXXX?*
    - *I am not sure I understand what you mean. Can you explain it again?*
  - **Manage** flow of the conversation, involving many voices from the class.
  - **Involve as many students** in the discussion as possible.
    - Be sure to **solicit** participation from students who do not have their hands raised, using *equity sticks*, *note cards*, *spinners*, or a *random name generator* for randomly selecting students to speak.
    - **Consider keeping track** of which students have spoken with a clipboard of the class roster, both to remember who has spoken and to ensure equitable participation.
  - **Hold** students accountable for listening to and understanding others' contributions, saying things like:
    - *Gloria thinks that XXXX. Tim, can you summarize what Gloria said, in your own words?*
  - **Provide** students credit for discussion participation as part of their grade.

Prepare to Compare & Discuss: Teacher Prep Checklist

For each Worked Example Pair, it is important you review the problem and its associated worksheets in advance before presenting it to the class. When prepping, keep the following checklist in mind:

- ✓ **Ensure you understand** each method in the WEP.
- ✓ **Read** the Big Idea message so you know where the discussion should lead by the conclusion of the exercise.
- ✓ **Review** the prompt on the *Discuss Connections* worksheet, and:
    - o **Add** extension questions that will push your students to dig deeper during the discussion, OR
    - o **Create** additional, supporting questions that will allow struggling students to grapple with the prompt more successfully.
- ✓ **Determine** when in the class you plan to present the WEP.
- ✓ **Make sufficient copies** of the worksheet(s) for participating students.

That's it! For each WEP, we don't anticipate more than 5-10 minutes of prep. Please remember to reach out to research staff if any questions or concerns come up during planning.

Differentiating Compare & Discuss Problems

We strongly believe, and our research supports, that Compare & Discuss problems can be an effective way to engage in mathematics for all learners. Below, you will find a general list of recommendations to keep in mind as you consider differentiating the Compare & Discuss problems to fit your students' needs.

DON'T:

- **Change** the examples such that they are a far removal from the implementation model.
- **Skip** whole chapters.
- **Change or adapt** the tests.
    o For research purposes, it is important every student takes the same test, even if content on the test was not covered in your class.
- **Eliminate** the side-by-side comparison of the solution methods.
- **Rush through/gloss over** the WEPs (don't save them for the last 5 minutes of class!).
    o If you are working with students and the Compare phase seems like it is moving quickly, that might not be a problem—it gives you more time to work on the Discuss phase, incorporating more extension questions for deeper discussion.

DO:

✓ **Plan ahead** with research staff.
✓ **Adapt** WEPs for content not covered, rather than skipping the examples altogether.
✓ **Blend** comparison types – types are not mutually exclusive (some can be both Why does it work? & Which is better?).
    o This may influence your extension questions for the Discuss phase.
✓ **Address** changes for later chapters with lower level classes (content in earlier chapters [1, 3, and 5] tends to be covered in all levels, but you may need to change/adapt content for later chapters [7, 9]).
✓ **Adapt** Which is correct?/How do they differ? WEPs for lower level classes.
    o Some students may be overwhelmed by a comparison with two different problems; others may struggle with determining which method is incorrect. Discuss with research staff ways to adapt these two comparison types for struggling students.

*Lastly, we encourage **creativity**! We're happy to work with you to find ways to incorporate the Compare & Discuss problems into your class as a yearlong theme (e.g. using **Holiday greeting cards, dress-up days**, etc.).*

Topic 4: Polynomials and Factoring- Overview

Section	Table of Contents (Page #)	WEP Type	Suggested Use
4.1	7	Which is better?	Mid-lesson
4.2	11	Why does it work?	End of lesson
4.3	15	Which is correct?	Beginning of lesson
4.4	19	How do they differ?	End of lesson
4.5	23	Which is correct?	Mid-lesson
4.6	27	Why does it work?	Mid-lesson
4.7	31	Which is correct?	Mid-lesson
4.8	35	Which is better?	Beginning of lesson
4.9	39	Which is better?	Review activity

Compare (8 minutes)

Prepare to Compare
- What is the problem asking?
- What is happening in the first method?
- What is happening in the second method?

Make Comparisons
- What are the similarities and differences between the two methods?
  - Which method is better?
  - Which method is correct?
  - Why do both methods work?
  - How do the problems differ?

Discuss (12minutes)

Prepare to Discuss (think, pair)
- How does this comparison help you understand this problem?
- How might you apply these methods to a similar problem?

Discuss Connections (share)
- What ideas would you like to share with the class?

Identify the Big Idea
- Can you summarize the Big Idea in your own words?

Topic 4.1: Adding and Subtracting Polynomials

WEP Type: Which is better?

Suggested use: Mid-lesson

Problem: Riley and Gloria were asked to simplify the polynomial

$$(4x^3 - 8x - 1) - (7x^2 - 3)$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why does Riley fill in the gaps with 0 terms?** **What did Riley and Gloria mean when they said that they "distributed the negative"?** **Why does Gloria rearrange the terms in her second step?**
Make Comparisons	**Which method do you think is better, Riley's "vertical" way or Gloria's "horizontal" way? Why?** *In this case, Gloria's "horizontal" way is better. The two expressions have different numbers of terms and are relatively short, so it takes fewer steps to distribute the negative and combine like terms horizontally. Riley has to fill in gaps with 0 terms to use his vertical method, creating an extra step for this problem.*
Prepare to Discuss (Think, Pair)	**If you were asked to simplify $(5x^4 - 7x^3 + x^2 - 8x + 3) + (10x^4 + 4x^3 - 6x^2 + 2x - 11)$, would you use Riley's "vertical" method or Gloria's "horizontal" method? Why?**
Discuss Connections (Share)	*Riley's "vertical" method would work best for this problem because there are many terms to keep track of in each expression when simplifying horizontally. Also, the expressions have the same number of terms, making it easy to line them up vertically to combine.*
Identify the Big Idea	**Is there a better way to simplify than horizontally?** *When combining polynomials, the vertical method makes it easier to see which terms should be combined. With this method, I might be less likely to make a mistake because I line up the like terms under each other, putting in terms with a zero coefficient if needed.*

Riley and Gloria were asked to simplify $(4x^3 - 8x - 1) - (7x^2 - 3)$.

Riley's "vertical" way

$$(4x^3 - 8x - 1) - (7x^2 - 3)$$

First, I rewrote the terms vertically.

$$\begin{array}{r} (4x^3 + 0x^2 - 8x - 1) \\ -(0x^3 + 7x^2 + 0x - 3) \end{array}$$

Then, I distributed the negative and combined the terms.

$$\begin{array}{r} 4x^3 + 0x^2 - 8x - 1 \\ -0x^3 - 7x^2 - 0x + 3 \\ \hline \end{array}$$

I combined like terms to simplify.

$$4x^3 - 7x^2 - 8x + 2$$

Gloria's "horizontal" way

$$(4x^3 - 8x - 1) - (7x^2 - 3)$$

First, I distributed the negative to the second term.

$$4x^3 - 8x - 1 - 7x^2 + 3$$

Then, I rearranged the terms.

$$4x^3 - 7x^2 - 8x - 1 + 3$$

I combined like terms to simplify.

$$4x^3 - 7x^2 - 8x + 2$$

What did Riley and Gloria mean when they said that they "distributed the negative"?

Which method do you think is better, Riley's "vertical" way or Gloria's "horizontal" way? Why?

Discuss Connections

If you were asked to simplify $(5x^4 - 7x^3 + x^2 - 8x + 3) + (10x^4 + 4x^3 - 6x^2 + 2x - 11)$ **would you use Riley's "vertical" method or Gloria's "horizontal" method? Why?**

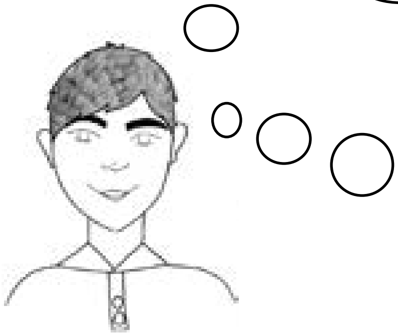
Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Riley and Gloria were asked to simplify $(4x^3 - 8x - 1) - (7x^2 - 3)$.



What did Riley and Gloria mean w… ley said that they … ributed the negative"?

Which method do you think is better, Riley's "vertical" way or Gloria's "horizontal" way? Why?

Topic 4.2: Multiplying Polynomials

WEP Type: Why does it work?
Suggested use: End of lesson
Problem: Gloria and Tim were asked to multiply

$$(x + 5)(x^2 + 4x + 2)$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Gloria and Tim multiply the polynomials?** **What does Tim mean when he says he "distributed the x^2 to (x + 5). Then [he] distributed the 4x, followed by the 2"?** **How is this different from what Gloria did?**
Make Comparisons	**What is similar about Gloria's "distribute (x + 5)" method and Tim's "distribute (x^2 + 4x + 2)" method? What is different? Is one method better than the other?** **Why do both methods work?** *Gloria and Tim both used distribution to solve this problem. Gloria distributed the $(x + 5)$ term, whereas Tim distributed the $(x^2 + 4x + 2)$ term. Both resulted in the same solution. In this example, Gloria and Tim had to do a similar amount of work to simplify, but oftentimes it is easier to distribute the shorter term.*
Prepare to Discuss (Think, Pair)	**Does it matter which polynomial you use to distribute? Why or why not?**
Discuss Connections (Share)	*You can choose to distribute either term when multiplying two expressions. The commutative property allows us to multiply in either direction.*
Identify the Big Idea	**Why do both ways work?** *The commutative property of multiplication allows you to distribute either polynomial first. The order of which you multiply the terms does not matter – you will get the same answer either way.*

Gloria and Tim were asked to multiply $(x + 5)(x^2 + 4x + 2)$.

Gloria's "distribute $(x + 5)$" way

First, I distributed the x to $(x^2 + 4x + 2)$. Then I distributed the 5.

I multiplied the terms.

I combined like terms to get my answer.

$$(x + 5)(x^2 + 4x + 2)$$

$$(x)(x^2 + 4x + 2) + (5)(x^2 + 4x + 2)$$

$$\downarrow$$

$$x^3 + 4x^2 + 2x + 5x^2 + 20x + 10$$

$$\downarrow$$

$$x^3 + 9x^2 + 22x + 10$$

Tim's "distribute $(x^2 + 4x + 2)$" way

First, I distributed the x^2 to $(x+5)$. Then I distributed the 4x, followed by the 2.

I multiplied the terms.

I combined like terms to get my answer.

$$(x + 5)(x^2 + 4x + 2)$$

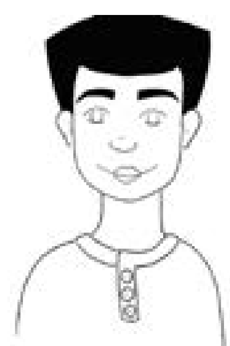
$$(x^2)(x + 5) + (4x)(x + 5) + (2)(x + 5)$$

$$\downarrow$$

$$x^3 + 5x^2 + 4x^2 + 20x + 2x + 10$$

$$\downarrow$$

$$x^3 + 9x^2 + 22x + 10$$



How did Gloria and Tim multiply the two polynomials?

What is similar about Gloria's "distribute $(x + 5)$" method and Tim's " distribute $(x^2 + 4x + 2)$" method? What is different? Is one method better than the other?

Discuss Connections

Does it matter which polynomial you use to distribute? Why or why not?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Gloria and Tim were asked to multiply $(x + 5)(x^2 + 4x + 2)$.

Why do both ways work?

The commutative property of multiplication allows me to distribute either polynomial first. The order which I multiply the terms does not matter - I get the same answer either way.

How did Gloria and Tim mult... ...wo polynomi...

What is similar about Gloria's "distribute $(x + 5)$" method and Tim's " distribute $(x^2 + 4x + 2)$" method? What is different? Is one method better than the other?

Topic 4.3: Solving Polynomial Equations Using the Zero Product Property

WEP Type: Which is correct?
Suggested use: Beginning of lesson
Problem: Riley and Gloria were asked to solve

$$8x^2 - 24x = 0$$

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why did Riley factor out the 8x?**
Make Comparisons	**What is the same or similar about Riley's "factor first" method and Gloria's "divide by x" method? What is different? Which method is correct?** *Riley and Gloria both found x = 3 as a solution to the equation. Their methods were different, however. Riley factored out the 8x first, and then used the zero product property to set both terms of the product equal to 0, resulting in two solutions. Gloria solved by dividing out the x at the end, resulting in only one solution, which is incorrect.*
Prepare to Discuss (Think, Pair)	**Can Riley and Gloria both be correct? How could you check to see if both of Riley's answers are correct?**
Discuss Connections (Share)	*Both Riley and Gloria found answers that were correct, but Gloria only found one of the two correct solutions. We can check to see if both of Riley's answers are correct by first plugging 0 into the original equation for x, and then repeating the process with 3. If both of these numbers result in a true statement when they are plugged into the original equation, it means that they are both correct answers.*
Identify the Big Idea	**How did Gloria's mistake happen?** *Gloria divided by a variable when she should have set each factor equal to zero and solved. This made her miss the solution when the value of the variable is equal to 0.*

Riley and Gloria were asked to solve $8x^2 - 24x = 0$.

Riley's "factor first" way

First, I factored out the 8x.

Then, I set 8x and (x -3) equal to 0 and solved.

Here are my answers.

$$8x^2 - 24x = 0$$

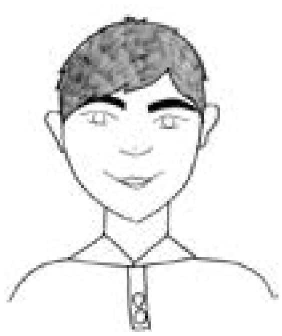
$$8x(x - 3) = 0$$

$$\downarrow$$

$$8x = 0 \text{ or } (x - 3) = 0$$

$$\downarrow$$

$$x = 0 \text{ or } x = 3$$



Gloria's "divide by x" way

$$8x^2 - 24x = 0$$

$$\begin{aligned} 8x^2 - 24x &= 0 \\ +24x \quad &+24x \\ 8x^2 &= 24x \end{aligned}$$

$$\downarrow$$

$$\frac{8x^2}{8x} = \frac{24x}{8x}$$

$$\downarrow$$

$$x = 3$$

First, I added 24x to both sides.

Then, I divided by 8x on both sides.

Here is my answer.

Why did Riley factor out the 8x?

What is the same or similar about Riley's "factor first" method and Gloria's "divide by x" method? What is different?

Discuss Connections

Can Riley and Gloria both be correct? How could you check to see if both of Riley's answers are correct?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Riley and Gl... were asked to s...e $8x^2 - 24x = 0$.

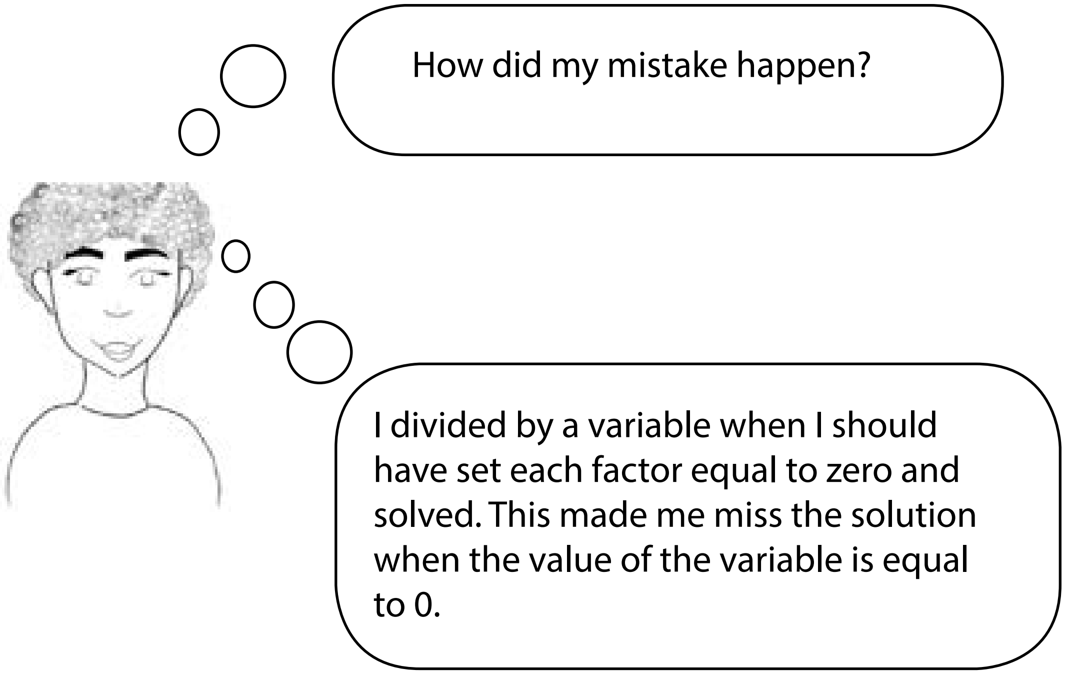
Ril... ... **way**

First, I factored out the 8...

The... set 8... (x -3) to 0 an... solved.

First, I added 24x...

...n, I ...ided by ... both



Why did Riley factor out the 8x?

What is the same or similar about Riley's "factor first" method and Gloria's "divide by x" method? What is different?

Topic 4.4: Using Factoring to Solve Equations and Simplify Expressions

WEP Type: How do they differ?
Suggested use: End of lesson
Problem: Gloria was asked to solve $(15x - 30)(8x + 24) = 0$, and Tim was asked to simplify $(15x - 30)(8x + 24)$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why did the 120 disappear in Gloria's solution, but not Tim's?** **Why did Tim multiply 4 and 2 to get 8, instead of adding 4 and 2 to get 6?**
Make Comparisons	**How are Gloria and Tim's methods similar? Why do their solutions differ?** **How do their problems differ?** *Gloria and Tim both used factoring in their methods. They also both factor 120 out of the terms. Their solutions differ because they were asked to do different things. Since Gloria was asked to solve, her solution includes the values of x. Because Tim was asked to simplify, his solution includes what was factored out of both terms.*
Prepare to Discuss (Think, Pair)	**What is the difference between solving an equation and simplifying an expression?**
Discuss Connections (Share)	*Solving an equation means finding a value for the variable that makes the equation true when you plug it in. Simplifying an expression means re-writing the expression in another, equivalent and often simpler form. We can factor both when we solve and when we simplify, but we are only looking for a value of x when we are asked to solve an equation.*
Identify the Big Idea	**What did we learn from comparing the two ways?** *Gloria's problem involved solving, which means finding what x is equal to. Tim's problem involved simplifying, which means re-writing the expression in a simpler form. Factoring can be used for both solving and simplifying.*

Gloria was asked to solve $(15x - 30)(8x + 24) = 0$, **and Tim was asked to simplify** $(15x - 30)(8x + 24)$**.**

Gloria's problem and method

First, I factored out the greatest common factor of each term.

$$(15x - 30)(8x + 24) = 0$$

$$15(x - 2)8(x + 3) = 0$$

$$\downarrow$$

$$120(x - 2)(x + 3) = 0$$

$$\downarrow$$

$$(x - 2)(x + 3) = 0$$

$$\downarrow$$

$$x - 2 = 0 \text{ or } x + 3 = 0$$

$$\downarrow$$

$$x = 2 \text{ or } x = -3$$

Then, I set the remaining terms equal to 0.

Here are my solutions.

Tim's problem and method

$$(15x - 30)(8x + 24)$$

$$5(3x - 6)4(2x + 6)$$

$$\downarrow$$

$$5 \bullet 3(x - 2)4 \bullet 2(x + 3)$$

$$\downarrow$$

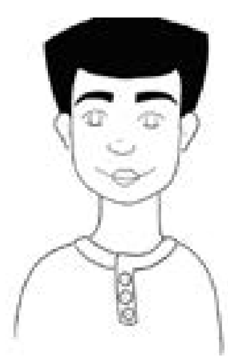
$$15(x - 2)8(x + 3)$$

$$\downarrow$$

$$120(x - 2)(x + 3)$$

First, I factored out a 5 and a 4 from the first and second terms. Then, I factored out an additional 3 and a 2 from the first and second terms.

I multiplied the numbers I factored out together.



Why did the 120 disappear in Gloria's solution, but not Tim's? Why did Tim multiply 4 and 2 to get 8, instead of adding 4 and 2 to get 6?

How are Gloria and Tim's methods similar? Why do their solutions differ?

Discuss Connections

What is the difference between solving an equation and simplifying an expression?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

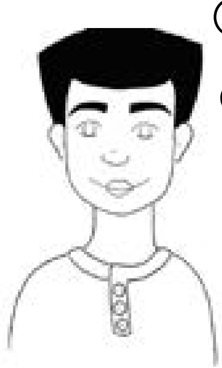
Gloria was asked to solve (15x ... 0)(8x + 24) = 0, an... im was asked to simplify ...20)(8x + 24)

Gloria's ...

First, I factored out a 5 and a 4 fro... and ... terms. I factore... out an additional 3 and a 2 from th... an...

I m... the nu... I factored out togethe...

First, I factored out ... 5 and a ...



What did I learn from comparing the two ways?

Gloria's problem involved solving, which means finding what x is equal to. My problem involved simplifying, which means re-writing the expression in a simpler form. Factoring can be used for both solving and simplifying.

Why did the 120 disappear in Glor... ...tion, but not Ti... ...y did Tim multiply 4 and 2 to get 8, instead of adding 4 and 2 ... get 6?

How are Gloria and Tim's methods similar? Why do their solutions differ?

Topic 4.5: Factoring to Solve Equations

WEP Type: Which is correct?
Suggested use: Mid-lesson
Problem: Layla and Riley were asked to use factoring to solve the equation $a^2 + 5a - 6 = -12$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why did Layla set her equation equal to 0? How did this change her equation?** **Why did Riley split his two equations in this way?** **How could you check to see if Layla or Riley's solutions are correct?**
Make Comparisons	**Which method is correct, Layla's "set equal to 0" method or Riley's "factor first" method?**
Prepare to Discuss (Think, Pair)	**Will Riley's "factor first" method ever get the right answer? Why or why not?**
Discuss Connections (Share)	*Solving by factoring is only a reliable strategy when the multiplied factors are equal to zero, as this allows for the use of the zero product property (when two or more factors multiply to get zero, at least one of the factors must be equal to zero). If two factors multiply to give a number other than zero (such as -12, as in Riley's "factor first" way), it is difficult to conclude anything about either of the factors, since there are so many ways that two numbers can be multiplied to arrive at -12.*
Identify the Big Idea	**How did Riley's mistake happen?** *Riley did not set the equation equal to zero. The zero product rule only applies when the multiplied expressions are set equal to zero.*

Layla and Riley were asked to use factoring to solve the equation $a^2 + 5a - 6 = -12$.

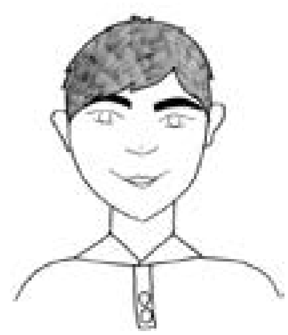
Layla's "set equal to 0" way	**Riley's "factor first" way**
$a^2 + 5a - 6 = -12$	$a^2 + 5a - 6 = -12$
$a^2 + 5a + 6 = 0$	
$(a + 2)(a + 3) = 0$	$(a + 6)(a - 1) = -12$
$a + 2 = 0$ or $a + 3 = 0$	$a + 6 = 6$ or $a - 1 = -2$
$a = -2$ or $a = -3$	$a = 0$ or $a = -1$

First, I set the equation equal to zero by adding 12 to both sides. Then, I factored.

I solved the equations to get my answers.

First, I factored.

Since 6 times -2 is -12, I set the first part equal to 6 and the second part equal to -2. Then I solved the equations to get my answers.



How could you check to see if Layla or Riley's solutions are correct?

Which method is correct, Layla's "set equal to 0" method or Riley's "factor first" method?

Discuss Connections

Will Riley's "factor first" method ever get the right answer? Why or why not?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Layla and Riley were asked to … **factoring to sol**… **he equation** $a^2 + 5a - 6 = -12$.

Layla' … **y**

First, I set the equation equal to ze… add… to bo… sides. I facto…

I solved t… equa… to…

First, I factored.

…al …d …ond …

How did my mistake happen?

I did not set the equation equal to zero. The zero product rule only applies when the multiplied expressions are set equal to zero.

How could you check to see if L… …ey's solution… …ect?

Which method is correct, Layla's "s… equal to 0" method or R…ley's "factor first" method?

Topic 4.6: Factoring Using Special Product Rules

WEP Type: Why does it work?
Suggested use: Mid-lesson
Problem: Gloria and Tim were asked to factor $x^4 - 16$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**What special product rule are Gloria and Tim using?** **Why did Tim choose to substitute in this way?**
Make Comparisons	**Gloria and Tim used different methods but arrived at the same answer. How?** *Gloria and Tim both used the special product rule to factor the difference of two squares. Tim's "substitution first" method is different because it allowed him to break the higher degree term into a smaller square, but he still ended up factoring a difference of two squares. Therefore, he arrived at the same answer as Gloria.*
Prepare to Discuss (Think, Pair)	**Use Tim's "substitution first" method on the problem $x^8 - 1$, substituting a for x^4.**
Discuss Connections (Share)	*For the problem $x^8 - 1$, we can use Tim's "substitution first" method like this:* $x^8 - 1$ $a = x^4$ $a^2 - 1$ $(a - 1)(a + 1)$ $(x^4 - 1)(x^4 + 1)$ $(x^2 - 1)(x^2 + 1)(x^4 + 1)$ $(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$ *When factoring, it is important to keep an eye out for special products, such as the difference of two squares.*
Identify the Big Idea	**Why does substituting a simpler expression into the problem work?** *Substituting a simpler expression in place of a more complicated expression can help us see special products. In this case, substituting a for x^4 might make the expression easier to see.*

Gloria and Tim were asked to factor $x^4 - 16$.

Gloria's "special product rule first" way

First, I factored the difference of two squares.

I then noticed there was another difference of two squares in my answer, so I factored that as well.

$$x^4 - 16$$

$$(x^2 - 4)(x^2 + 4)$$

$$\downarrow$$

$$(x - 2)(x + 2)(x^2 + 4)$$

Tim's "substitution first" way

First, I used a variable to substitute for x^2. I factored using this variable.

I then plugged the x^2 back in for my variable.

I factored again.

$$x^4 - 16$$

$$a = x^2$$

$$\downarrow$$

$$a^2 - 16$$

$$\downarrow$$

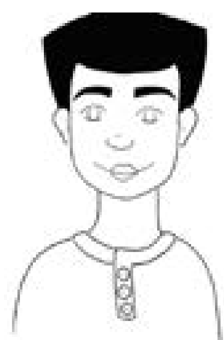
$$(a - 4)(a + 4)$$

$$\downarrow$$

$$(x^2 - 4)(x^2 + 4)$$

$$\downarrow$$

$$(x - 2)(x + 2)(x^2 + 4)$$



What special product rule are Gloria and Tim using?

Gloria and Tim used different methods, but arrived at the same answer. How?

Discuss Connections

Use Tim's "substitution first" method on the problem $x^8 - 1$**, substituting** a **for** x^4.

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Gloria ar... ere aske... or x^4 - 16.

Gloria's "sp... " way

Why does substituting a simpler expression into the problem work?

Substituting a simpler expression in place of a more complicated expression can help us see special products. In this case, substituting *a* for x^4 might make the expression easier to see.

What special product rule are G... d Tim using?

Gloria and Tim used different methods, but arrived at the same answer. How?

Topic 4.7: Solving Quadratic Equations Using Factoring and Isolating the Variable

WEP Type: Which is correct?
Suggested use: Mid-lesson
Problem: Layla and Riley were asked to solve $x^2 + 6x + 9 = x^2 + 4x + 3$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**What did Layla and Riley mean when they said they canceled out terms in their equations? Is it OK to cancel the way that Riley did? Why or why not?** **How did Riley cancel out the x^2 in his first step?** **How did Layla cancel out an (x + 3) on both sides?**
Make Comparisons	**Which method is correct, Layla or Riley's? How do you know?** *Riley's method is correct. Layla factored before the equation was set equal to 0, so her method resulted in an incorrect solution. Riley first worked to get the variables on one side of the equation, and in doing so, the quadratic term cancelled out. So, he was able to solve by just isolating the remaining variable.*
Prepare to Discuss (Think, Pair)	**For the problem (x + 1)(x + 2) = (x + 1)(x + 4), what solution would Riley's way get that Layla's way would not get?**
Discuss Connections (Share)	*Riley's way would give us a solution of $x = -1$. By eliminating the $(x + 1)$ from both sides using division, Layla does not consider that $(x + 1)$ could equal 0.* *We can solve using Riley's way like this:* $(x + 1)(x + 2) = (x + 1)(x + 4)$ $x^2 + 3x + 2 = x^2 + 5x + 4$ $3x + 2 = 5x + 4$ $-2 = 2x$ $x = -1$
Identify the Big Idea	How did Layla's mistake happen? *In the original problem, when she cancelled out (x + 3) by dividing both sides of the equation by (x +3), she didn't consider whether (x + 3) could be equal to zero.*

Layla and Riley were asked to solve $x^2 + 6x + 9 = x^2 + 4x + 3$.

Layla's way	**Riley's way**

Layla's way

First, I factored each side of the equation. Then, I got rid of an (x + 3) by canceling it out.

I subtracted to get the variable on one side.

Since 3 does not equal 1, there is no solution.

$$x^2 + 6x + 9 = x^2 + 4x + 3$$

$$(x+3)^2 = (x+3)(x+1)$$

$$\downarrow$$

$$(x+3) = (x+1)$$

$$\downarrow$$

$$x + 3 = x + 1$$
$$3 = 1$$

$$\downarrow$$

No solution

Riley's way

$$x^2 + 6x + 9 = x^2 + 4x + 3$$

$$6x + 9 = 4x + 3$$

$$\downarrow$$

$$2x + 9 = 3$$

$$\downarrow$$

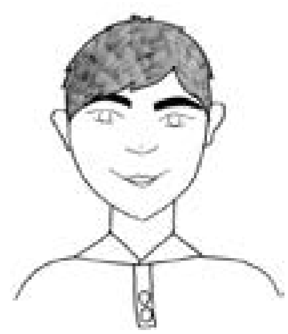
$$2x = -6$$

$$\downarrow$$

$$x = -3$$

First, I canceled out the x^2 term on both sides of the equation. I then solved for the remaining variable.

Here is my answer.



What did Layla and Riley mean when they said they canceled out terms in their equations? Is it OK to cancel the way that Riley did? Why or why not?

Which method is correct, Layla or Riley's? How do you know?

Discuss Connections

For the problem $(x + 1)(x + 2) = (x + 1)(x + 4)$, what solution would Riley's way get that Layla's way would not get?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Layla and Riley were asked to solve $x^2 + 6x + 9 = x^2 + 4x + 3$.

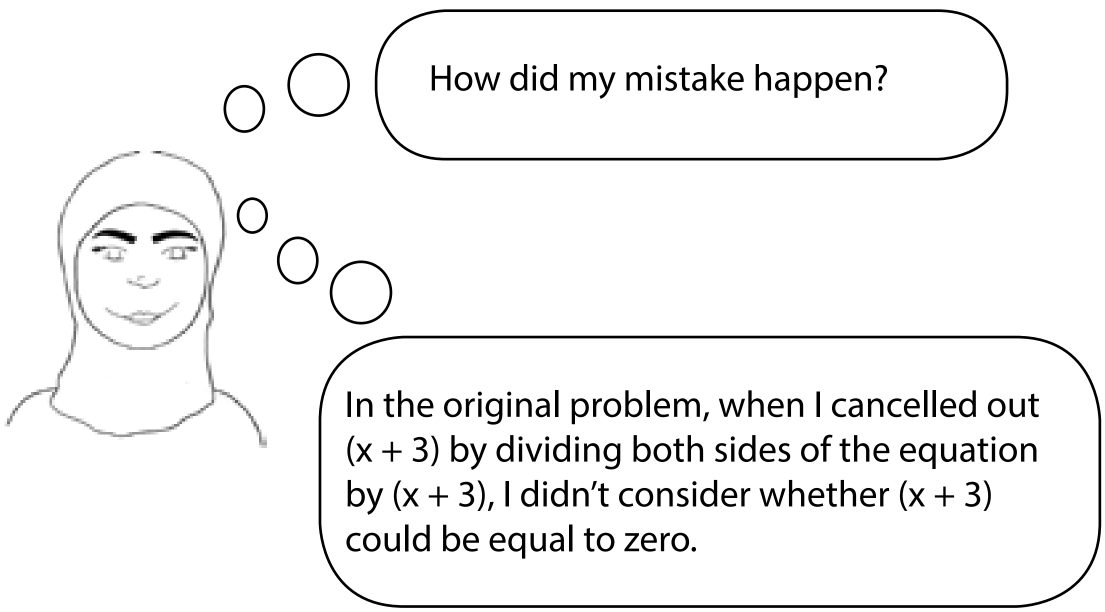
Layla's way	Riley's way

First, I factored each side of the equation. The... rid o... (x + 3)... cancell... out.

I subtrac... to get... v...

S... not e... there is n... solution.

First, I canceled out the x^2 *term on ... ...ed ...hing ...ble.*



What did Layla and Riley mea... ...t terms in their equations? Is it OK to cancel the way that Riley ... why n...

Which method is correct, Layla o... ...y's? How do you k...

Topic 4.8: Factoring Harder Trinomials

WEP Type: Which is better?
Suggested use: Beginning of lesson
Problem: Tim and Emma were asked to factor $6x^3 - 18x^2 + 12x$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Tim and Emma factor the trinomial?** **Why was Tim left with (x – 2) when factoring the second half of the trinomial instead of (x + 2)?** **Why did Emma have to factor out an additional 3 in the first term of her third step?**
Make Comparisons	**How did Tim and Emma get the same answer if they used different methods?** *Tim and Emma both solved by factoring the problem completely – they just approached it differently. Emma started with factoring out a common factor first. Tim solved by factoring out the greatest common factor first. This left him with a quadratic trinomial that did not need to be factored any further, resulting in fewer steps. Both methods are correct and result in the same solution.*
Prepare to Discuss (Think, Pair)	**What would be the GCF that Tim should factor out first on the problem** $4x^4 - 8x^3 - 60x^2$?
Discuss Connections (Share)	*The greatest common factor that Tim should factor out first is* $4x^2$. *We can solve the problem like this:* $4x^4 - 8x^3 - 60x^2$ $4x^2(x^2 - 2x - 15)$ $4x^2(x - 5)(x + 3)$ *When looking for the GCF, it is important to look at both the constants and coefficients, as well as the degree of the variable terms.*
Identify the Big Idea	**How do you know if you chose a good way to solve this problem?** *When you factor out the GREATEST common factor, this makes any remaining factoring you have to do easier.*

Tim and Emma were asked to factor $6x^3 - 18x^2 + 12x$.

Tim's "factor out the GCF" way

$6x^3 - 18x^2 + 12x$

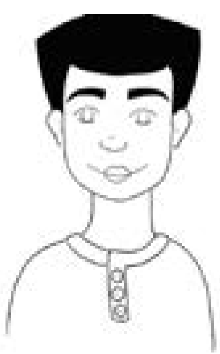
First, I factored out the greatest common factor of 6x.

$6x(x^2 - 3x + 2)$

Then, I factored the trinomial.

Here is my answer.

$6x(x - 1)(x - 2)$



Emma's "factor out a common factor" way

$6x^3 - 18x^2 + 12x$

First, I factored out a common factor of 2x.

$2x(3x^2 - 9x + 6)$

I then factored the trinomial.

$2x(3x - 3)(x - 2)$

I noticed one factored pair needed to be factored further.

$2x \bullet 3(x - 1)(x - 2)$

Here is my answer.

$6x(x - 1)(x - 2)$

How did Tim and Emma factor the trinomial?

How did Tim and Emma get the same answer if they used different methods?

Discuss Connections

What would be the GCF that Tim should factor out first on the problem $4x^4 - 8x^3 - 60x^2$**?**

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

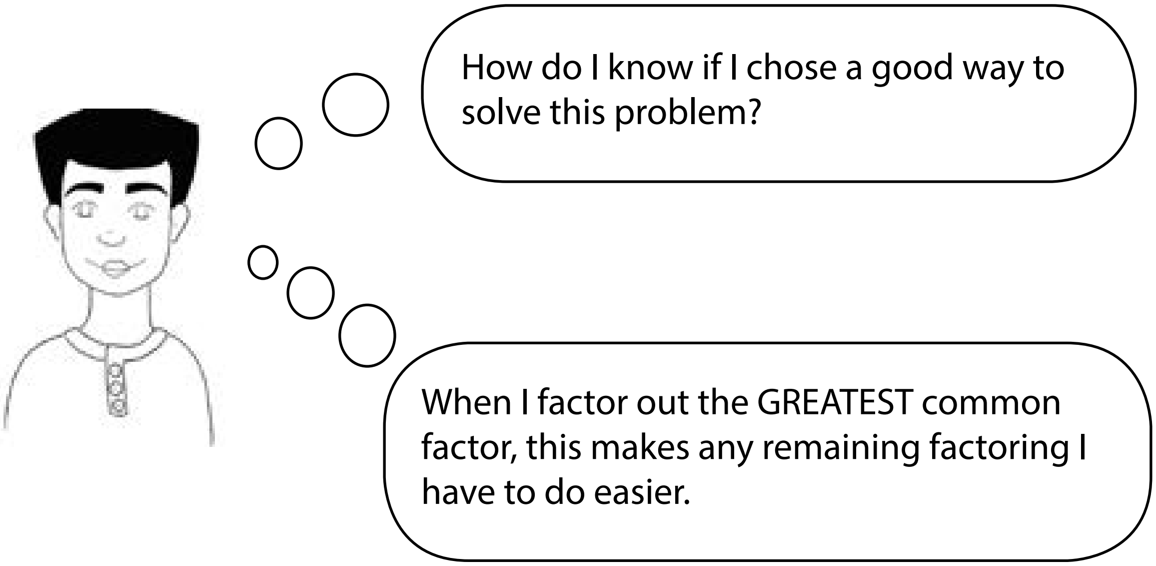
Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Tim and Em **were asked to f** $6x^3 - 18x^2 + 12x$.

Ti **actor** **non factor"**

First, I factored out the

First, I factored out



How did Tim and Emma fa nial?

How did Tim and Emma get t e same answer if they us d different methods?

Topic 4.9: Choosing a Method to Factor Polynomials

WEP Type: Which is better?
Suggested use: Review activity
Problem: Find a partner. Each of you will factor $x^2 + 5x + 6 = x^2 + 10x + 21$ using a *different* method. Show the work for your method.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Which methods did you and your partner use to solve the equation?** *Facilitator Note: Identify the methods used by asking partners to present out their work. Keep track of common methods used to guide discussion towards why that method may have been used most, and why other methods were used less frequently.*
Make Comparisons	**What are the advantages and disadvantages of each method? Which method do you think is better for solving this problem?** *Facilitator Note: Ask partners to pair-share the advantages and disadvantages to each method they used before having them share out for whole group discussion. Consider asking students to come to the board to contribute to a pros/cons T-chart for different methods used.*
Prepare to Discuss (Think, Pair)	**Create a problem where Partner A's way would work better. Then create a problem where Partner B's way would work better.**
Discuss Connections (Share)	*Facilitator Note: Answers may vary significantly at this point, and that's ok! Encourage students to work with their partners to identify problems that work better for each method they used, circulating and assisting groups as needed. Ask volunteers to share their ideas with the class. For whole group discussion, ask guided questions about how groups created the problems and what characteristics of the problems lend themselves to certain methods over others.*
Identify the Big Idea	**What do you think? Fill in your reasons why your method is a good one for this problem.** *There are multiple ways to factor a polynomial. For this problem, taking out common factors or factoring by grouping are easier than using the trial and error method. Regardless, each method will result in the same solution because each helps you separate the common factors in the polynomial. If you look at the features of the problem first, you can find which way will be easiest.* *Facilitator Note: Encourage students to share their opinions by listing out all the methods they used and taking a class vote on which method is best. This is a suggested takeaway for this type of problem, though students may have other ideas! Ask volunteers to justify their responses for which method is best.*

Find a partner. Each of you will factor the polynomial using a *different* method. Write your name in the space below, and show the work for your method.

$x^2 + 5x + 6 = \ x^2 + 10x + 21$

_________________________**'s way**	_________________________**'s way**

Describe the methods used by you and your partner to factor the polynomial.

What are the advantages and disadvantages of each method? Which method do you think is better for solving this problem?

Discuss Connections

Create a problem where Partner A's way would work better. Then create a problem where Partner B's way would work better.

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Find a partner. Each of you will factor the polynomial using a *different* method. Write your name in the space below, and show the work for your method.

x x + 6 = $x^2 + 10x$

________ 's way

What do you think? Fill in your reasons why your method is a good one for this problem.

Describe t ods used polynom

What are the advantages and s of ea hich method do you think is better for solving this problem.