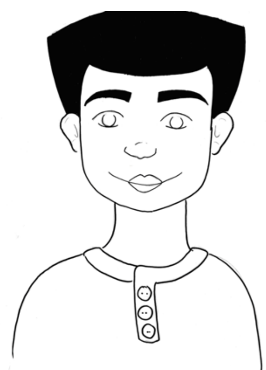
Compare & Discuss Problems

Topic 2: Functions and Graphing Linear Equations



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Implementation Checklist

Compare	**Prepare to Compare** Students took time to understand what the problem was asking and understand both methods.	☐ Yes ☐ No
	Make Comparisons Students identified mathematical similarities and differences between the two methods.	☐ Yes ☐ No
Discuss	**Prepare to Discuss** Think: Students spent around 1 minute thinking independently about the worksheet prompts.	☐ Yes ☐ No
	Pair: Students spent around 2 minutes working in pairs or small groups discussing the worksheet prompts.	☐ Yes ☐ No
	Discuss Connections Share: A 3-6 minute whole-class conversation occurred where students discussed connections that included question asking and answering by the teacher and students. • Most students were involved in this whole class conversation. • The teacher asked follow-up questions in response to students' thinking, such as "Why do you think that's true?", "Do you agree or disagree? Why?", "Can you say more about that?", and "What did you like about their answer?".	☐ Yes ☐ No
	Identify the Big Idea The teacher showed the Big Idea page to the class to provide a clear, explicit statement of the Big Idea. Students identified the Big Idea and summarized it in their own words.	☐ Yes ☐ No

Timing	At least 8 minutes were spent in the Compare phase and at least 12 minutes were spent in the Discussion phase. Students spent more than half their time in the Discussion phase.	☐ Yes ☐ No

Compare & Discuss: Algebra 1 PD Institute
Discussion Resources

<u>Why have a mathematical discussion?</u>

- To deepen students' understanding of the mathematical content.
- To enhance student engagement and interest in mathematics.

<u>What should a teacher do to have a good mathematical discussion?</u>

- **<u>BEFORE</u>** the discussion starts:
  - Thoroughly **solve** the problems that will be discussed.
  - **Anticipate** student responses, errors, and difficulties.
  - **Plan** questions to ask, as well as problem extensions to use.
- **<u>DURING</u>** the discussion:
  - **Ask** lots of open-ended questions, using the following question stems to spark and continue conversation:
    - *Do you agree with Layla? Why?*
    - *Can you summarize what Riley said?*
    - *Can you give another example?*
    - *Can you describe that in more detail?*
    - *What do you mean by XXXX?*
    - *How did you do that?*
    - *What might be confusing about this example?*
  - **Re-voice** and **summarize** student contributions to keep the conversation going, saying things like:
    - *What I am hearing is XXXX. Is that what you mean?*
    - *Are you saying XXXX?*
    - *I am not sure I understand what you mean. Can you explain it again?*
  - **Manage** flow of the conversation, involving many voices from the class.
  - **Involve as many students** in the discussion as possible.
    - Be sure to **solicit** participation from students who do not have their hands raised, using *equity sticks*, *note cards*, *spinners*, or a *random name generator* for randomly selecting students to speak.
    - **Consider keeping track** of which students have spoken with a clipboard of the class roster, both to remember who has spoken and to ensure equitable participation.
  - **Hold** students accountable for listening to and understanding others' contributions, saying things like:
    - *Gloria thinks that XXXX. Tim, can you summarize what Gloria said, in your own words?*
  - **Provide** students credit for discussion participation as part of their grade.

Prepare to Compare & Discuss: Teacher Prep Checklist

For each Worked Example Pair, it is important you review the problem and its associated worksheets in advance before presenting it to the class. When prepping, keep the following checklist in mind:

- ✓ **Ensure you understand** each method in the WEP.
- ✓ **Read** the Big Idea message so you know where the discussion should lead by the conclusion of the exercise.
- ✓ **Review** the prompt on the *Discuss Connections* worksheet, and:
  - o **Add** extension questions that will push your students to dig deeper during the discussion, OR
  - o **Create** additional, supporting questions that will allow struggling students to grapple with the prompt more successfully.
- ✓ **Determine** when in the class you plan to present the WEP.
- ✓ **Make sufficient copies** of the worksheet(s) for participating students.

That's it! For each WEP, we don't anticipate more than 5-10 minutes of prep. Please remember to reach out to research staff if any questions or concerns come up during planning.

Differentiating Compare & Discuss Problems

We strongly believe, and our research supports, that Compare & Discuss problems can be an effective way to engage in mathematics for all learners. Below, you will find a general list of recommendations to keep in mind as you consider differentiating the Compare & Discuss problems to fit your students' needs.

DON'T:

- **Change** the examples such that they are a far removal from the implementation model.
- **Skip** whole chapters.
- **Change or adapt** the tests.
    o For research purposes, it is important every student takes the same test, even if content on the test was not covered in your class.
- **Eliminate** the side-by-side comparison of the solution methods.
- **Rush through/gloss over** the WEPs (don't save them for the last 5 minutes of class!).
    o If you are working with students and the Compare phase seems like it is moving quickly, that might not be a problem—it gives you more time to work on the Discuss phase, incorporating more extension questions for deeper discussion.

DO:

✓ **Plan ahead** with research staff.
✓ **Adapt** WEPs for content not covered, rather than skipping the examples altogether.
✓ **Blend** comparison types – types are not mutually exclusive (some can be both Why does it work? & Which is better?).
    o This may influence your extension questions for the Discuss phase.
✓ **Address** changes for later chapters with lower level classes (content in earlier chapters [1, 3, and 5] tends to be covered in all levels, but you may need to change/adapt content for later chapters [7, 9]).
✓ **Adapt** Which is correct?/How do they differ? WEPs for lower level classes.
    o Some students may be overwhelmed by a comparison with two different problems; others may struggle with determining which method is incorrect. Discuss with research staff ways to adapt these two comparison types for struggling students.

*Lastly, we encourage **<u>creativity</u>**! We're happy to work with you to find ways to incorporate the Compare & Discuss problems into your class as a yearlong theme (e.g. using **Holiday greeting cards, dress-up days**, etc.).*

Topic 2: Functions and Graphing Linear Equations- Overview

Section	Table of Contents (Page #)	WEP Type	Suggested Use
2.1	7	Why does it work?	Mid-lesson
2.2	11	Why does it work?	Mid-lesson
2.3	15	How do they differ?	Beginning of Lesson
2.4	19	Which is correct?	Mid-lesson
2.5	23	Why does it work?	Mid-lesson
2.6	27	Which is better?	End of Lesson
2.7	31	Which is better?	Beginning of Lesson
2.8	35	Which is correct?	Mid-lesson

Compare (8 minutes)

Prepare to Compare
- What is the problem asking?
- What is happening in the first method?
- What is happening in the second method?

Make Comparisons
- What are the similarities and differences between the two methods?
  - Which method is better?
  - Which method is correct?
  - Why do both methods work?
  - How do the problems differ?

Discuss (12minutes)

Prepare to Discuss (think, pair)
- How does this comparison help you understand this problem?
- How might you apply these methods to a similar problem?

Discuss Connections (share)
- What ideas would you like to share with the class?

Identify the Big Idea
- Can you summarize the Big Idea in your own words?

Topic 2.1: Functions

WEP Type: Why does it work?

Suggested Use: Mid-lesson

Problem: Riley and Gloria were given the set of ordered pairs {(–3, 6), (2, 5), (3, 1), (2, 4), (5, 1)}, and asked to determine if the relation is a function.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Riley determine if the relation was a function?** **How did Gloria determine if the relation was a function?** • Riley circles the 2's. Why? • What if the two 2's had both pointed to the number 5? • Why does Riley not worry that there are two 1's in the range? • How did Gloria know where to draw her vertical line?
Make Comparisons	**Why do both methods work?** **Why does the vertical line test tell us the same thing as the table of values?** *Both help you determine if there is a unique output (y) for every given input (x).*
Prepare to Discuss (Think, Pair)	**Why does the vertical line test work?**
Discuss Connections (Share)	*If a vertical line intersects the graph of a relation more than once, this means that there are two (or more) points on the graph that have the same x value but different y values. When this occurs, this means that the relation is not a function.*
Identify the Big Idea	**Why does the vertical line test work?** *If a vertical line intersects the graph more than once, it shows that there are two points on the graph that have the same x value but different y values, so it is not a function.*

Riley and Gloria were given the set of ordered pairs

$\{(-3, 6), (2, 5), (3, 1), (2, 4), (5, 1)\}$,

and asked to determine if the relation is a function.

Riley's "make a table" way

I made a table.

x (domain)	y (range)
-3	6
2	5
2	4
3	1
5	1

I saw that 2 in the domain is paired with both a 5 and a 4 in the range.

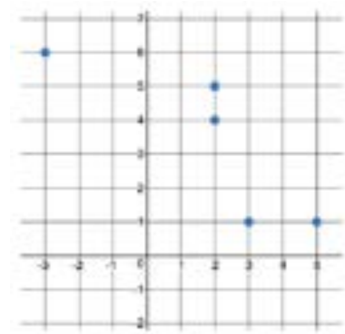
This means the relation is not a function.

x (domain)	y (range)
-3	6
2 →	5
2 →	4
3	1
5	1

Not a function

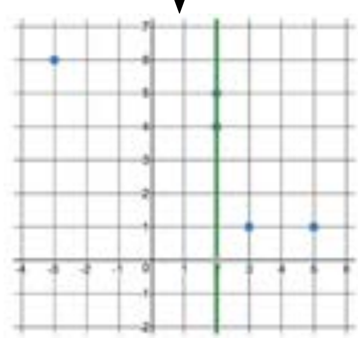
Gloria's "graph and vertical line test" way

I graphed the ordered pairs.



I found a vertical line that intersected two of the points.

This means the relation is not a function.



Not a function

How did Riley determine if the relation was a function? How did Gloria determine if the relation was a function?

Why do both methods work? Why does the vertical line test tell us the same thing as the table of values?

Discuss Connections

Why does the vertical line test work?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

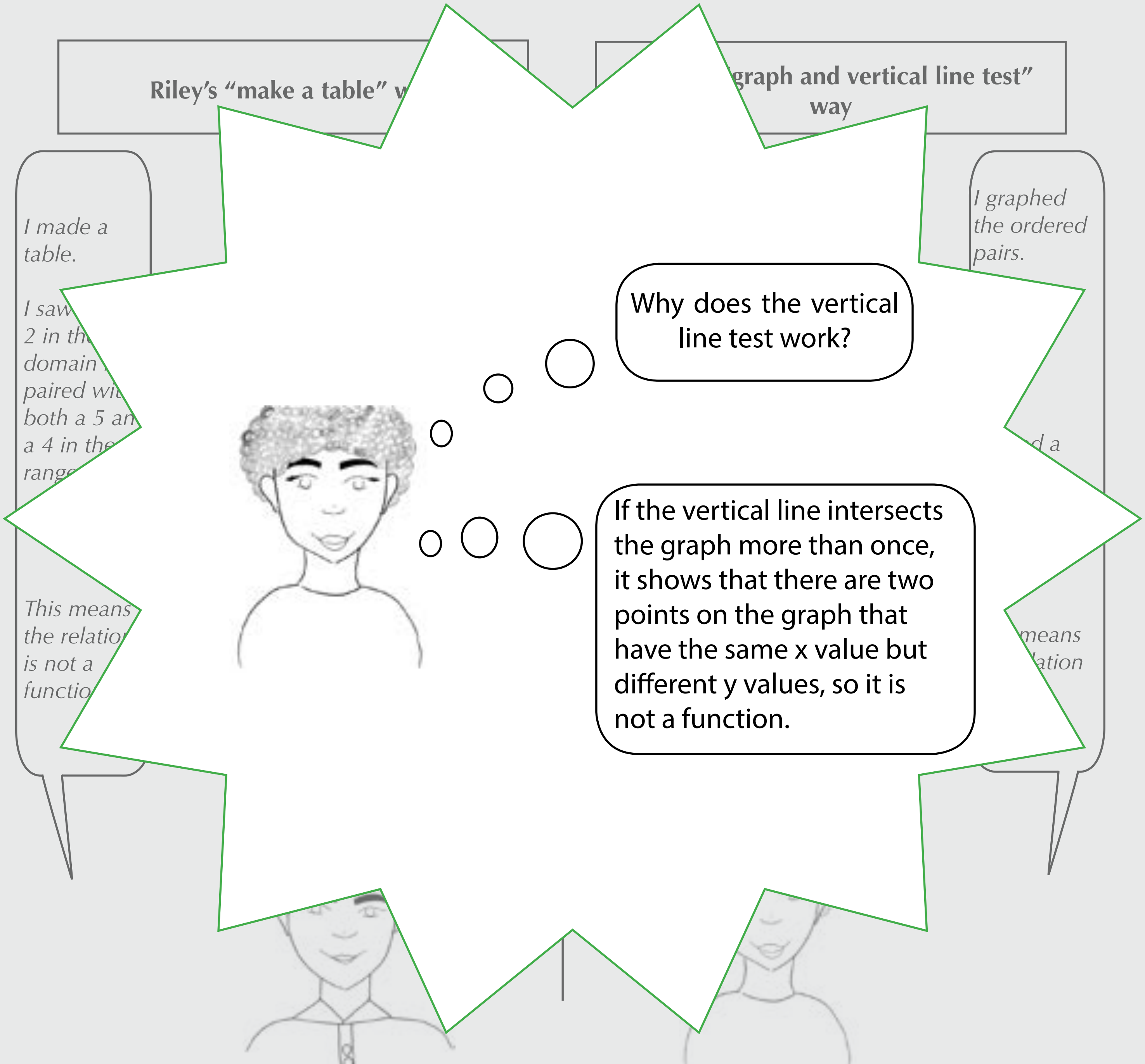
Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Riley and Gloria were given the set of ordered pairs

{(–3, 6), (2, 5), (3, 1), (2, 4), (5, 1)},

and asked to determine if the relation is a function.



How did Riley determine if the relation was a function? How did Gloria determine if the relation was a function?

Why do both methods work? Why does the vertical line test tell us the same thing as the table of values?

Topic 2.2: Linear Functions

WEP Type: Why does it work?
Suggested Use: Mid-lesson
Problem: Emma and Layla were asked if the points in the table could represent a linear function.

x	3	5	6
y	20	30	35

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Layla know to write "25"?** **What does Emma mean when she says x and y change "twice as much"?** **How did Emma know that the rate of change was constant?**
Make Comparisons	**Why do both "analyze the table" methods work?** **How do Emma's and Layla's first steps both show they are thinking about "rate of change"?**
Prepare to Discuss (Think, Pair)	**Fill in the empty areas in the table to create a linear function.**

X	2	3	
Y		10	30

Phase	Guiding Discussion Questions and Implementation Notes
Discuss Connections (Share)	*There are many possible correct answers. For example, if the missing x value is 4, this means that as x goes up by 1, y goes up by 20 (making the missing y value to be -10). If the missing x value is 5, this means that as x goes up by 1, y goes up by 10 (making the missing y value to be 0).*
Identify the Big Idea	**What do these examples show us about looking for constant rates of change?** *Rate of change is a relationship between the change in y and change in x. You have to pay attention to changes in both x and y to figure out if a pattern is linear.*

Emma and Layla were asked if the points in the table could represent a linear function.

x	3	5	6
y	20	30	35

Emma's "analyze the table" way

I found the differences between the x values and y values in the table.

When x changes twice as much, the y also changes twice as much.

Since the rate of change is constant, I think this is linear.

+2 +1

x	3	5	6
y	20	30	35

+10 +5

x by 2, y by 10
x by 1, y by 5

Yes, linear.

Layla's "analyze the table" way

4

x	3	5	6
y	20	30	35

25

y always goes up by 5

Yes, linear.

I noticed that 4 was missing in the table, so I filled it in.

Now the change in y is constant as x goes up.

Since the rate of change is constant, I think this is linear.

How did Emma know that the rate of change was constant?

Why do both methods work? How do Emma's and Layla's first steps both show they are thinking about "rate of change"?

Discuss Connections

Fill in the empty areas in the table to create a linear function. Use Emma and Layla's ways to justify a constant rate of change.

X	2	3	
Y		10	30

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

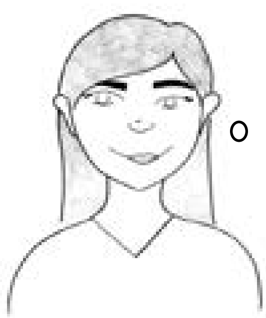
Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Emma and Layla were asked if the points in the table could represent a linear function.

Why do we need to pay attention to both x and y when figuring out the rate of change for a linear function?



Rate of change is a relationship between the change in y and the change in x. In a linear function, the rate that y changes as compared to the rate that x changes is constant.

How did Emma know that the rate of change was constant?

Why do both methods work? How do Emma's and Layla's first steps both show they are thinking about "rate of change"?

Topic 2.3: Function Notation

WEP Type: How do they differ?
Suggested Use: Beginning of lesson
Problem: Gloria and Tim were solving the problem f(x) = 4x + 1 to find f(2).

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Gloria know to find 2 on the x-axis instead of the y-axis?**
Make Comparisons	**Did Gloria and Tim get the same answer? How do you know?**
Prepare to Discuss (Think, Pair)	**Use Gloria's "graphing" way and Tim's "function notation" way to find where f(x) = 13 using the same equation f(x) = 4x + 1.**
Discuss Connections (Share)	*Using Tim's way, I would plug 13 in for x in the function f(x), and the answer f(13) would give me the y-value for when the x-value is 13. If I used Gloria's way, I would graph the line f(x) = 4x + 1 and find the corresponding y-value for the ordered pair when x is 13.*
Identify the Big Idea	**What do we learn from comparing the two ways?** *We can use function notation as well as x's and y's to write and graph linear functions. Both f(x) and y refer to the output of the function, when x is the input.*

Gloria and Tim were solving the problem
$f(x) = 4x + 1$
to find $f(2)$.

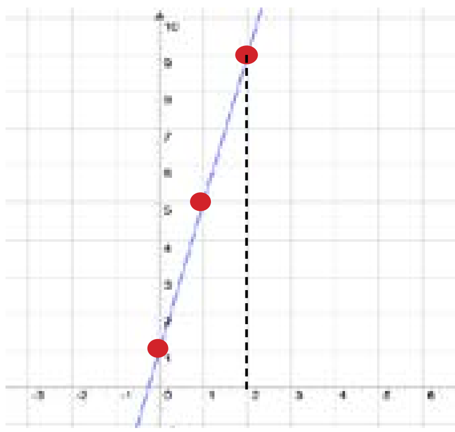
Gloria's "graphing" way

The equation is in slope-intercept form, so I know that 4 is the slope and 1 is the y-intercept.

I plotted the y-intercept then continued to plot points using the slope.

I got (2,9) as my answer.

$f(x) = 4x + 1$



(2,9)

Tim's "function notation" way

$f(x) = 4x + 1$

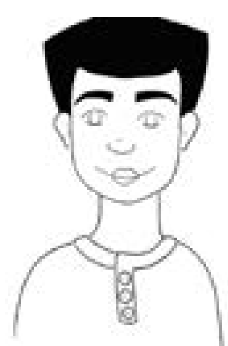
$f(2) = 4(2) + 1$

$f(2) = 8 + 1$

$f(2) = 9$

I am solving for the ouput, and I know 2 is the input.

I got f(2)= 9 as my answer.



How did Gloria know to find 2 on the x-axis instead of the y-axis?

Did Gloria and Tim get the same answer? How do you know?

Discussion Connections

Use Gloria's "graphing" and Tim's "function notation" ways to find where f(x) = 13.

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Gloria and Tim were solving the problem $f(x) = 4x + 1$ to find $f(2)$.

Gl ia's "graphing" | "function n on" way

The equation is in slope- i fo is t and y-inte

I got (2

I am solving for the ouput, and I know 2 t.

f(2)= 9 my answer

What did I learn from comparing the two ways?

We can use function notation as well as x's and y's to write and graph linear functions. Both $f(x)$ and y refer to the output of the function, when x is the input.

How did Gloria know to find 2 axis instea axis?

Did Gloria and Tim get the same nswer? How do you k w?

Topic 2.4: Graphing Linear Equations in Standard Form

WEP Type: Which is correct?
Suggested Use: Mid-lesson
Problem: Layla and Riley were asked to graph x = 5 and y = 2.

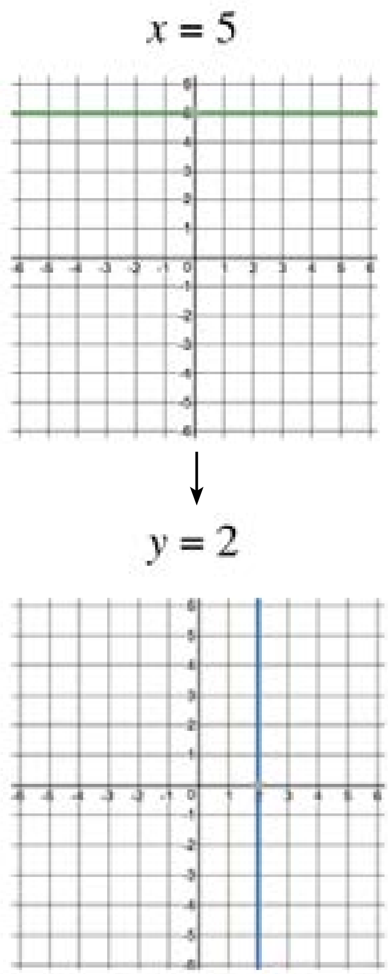
Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Layla decide where to draw her lines? What does she mean when she says she "just moved up 5 on the x-axis"?** **How did Riley decide where to draw his lines?** **Why is Layla's method called the "move the axis" way?** **Why is Riley's method called the "think of points" way?**
Make Comparisons	**Which method is correct?** **How could you convince Layla or Riley that their way is not right?** **What can you do to help yourself remember not to make the same mistake as Layla?**
Prepare to Discuss (Think, Pair)	**Where do the lines x = 4 and y = 3 intersect? Explain how you can find where they cross both by graphing and without graphing.**
Discuss Connections (Share)	*To find the point of intersection by graphing, we can graph the vertical line x = 4 and the horizontal line y = 3 and note that they cross where x is 4 and y is 3, or (4, 3). Without graphing, the point that lies on both x = 4 and y = 3 is where x is 4 and y is 3, or (4, 3).*
Identify the Big Idea	**How did Layla's mistake happen?** *Layla moved the x- or y-axis, but instead she should have thought about what an equation of a line means. The equation x = 5 means that every point on the line has an x-coordinate of 5.*

Layla and Riley were asked to graph x = 5 **and** y = 2**.**

Layla's "move the axis" way

Since x is the x axis, and since x = 5, I just moved up 5 on the x axis.

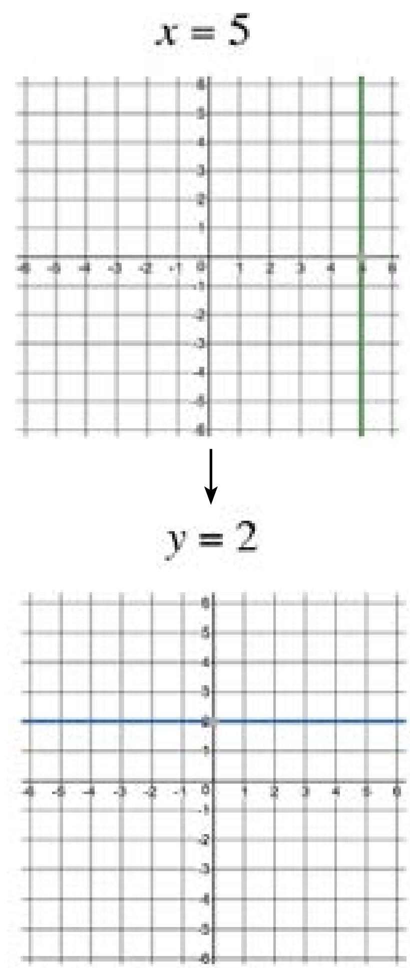
Since y is the y axis, and y is 2, I moved up 2 on the y axis.

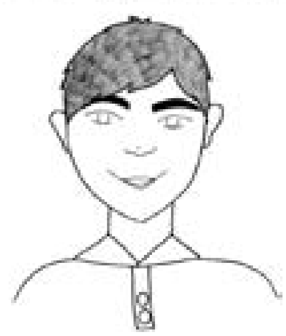


Riley's "think of points" way

x = 5 means that the x coordinate is always 5, as in (5, 1) (5, 2), etc. So I drew this line.

y = 2 means that y is always 2, as in (0, 2) (1, 2), etc. So I drew this line.





Why is Layla's method called the "move the axis" way? Why is Riley's called the "think of points" way?

Which method is correct?

Discuss Connections

Where do the lines $x = 4$ **and** $y = 3$ **intersect? Explain how you can find where they cross both by graphing and without graphing.**

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Layla and Riley w… asked to graph … 5 and y = 2.

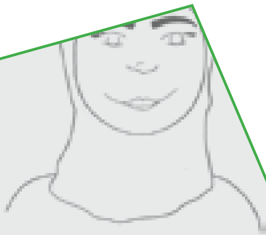
Layla's "… | **…ay**

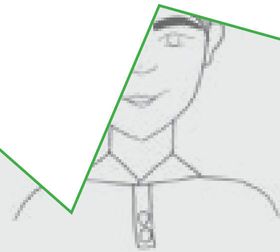
x = 5 means that … coord… is alway… as in (5, … (5, 2), etc. I drew this line.

y = 2 … that y is always 2, as in (0, 2) (… 2), etc. S… I drew … line.

How did my mistake happen?

I moved the x or y-axis, but instead, I should have thought about what an equation of a line means. For example, the equation x = 5 means that every point on the line has an x- coordinate of 5.





Why is Layla's method called the "move the axis" way? Why is Riley's called the "think of points" way?

Which method is correct?

Topic 2.5: Graphing Linear Equations in Slope-Intercept Form

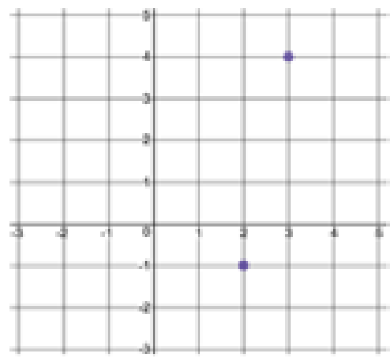
WEP Type: Why does it work?
Suggested Use: Mid-lesson
Problem: Tim and Emma were asked to find the slope of the line passing through (3, 4) and (2, –1).

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Emma know where to plug in each number in the slope formula?** **Could Emma have used (3, 4) as (x_2, y_2) instead of (x_1, y_1)?** **Tim counted the spaces between the two points, beginning at the point (2, -1). Would Tim have gotten the same answer by starting from the other point, (3, 4)?**
Make Comparisons	**Why do both methods work?** **How does each method show that slope is "rise over run"?**
Prepare to Discuss (Think, Pair)	**Graph a line with a zero slope, then use Tim's "graph" way and Emma's "formula" way to show why the slope is zero.**
Discuss Connections (Share)	*Horizontal lines have zero slope. Looking at a graph of a horizontal line using Tim's way, the "rise" is always 0, regardless of the "run." So the slope, rise/run, is always 0/run or just 0. Using Emma's way and the slope formula, any two points on a horizontal line have the same y value. So computing the change in y values would always be zero, which makes the slope zero.*
Identify the Big Idea	**Why does graphing the points work?** *Counting how far up and over it is from one point to the next point is the same as finding the change in y over the change in x in the slope formula.*

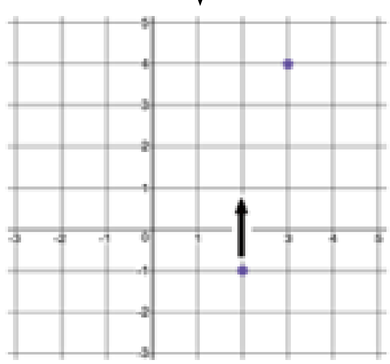
Tim and Emma were asked to find the slope of the line passing through (3, 4) and (2, –1).

Tim's "graph" way

First I plotted the two points on a graph.

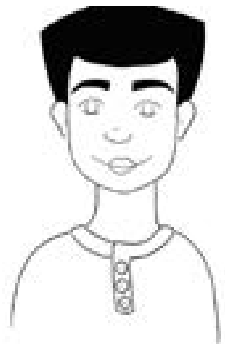


I started at the bottom and counted up 5 and over 1.



The slope is 5.

$$m = \frac{5}{1} = 5$$



Emma's "formula" way

I wrote the formula for slope.

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}$$

I plugged in the points.

$$m = \frac{-1 - 4}{2 - 3}$$

I simplified the numerator and denominator.

$$m = \frac{-5}{-1}$$

The slope is 5.

$$m = 5$$

Tim counted the spaces between the two points, beginning at the point (2, –1). Would Tim have gotten the same answer by starting from the other point, (3, 4)?

Why do both methods work? How does each method show that slope is "rise over run"?

Discuss Connections

Graph a line with zero slope, then use Tim's "graph" way and Emma's "formula" way to show why the slope is 0.

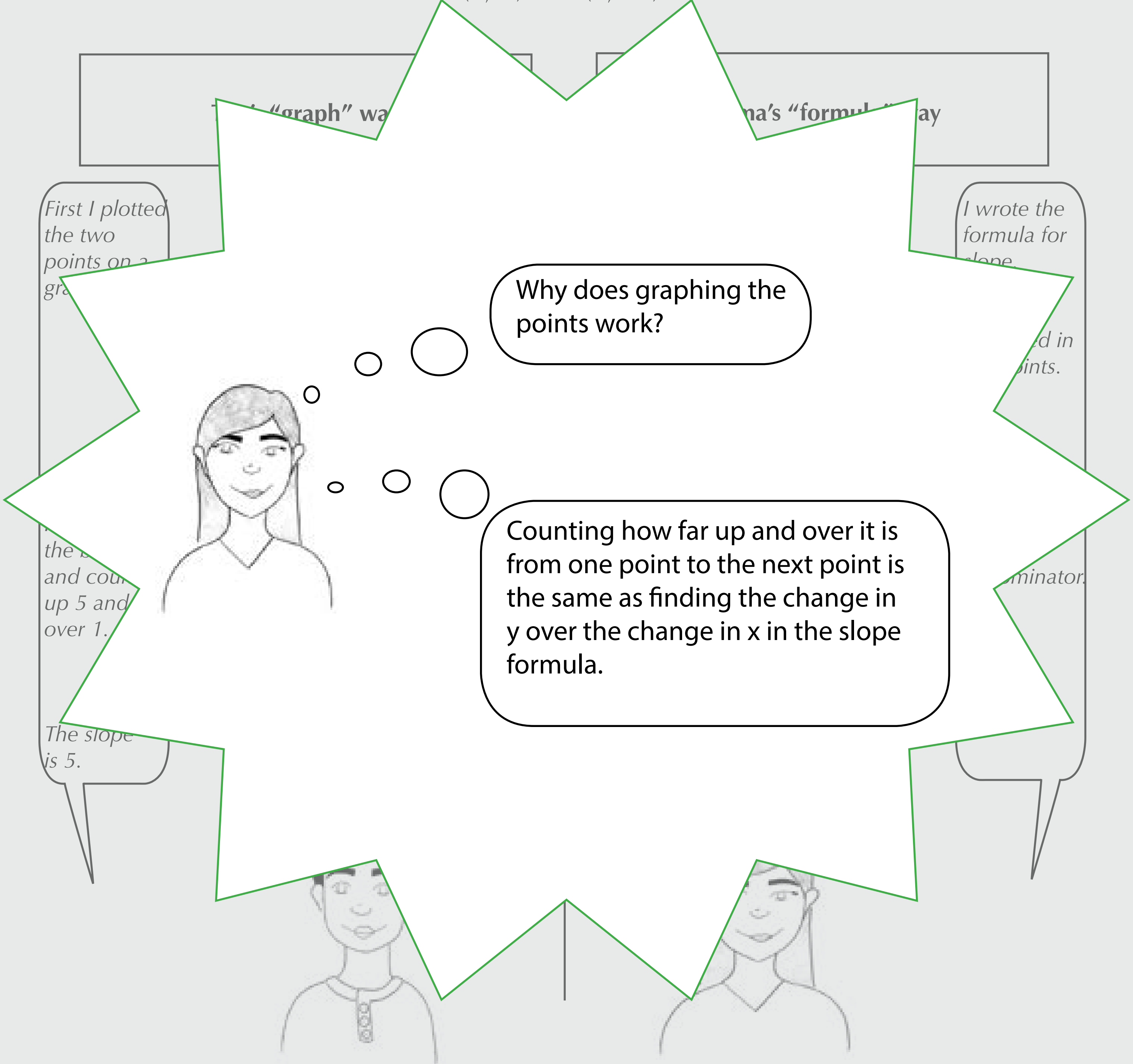
Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Tim and Emma were asked to find the slope of the line passing through $(3, 4)$ and $(2, -1)$.



Tim counted the spaces between the two points, beginning at the point (2, –1). Would Tim have gotten the same answer by starting from the other point, (3, 4)?

Why do both methods work? How does each method show that slope is "rise over run"?

Topic 2.6: Graphing Linear Equations in Slope-Intercept Form
WEP Type: Which is better?
Suggested Use: End of lesson
Problem: Riley and Gloria were asked to graph the equation $3x - 2y = 6$.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Riley graph the line?** **Why did Gloria solve the equation for y as a first step?** • What does Gloria mean by "[I] used rise over run to get more points"?
Make Comparisons	**Which method is better?** *For this problem, it is better to use Riley's "x and y intercepts" method. If the equation is given in standard form, it is easier to find the x and y-intercepts, plot those two points, and then connect them with a line.*
Prepare to Discuss (Think, Pair)	**If the original problem had been to graph 3x – 2y = 3, would you use Riley's "x and y intercepts" way or Gloria's "slope-intercept" way? Why?**
Discuss Connections (Share)	*In the line 3x – 2y = 3, Riley's way would tell us that the x-intercept is 1 and the y-intercept is -3/2. Given that it is easy to graph these two values, Riley's way might be easier.*
Identify the Big Idea	**How do you know if finding the x- and y-intercepts is a good way to graph this equation?** *The equation is in standard form, and 6 is divisible by 3 and -2, so finding the intercepts is easy.*

Riley and Gloria were asked to graph the equation $3x - 2y = 6$.

Riley's "x- and y-intercepts" way

$$3x - 2y = 6$$

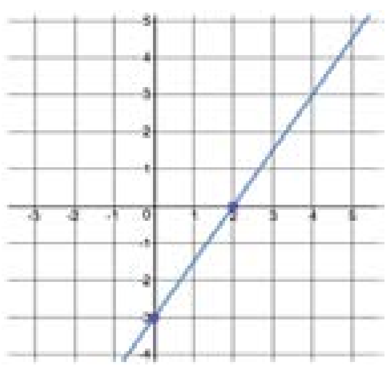
First I found the x-intercept by plugging in 0 for y.

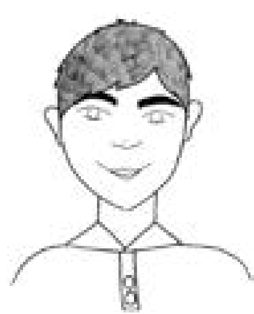
$$3x - 2(0) = 6$$
$$3x = 6$$
$$x = 2$$
x-intercept: (2, 0)

Then I found the y-intercept by plugging in 0 for x.

$$3(0) - 2y = 6$$
$$-2y = 6$$
$$y = -3$$
y-intercept: (0, −3)

I plotted the intercepts and connected them.





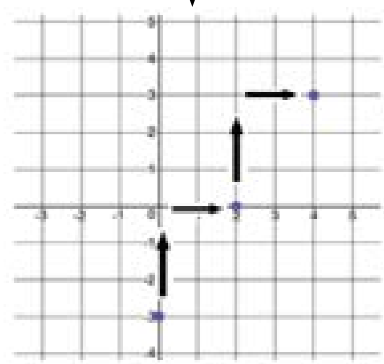
Gloria's "slope-intercept" way

$$3x - 2y = 6$$

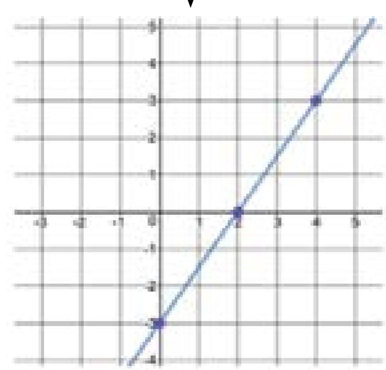
I solved for y to put the equation in y = mx + b form.

$$-2y = -3x + 6$$
$$y = \frac{3}{2}x - 3$$

I graphed the y-intercept of -3 then used rise over run to get more points.



I connected the points to get the line.



How did Riley graph the line? Why did Gloria solve the equation for y as a first step?

Which method is better?

Discuss Connections

If the original problem had been to graph $3x - 2y = 3$, **would you use Riley's "x- and y-intercepts" way or Gloria's "slope-intercept" way? Why?**

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Riley and Gloria were asked to graph the equation $3x - 2y = 6$.

Riley's "x- and y-intercepts" wa

Cl 's "slope-intercept" way

I solved for y to put the equation in y = mx + b form.

I graph y-interc -3 then us rise over ru to get more points.

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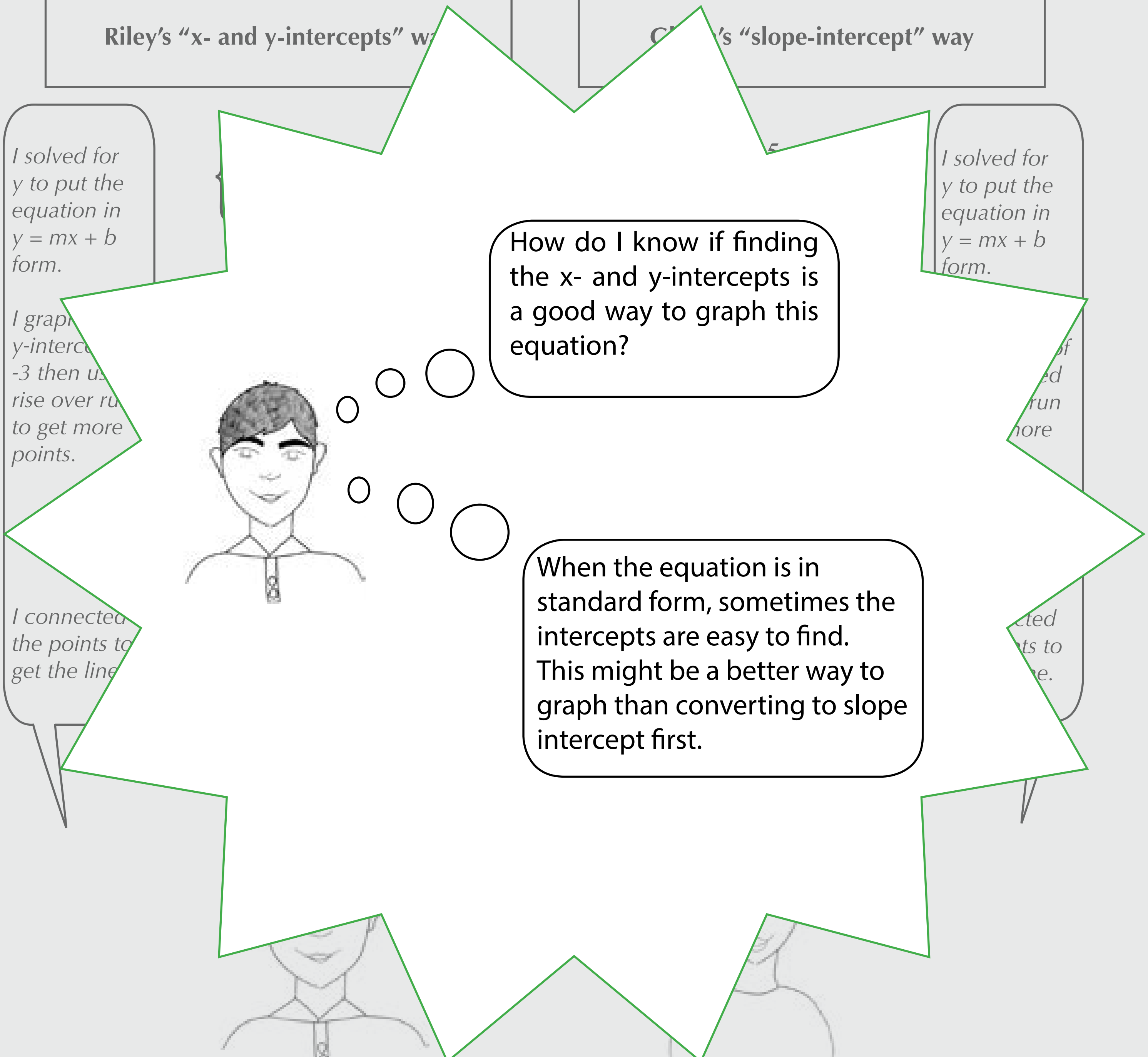


How did Riley graph the line? Why did Gloria solve the equation for y as a first step?

Which method is better?

Topic 2.7: Writing Equations in Slope-Intercept Form

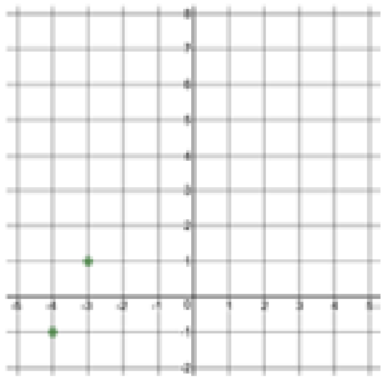
WEP Type: Which is better?
Suggested Use: Beginning of lesson
Problem: Gloria and Tim were asked to find the y-intercept of the line connecting the two points (-3, 1) and (-4, -1).

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Why did Tim use the point (-4, -1) in the equation to find b? What if he had used (-3, 1)?** • What does Gloria mean when she says she "followed the up 2, right 1 pattern"?
Make Comparisons	**Which method is better?** **Even though Gloria and Tim did different steps, why did they both get the same answer?** • Where does the slope of 2 show up in Gloria's " graphing" method? *Both methods are correct, so they will result in the same answer*
Prepare to Discuss (Think, Pair)	**If the points were changed to (3, -4) and (4, 2), find the y-intercept. Did you use Gloria's "graphing" way or Tim's "algebraic" way, and which is better?**
Discuss Connections (Share)	*For the points (3, -4) and (4, 2), if we tried to use Gloria's graphing way, we would find that it is difficult to find the y-intercept using graphing. (It is (0, -22)). So Riley's algebraic way is easier.*
Identify the Big Idea	**Is there a better way to find the y-intercept than Gloria's "graphing" way?** *While graphing always works, sometimes it is hard because the numbers are big and not whole numbers. It might be better to use the algebraic way instead.*

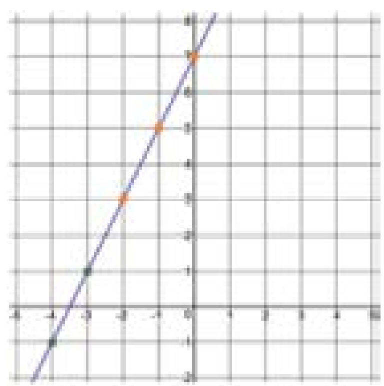
Gloria and Tim were asked to find the y-intercept of the line connecting the two points (–3, 1) and (–4, –1).

Gloria's "graphing" way

I plotted the two points.



I followed the up 2, right 1 pattern to get more points until I crossed the y-axis.



The y-intercept is at (0, 7)

Tim's "algebraic" way

I'll write the equation in slope intercept form. First I need to find the slope.

y = mx + b

$$m = \frac{-1 - 1}{-4 - (-3)} = \frac{-2}{-1} = 2$$

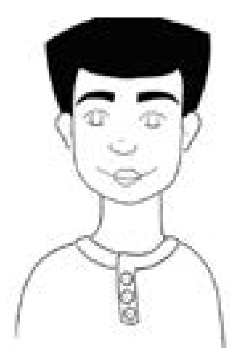
So far, I know m.

$$y = 2x + b$$

I used (-4,-1) in the equation to find b.

$$-1 = 2(-4) + b$$
$$-1 = -8 + b$$
$$7 = b$$

The y-intercept is at (0, 7)



Why did Tim use the point (–4, –1) in the equation to find b? What if he had used (–3, 1)?

Which method is better? Even though Gloria and Tim did different steps, why did they both get the same answer?

Discuss Connections

If the points were changed to (3, -4) **and** (4, 2) **find the y-intercept. Did you use Gloria's "graphing" way or Tim's "algebraic" way, and which is better?**

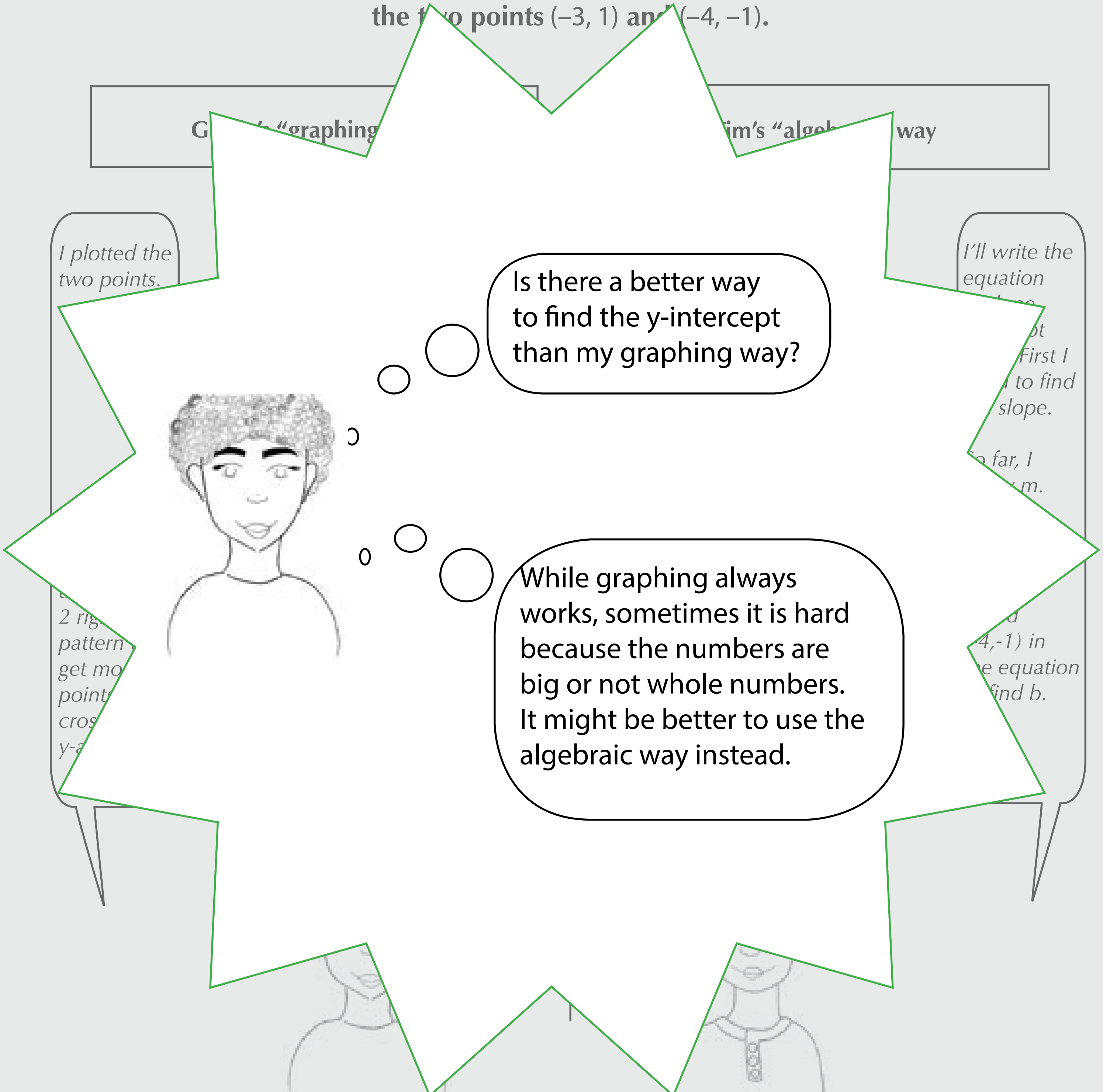
Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Gloria and Tim were asked to find the y-intercept of the line connecting the … points (–3, 1) an… (–4, –1).



Why did Tim use the point (–4, –1) in the equation to find b? What if he had used (–3, 1)?

Which method is better? Even though Gloria and Tim did different steps, why did they both get the same answer?

Topic 2.8: Writing Equations in Point-Slope Form

WEP Type: Which is correct?

Suggested Use: Mid-lesson

Problem: Layla and Riley were asked to write an equation for the line through (-2, 5) and (6, 1) using point-slope form.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**Does it matter which point is (x_1, y_1) versus (x_2, y_2) when finding the slope? Could they have switched the order of the points in their slope calculation?**
Make Comparisons	**Which method is correct?** • What is the x-intercept and y-intercept of Layla's line? • What is the x-intercept and y-intercept of Riley's line? *Layla's and Riley's methods are both correct. There are many ways of verifying this. The most straightforward way to verify is to rewrite both of these equations in slope-intercept form.*
Prepare to Discuss (Think, Pair)	**Write two (or more) different equations for the line that goes through (4, 1) and (2, -4).**
Discuss Connections (Share)	*The slope of the line through these two points is 5/2. Using the point (4, 1) in point-slope form, the equation of the line is $y - 1 = 5/2(x - 4)$. Using the point (2, -4), the equation of the line is $y + 4 = 5/2(x - 2)$. Both of these equations are equivalent to the slope-intercept form of this line, $y = (5/2)x - 9$.*
Identify the Big Idea	**Why do both ways work?** *Given two points, we can always draw a unique line. So, it doesn't matter which point you use in finding the equation in point-slope form—you will get the same line either way.*

Layla and Riley were asked to write an equation for the line through (–2, 5) and (6, 1) using point-slope form.

Layla's "using (–2, 5)" way

First I found the slope.

$$m = \frac{1 - 5}{6 - (-2)} = \frac{-4}{8} = \frac{-1}{2}$$

$$y - y_1 = m(x - x_1)$$

Then I used (–2, 5) in the equation.

$$y - 5 = \frac{-1}{2}(x - (-2))$$

$$y - 5 = \frac{-1}{2}(x + 2)$$

The equation is
$$y - 5 = \frac{-1}{2}(x + 2)$$

Riley's "using (6, 1)" way

First I found the slope.

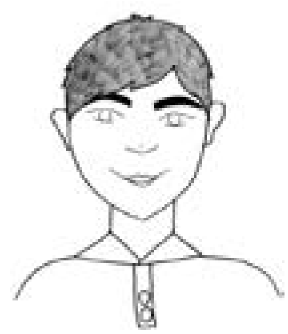
$$m = \frac{1 - 5}{6 - (-2)} = \frac{-4}{8} = \frac{-1}{2}$$

$$y - y_1 = m(x - x_1)$$

Then I used (6, 1) in the equation.

$$y - 1 = \frac{-1}{2}(x - 6)$$

The equation is
$$y - 1 = \frac{-1}{2}(x - 6)$$



Does it matter which point is (x_1, y_1) versus (x_2, y_2) when finding the slope? Could they have switched the order of the points in their slope calculation?

Which method is correct?

Discuss Connections

Write two (or more) different equations for the line that goes through (4, 1) and (2, -4).

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words:

Layla and Riley were asked to write an equation for the line through (–2, 5) and (6, 1) using point-slope form.

Layla's "using (–2, 5)" w	's "using (6, 1)" way
First I found the slope.	*First I found the slope.*
Then I ... (–2, 5) ... in the equation.	*... on.*

Why do both ways work?

Given two points, we can always draw a unique line. So, it doesn't matter which point you use in finding the equation in point-slope form. You will get the same line either way.

Does it matter which point is (x_1, y_1) ... sus (x_2, y_2) when ... g the slope? Could they have switched the order of the points in ... eir slope calculation?

Which method is correct?