Compare & Discuss Problems

Topic 1: Linear Equations

Implementation Checklist

Phase	Step	Yes	No
Compare	**Prepare to Compare** Students took time to understand what the problem was asking and understand both methods.	☐ Yes	☐ No
	Make Comparisons Students identified mathematical similarities and differences between the two methods.	☐ Yes	☐ No
Discuss	**Prepare to Discuss** Think: Students spent around 1 minute thinking independently about the worksheet prompts.	☐ Yes	☐ No
	Pair: Students spent around 2 minutes working in pairs or small groups discussing the worksheet prompts.	☐ Yes	☐ No
	Discuss Connections Share: A 3-6 minute whole-class conversation occurred where students discussed connections that included question asking and answering by the teacher and students. • Most students were involved in this whole class conversation. • The teacher asked follow-up questions in response to students' thinking, such as "Why do you think that's true?", "Do you agree or disagree? Why?", "Can you say more about that?", and "What did you like about their answer?".	☐ Yes	☐ No
	Identify the Big Idea The teacher showed the Big Idea page to the class to provide a clear, explicit statement of the Big Idea. Students identified the Big Idea and summarized it in their own words.	☐ Yes	☐ No

Phase	Criterion	Yes	No
Timing	At least 8 minutes were spent in the Compare phase and at least 12 minutes were spent in the Discussion phase. Students spent more than half their time in the Discussion phase.	☐ Yes	☐ No

Compare & Discuss: Algebra 1 PD Institute
Discussion Resources

<u>Why have a mathematical discussion?</u>

- To deepen students' understanding of the mathematical content.
- To enhance student engagement and interest in mathematics.

<u>What should a teacher do to have a good mathematical discussion?</u>

- **<u>BEFORE</u>** the discussion starts:
  - Thoroughly **solve** the problems that will be discussed.
  - **Anticipate** student responses, errors, and difficulties.
  - **Plan** questions to ask, as well as problem extensions to use.
- **<u>DURING</u>** the discussion:
  - **Ask** lots of open-ended questions, using the following question stems to spark and continue conversation:
    - *Do you agree with Layla? Why?*
    - *Can you summarize what Riley said?*
    - *Can you give another example?*
    - *Can you describe that in more detail?*
    - *What do you mean by XXXX?*
    - *How did you do that?*
    - *What might be confusing about this example?*
  - **Re-voice** and **summarize** student contributions to keep the conversation going, saying things like:
    - *What I am hearing is XXXX. Is that what you mean?*
    - *Are you saying XXXX?*
    - *I am not sure I understand what you mean. Can you explain it again?*
  - **Manage** flow of the conversation, involving many voices from the class.
  - **Involve as many students** in the discussion as possible.
    - Be sure to **solicit** participation from students who do not have their hands raised, using *equity sticks*, *note cards*, *spinners*, or a *random name generator* for randomly selecting students to speak.
    - **Consider keeping track** of which students have spoken with a clipboard of the class roster, both to remember who has spoken and to ensure equitable participation.
  - **Hold** students accountable for listening to and understanding others' contributions, saying things like:
    - *Gloria thinks that XXXX. Tim, can you summarize what Gloria said, in your own words?*
  - **Provide** students credit for discussion participation as part of their grade.

Prepare to Compare & Discuss: Teacher Prep Checklist

For each Worked Example Pair, it is important you review the problem and its associated worksheets in advance before presenting it to the class. When prepping, keep the following checklist in mind:

- ✓ **Ensure you understand** each method in the WEP.
- ✓ **Read** the Big Idea message so you know where the discussion should lead by the conclusion of the exercise.
- ✓ **Review** the prompt on the *Discuss Connections* worksheet, and:
    - **Add** extension questions that will push your students to dig deeper during the discussion, OR
    - **Create** additional, supporting questions that will allow struggling students to grapple with the prompt more successfully.
- ✓ **Determine** when in the class you plan to present the WEP.
- ✓ **Make sufficient copies** of the worksheet(s) for participating students.

That's it! For each WEP, we don't anticipate more than 5-10 minutes of prep. Please remember to reach out to research staff if any questions or concerns come up during planning.

Differentiating Compare & Discuss Problems

We strongly believe, and our research supports, that Compare & Discuss problems can be an effective way to engage in mathematics for all learners. Below, you will find a general list of recommendations to keep in mind as you consider differentiating the Compare & Discuss problems to fit your students' needs.

DON'T:

- **Change** the examples such that they are a far removal from the implementation model.
- **Skip** whole chapters.
- **Change or adapt** the tests.
  o For research purposes, it is important every student takes the same test, even if content on the test was not covered in your class.
- **Eliminate** the side-by-side comparison of the solution methods.
- **Rush through/gloss over** the WEPs (don't save them for the last 5 minutes of class!).
  o If you are working with students and the Compare phase seems like it is moving quickly, that might not be a problem—it gives you more time to work on the Discuss phase, incorporating more extension questions for deeper discussion.

DO:

✓ **Plan ahead** with research staff.
✓ **Adapt** WEPs for content not covered, rather than skipping the examples altogether.
✓ **Blend** comparison types – types are not mutually exclusive (some can be both Why does it work? & Which is better?).
  o This may influence your extension questions for the Discuss phase.
✓ **Address** changes for later chapters with lower level classes (content in earlier chapters [1, 3, and 5] tends to be covered in all levels, but you may need to change/adapt content for later chapters [7, 9]).
✓ **Adapt** Which is correct?/How do they differ? WEPs for lower level classes.
  o Some students may be overwhelmed by a comparison with two different problems; others may struggle with determining which method is incorrect. Discuss with research staff ways to adapt these two comparison types for struggling students.

*Lastly, we encourage **<u>creativity</u>**! We're happy to work with you to find ways to incorporate the Compare & Discuss problems into your class as a yearlong theme (e.g. using **Holiday greeting cards, dress-up days**, etc.).*

Topic 1: Solving Linear Equations- Overview

Section	Table of Contents (Page #)	WEP Type	Suggested Use
1.1	7	Which is correct?	Beginning of lesson
1.2	11	Why does it work?	Beginning of lesson
1.3	15	How do they differ?	Mid-lesson
1.4	19	Why does it work?	Mid-lesson
1.5	23	Which is better?	Mid-lesson
1.6	27	Which is correct?	Mid-lesson
1.7	31	Which is better?	Mid-lesson
1.8	35	Which is better?	Beginning of Lesson
1.9	39	Which is correct?	Beginning of Lesson

Compare (8 minutes)

Prepare to Compare

- What is the problem asking?
- What is happening in the first method?
- What is happening in the second method?

Make Comparisons

- What are the similarities and differences between the two methods?
  - Which method is better?
  - Which method is correct?
  - Why do both methods work?
  - How do the problems differ?

Discuss (12minutes)

Prepare to Discuss (think, pair)

- How does this comparison help you understand this problem?
- How might you apply these methods to a similar problem?

Discuss Connections (share)

- What ideas would you like to share with the class?

Identify the Big Idea

- Can you summarize the Big Idea in your own words?

Topic 1.1: Solving Simple Equations

WEP Type: Which is correct?

Suggested Use: Beginning of lesson

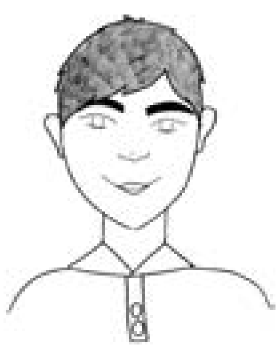
Problem: Riley and Emma were asked to find the value of x in the equation: 5 + 3 = x + 2

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Riley's "add up" way get 8 as the answer?** **How did Emma's "equalize" way get 6 as the answer?**
Make Comparisons	**Which method is correct?**
Prepare to Discuss (Think, Pair)	**Riley is using an incorrect definition of the equal sign. What might it be?** **What is the correct definition of the equal sign?**
Discuss Connections (Share)	*Riley incorrectly thinks that the equal sign means "compute the answer" like the equal sign button on a calculator. So when he sees 5 + 3 =, he thinks the answer is 8. Emma correctly thinks the equal sign means that the quantity on the left side is equivalent to the quantity on the right side. Since 5 + 3 is 8 on the left side, the right side (x + 2) must add up to 8 too.*
Identify the Big Idea	**How did Riley's mistake happen?** *I added 5 and 3 and got 8 instead of finding what number plus 2 adds to 8. I didn't make sure both sides of the equation were worth the same amount.*

Riley and Emma were asked to find the value of x in the equation:

$5 + 3 = x + 2$

Riley's "add up" way	**Emma's "equalize" way**
$5 + 3 = x + 2$	$5 + 3 = x + 2$
x is 8	x is 6
Since $5 + 3 = x$, I know x is 8	*Since $5 + 3$ is the same as $x + 2$, I know x is 6*



How did Riley's "add up" way get 8 as the answer? How did Emma's "equalize" way get 6 as the answer?

Which method is correct?

Discuss Connections

Riley is using an incorrect definition of the equal sign. What might it be? What is the correct definition of the equal sign?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words:

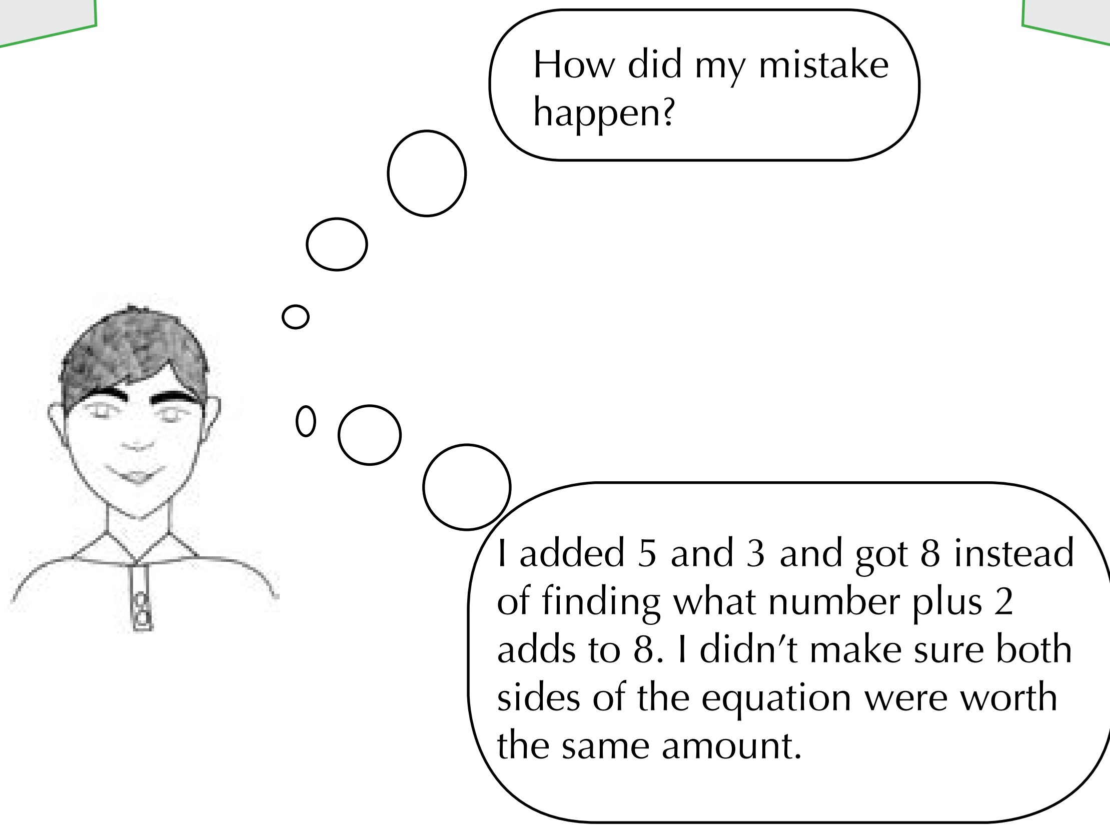
Riley and Emma were a... d to find the va... of x in the equation:

... + 3 = x + 2 ...

Riley's "add up" w...	...'s "equalize" way

Since 5 + ... = x, I l... is 8

... 3 is



How did Rile... answer ...ay get 6 as the

Which meth... correct?

Topic 1.2: Solving Simple Equations

WEP Type: Why does it work?
Suggested Use: Beginning of lesson
Problem: Riley and Gloria were asked to solve x + 5 = 10.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Riley solve the equation? How did Gloria solve the equation?** • Why did Gloria subtract 10 from both sides of the equation? Is this step correct?
Make Comparisons	**Why do both methods work?** *Riley and Gloria are both using properties of equality to solve their equations, so they both arrived at the same solution. The difference is that they subtracted different numbers from both sides of the equation in their first steps.*
Prepare to Discuss (Think, Pair)	**For this equation, does it matter which number is added to or subtracted from both sides? Why or why not?**
Discuss Connections (Share)	*No, it doesn't matter, because the Addition and Subtraction Properties of Equality allow us to add or subtract any numbers from both sides without changing the answer.*
Identify the Big Idea	**Why does subtracting the same number from both sides of the equation work?** *The equal sign means both sides are worth the same amount. So you can always add or subtract any number from both sides of the equation without changing its answer.*

Riley and Gloria were asked to solve $x + 5 = 10$**.**

Riley's "subtract 5 from both sides" way

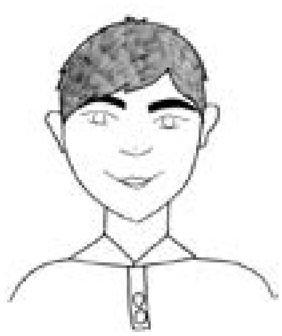
$x + 5 = 10$

I subtracted 5 from both sides.

$x + 5 - 5 = 10 - 5$

Then I simplified to get my answer.

$x = 5$



Gloria's "subtract 10 from both sides" way

$x + 5 = 10$

I subtracted 10 from both sides to get 0 on the right.

$x + 5 - 10 = 10 - 10$

Then I simplified.

$x - 5 = 0$

I added 5 to both sides.

$x - 5 + 5 = 0 + 5$

Here is my answer.

$x = 5$

How did Riley solve the equation? How did Gloria solve the equation?

Why do both methods work?

Discuss Connections

For this equation, does it matter which number is added to or subtracted from both sides? Why or why not?

Think, Pair. First, think about the question(s) above independently, Then, get with a partner and discuss you answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words:

Riley and Gloria were asked to solve $x + 5 = 10$.

Why does subtracting the same number from both sides of the equation work?

The equal sign means both sides are worth the same amount. So I can always add or subtract any number from both sides of the equation without changing its answer.

How did Riley solve the equat… …w did Gloria s… …e equation?

Why do both methods work?

Topic 1.3: **Solving Multi-Step Equations**
WEP Type: How do they differ?
Suggested Use: Mid-lesson
Problem: Emma was asked to solve 2x = 10; Layla was asked to solve 2(x + 1) = 10

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**What is the first step of Layla's "use (x+1) as the variable" way?** **Why is Layla's way called the "use (x+1) as the variable" way?**
Make Comparisons	**How are the two ways similar? How are the two ways different?** *Both problems state that 2 times 'something' is equal to 10, where 'something' is the unknown quantity or variable that we are solving for. For Emma, this variable is x, but for Layla, this variable is (x + 1). Note also that both ways begin with the same mathematical operation of dividing by 2 to both sides of the equation.*
Prepare to Discuss (Think, Pair)	**On what kinds of equations would Layla's "use (x+1) as the variable" way work?** **Create a new equation that can be solved using this way.**
Discuss Connections (Share)	*Answers will vary. Any equation will work that includes a coefficient multiplied by a composite variable. Make sure students understand that a composite variable doesn't need to be (x + 1). For instance, 25(y + 67) = 75 would work. Showing students an example with large numbers might emphasize when using Layla's method can be very useful.*
Identify the Big Idea	**What did you learn from comparing the two ways?** *The unknown in an equation is called a variable. Sometimes we can treat an expression such as (x + 1) as the variable. This might help to solve an equation in a better way.*

Emma was asked to solve 2x = 10; Layla was asked to solve 2(x + 1) = 10.

Emma's "use x as the variable" way

I divided both sides by 2.

I got 5.

$$2x = 10$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$\downarrow$$

$$x = 5$$

Layla's "use (x+1) as the variable" way

$$2(x + 1) = 10$$

$$\frac{2(x + 1)}{2} = \frac{10}{2}$$

$$\downarrow$$

$$x + 1 = 5$$

$$\downarrow$$

$$x = 4$$

First, I divided both sides by 2.

Then I subtracted 1 from both sides.

I got 4.

What is the first step of Layla's "use (x+1) as the variable" way? Why is Layla's way called the "use (x+1) as the variable" way?

How are the two ways similar? How are the two ways different?

Discuss Connections

On what kinds of equations would Layla's "use (x+1) as the variable" way work? Create a new equation that can be solved using this way.

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words:

Emma was asked to solve 2x = 10; Layla was asked to solve 2(x + 1) = 10

Emma's "use x as the variable"

(x+1) as the variable" way

I divided both sides by 2.

I got 5.

What did I learn from comparing the two ways?

The unknown in an equation is called a variable. Sometimes we can treat an expression such as (x + 1) as the variable. This might help to solve an equation in a better way.

What is the first step of Layla's way

How are the two ways similar? How are the two ways different?

Topic 1.4: Solving Multi-Step Equations

WEP Type: Why does it work?
Suggested Use: Mid-lesson
Problem: Emma and Layla were asked to solve 8(x + 1) – 4 = 12.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**What did Emma and Layla do to solve their equations? What do you think "composite variable" means in the name of Emma's way?** • How did Layla get her first step?
Make Comparisons	**Why do both methods work?** **What do you think is the most important difference between Emma's "composite variable" method and Layla's "use the distributive property" method?** *Emma treated the expression in the parenthesis as a quantity, and Layla used the distributive property to isolate the variable. Both methods were correct and resulted in the same answer.*
Prepare to Discuss (Think, Pair)	**Can you write an equation for which Emma's "composite variable" method will be better than Layla's "use the distributive property" method?** **What characteristics of that equation made you choose it?** • On what kinds of equations would Emma's way work?
Discuss Connections (Share)	*Emma's way works for equations that have the same composite variables, or expressions with terms and variables, on both sides of the equation. Emma's way works better for equations that have composite variables on both sides of the equation that would otherwise require multiple steps to multiply out using the distributive property.*
Identify the Big Idea	**Why does the composite variable work?** *Any expressions with a variable can be treated as a single quantity (a **composite variable**). As long as the properties of equality are maintained, using composite variables can save steps.*

Emma and Layla were asked to solve $8(x + 1) - 4 = 12$.

Emma's "composite variable" way

$$8(x + 1) - 4 = 12$$

First I added 4 to both sides.

$$8(x + 1) - 4 = 12$$
$$\quad +4 \quad +4$$

Then I divided by 8.

$$\frac{8(x + 1)}{8} = \frac{16}{8}$$

I subtracted 1 from both sides.

$$x + 1 = 2$$
$$\quad -1 \quad -1$$

I got 1.

$$x = 1$$

Layla's "use the distributive property" way

$$8(x + 1) - 4 = 12$$

First, I distributed.

$$8x + 8 - 4 = 12$$

Then I subtracted 4 from both sides.

$$8x + 4 = 12$$
$$\quad -4 \quad -4$$

I divided by 8.

$$\frac{8x}{8} = \frac{8}{8}$$

I got 1.

$$x = 1$$

What did Emma and Layla do to solve their equations? What do you think "composite variable" means in the name of Emma's way?

Why do both methods work? What do you think is the most important difference between Emma's "composite variable" method and Layla's "use the distributive property" method?

Discuss Connections

Can you write an equation for which Emma's "composite variable" method will be better than Layla's "use the distributive property" method? What characteristics of that equation made you choose it?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Emma and Layla were asked to solve 8(x + 1) - 4 = 12**.**

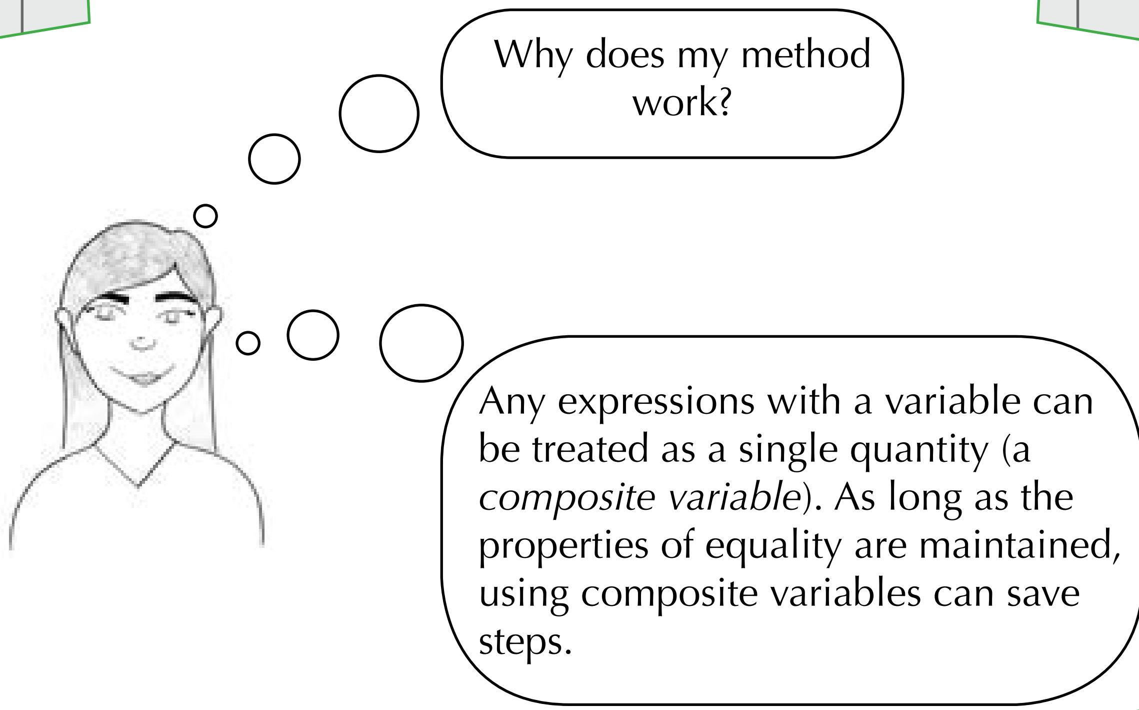
Emma's ...mposite varia...

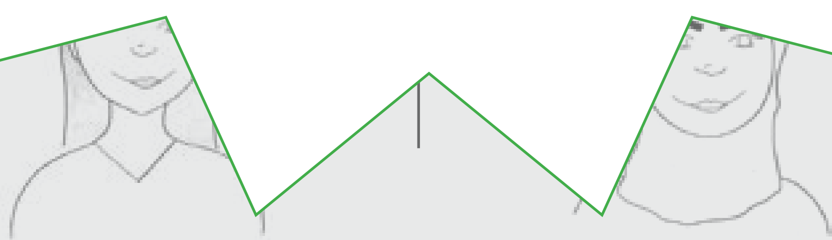
...the distributive property" wa...

First ... 4 to b... sides.

I subt... 1 from b... sides.

I g...





What did Emma and Layla do to solve their equations?

Why do both methods work? What do you think is the most important difference between Emma's method and Layla's method?

Topic 1.5: Solving Multi-Step Equations
WEP Type: Which is better?
Suggested Use: Mid-lesson
Problem: Gloria and Tim were asked to solve 5(x + 3) = 20.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Gloria and Tim find the solution to the equation?** • How did Gloria get her first step? How did Tim get his first step?
Make Comparisons	**Which method is better? What are some important differences between Gloria's "distribute first" method and Tim's "divide first" method?** *One important difference is that they use different first steps, where Gloria distributes first and Tim divides first. Tim's method is better because it results in fewer steps.*
Prepare to Discuss (Think, Pair)	**If solving 5x(x + 2) + 7 = 12, which method would be better? Why?**
Discuss Connections (Share)	*I would use Tim's "divide first" way after subtracting 7 from both sides because dividing 5 from both sides is an easy next step that results in fewer calculations to solve for x.*
Identify the Big Idea	**Is there a better way to solve this problem than distributing first?** *Treating the expression as a quantity and dividing first is faster than using the distributive property on this problem. It's faster because 20 is divisible by 5.*

Gloria and Tim were asked to solve $5(x + 3) = 20$.

Gloria's "distribute first" way

$5(x + 3) = 20$

First I distributed.

$5x + 15 = 20$

Then I subtracted on both sides.

$5x + 15 = 20$
$\quad -15 \quad -15$

I divided by 5.

$\frac{5x}{5} = \frac{5}{5}$

Here is my answer.

$x = 1$

Tim's "divide first" way

$5(x + 3) = 20$

First I divided by 5.

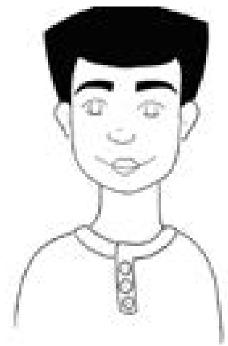
$\frac{5(x + 3)}{5} = \frac{20}{5}$

Then I subtracted from both sides.

$x + 3 = 4$
$\quad -3 \quad -3$

Here is my answer.

$x = 1$



How did Gloria and Tim find the solution to the equation?

Which method is better? What are some important differences between Gloria's "distribute first" method and Tim's "divide first" method?

Discuss Connections

If solving 5(x+2) + 7 = 12, which method would be better? Why?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words:

Gloria and Tim were asked to solve $5(x + 3) = 20$**.**

Gloria's "distribute firs

m's "divide first" way

First I distributed.

First I divided by 5.

Th subt on bo sides.

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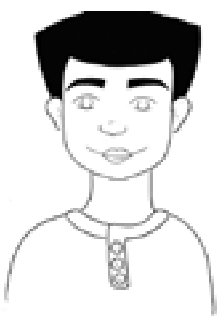


Is there a better way to solve this problem than distributing first?

Treating the expression as a quantity and dividing first is faster than using the distributive property on this problem. It's faster because 20 is divisible by 5.

How did Gloria and Tim find the solution to the equation?

Which method is better? What are some important differences between Gloria's method and Tim's method?

Topic 1.6: **Combining Like Terms**
WEP Type: Which is correct?
Suggested Use: Mid-lesson
Problem: Emma and Layla were asked to simplify the expression 2(x + 1) + 3(x + 6)

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**What were the like terms combined in Emma's "distribute first" way? What were the like terms combined in Layla's "combine like terms" way?** *Combining like terms is an important step in solving most equations. It might be hard for students to see that (x + 1) and (x + 6) are not like terms. While common examples of like terms are 2x and 3x—or 4y and 7y—it is also true that 3(x + 6) and 2(x + 6) can be combined as like terms. 3(x + 6) really says 3 times some number, and 2(x + 6) says 2 times the same number. These can be combined to be 5 times the same number. But you can't combine 2 times some number (x + 1) and 3 times some other number (x + 6).*
Make Comparisons	**Which method is correct? For the incorrect method, what needs to be done to make it correct?** • Can you change one number in the original problem so that Layla's "combine like terms" way works? *Changing (x + 1) to (x + 6) or changing (x + 6) to (x + 1) would make it possible to combine like terms.*
Prepare to Discuss (Think, Pair)	**Write a definition of "like terms" and give several examples. Would your definition help avoid the mistake made in Layla's "combine like terms" way?**
Discuss Connections (Share)	*"Like terms" are terms that have the same variables, including composite variables like (x + 6). Examples of like terms include 2(y + 3) and 6(y + 3) and 7xy and 4xy. To be like terms, the variables need to match exactly, which is why 2(x + 1) and 3(x + 6) are not like terms.*
Identify the Big Idea	**How did Layla's mistake happen?** *She combined terms that were not like terms. If the problem had been 3(x + 6) + 2(x + 6) she could have combined these like terms to get 5(x + 6).*

Emma and Layla were asked to simplify the expression 2(x + 1) + 3(x + 6)

Emma's "distribute first" way

$2(x + 1) + 3(x + 6)$

$2x + 2 + 3x + 18$

↓

$5x + 20$

First, I distributed the parentheses.

Then, I combined like terms to get 5x + 20.

Layla's "combine like terms" way

$2(x + 1) + 3(x + 6)$

$5(2x + 7)$

↓

$10x + 35$

First, I combined what was in the parentheses.

Then, I distributed the parentheses to get 10x + 35.

What were the like terms in Emma's "distribute first" way? What were the like terms combined in Layla's "combine like terms" way?

Which method is correct? For the incorrect method, what needs to be done to make it correct?

Discuss Connections

Write a definition of "like terms" and give several examples. Would your definition help avoid the mistake made in Layla's "combine like terms" way?

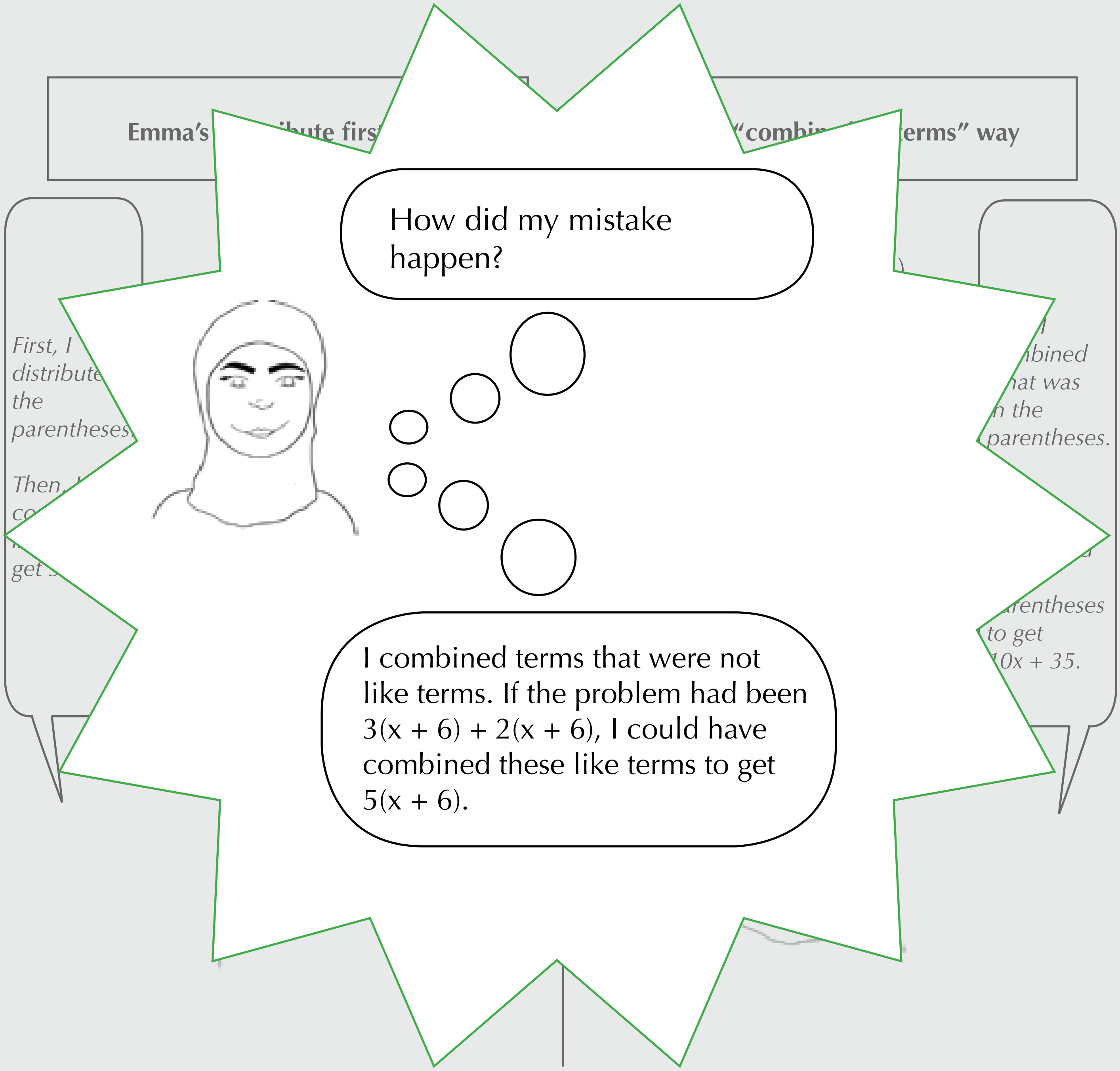
Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words:

Emma and Layla were asked to simplify the expression 2(x + 1) + 3(x + 6)



What were the like terms in Emma's "distribute first" way? What were the like terms combined in Layla's "combine like terms" way?

Which method is correct? For the incorrect method, what needs to be done to make it correct?

Topic 1.7: Solving Multi-Step Equations with Variables on Both Sides of the Equation

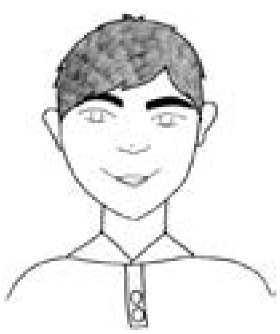
WEP Type: Which is better?
Suggested Use: Mid-lesson
Problem: Riley and Gloria were asked to solve 5(n + 6) = 2(n + 6) + 6

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Riley and Gloria solve the equation?** • What does Gloria mean when she says she subtracted the quantity 2(n + 6) from both sides in her first step?
Make Comparisons	**Which method is better? What are some important differences between Riley's "distribute first" method and Gloria's "composite variable" method?** *One important difference is that Gloria and Riley have different first steps in their methods, which results in Gloria's method having fewer steps; therefore, Gloria's method is better.*
Prepare to Discuss (Think, Pair)	**Come up with another problem where the composite variable method will work. Then solve it using the distributive property. Which method is better?**
Discuss Connections (Share)	*Answers will vary. Look for examples like 3(x + 7) = 5(x + 7) – 8. Gloria's method will always be faster when compared to distribution.*
Identify the Big Idea	**How do you know if using composite variables is a good way to solve this problem?** *The expression (n + 6) is on each side of the equation. It saves steps to subtract 2(n + 6) first so you don't have to distribute the 2.*

Riley and Gloria were asked to solve $5(n + 6) = 2(n + 6) + 6$**.**

Riley's "distribute first" way

	$5(n + 6) = 2(n + 6) + 6$
First, I distributed.	$5n + 30 = 2n + 12 + 6$
Then I moved the variable to one side of the equation.	$5n + 30 = 2n + 18$ $-2n \quad\quad -2n$
I subtracted from both sides.	$3n + 30 = 18$ $-30 \quad -30$
I divided by 3.	$\frac{3n}{3} = \frac{-12}{3}$
Here's my answer.	$n = -4$



Gloria's "composite variable" way

$5(n + 6) = 2(n + 6) + 6$	
$5(n + 6) = 2(n + 6) + 6$ $-2(n + 6) \quad -2(n + 6)$	*First, I subtracted the quantity 2(n + 6) from both sides.*
$\frac{3(n + 6)}{3} = \frac{6}{3}$	*Then I divided by 3.*
$n + 6 = 2$ $-6 \quad -6$	*I subtracted from both sides.*
$n = -4$	*Here's my answer.*

How did Riley and Gloria solve the equation?

Which method is better? What are some important differences between Riley's"distribute first" method and Gloria's "composite variable" method?

Discuss Connections

Come up with another problem where the composite variable method will work. Then solve it using the distributive property. Which method is better?

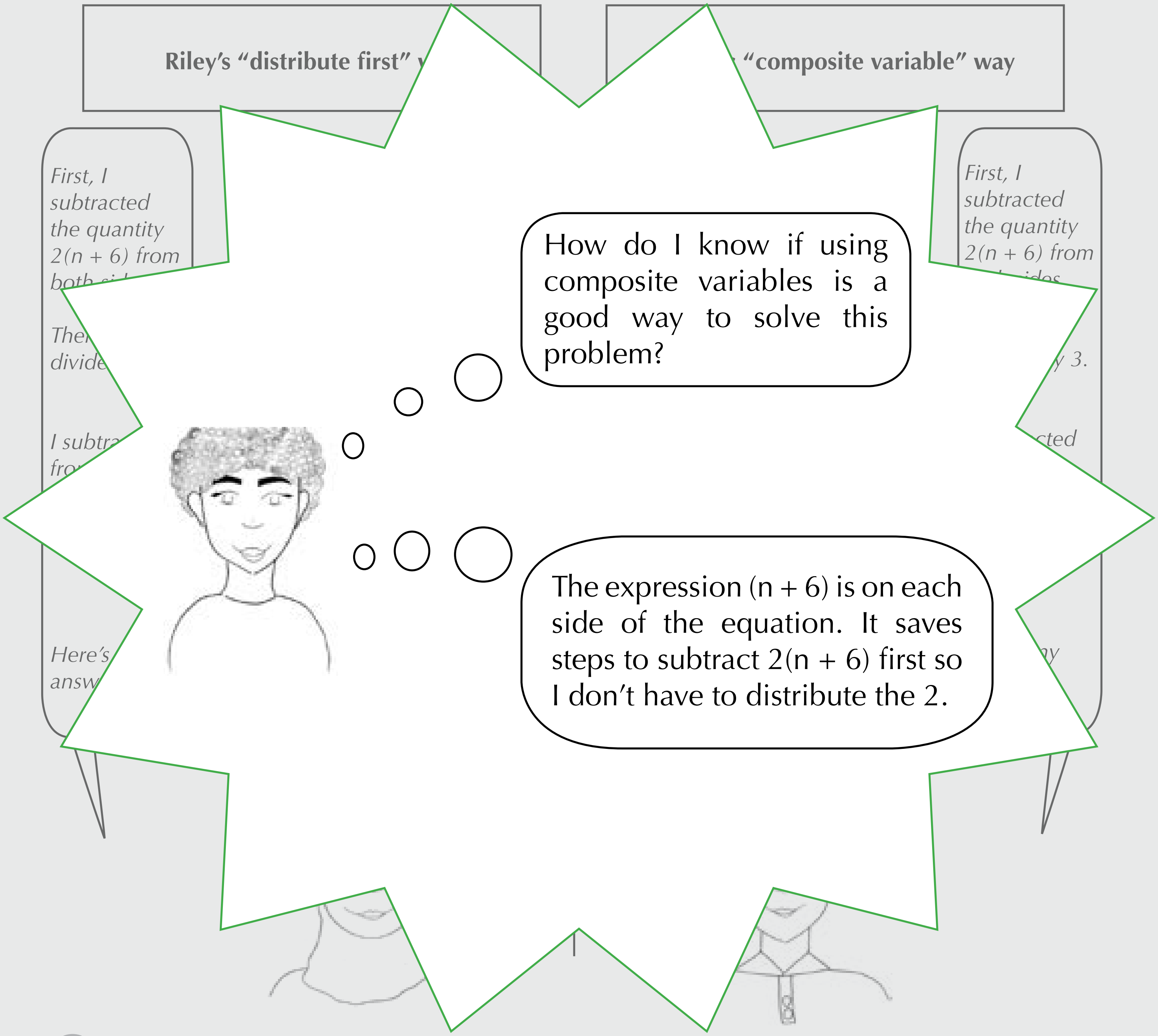
Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Riley and Gloria were asked to solve $5(n + 6) = 2(n + 6) + 6$**.**



How did Riley and Gloria solve the equation?

Which method is better? What are some important differences between Riley's "distribute first" method and Gloria's "composite variable" method?

Topic 1.8: Transforming Literal Equations
WEP Type: Which is better?
Suggested Use: Beginning of lesson
Problem: Emma and Layla were asked to solve 2a + 14 = b for *a*, given *b* = 4 and *b* = 8.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Emma and Layla solve the equation for *a*?** • What did Emma mean when she says she divided to solve for *a*?
Make Comparisons	**Which method is better? What is an important difference between Emma's "solve for *a* first" method and Layla's "plug in the value first" method?** *Emma is solving the literal equation first, whereas Layla is plugging the value for b into the literal equation first. Because of the way the literal equation is written, Emma's "solve for a first" way is better and results in fewer steps.*
Prepare to Discuss (Think, Pair)	**If Emma and Layla needed to solve the equation given many different values for *b*, which method would you use? Why?**
Discuss Connections (Share)	*I would use Emma's "solve for a first" method. This way, you only need to manipulate the literal equation once, and you can continue plugging in values for* b *into the equation that is already solved for* a. *In the long run, this will result in fewer steps than doing Layla's method for each given value of* b.
Identify the Big Idea	**Is there a better way to find *a* than by plugging in the values for *b*?** *Solving the equation for the variable you're looking for first saves you steps because you only have to manipulate the equation once. It's faster to plug in the given values from there, especially when you're given multiple values.*

Emma and Layla were asked to solve $2a + 14 = b$ **for *a*, given *b* = 4 and *b* = 8.**

Emma's "solve for *a* first" way

First, I subtracted 14 from both sides.

Then I divided by 2.

I simplified to solve the equation for a. Then I plugged 4 and 8 in for b.

Here are my answers.

$$2a + 14 = b$$

$$\begin{aligned} 2a + 14 &= b \\ -14 \quad & \quad -14 \end{aligned}$$

$$\frac{2a}{2} = \frac{b - 14}{2}$$

$$a = \frac{b}{2} - 7$$

$$a = \frac{4}{2} - 7, \quad a = \frac{8}{2} - 7$$

$$a = -5, \quad a = -3$$

Layla's "plug in the value first" way

First, I plugged 4 in for b. Then I subtracted 14 from both sides of the equation. Next, I divided both sides by 2 to get my first answer.

Then I plugged 8 in for b and solved.

Here's my second answer.

$$2a + 14 = b$$

$$\begin{aligned} 2a + 14 &= 4 \\ -14 \quad & \quad -14 \end{aligned}$$

$$\frac{2a}{2} = \frac{-10}{2}$$

$$a = -5$$

$$\begin{aligned} 2a + 14 &= 8 \\ -14 \quad & \quad -14 \end{aligned}$$

$$\frac{2a}{2} = \frac{-6}{2}$$

$$a = -3$$

How did Emma and Layla solve the equation for *a*?

Which method is better? What is an important difference between Emma's "solve for *a* first" method and Layla's "plug in the value first" method?

Discuss Connections

If Emma and Layla needed to solve the equation given many different values for b, which method would you use? Why?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

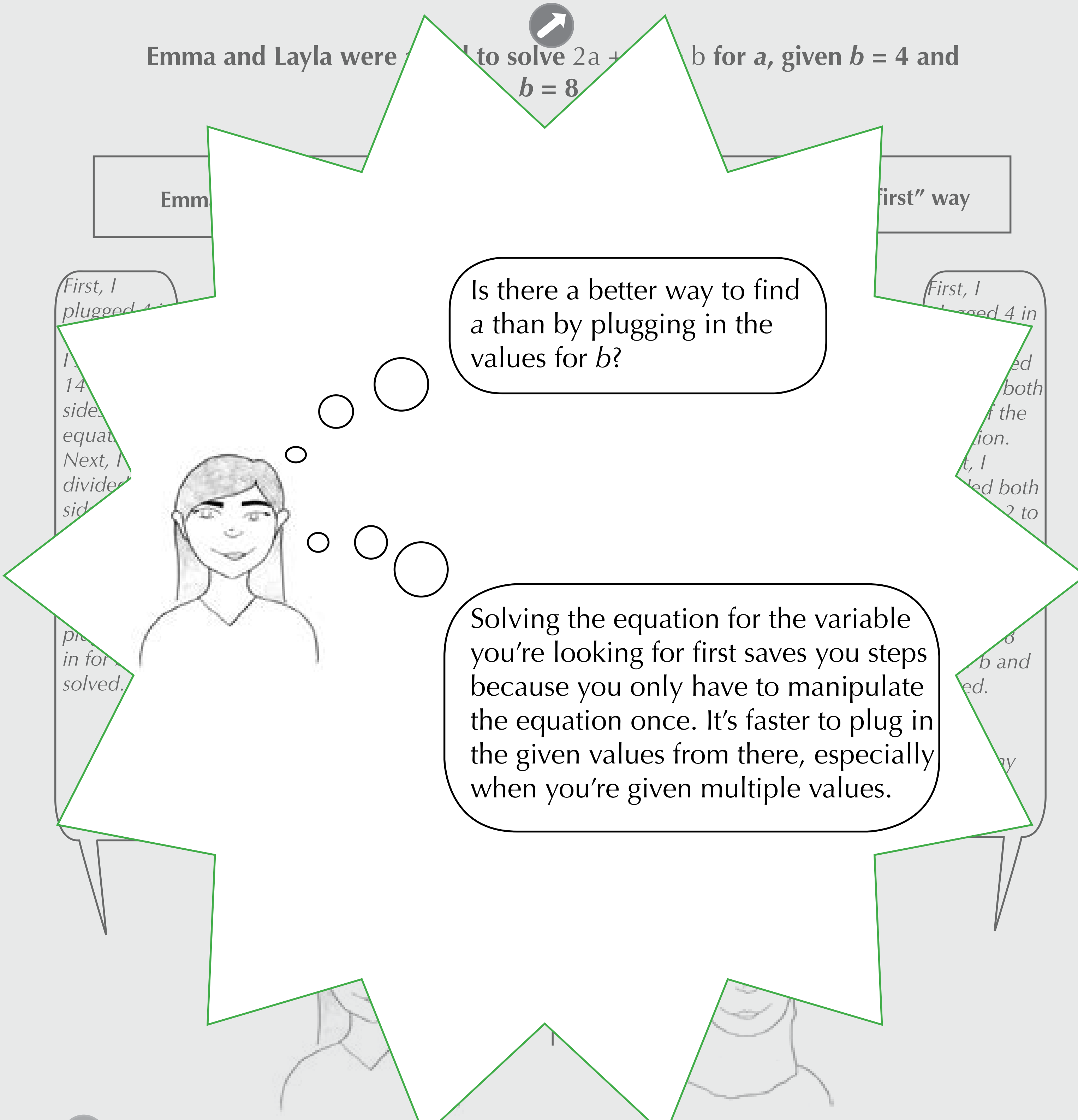
Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Emma and Layla were … to solve 2a + … b for *a*, given *b* = 4 and *b* = 8

Emm… | **…first" way**



How did Emma and Layla solve the equation for *a*?

Which method is better? What is an important difference between Emma's "solve for *a* first" method and Layla's "plug in the value first" method?

Topic 1.9: Transforming Literal Equations

WEP Type: Which is correct?
Suggested Use: Beginning of lesson
Problem: Gloria and Tim were asked to solve 16x + 9 = 9y – 2x for *x*.

Phase	Guiding Discussion Questions and Implementation Notes
Prepare to Compare	**How did Gloria and Tim solve their equations for *x*?** • Why did Gloria and Tim add 2x to both sides, and then subtract 9 from both sides? • How did Gloria and Tim simplify in their last step?
Make Comparisons	**Which method is correct?** • Identify the step that Gloria and Tim did differently. *Gloria and Tim's methods are the same, but Tim makes an error in his third step when he rewrites the equation after subtracting 9 from both sides. He rewrites the equation as 18x = 9 – 9y, when it should be 18x = 9y – 9.*
Prepare to Discuss (Think, Pair)	**Tim substitutes 1 into both his answer and Gloria's answer, and for both the result is *x* = 0. Tim concludes that this means that both answers are correct. Is Tim's reasoning correct? Why or why not?**
Discuss Connections (Share)	*Tim's reasoning would be correct if 1 was not a number with a special property. You can check to ensure your equations are equivalent by plugging in the same values for the variable, but with caution—numbers like 0 and 1 often result in the same answers, even for equations that are not equivalent!*
Identify the Big Idea	**How did Tim's mistake happen?** *Gloria's way helped him see that he swapped 9 and 9y's order when he subtracted 9 from both sides to isolate x. This made the signs wrong.*

Gloria and Tim were asked to solve $16x + 9 = 9y - 2x$ **for *x*.**

Gloria's "isolate x" way

I got the x's on the same side of the equal sign.

Then I began to isolate x.

I divided by 18.

I simplified to find my answer.

$$16x + 9 = 9y - 2x$$

$$\begin{aligned} 16x + 9 &= 9y - 2x \\ +2x \quad & \quad\quad\;\; +2x \end{aligned}$$

$$\downarrow$$

$$\begin{aligned} 18x + 9 &= 9y \\ -9 \;\; & \;\; -9 \end{aligned}$$

$$\downarrow$$

$$\frac{18x}{18} = \frac{9y - 9}{18}$$

$$\downarrow$$

$$x = \frac{1}{2}y - \frac{1}{2}$$

Tim's "isolate x" way

$$16x + 9 = 9y - 2x$$

$$\begin{aligned} 16x + 9 &= 9y - 2x \\ +2x \quad & \quad\quad\;\; +2x \end{aligned}$$

$$\downarrow$$

$$\begin{aligned} 18x + 9 &= 9y \\ -9 \;\; & \;\; -9 \end{aligned}$$

$$\downarrow$$

$$\frac{18x}{18} = \frac{9 - 9y}{18}$$

$$\downarrow$$

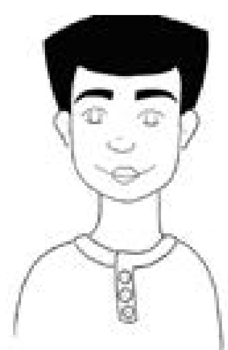
$$x = \frac{1}{2} - \frac{1}{2}y$$

I got the x's on the same side of the equal sign.

Then I began to isolate x.

I divided by 18.

I simplified to find my answer.



How did Gloria and Tim solve their equations for x?

Which method is correct?

Discuss Connections

Tim substitues 1 into both his answer and Gloria's answer, and for both the result is x = 0. Tim concludes that this means that both answers are correct. Is Tim's reasoning correct? Why or why not?

Think, Pair. First, think about the question(s) above independently. Then, get with a partner and and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response

Big Idea. When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.

Gloria and Tim were asked to solve $16x + 9 = 9y - 2x$ **for** x**.**

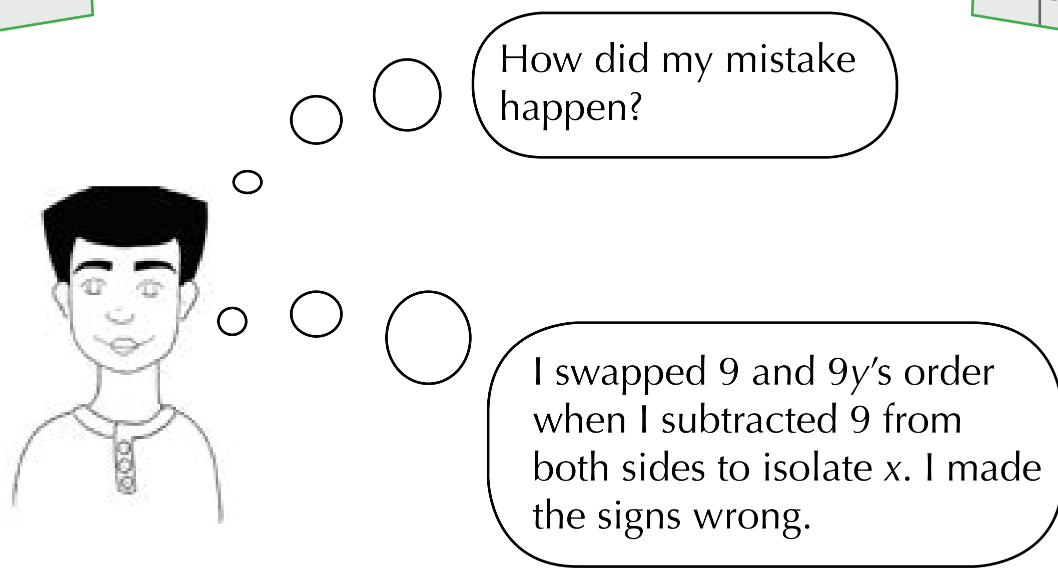
Gloria's "isolate x" way

Tim's "isolate x" way

I got the x's on the same side of the equal sign.

I divided by 18.

I got the x's on the same side of the equal sign.



How did Gloria and Tim solve their equations for x?

Which method is correct?