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# Effects of Encouraging Comparison and Explanation of Multiple Strategies on Instructional Practices in Algebra Classrooms

Recent theories of algebra learning have focused on improving conceptual knowledge (Kieran, 1992), but algebra requires both conceptual and procedural knowledge. Conceptual knowledge is the knowledge of abstract, general principles (e.g., Byrnes & Wasik, 1991), and procedural knowledge is the knowledge of mathematical strategies (e.g., Baroody, Feil, & Johnson, 2007; Byrnes & Wasik, 1991). Past theories on algebra learning have not presented an explicit focus on flexibility in the use of symbolic strategies. Procedural flexibility, the ability to know multiple strategies for solving a problem and select the most appropriate strategy, is an important component of successful mathematics learning (Star & Rittle-Johnson, 2008; Woodward et al., 2012). Experts in mathematics have procedural flexibility (Dowker, 1992; Star & Newton, 2009), and learners with procedural flexibility are more likely to adapt strategies to solve unfamiliar problems and have higher conceptual knowledge (Blöte, Van der Burg, & Klein, 2001; Hiebert et al., 1996). Consequently, we developed a theory of algebra learning that emphasizes the importance of multiple strategies, comparison and explanation. Comparing multiple strategies can improve learning in many domains, including mathematics (Alfieri, Nokes-Malach, & Schunn, 2013; Gentner, Loewenstein, & Thompson, 2003; Rittle-Johnson, Star, & Durkin, 2009). Recent evidence from classroom studies indicates that comparison can be beneficial for algebra learning (Authors, 2007, 2009, 2012). In fact, comparison may be particularly effective when paired with explanation, as generating explanations improves learning across several topics and age groups (Aleven & Koedinger, 2002; Hodds, Alcock, & Inglis, 2014; Rittle-Johnson, 2006).

Based on evidence from short-term classroom studies, we developed a supplemental algebra curriculum comprised of worked example pairs and explanation prompts (Authors, 2015). Each worked example pair was designed to explicitly support comparison of multiple strategies, and explanation prompts were designed to encourage mathematics conversations in the classroom. Classroom discussions that allow students to generate explanations and build on one another's responses can improve learning (e.g., Lampert, 1990; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005), but teachers often struggle to ask more challenging, open-ended questions (Star, Newton, et al., 2015). In fact, teachers in the U.S. often have students explain simple aspects of comparison but explain the more challenging aspects themselves (Hiebert et al., 2003; Richland, Stigler, & Holyoak, 2012). With our approach, we hoped to increase comparison and interactive discussions in algebra classrooms with carefully designed worked example pairs and explanation prompts, as well as professional development opportunities.

### *Current Study*

In the current paper, we investigate how use of our materials designed to encourage comparison and discussion in algebra classrooms affected classroom instruction. Teachers were provided with worked example pairs and explanation prompts, and these materials were used during four chapters spread throughout the school year. A subset of these lessons were filmed and coded for key instructional practices described in more detail below, and we coded segments of lessons when teachers were and were not using our materials.

**Method**

*Participants*

In this pilot year of data collection, four Algebra I teachers from a school district in suburban Massachusetts used our materials. One teacher taught $8^{th}$ grade and three teachers taught $9^{th}$ grade, and their classes covered a wide range of student ability levels. Thirty lessons across all four teachers were videotaped throughout the year depending on the teachers' schedules and availability: 3 lessons from Teacher 1, 21 lessons from Teacher 2, 5 lessons from Teacher 3, and 1 lesson from Teacher 4. Thus far, we have coded 12 of these videos, and this preliminary coding will be used in the current paper.

*Materials*

The teachers were provided with comparison materials that tightly aligned with four chapters in their textbooks: Functions and Graphing Linear Equations, Solving Systems of Linear Equations, Polynomials and Factoring, and Solving Quadratic Equations. For each chapter, teachers were provided with 8 to 16 worked example pairs, depending on the goals of that chapter, which corresponded to lessons within the unit. The worked example pairs were similar to those used in past research and showed the work of two hypothetical students who solved a math problem followed by prompts for explanation (Authors, 2015, see Figure 1). The prompts were designed to help students make comparisons and scaffold discussion. For each worked example pair, we also provided a Big Idea page where the hypothetical students identified the learning goal. Each Big Idea page was designed to provide an explicit summary of the example pair's instructional goal because past research suggests that direct instruction helps support student comparisons (Schwartz & Bransford, 1998).

*Coding Scheme*

To investigate how teachers were using comparison and discussion in their classrooms both with and without our materials, videos were coded according to a detailed coding scheme adapted from a previous scheme on algebra instruction (Litke, 2015). Each video was broken into 7.5 minute segments to be coded because past work suggested that this was a short enough time to keep track of practices but long enough to see meaningful instructional interactions and have reliability (Litke, 2015). Each segment was coded for Making Sense of Procedures, Supporting Procedural Flexibility, Comparing Multiple Procedures, Teacher Questioning, Student Responding, and Interactions, as well as whether our materials were used and whether students were given the opportunity to produce written explanations (see Table 1 for details). Each code was rated on a scale from 1 to 4, with 1 being the lowest level and 4 being the highest level. For instance, for the Interactions code, a 1 was coded if the teacher was the only participant in a discussion and did not ask any questions, a 2 was coded if a teacher asked one student a question, a 3 was coded if a teacher asked multiple students a question but the students did not build on each other's thinking, and a 4 was coded if the teacher or students asked questions that provided an opportunity for students to build on one another's thinking.

*Data Analysis*

We looked at the percentage of videos in which each rating occurred (e.g., what percentage of videos contained a comparison of multiple procedures) and calculated the mean rating for each coding category (e.g., the mean Interaction rating across video segments). We did this looking separately at those segments where teachers used our materials versus those segments where they proceeded with their normal classroom instruction. Due to the small number of videos coded so far, here we provide a descriptive comparison of practices with and without our materials.

**Results**

Of the 79 segments coded from 12 videos, 59 did not involve the teachers using our materials and 20 did. When teachers used our materials, they more often compared strategies, had higher levels of supporting procedural flexibility, and had higher levels of teacher questioning and student responding than when not using our materials (Table 2). Making sense of procedures was similar whether teachers used our materials or not. Teachers only explicitly compared strategies when using our materials (80% of segments with our materials). This manipulation check indicated that teachers were using the comparisons of the worked example pairs as intended and confirmed past work that teachers are unlikely to explicitly compare strategies unless provided with supports to do so (Star, Pollack, et al., 2015). In addition, teachers were much more likely to have instructional time where procedural flexibility was the main focus when using our materials (a 2.60 vs. 1.46 rating). When teachers used our materials that provided prompts to encourage mathematical conversations, their level of questioning increased dramatically. Teachers were much less likely to ask "why" and open-ended questions without our materials and instead spent most of their questioning time asking "what" and "how" questions. Similarly, students had higher level responses when using our materials, though the difference was not as large. Improving the level of interaction in the classroom with our materials was more difficult. Teachers had a mean interaction rating of 2.55 with our materials and a score of 2.02 without our materials. Thus, the level of interaction in these classrooms was often weak, with teachers mainly asking low level questions to one student at a time. In fact, only a little less than 10% of segments overall included students building on one another's thinking in a mathematical conversation. Finally, students wrote explanations about half of the time (55%) when using our materials but never did so when not using our materials. Thus, without explicit scaffolds to help students generate written explanations, such as our prompts, students were rarely given the opportunity to write down their own explanations.

**Discussion**

Generally, our materials worked as intended to help teachers compare strategies, support procedural flexibility, and increase teacher questioning and student responding. Although these practices were higher when teachers were using our materials, the levels of these things still were not particularly high, falling between a 2 and 3 rating on average. Increasing teachers' level of interaction in the classroom was more difficult, even with our materials. Teachers often find it difficult to engage in mathematical conversations that help students build on each other's thinking, but it is possible to ask students to compare their own ideas to others' ideas. Using specific language can encourage students to monitor their own and each other's ideas (Webb et al., 2014).

As this was the pilot year of our project, not much support was provided to teachers outside of being provided with our materials. For the first full year of the project, we have since provided a professional development training during the summer to give teachers more guidelines and supports for using comparison and discussion effectively with the worked example pairs. During this professional development, we emphasized an implementation model that incorporated think-pair-share work to encourage multiple students participating in a discussion of mathematical ideas. A large portion of the professional development was spent practicing the facilitation of mathematical conversations and providing ideas of question stems and strategies to sustain a discussion. Teachers will also receive "just in time" professional development throughout the school year before each chapter in which they use our materials where these ideas will be revisited as well. Hopefully with these supports, teachers will be able to increase their level of interaction. Future work will determine whether increasing these instructional practices in algebra classrooms will lead to improved student outcomes or mediate the effects of our curriculum on student learning. Consequently, using our materials did help teachers improve their instructional practices, but further supports are needed to continue developing these practices in classrooms to improve students' procedural flexibility and learning.

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Table 1 *Coding Scheme Categories*

| Category | Description | Example of a 1 Rating | Example of a 4 Rating |
|---|---|---|---|
| Making Sense of Procedures | Teacher or students make meaning of individual steps of or a solution generated by a procedure, attend to the mathematical goals or properties underlying a procedure, or attend to why a procedure holds. | A procedure is presented like a recipe to be followed with no attention to meaning. | The focus of the segment is why you plug in x=0 into a linear equation when finding the y-intercept. |
| Supporting Procedural Flexibility | The degree to which teachers present procedures to afford students the opportunity to notice multiple procedures to solve the same problem, attend to applicability of a procedure, attend to key conditions of steps within a procedure to understand its efficiency, or compare multiple procedures for their affordances and limitations. | A procedure is presented without any attention to its affordances/ limitations or alternative procedures. | Throughout the segment, the students compare when one procedure might be faster than another. |
| Teacher Questioning | The highest level question the teacher uses during the segment, and a student must respond in some way to the question in order for it to be considered. Question levels range from 1) "What" questions with simple one-word responses to 2) "How" questions about the steps of a procedure to 3) "Why" questions about understanding why an answer was correct or a procedure was a good choice to 4) "Open-ended" questions that allow students to share ideas, elaborate on others' ideas, and generalize. | A teacher asks, "What is the answer?" | A teacher asks, "Can you generate another problem where Riley's method could not be used?" |
| Student Responding | The highest level question or response a student generates during the segment. Response levels range from 1) answering with simple declarative information such as a numerical answer to 2) describing the steps of a procedure to 3) explaining their understanding such as justifying why a procedure is a good choice to 4) explaining a generalization or their understanding. | The student says, "The answer is x = 5." | The student states an explanation of why he or she does not agree with another student's choice of procedure. |
| Interactions | The highest level of student interaction defined as the opportunity to verbally share ideas regarding mathematical procedures and/or content, as elicited by the questioning. | The teacher is the only participant in the conversation. | Multiple students respond to the same question, and the teacher prompts them to build on each other's thinking. |
| Comparing Multiple Procedures | The teacher or students analyze multiple, different procedures for the same problem, noting the affordances or limitations of each. Rated as a Yes or No. | n/a | n/a |
| Use of Written Explanations | Students have an opportunity to produce written explanations. Rated as a Yes or No. | n/a | n/a |

Table 2 *Mean Coding Ratings across Videos With and Without Our Materials*

| Code | Mean Rating (1 to 4 scale) With Materials | Mean Rating (1 to 4 scale) Without Materials |
|---|---|---|
| Making Sense of Procedures | 2.30 | 1.95 |
| Supporting Procedural Flexibility | 2.60 | 1.46 |
| Teacher Questioning | 2.50 | 1.34 |
| Student Responding | 2.45 | 1.60 |
| Interactions | 2.55 | 2.02 |

| Code | Percentage of Segments With Materials | Percentage of Segments Without Materials |
|---|---|---|
| Comparing Multiple Procedures | 80.00% | 8.47% |
| Use of Written Explanations | 55.00% | 0.00% |

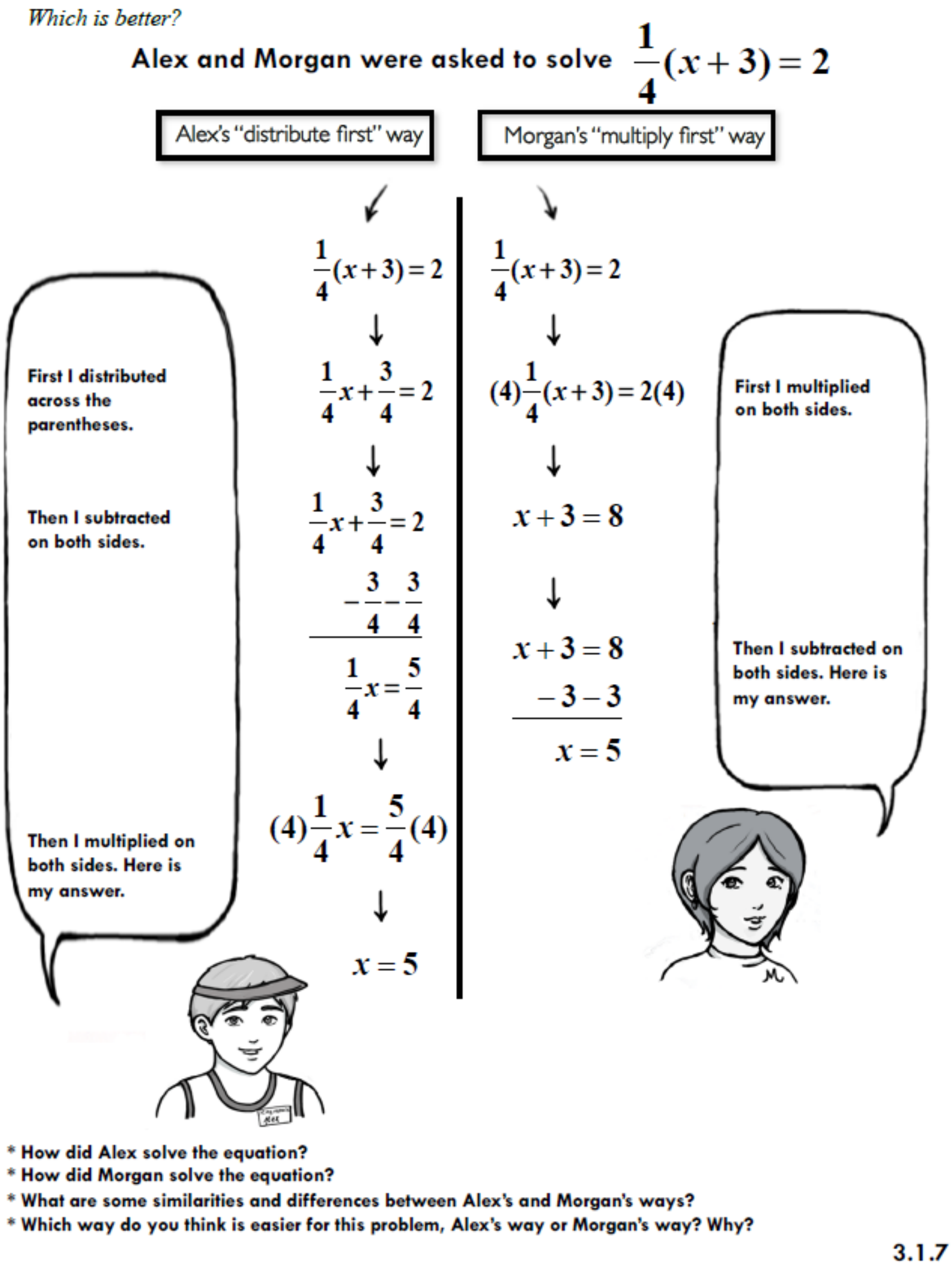


Figure 1. *Sample Worked Example Pair*