PROCEDURAL FLEXIBILITY FOR ALGEBRA: ASSESSMENT DEVELOPMENT

ZHANG, Yinghao, Vanderbilt University, USA
FINE, Stephanie, Vanderbilt University, USA
LOEHR, Abbey, Vanderbilt University, USA
STAR, Jon R., Harvard University, USA
RITTLE-JOHNSON, Bethany, Vanderbilt University, USA

Students need to develop procedural flexibility - knowing multiple strategies for solving a problem and selecting the most appropriate strategy for a given problem (Star, 2005; Star & Newton, 2009). This paper focuses on the development of a new measure of procedural flexibility for algebra.

Students in the United States completed our assessment near the end of their Algebra I course (N = 77; M age = 15.0 years). The assessment had three subtests: procedural flexibility (see Figure 1 for sample items), conceptual knowledge and procedural knowledge.

Students' procedural flexibility scores (m = 42%) were correlated with their conceptual knowledge scores (m = 60%, r (75) = .46) and procedural knowledge scores (m = 46%, r (75) = .43). Internal consistency on the three subscales was moderate. Procedural flexibility scores were also correlated with a previous standardized math test (r (75) = .47). We also examined students' errors, as described in Table 1. Efficiency errors accounted for 42% of responses, while correctness errors occurred on 13% of trials.

Our procedural flexibility assessment is both valid and reliable. Although students demonstrated some flexibility, U.S. students may need additional instructional support to develop more robust procedural flexibility.

21) Below is the beginning of Gabriella's, Jamal's, and Nadia's work in solving the equation $x+4=12$. Which way(s) would be a mathematically okay way(s) to start solving the problem? (Circle the letter for the mathematically okay way(s)).

a. Jamal's way
b. Nadia's way
c. Gabriella's and Jamal's ways
d. Gabriella's, Jamal's, and Nadia's ways

Gabriella's way:	Jamal's way:	Nadia's way:
$x+4-12=12-12$	$x+4-4=12-4$	$x+4-x=12-x$

22) Jamal solved the following problem:

$$\begin{cases} x+2y=9 \\ 3x-2y=11 \end{cases}$$

This is how Jamal started the problem:

$$x+2y=9$$
$$2y=9-x$$
$$y=\frac{9-x}{2}$$

22b. Do you think this is a good way to start this problem? **Circle one:**
a. Very good way
b. Mathematically OK, but not a very good way
c. Not OK mathematically

27) On a timed test, which would be the BEST way to start factoring the trinomial $6x^2 + 13x - 5$? (Circle the letter for the best way.)

a. Gabriella's way:
$6 \cdot -5 = -30$

Factors of -30	**Sum of Factors**
1, -30	1 + -30 = -29
-1, 30	-1 + 30 = 29
2, -15	2 + -15 = -13
-2, 15	-2 + 15 = 13

$6x^2 - 2x + 15x - 5$

b. Jamal's way:
$6 \cdot 13 = 78$

Factors of 78	**Sum of Factors**
1, 78	1 + 78 = 79
2, 39	2 + 39 = 41
3, 26	3 + 26 = 29
6, 13	6 + 13 = 19

c. Nadia's way:

Factors of 6	**Factors of -5**	**Factorization**
1, 6	1, -5	$(x + 1)(6x - 5)$
1, 6	5, -1	$(x + 5)(6x - 1)$
6, 1	1, -5	$(6x + 1)(x - 5)$
…		

28) On a timed test, which woud be the BEST way to start solving the equation below? (Circle the letter for the best way).

$$(7x+5)^2=64$$

a. Gabriella's way:	b. Jamal's way:	*c. Nadia's way:*
$(7x+5)(7x+5)=64$	$(7x)^2+2(7x)(5)+5^2=64$	$\sqrt{(7x+5)^2}=\sqrt{64}$

Figure 1. Sample Procedural Flexibility Items. Correct Answer is in Italics.

References

Star, J.R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36*(5), 404-411.

Star, J.R., & Newton, K.J. (2009). The nature and development of experts' strategy flexibility for solving equations. *ZDM - The International Journal on Mathematics Education*, *41*, 557-567.