1 Take-Home Problems

1. Show that the number of partitions into odd parts is the same as the number of partitions into distinct parts. For example, the partitions of 9 into odd parts are $9, 7 + 1 + 1, 5 + 3 + 1, 5 + 1 + 1 + 1, 3 + 3 + 3 + 1 + 1, 3 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$ and the partitions of 9 into distinct parts are $9, 8 + 1, 7 + 2, 6 + 3, 6 + 2 + 1, 5 + 4, 5 + 3 + 1, 4 + 3 + 2$. There are 8 partitions of each type.

2. (Andrews) The number of partitions of $n$ in which only odd parts may be repeated equals the number of partitions of $n$ in which no part appears more than three times. For example, the partitions of $n = 5$ of the first type are $5, 4 + 1, 3 + 2, 3 + 1 + 1, 2 + 1 + 1 + 1 + 1 + 1 + 1$, and the partitions of 5 of the second type are $5, 4 + 1, 3 + 2, 3 + 1 + 1, 2 + 2 + 1, 2 + 1 + 1 + 1$. In both cases, there are 6 such partitions.

3. Prove that the greatest common divisor of $F_n$ and $F_{n+1}$ is 1 for all $n$.

4. (For those who know what mathematical induction is:) Find a formula for the sum of the first $n$ Fibonacci numbers. (*Hint:* Try writing down the first few sums and see if you can come up with a guess for the answer. Then trying proving the result by induction.)

5. Define a sequence by $a_0 = 0$ and $a_{n+1} = 2a_n + 1$ for $n \geq 0$. Can you write down a closed formula for $a_n$? (*Hint:* Consider $GF(a_n)$ as we did when trying to find $F_n$.)