1. On April 1, 1975, Martin Gardner published a "counterexample" to the Four Color Theorem (given on the last page). Was he correct? If so, explain why. If not, draw a four coloring of it.

2. We showed that the complete graph on five vertices K_5 (i.e. the graph with five vertices such that every two vertices are connected by an edge) is not planar. However, we can draw graphs on surfaces other than the plane! A "torus" is a donut-shaped object. See if you can draw this graph on the torus without any of the edges intersecting! For those of you who know what a Möbius band is and are able to finish the torus problem, try drawing this graph on this band next.

3. There is a country of with many little lakes. The total area of lakes is less than 1 square mile. Show that one can build a square grid of phone poles so that every square is of area 1 square mile, every point of the country is within 1 m from a pole, and no pole is in a lake.

4. Recall that we proved Euler's formula for a connected planar graph:

$$|V| - |E| + |F| = 2,$$

where $|V|$ is the number of vertices, $|E|$ is the number of edges, and $|F|$ is the number of faces of a graph. In order to prove that K_5 is not planar, we showed that if the number of vertices is greater than 2, then

$$|E| \leq 3|V| - 6.$$

We will now prove something with a similar graph. Now, consider the following graph:

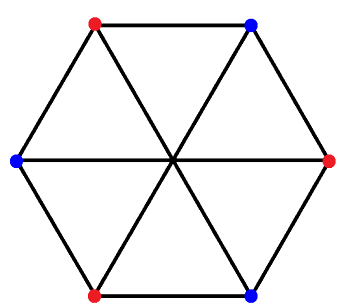

Figure 1: $K_{3,3}$

This is called $K_{3,3}$. Every *cycle* in this graph has at least four vertices. In other words, starting with a given vertex and traveling along the edges, you have to go through at least three other vertices to get back to the original one. This means that if we can draw this graph in the plane, every face must be bounded by at least 4 edges.

a) Use this to show that $4|F| \geq 2|E|$.

b) Now, in order to show that we cannot draw this in the plane, suppose that we could. Use Euler's formula and part (a) to show that

$$|E| \leq 2|V| - 4.$$

c) Count the number of edges and vertices in $K_{3,3}$ to conclude that this is a contradiction.

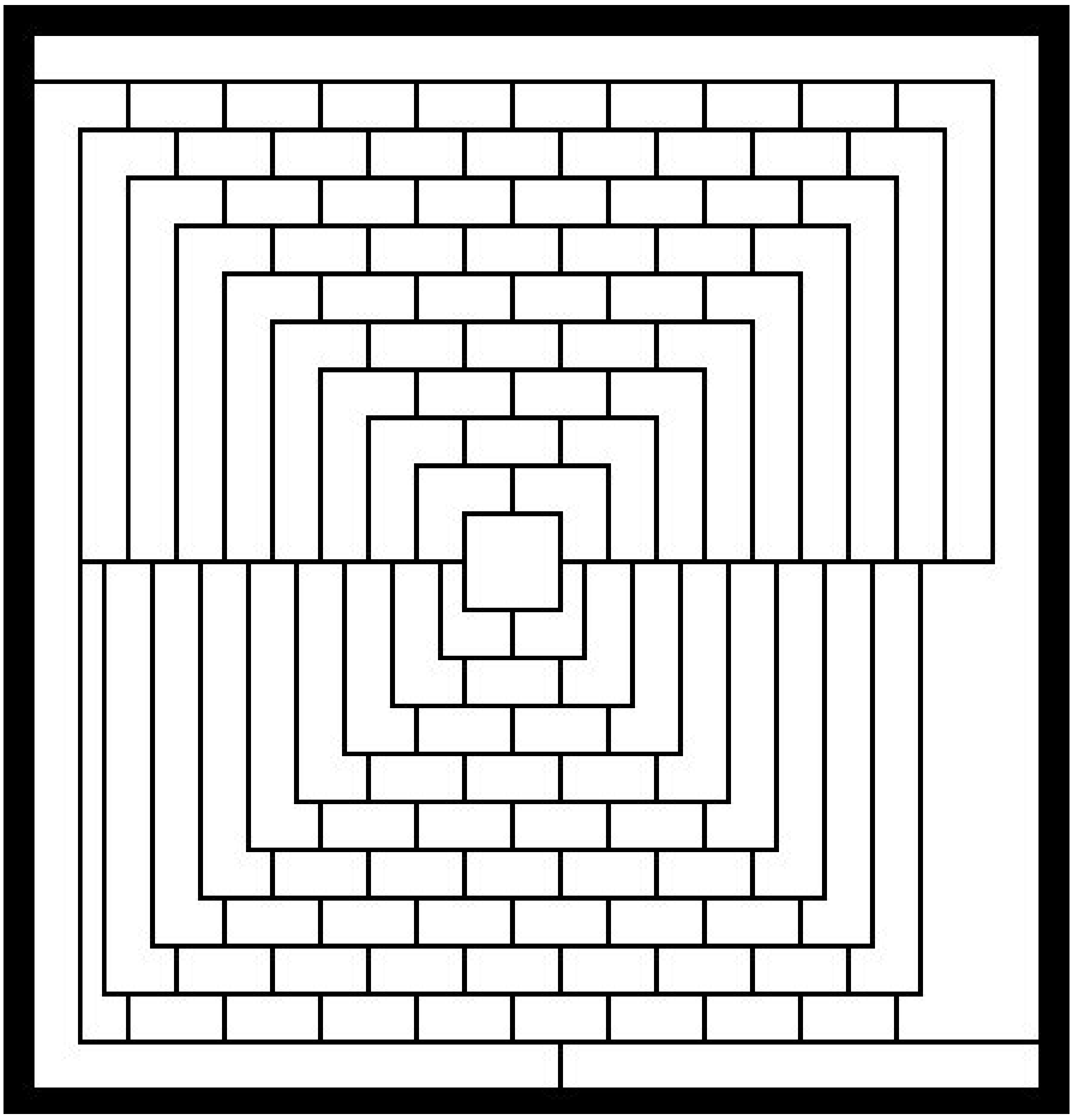

Figure 2: Gardner's Example