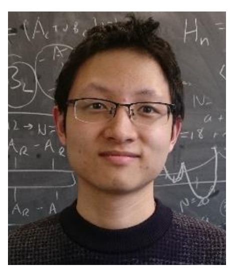
emergence of conformal symmetry in critical quantum spin chains and tensor networks

NCGOA 2019
Vanderbilt University

Ashley Milsted

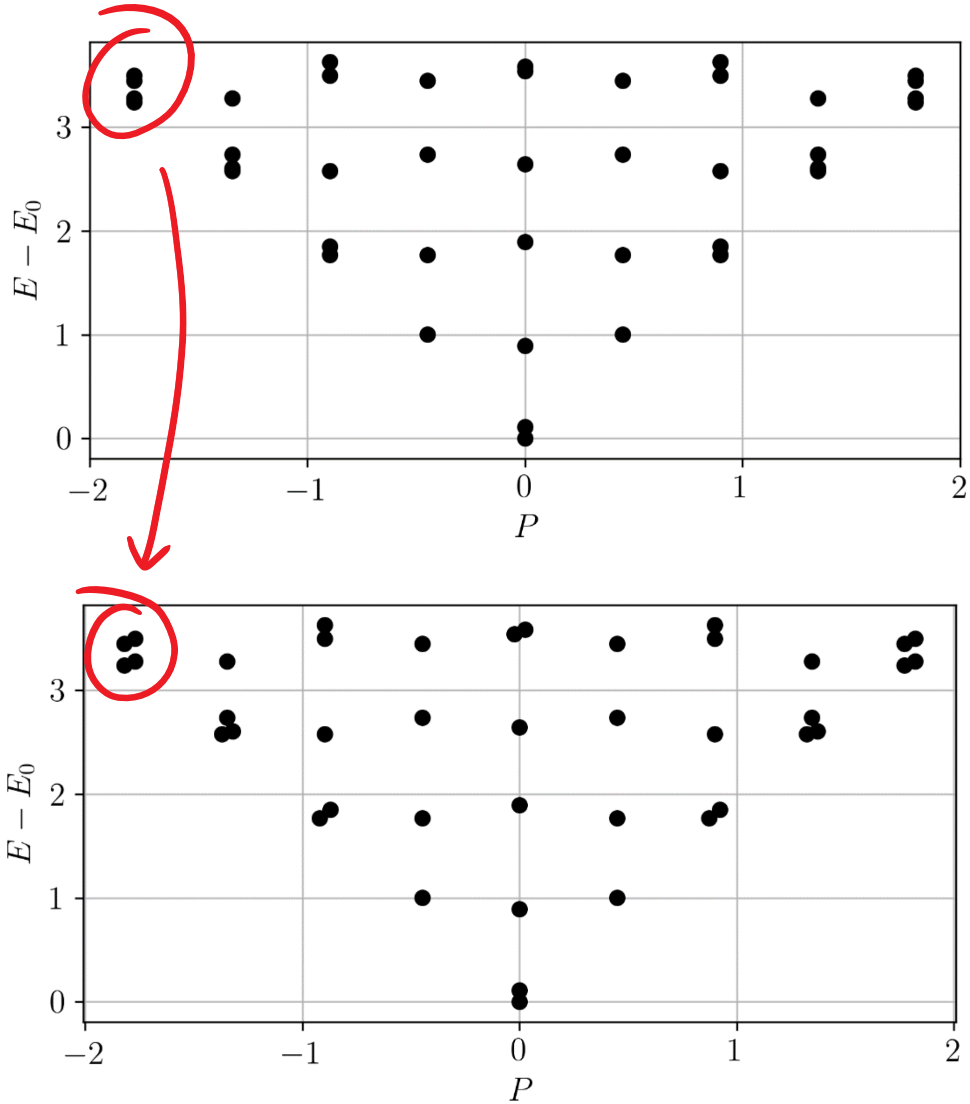
Yijian Zou, Guifre Vidal



PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

TENSOR NETWORKS INITIATIVE

Simons Collaboration on the Many Electron Problem



ABUSE OF X-AXES FOR THE NEXT 33 SLIDES

critical spin chain

1+1D CFT

$$H = \sum_{j=1}^{N} h_j$$

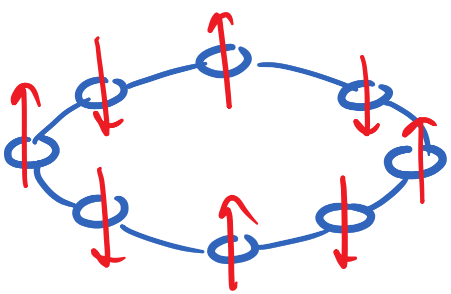
central charge: c

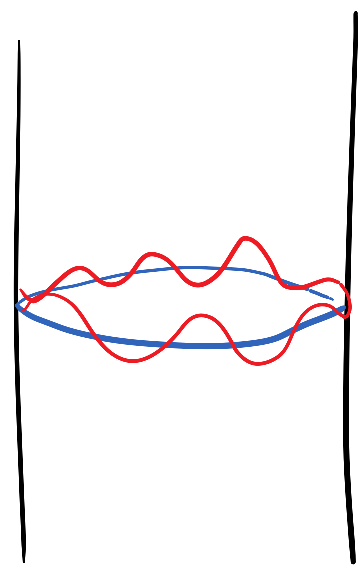
primary fields: Δ_ϕ, s_ϕ

OPE coefficients: $C^{\phi_1}_{\phi_2 \phi_3}$

(3-point correlators)

on the circle



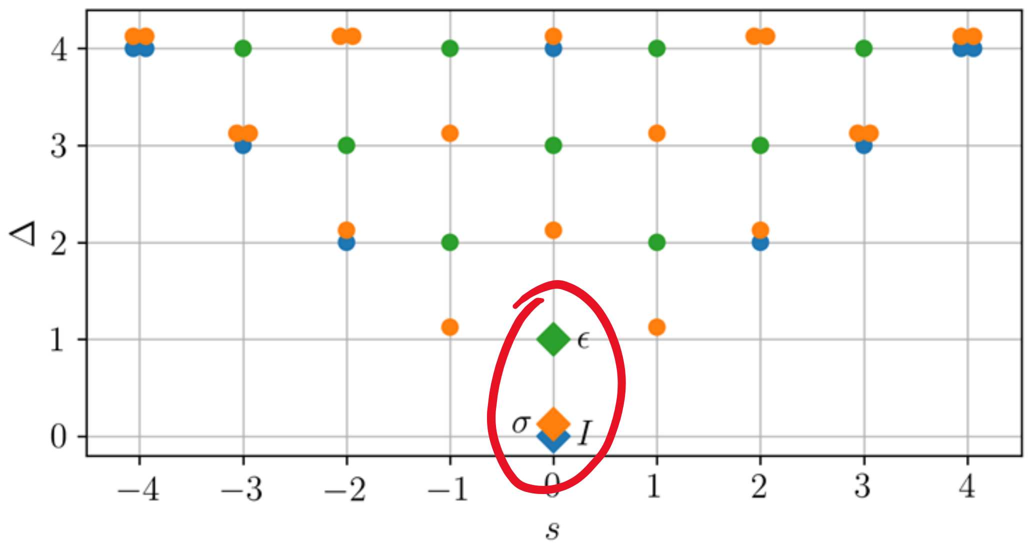


Belavin, Polyakov, Zamolodchikov, Nucl. Phys. B **241**, 333 (1984)

conformal data: 1+1D Ising CFT

central charge: $c = \frac{1}{2}$

scaling operator dimensions



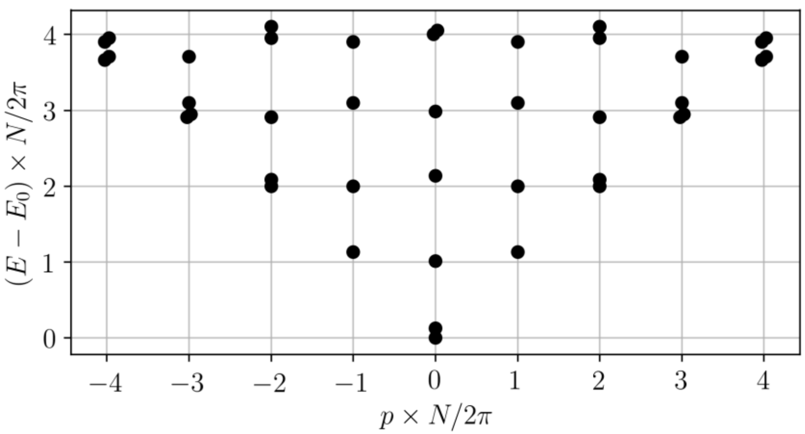
primary fields

ϕ	Δ	s
I	0	0
σ	1/8	0
ϵ	1	0

nonzero OPE coefficients: $C_{\sigma\sigma}^{\epsilon} = \frac{1}{2}$

(not involving I)

critical spin chain



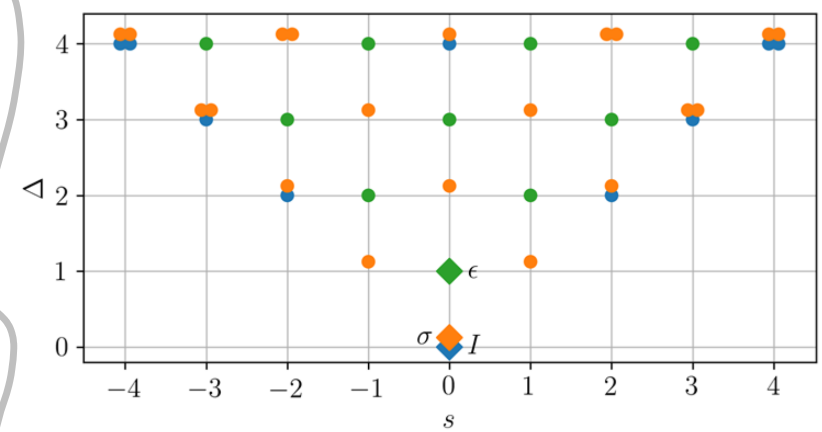
lattice energy-momentum spectrum

$$E_\alpha = A + \frac{B}{N}(\Delta_\alpha - \frac{c}{12}) + \mathcal{O}(N^{-x})$$

$$P_\alpha = \frac{2\pi}{N} s_\alpha$$

Cardy, Blöte, Nightingale, Affleck (1986)

1+1D CFT



CFT energy-momentum spectrum

$$E_\alpha = A + \frac{B}{N}(\Delta_\alpha - \frac{c}{12})$$

$$P_\alpha = \frac{2\pi}{N} s_\alpha$$

critical spin chain

$$\phi_j$$

lattice primary field operators

$$\exp\left(\sum_n \left[a_n L_n + \overline{a}_n \overline{L}_n\right]\right)$$

"lattice conformal transformations"

e.g. Koo & Saleur, 1994

1+1D CFT

$$\phi^{CFT}(x)$$

primary field operators

$$\exp\left(\sum_n \left[a_n L_n^{\mathrm{CFT}} + \overline{a}_n \overline{L}_n^{\mathrm{CFT}}\right]\right)$$

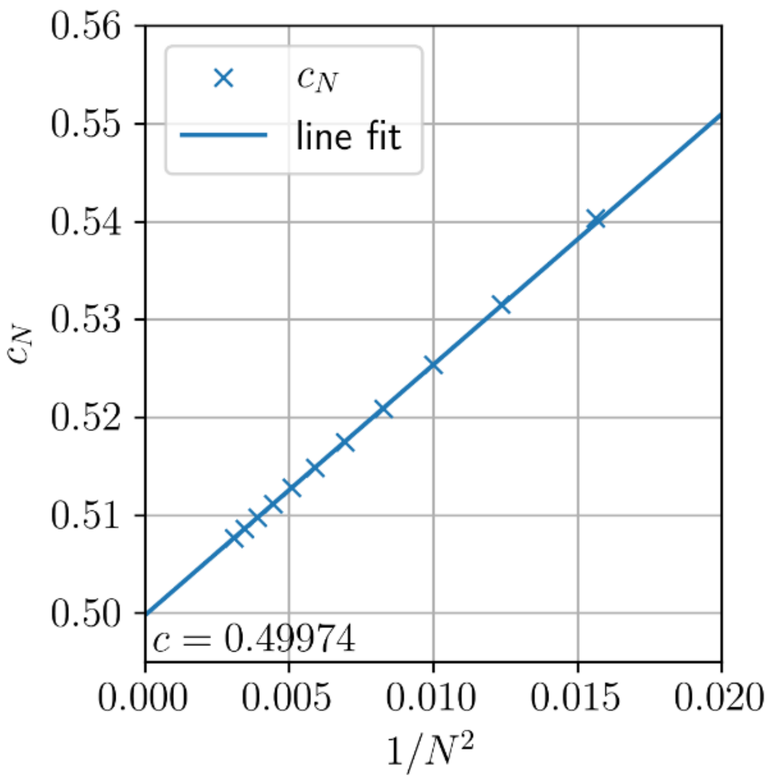
conformal transformations acting on the CFT Hilbert space

$$\langle\phi_\alpha|\mathcal{O}|\phi_\beta\rangle \xrightarrow[N\to\infty]{} \langle\phi_\alpha^{CFT}|\mathcal{O}^{CFT}|\phi_\beta^{CFT}\rangle$$

example

critical Ising spin chain

$$c_N \equiv 2\langle I|L^{\dagger}_{-2}L_{-2}|I\rangle$$



$$N \leq 18$$

1+1D Ising CFT

$$c = 2\ \langle I^{CFT}|L^{CFT\dagger}_{-2}L^{CFT}_{-2}|I^{CFT}\rangle$$

$$c_{Ising} = \frac{1}{2}$$

outline

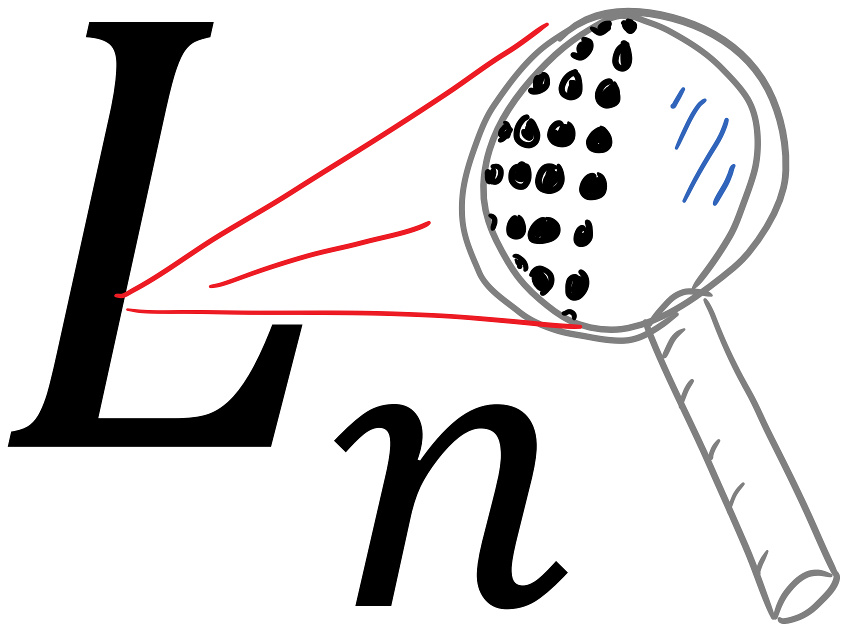
We show how “**lattice Virasoro generators”** can be used to...

1. ...systematically identify low-energy eigenstates of critical spin chains with energy eigenstates in the CFT,

2. find “lattice primary field operators” that approximately obey the correct operator algebra,

3. identify emergent conformal transformations in Tensor Networks (e.g. MERA) that describe critical systems.

“detailed emergence of conformal symmetry in lattice systems”



lattice Virasoro generators

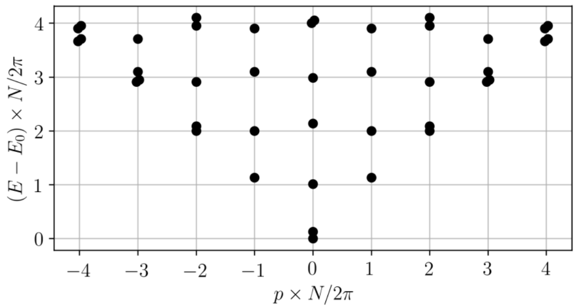
and extracting conformal data

AM, G. Vidal, Phys. Rev. B 96 245105 (2017)
Y. Zou, **AM**, G. Vidal, PRL 121, 230402 (2018)
Y. Zou, **AM**, G. Vidal, arXiv:1901.06439 (2019)

extracting conformal data

critical spin chain

low-energy spectrum



$$E_\alpha = A + \frac{B}{N}(\Delta_\alpha - \frac{c}{12}) + \mathcal{O}(N^{-x})$$

$$P_\alpha = \frac{2\pi}{N} s_\alpha$$

1+1D CFT

central charge: c

some Δ_ϕ, s_ϕ ?

primary fields: Δ_ϕ, s_ϕ

OPE coefficients: $C^{\phi_1}_{\phi_2\phi_3}$

(3-point correlators)

identifying primary states on the lattice

critical spin chain

$$H_n \propto \sum_{j=1}^{N} e^{-inj\frac{2\pi}{N}} h_j$$

"lattice Virasoro generators"
distinguish
"lattice primary states"

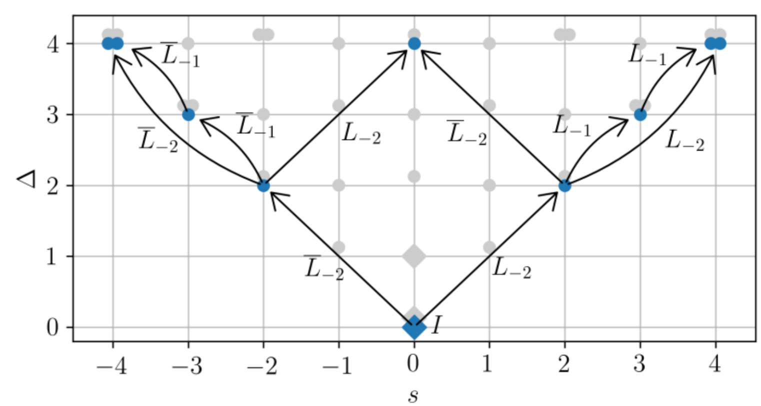
Koo & Saleur (1994)
AM & G. Vidal, PRB 96, 245105 (2017)

1+1D CFT

$$H_n^{CFT} \propto \int \mathrm{d}x\, e^{-inx\frac{2\pi}{N}} h^{CFT}(x)$$

$$H_n^{CFT} \equiv L_n^{CFT} + \overline{L}_{-n}^{CFT}$$

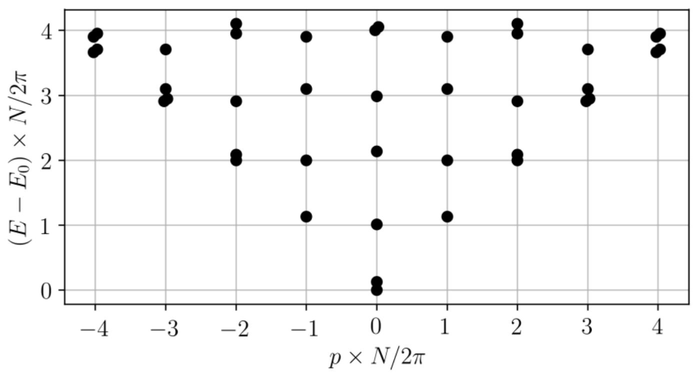
Virasoro generators (ladder operators)
distinguish primary states

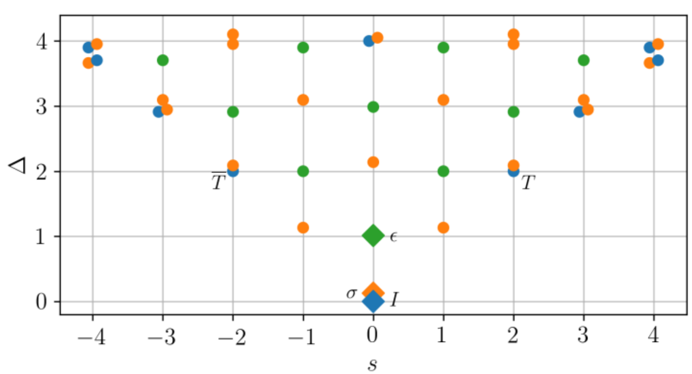


extracting conformal data

critical spin chain

low-energy spectrum





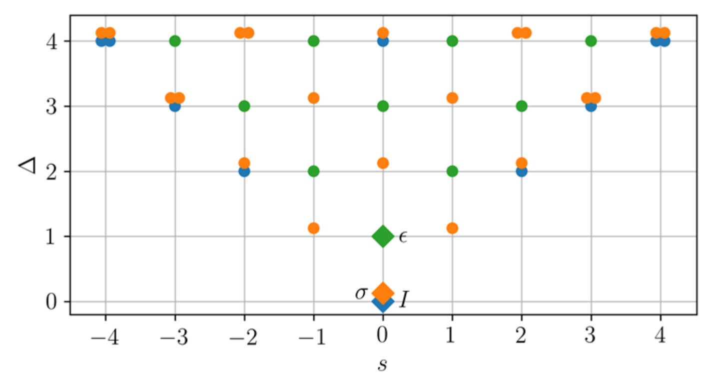
1+1D CFT

central charge: c

primary fields: Δ_ϕ, s_ϕ

OPE coefficients: $C^{\phi_1}_{\phi_2\phi_3}$

(3-point correlators)



also: identification of descendant states

lattice primary operators and OPE coefficients

critical spin chain

$$C^{\phi_1}_{\phi_2\phi_3} \approx \langle \phi_1 | \phi_2 | \phi_3 \rangle$$

=?

$$\phi_j$$

find **lattice** primary field operators **variationally**

$$\langle \phi^{(n,m)} | \; \phi \; | I \rangle$$

$\approx$

1+1D CFT

$$C^{\phi_1}_{\phi_2\phi_3} = \langle \phi_1^{CFT} | \phi_2^{CFT} | \phi_3^{CFT} \rangle$$

$$\phi^{CFT}(x)$$

primary field operators

$$\langle \phi^{(n,m)\,CFT} | \; \phi^{CFT} \; | I^{CFT} \rangle$$

accuracy ~ **10^{-7}**

(for Ising model)

YZ, AM, G. Vidal, arXiv:1901.06439 (2019)

extracting conformal data

critical spin chain

1+1D CFT

low-energy spectrum

central charge: c

primary fields: Δ_ϕ, s_ϕ

OPE coefficients: $C_{\phi_2\phi_3}^{\phi_1}$

(3-point correlators)

lattice Virasoro generators

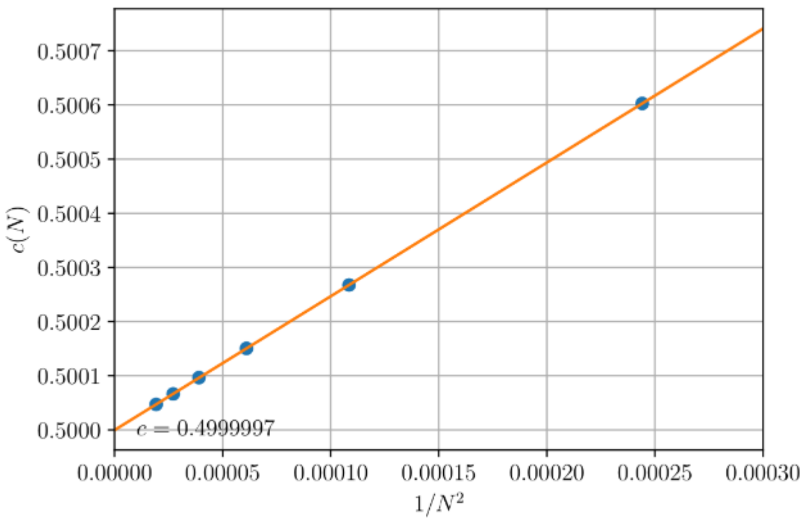
AM & G. Vidal (2017)

lattice primary operators

YZ, **AM,** G. Vidal, arXiv:1901.06439 (2019)

extends to **richer chiral symmetries**
e.g. superconformal, affine Lie + Virasoro

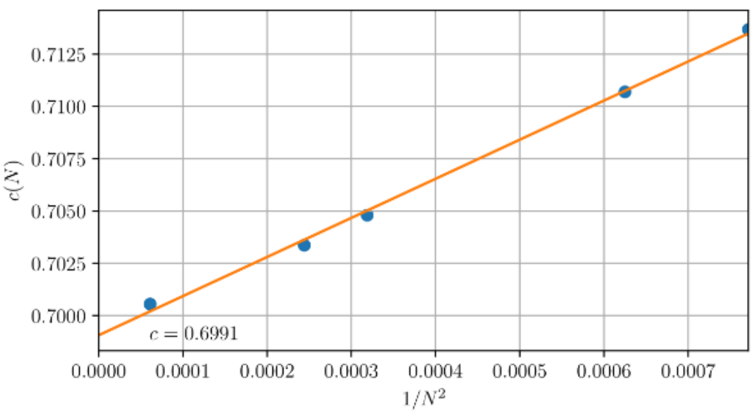
extracting *precise* conformal data using puMPS



Critical Ising model

	exact	puMPS	error
c	0.5	0.4999997	10^{-7}
Δ_σ	0.125	0.1249995	10^{-7}
Δ_ε	1	0.9999994	10^{-7}
$\Delta_{\partial\bar{\partial}\sigma}$	2.125	2.12501	10^{-5}
$\Delta_{\partial\bar{\partial}\varepsilon}$	3	3.00002	10^{-5}
$\Delta_{T\bar{T}}$	4	4.007	10^{-3}

$N \leq 228$



OF model, TCI point

	exact	puMPS	error
c	0.7	0.6991	10^{-4}
Δ_σ	0.075	0.07492	10^{-5}
Δ_ε	0.2	0.2001	10^{-4}
$\Delta_{\sigma'}$	0.875	0.8747	10^{-4}
$\Delta_{\varepsilon'}$	1.2	1.203	10^{-3}
$\Delta_{\varepsilon''}$	3.0	3.002	10^{-3}

$N \leq 128$

YZ, **AM,** G. Vidal, PRL 121, 230402 (2018)

the story so far

critical quantum spin chain Hamiltonian

⇓

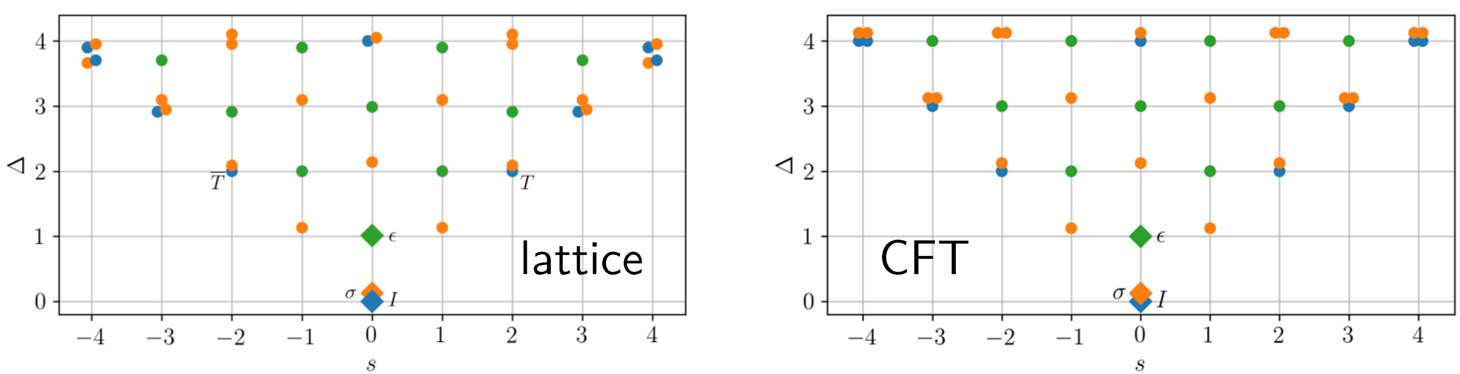
lattice Virasoro generators + low-energy eigenstates

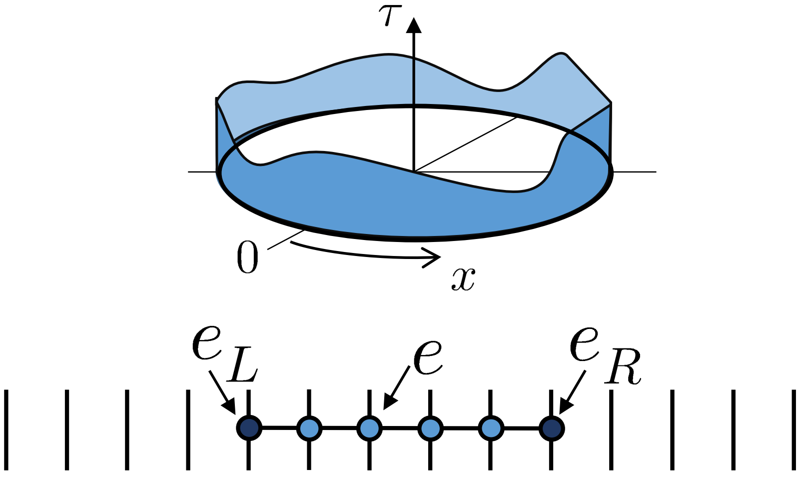
⇓

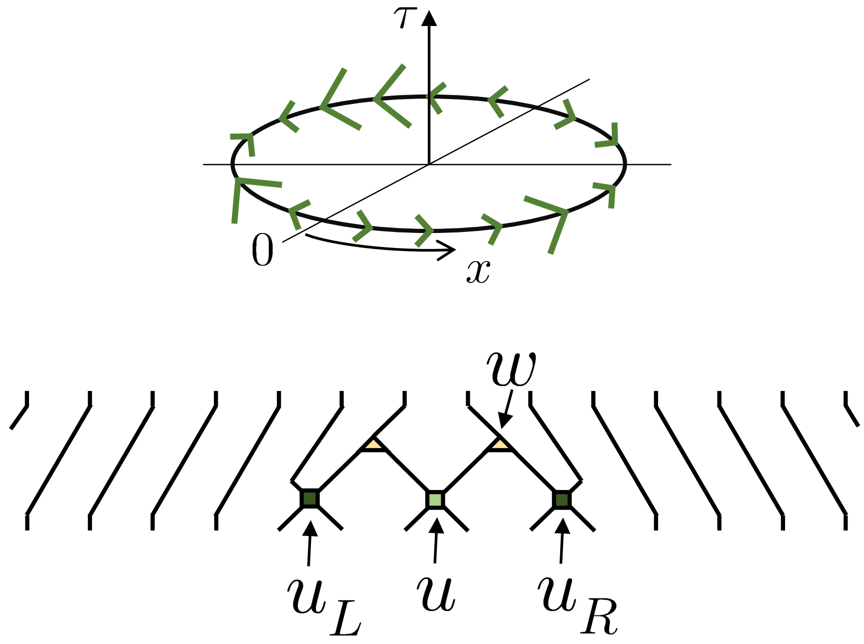
complete conformal data

also:

identification of each low-energy lattice eigenstate with a CFT scaling operator/state







tensor networks as conformal transformations

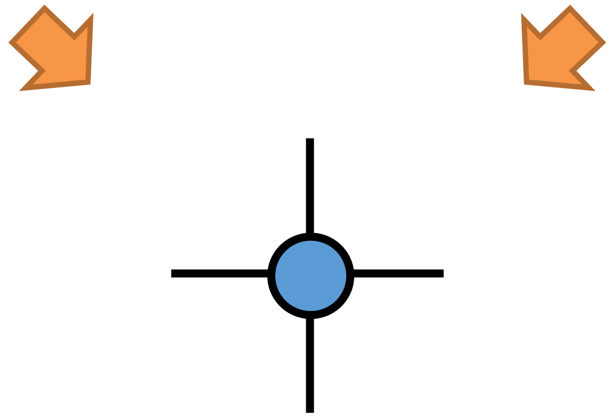
AM & G. Vidal

arXiv: 1805.12524, 1807.02501, 1812.00529 (2018)

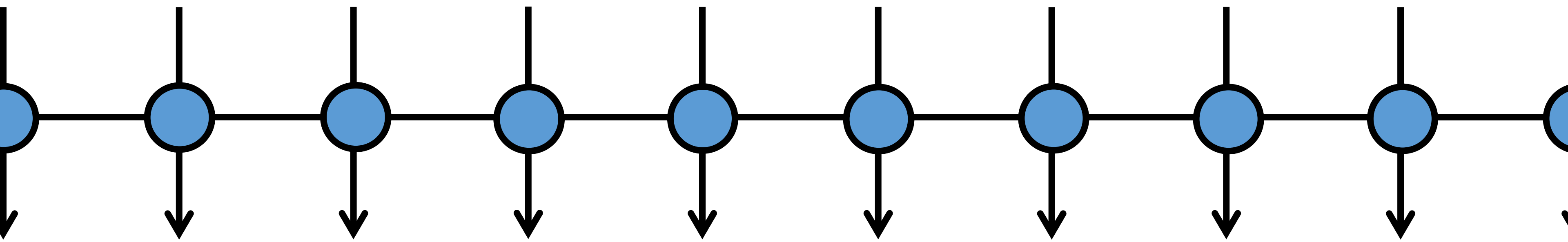
tensor networks are **powerful computational tools** for characterizing critical systems (see e.g. DMRG, MERA)

we identify **emergent conformal transformations** in **Tensor Networks** (e.g. MERA) that describe critical systems

$$H = \sum_{j=1}^{N} h_j \qquad\qquad Z = \sum_{c} \exp(-\beta E(c))$$

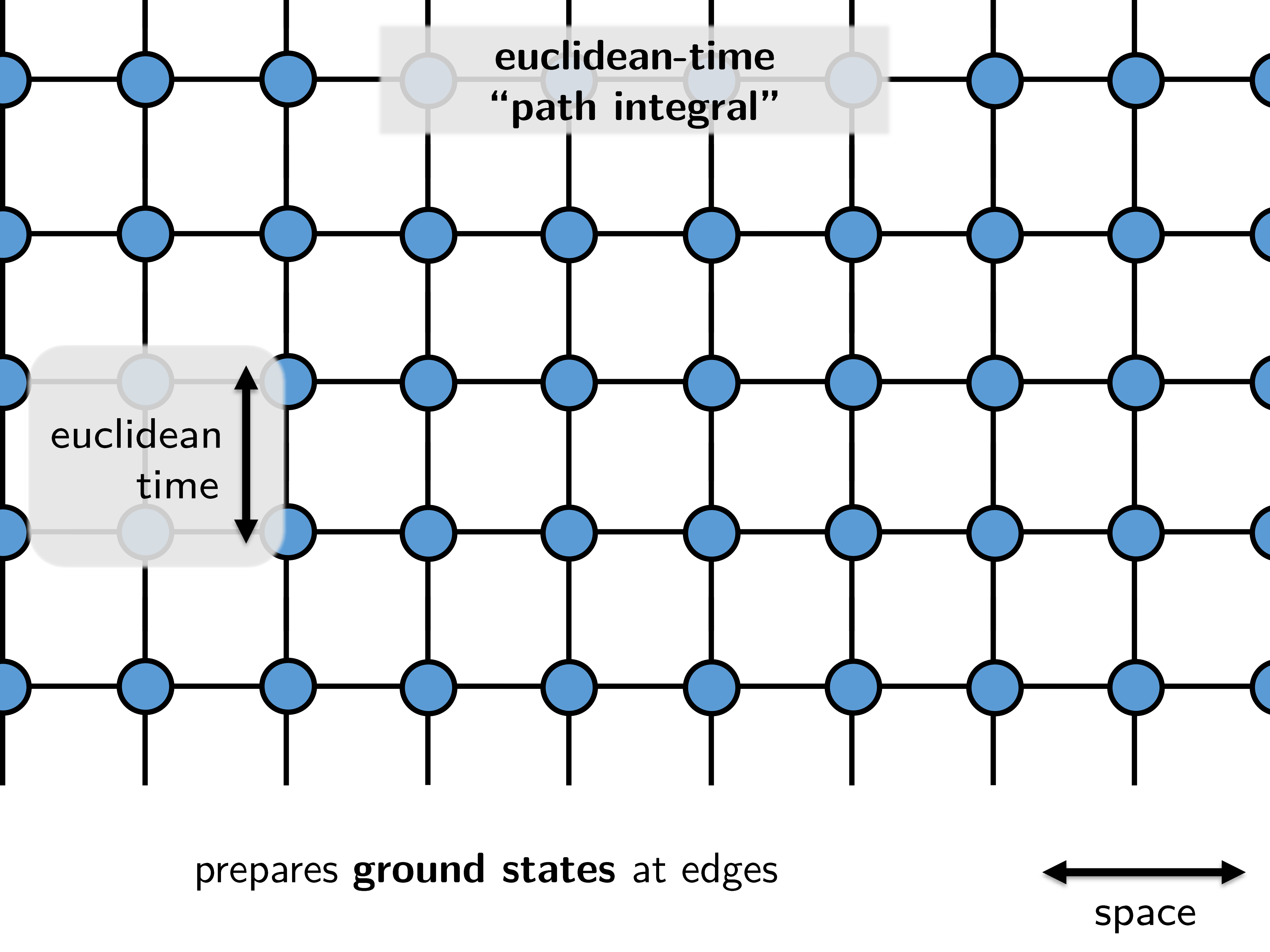


“euclideon” tensor

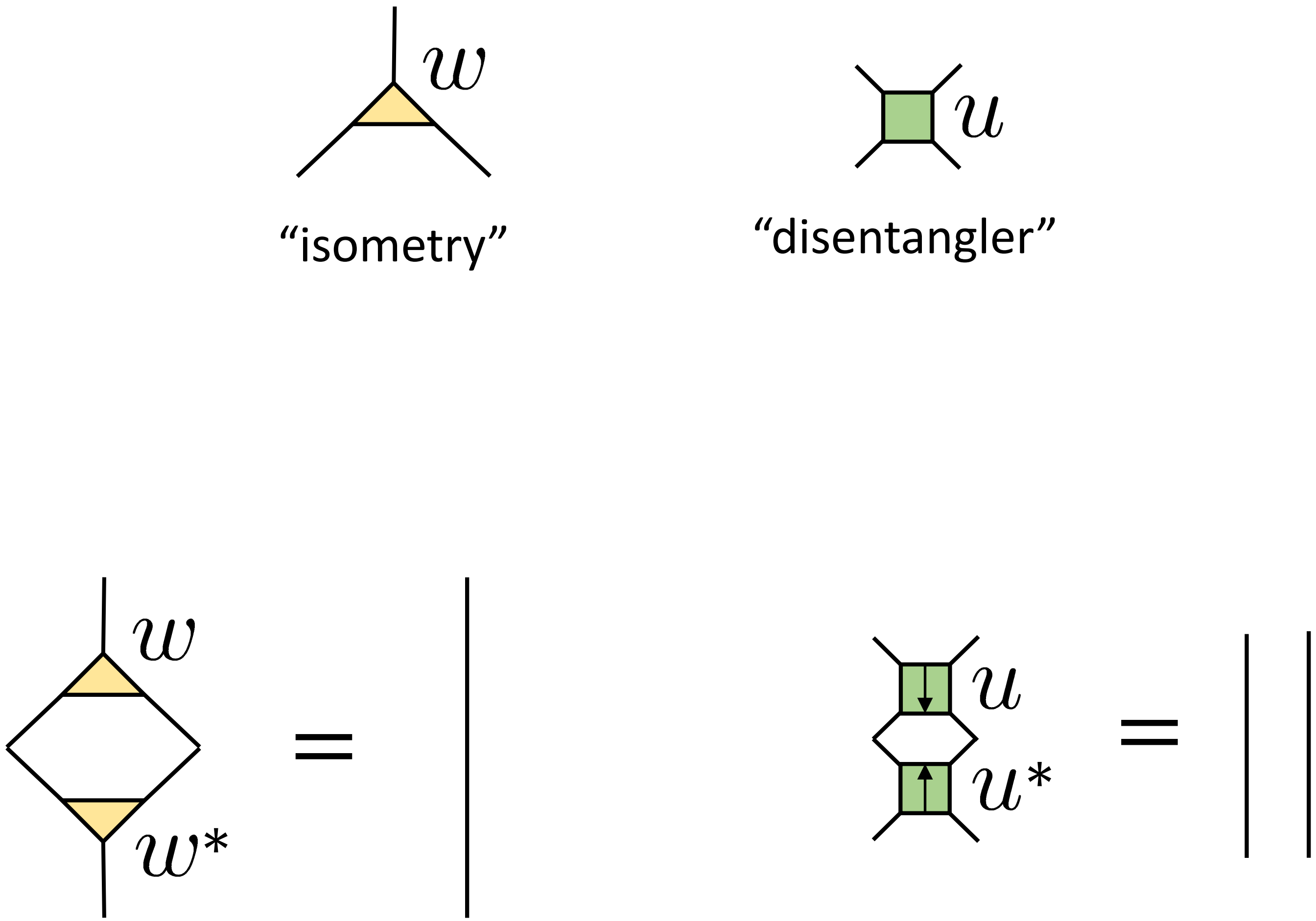


$$\exp(-\delta H)$$

(transfer matrix / euclidean-time propagator)


euclidean-time
"path integral"
euclidean
time
prepares **ground states** at edges
space

MERA

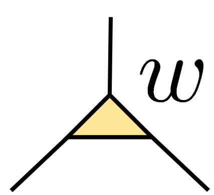


MERA

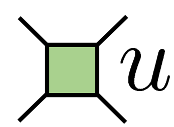
$$H = \sum_{j=1}^{N} h_j$$

$$Z = \sum_{c} \exp(-\beta E(c))$$

variational optimization

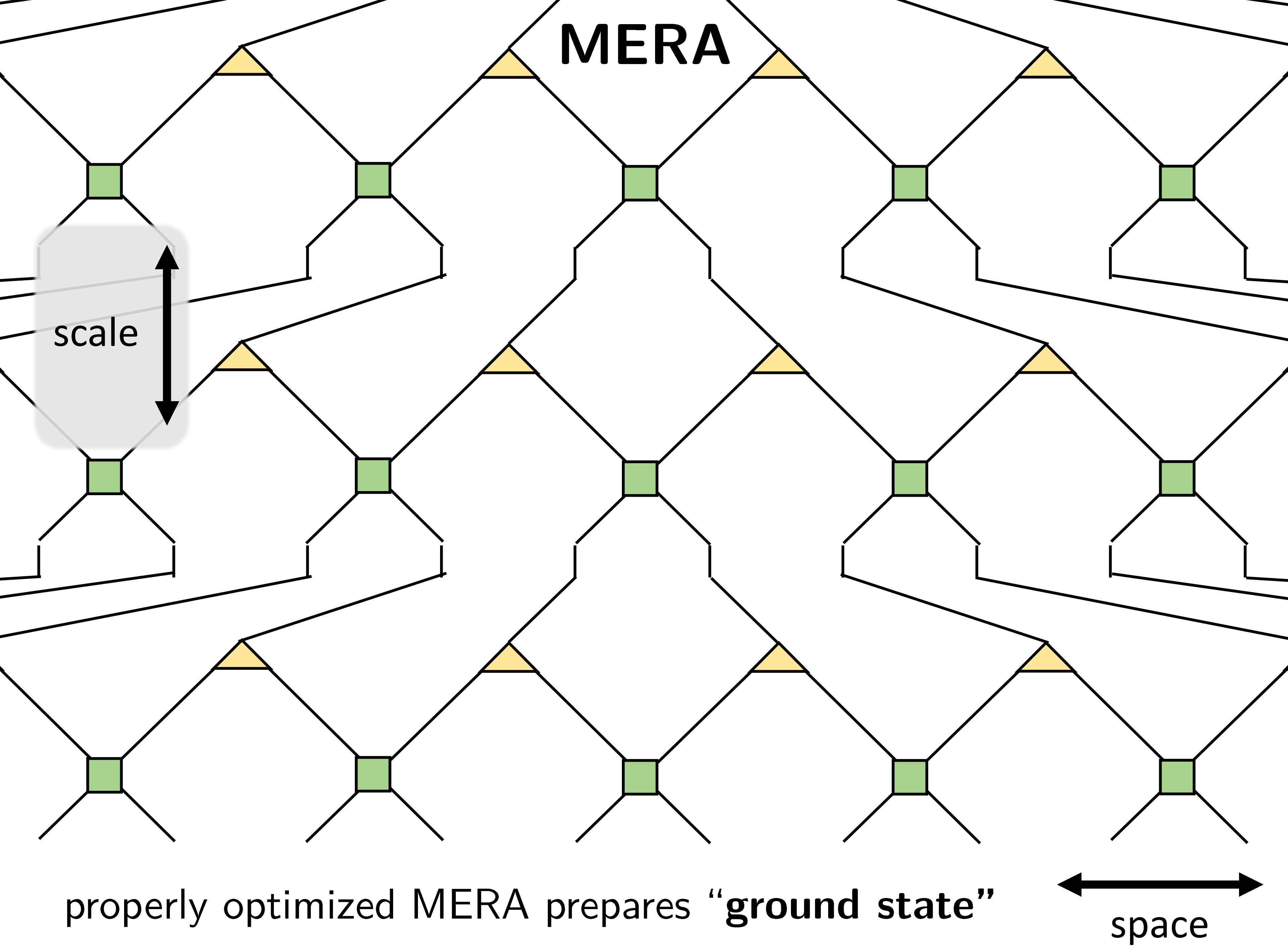


"isometry"



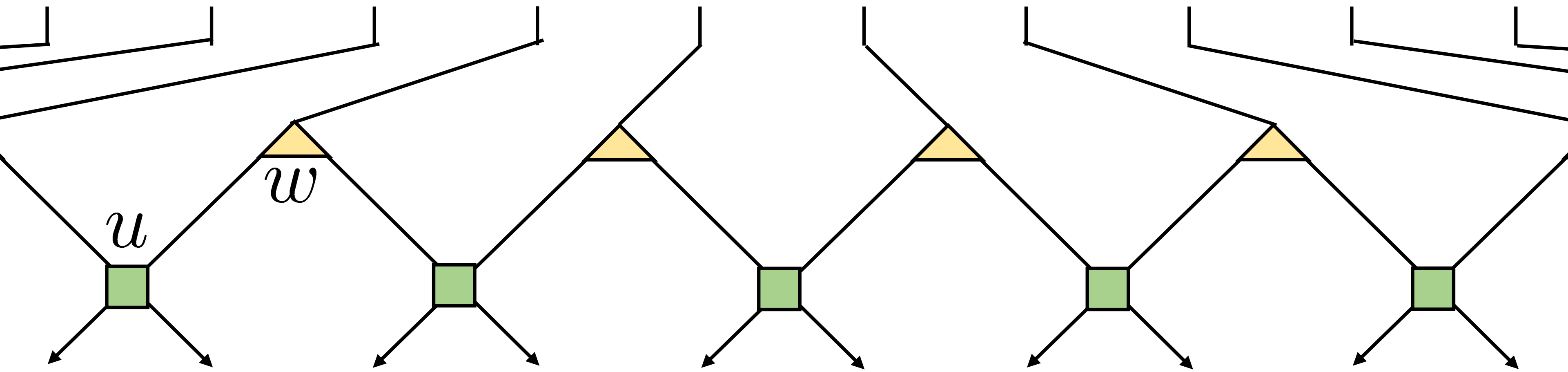
"disentangler"

MERA



properly optimized MERA prepares “**ground state**”

MERA



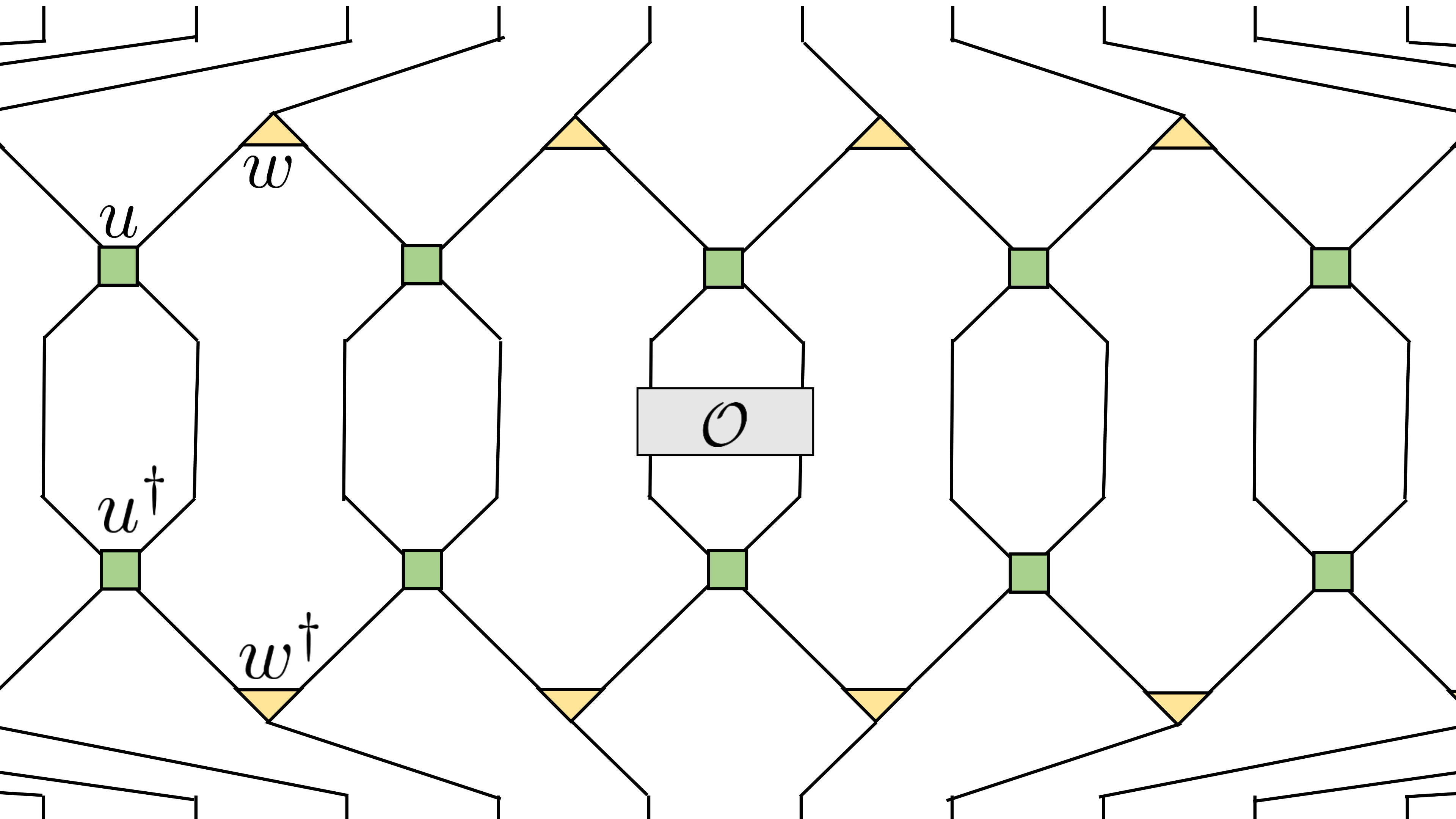
a layer of **MERA** behaves like a **dilation**

$$\sim \exp(-\mathrm{i}sD)$$

G. Vidal 2007, R. Pfeifer et al. 2009

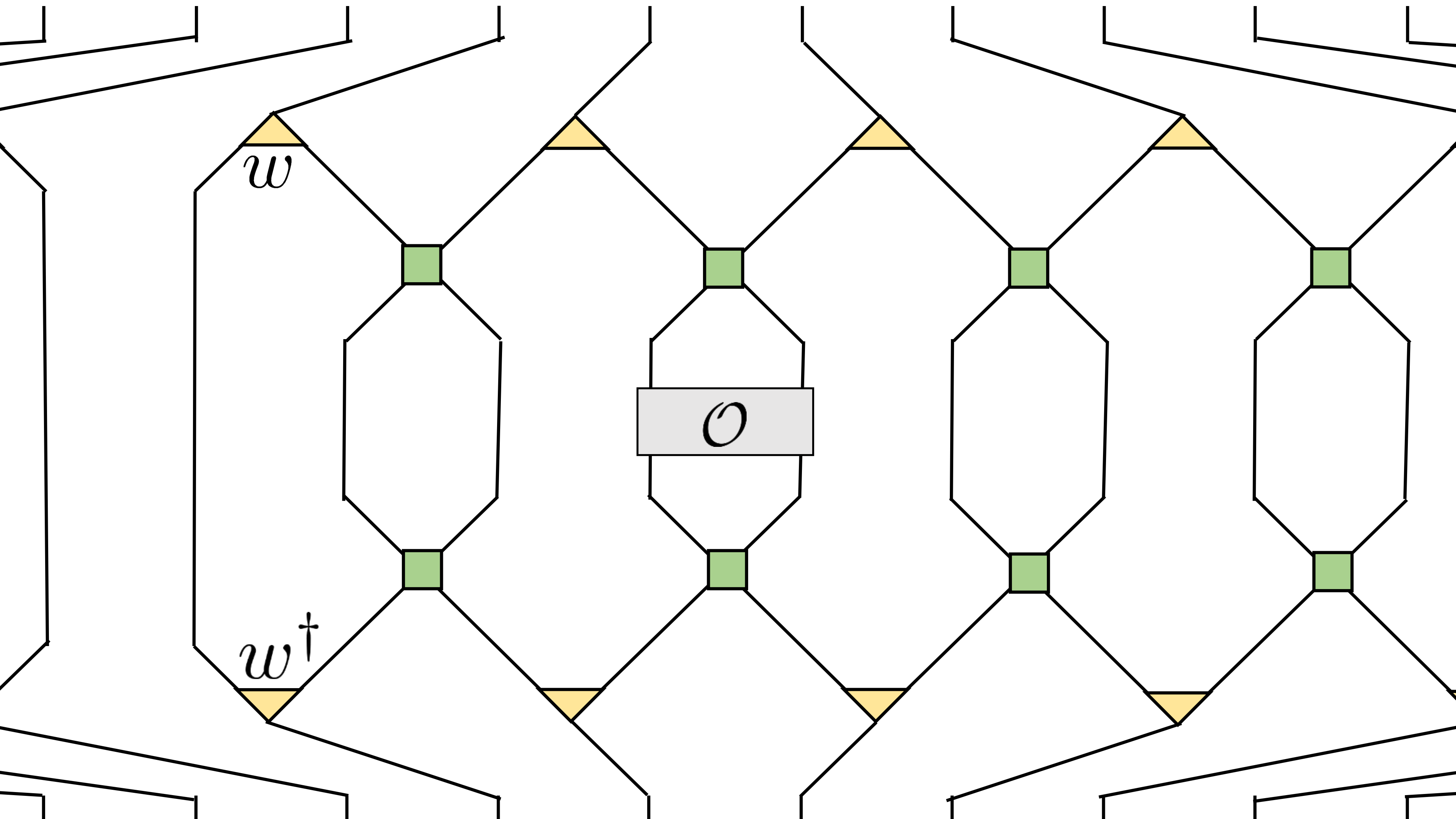
MERA

as a coarse-graining map on operators



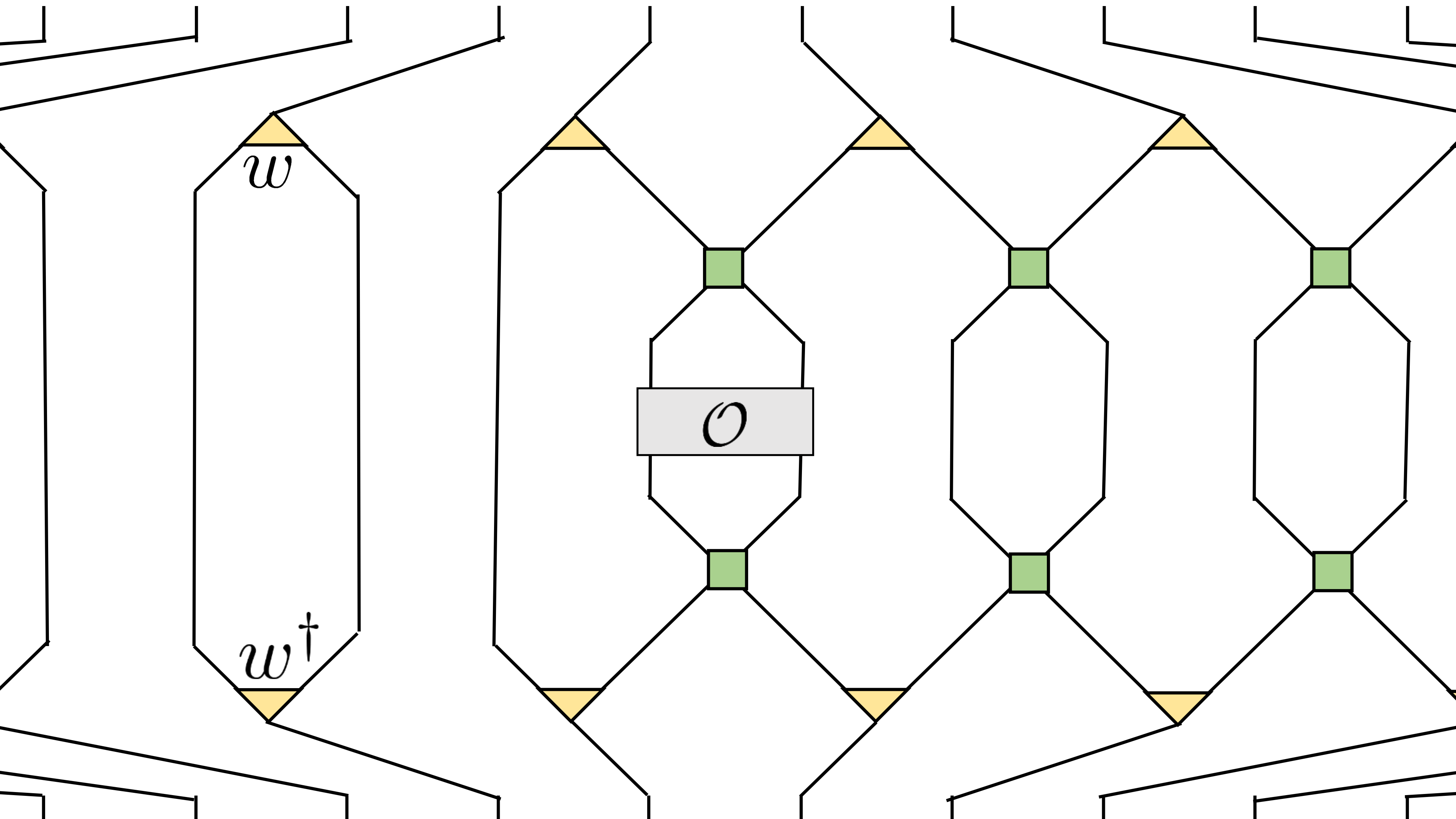
MERA

as a coarse-graining map on operators



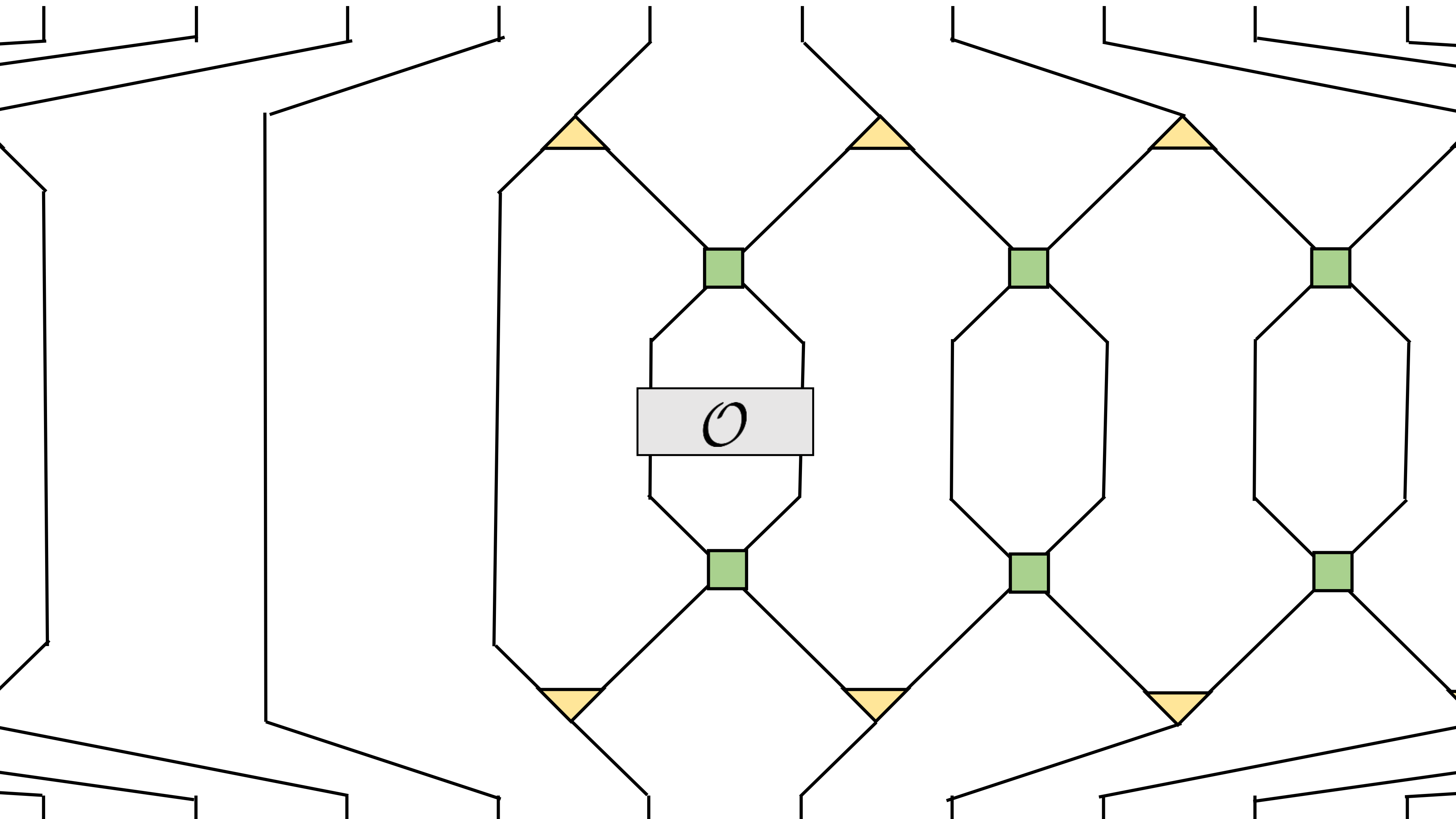
MERA

as a coarse-graining map on operators



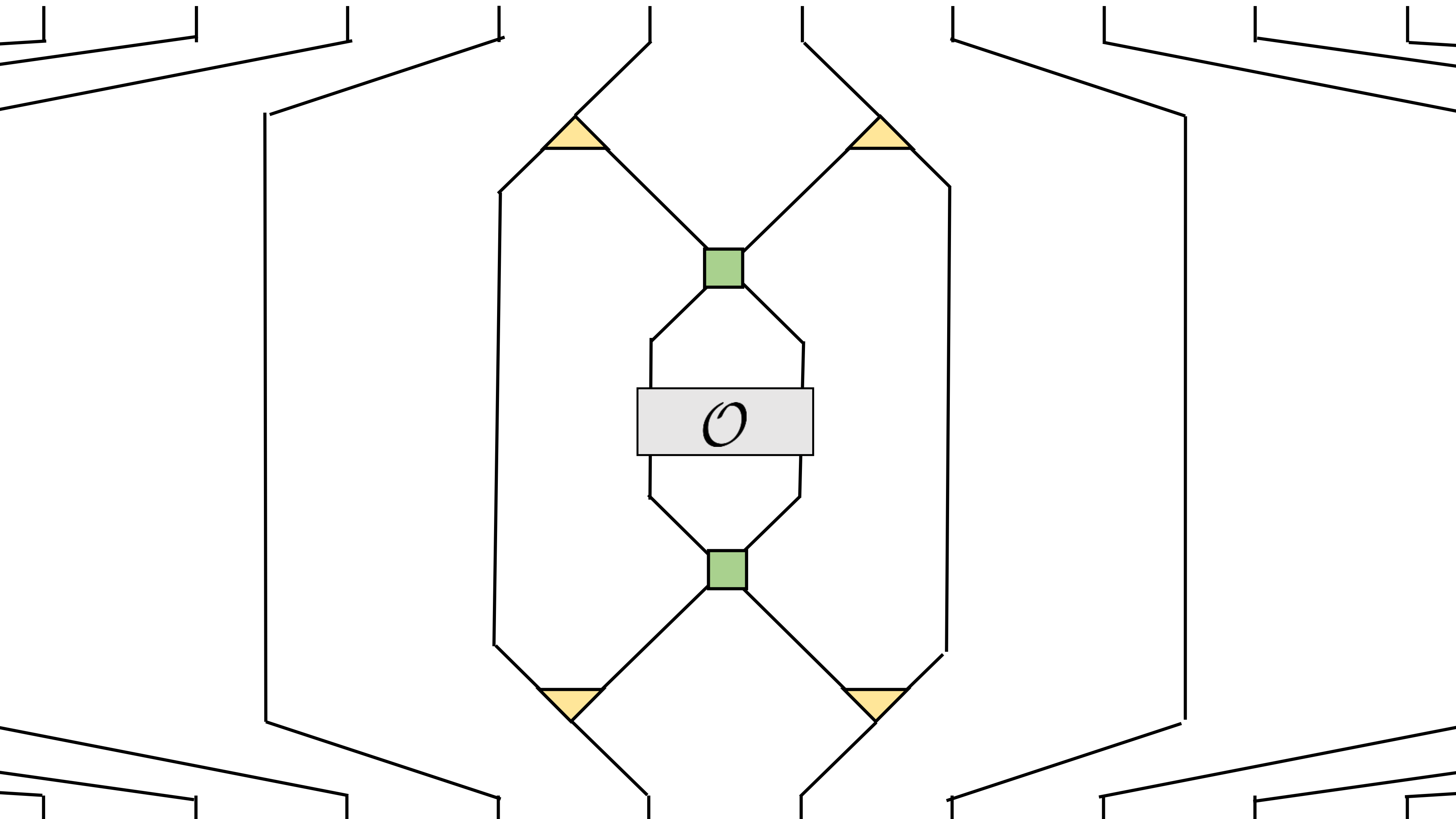
MERA

as a coarse-graining map on operators



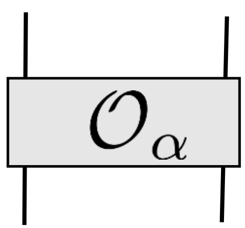
MERA

as a coarse-graining map on operators

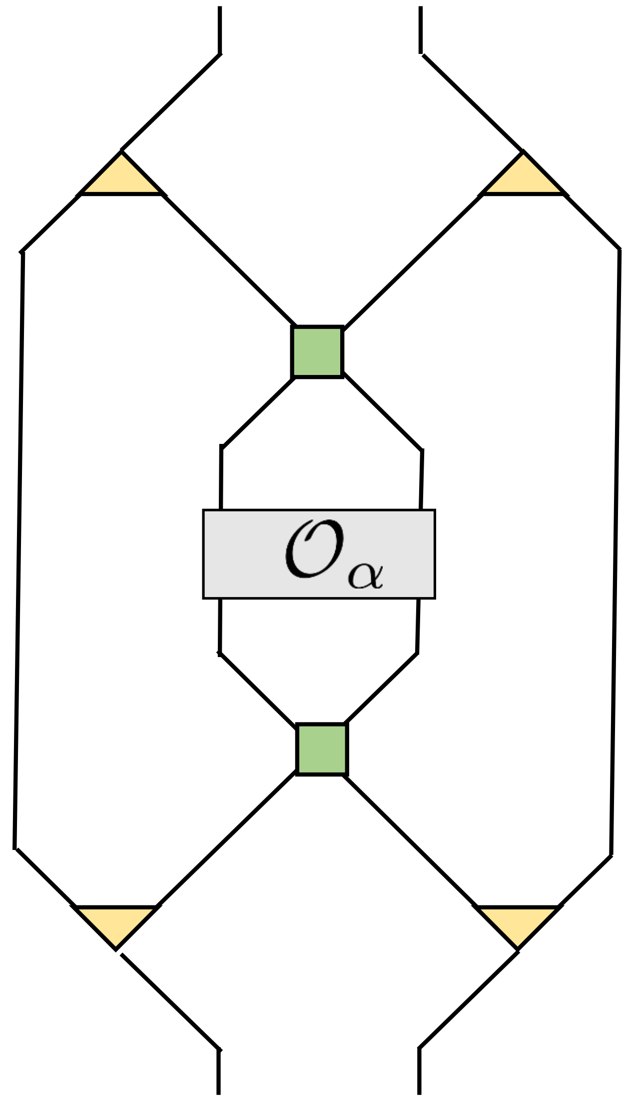


MERA

critical spin chains

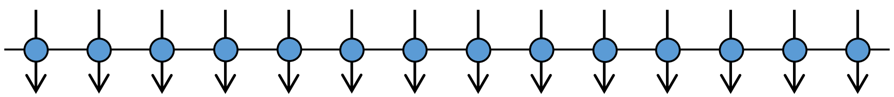


$2^{-\Delta_\alpha} =$



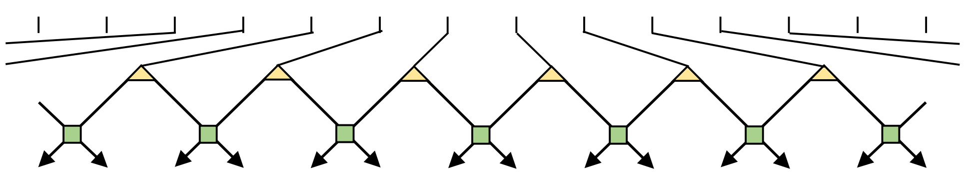
Δ_α : CFT scaling dimensions (approx.)

critical spin chain **1+1D CFT**



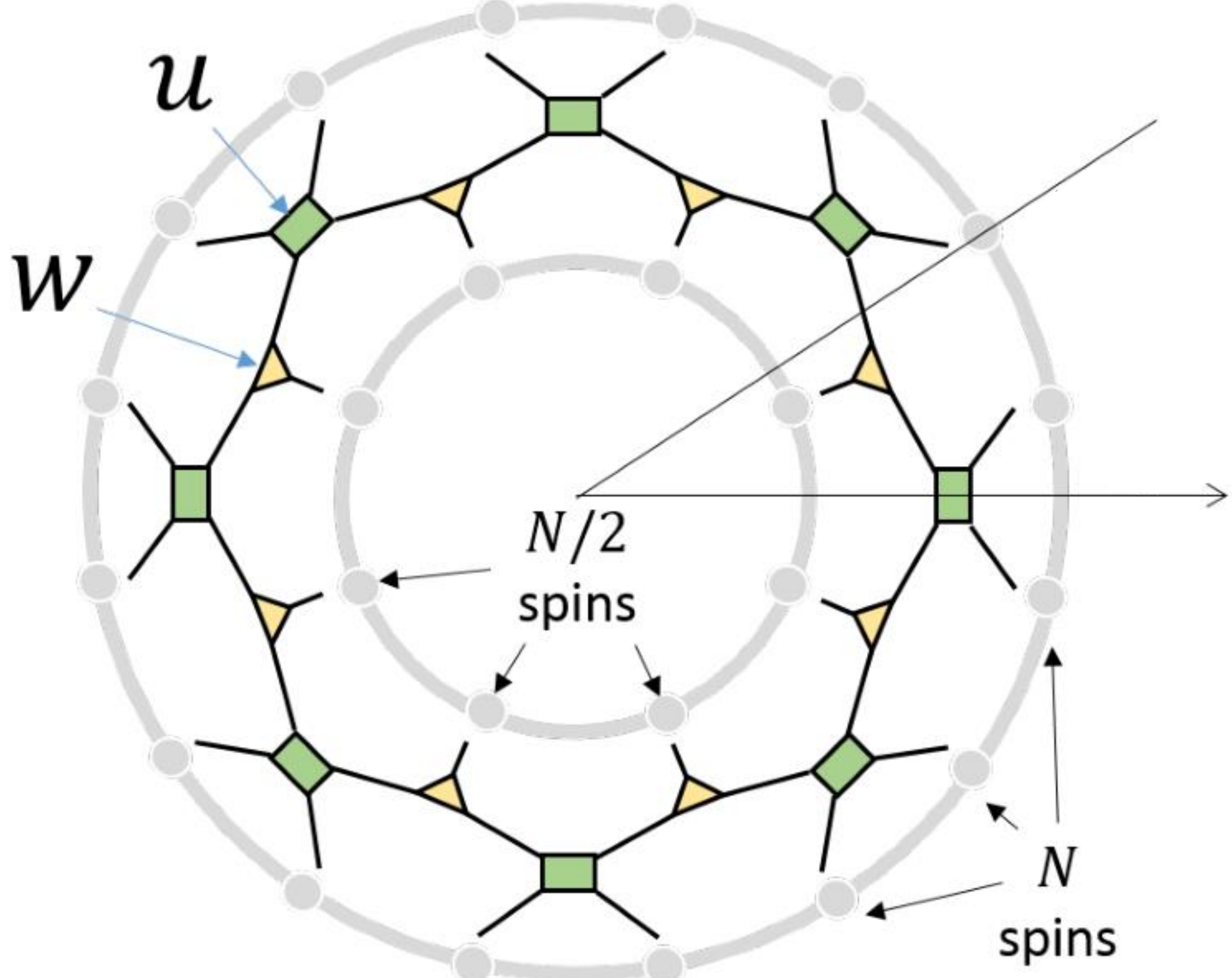
$$\tau \to \tau + \delta$$

(time) translation



$$x \to x + sx$$

dilation



u
w
$N/2$
spins
N
spins

Hamiltonian

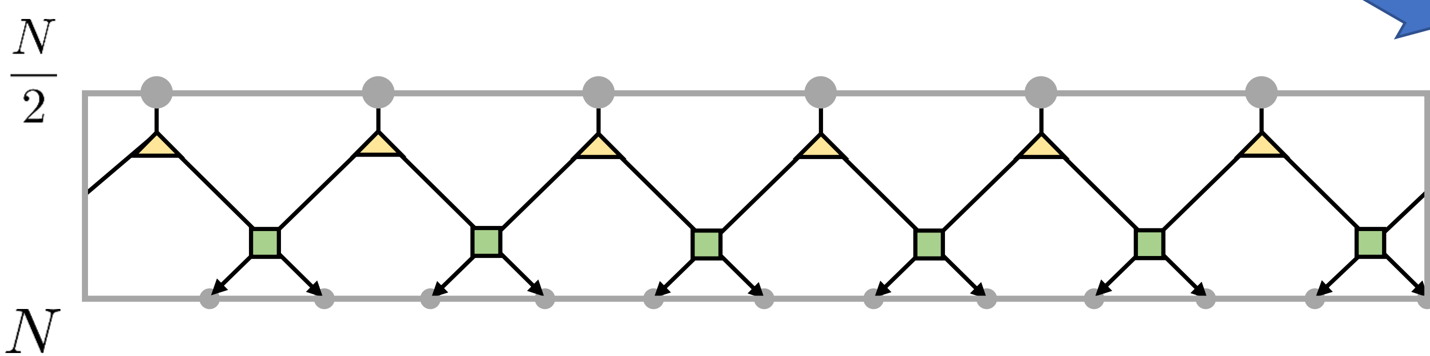
low-energy eigenstates

$$H = \sum_{j=1}^{N/2} h_j \quad \Rightarrow \quad \{|\phi_\alpha\rangle\}^{(N/2)}$$

lattice Virasoro

CFT energy eigenstates

$$\{|\phi_\alpha\rangle\}^{\mathrm{CFT}}$$

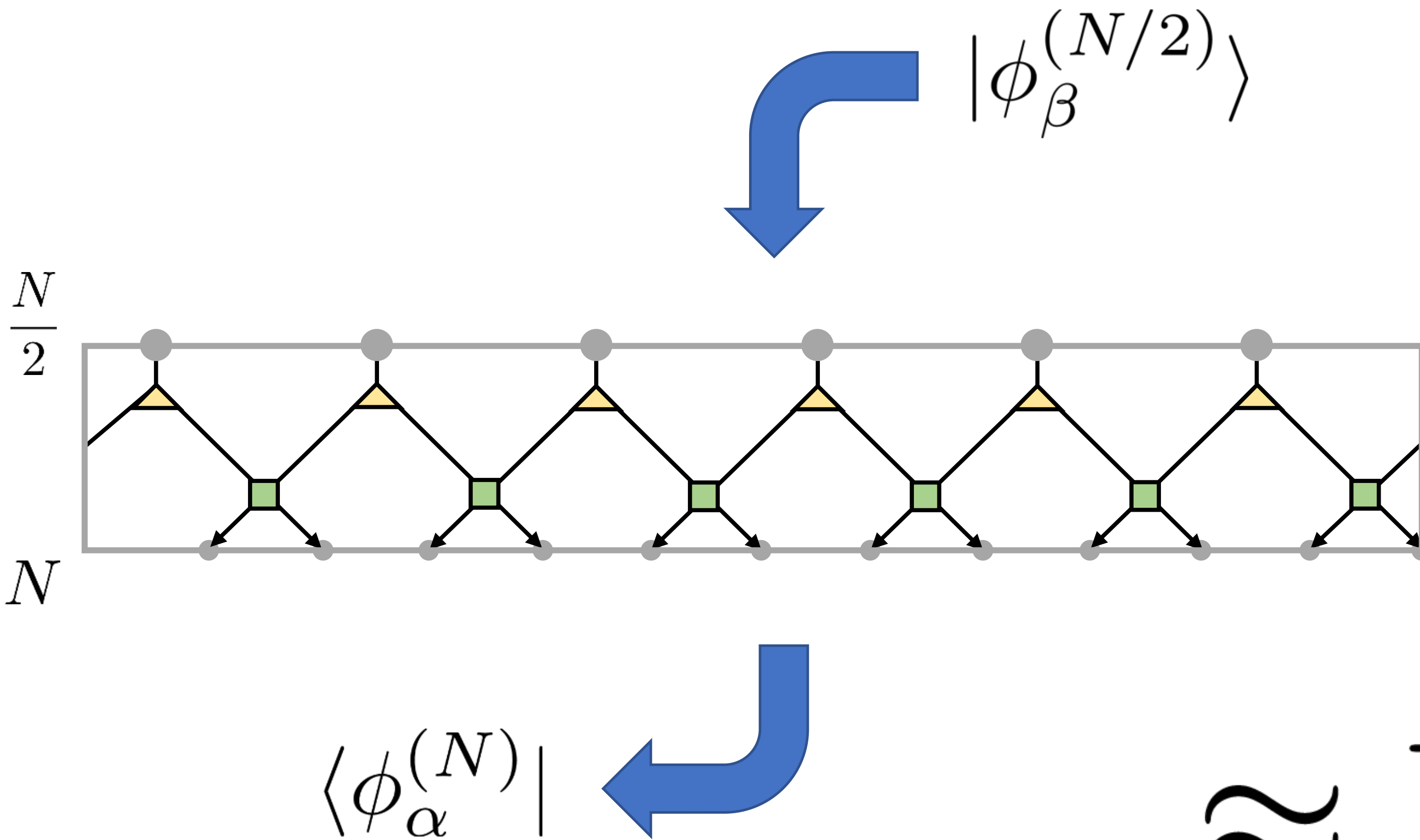


$$H = \sum_{j=1}^{N} h_j \quad \Rightarrow \quad \{|\phi_\alpha\rangle\}^{(N)}$$

lattice Virasoro

(fix relative phases using lattice Virasoro / primary operators)

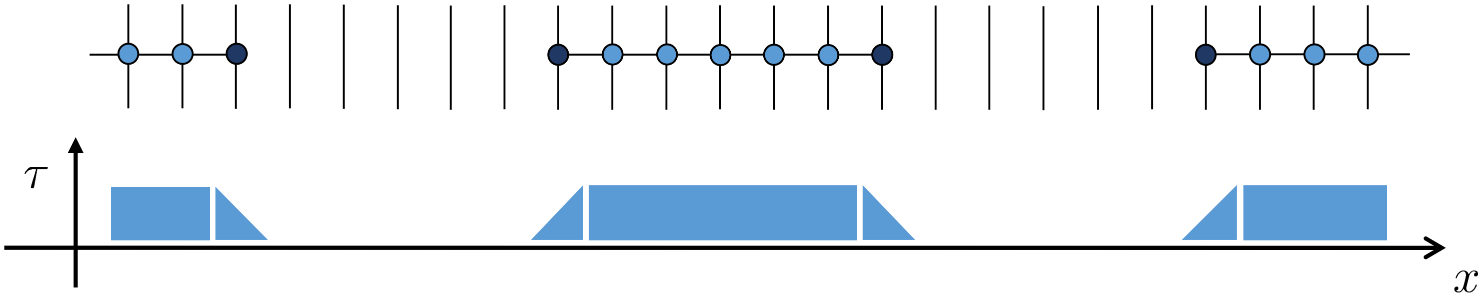
matrix elements in low-energy subspace



$$\approx \mathbb{I}_{\alpha\beta}$$

(critical Ising numerics)

AM & G. Vidal, arXiv: 1812.00529 (2018)

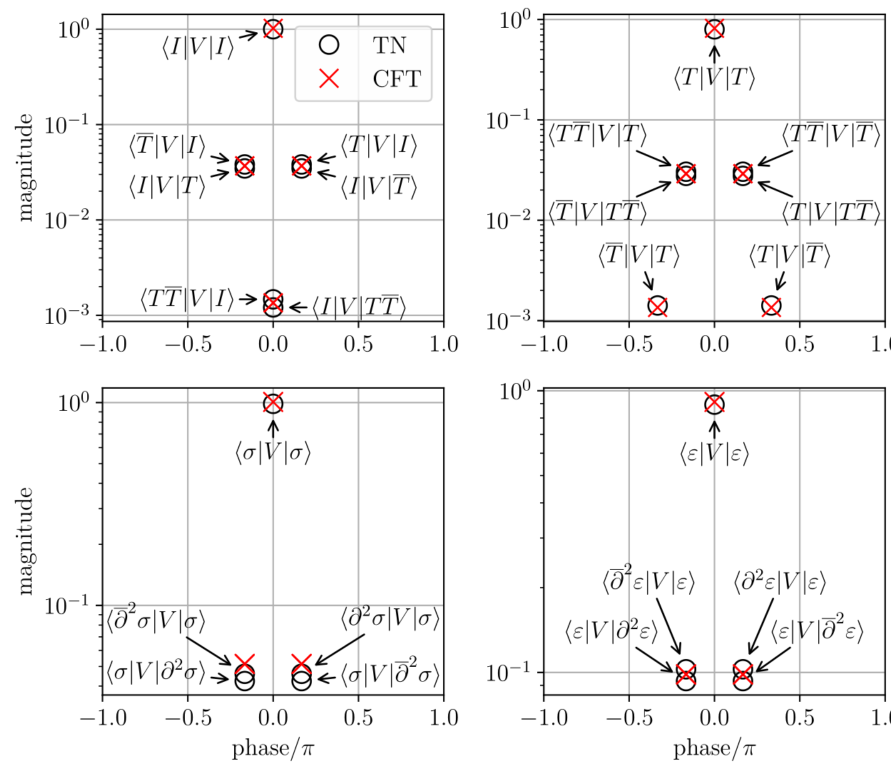


local euclidean evolution (translation)

compare with:

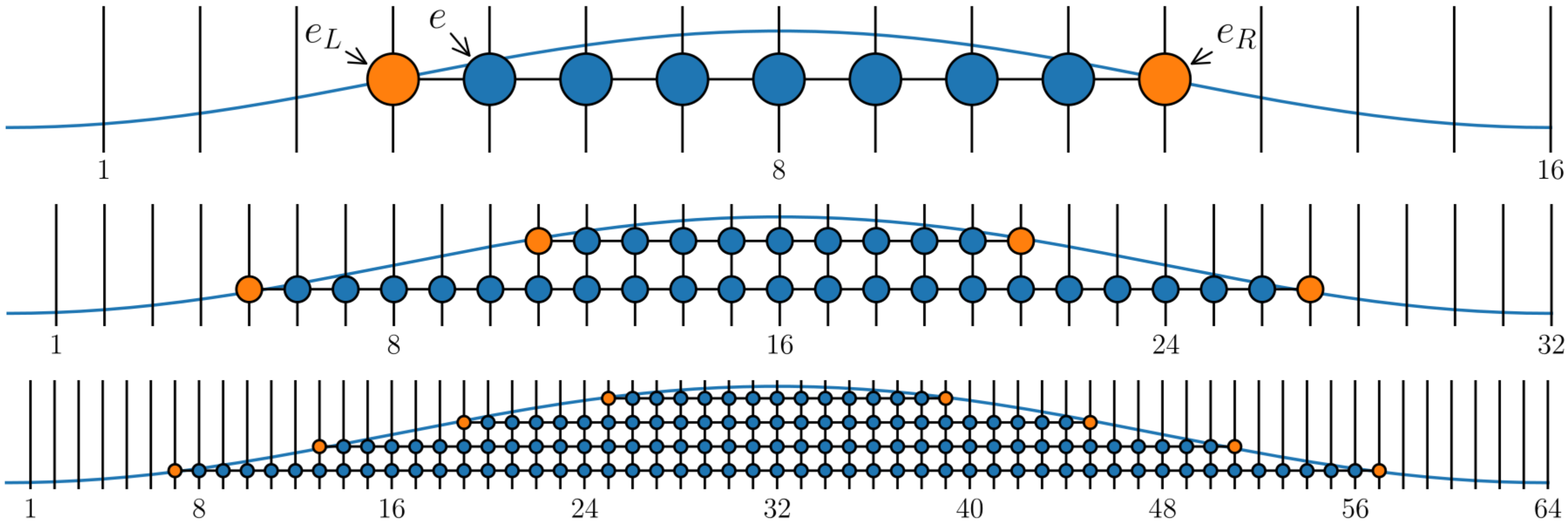
$$\exp\left(-\frac{2\pi}{N}\sum_{n=-N/2}^{N/2} a_n(L_n+\overline{L}_{-n})\right)$$

$$a_n = \frac{1}{4}\,\mathrm{sinc}\left(\frac{n}{4}\right)\,\mathrm{sinc}\left(\frac{n}{N}\right)\left(1+e^{in\pi}\right)$$

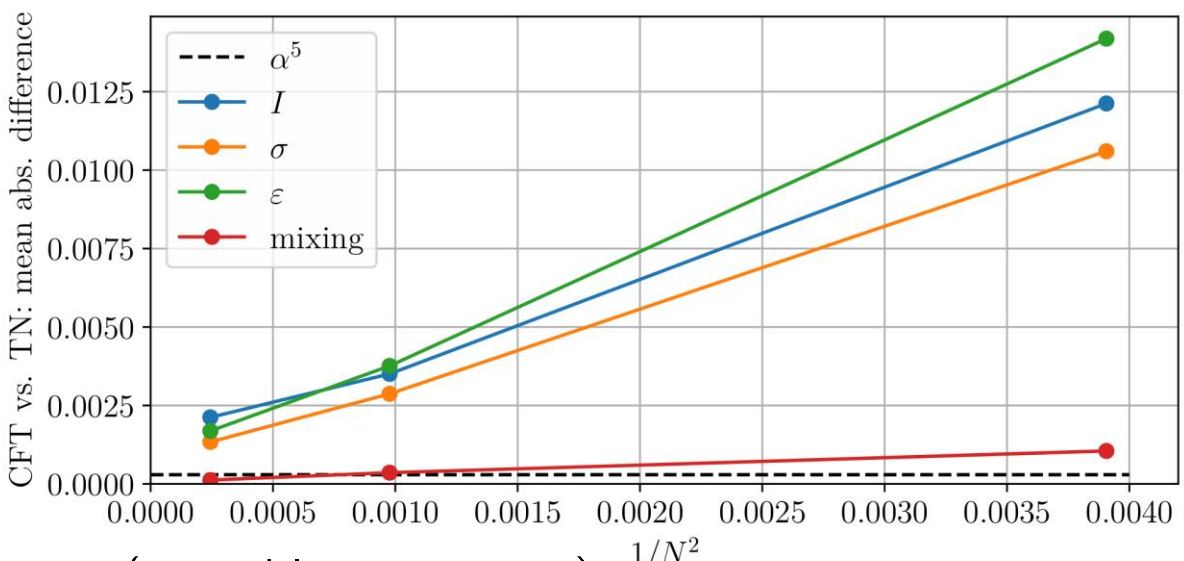


AM & G. Vidal, arXiv: 1805.12524 (2018)

(critical Ising numerics)

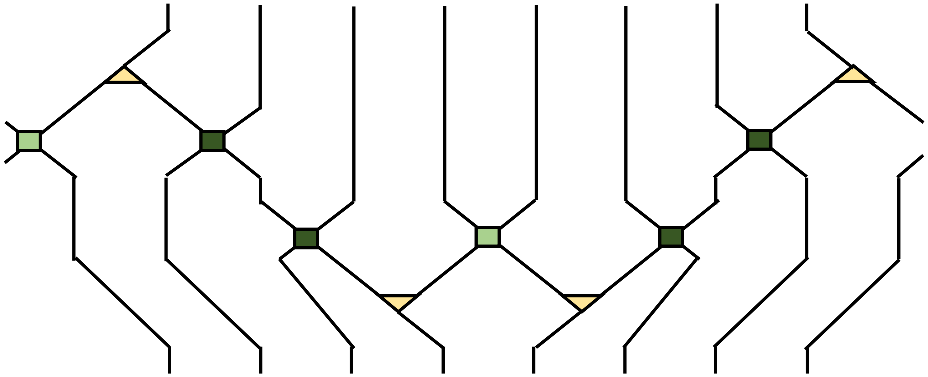


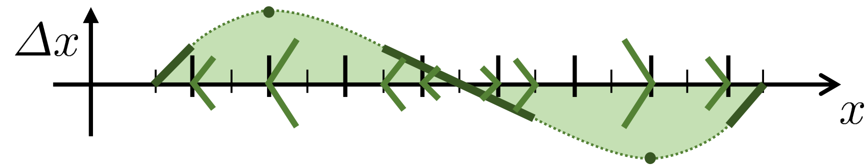
refinement of nonuniform Euclidean evolution



(critical Ising numerics)

AM & G. Vidal, arXiv: 1807.02501 (2018)



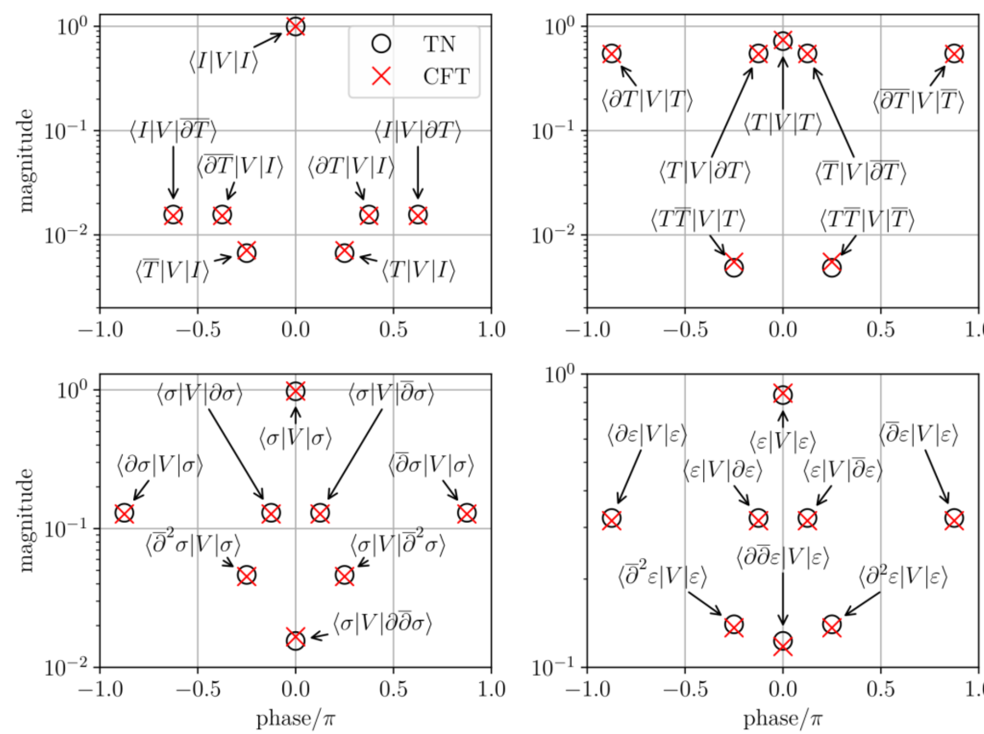


local rescaling of space

compare with:

$$\exp\left(-i\frac{2\pi}{N}\sum_n b_n P_n\right)$$

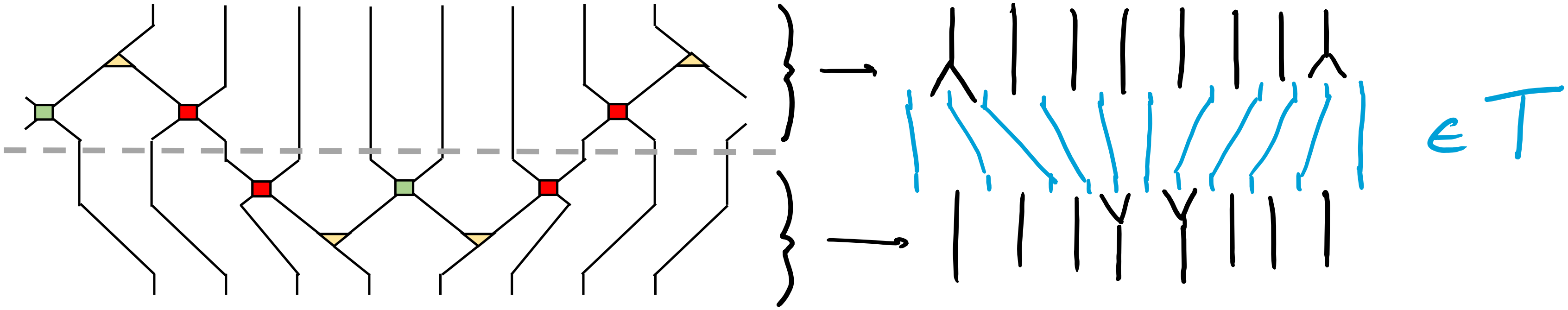
$b_1 = 0.5, \quad b_3 = 0.0275,$
otherwise $b_n = 0$



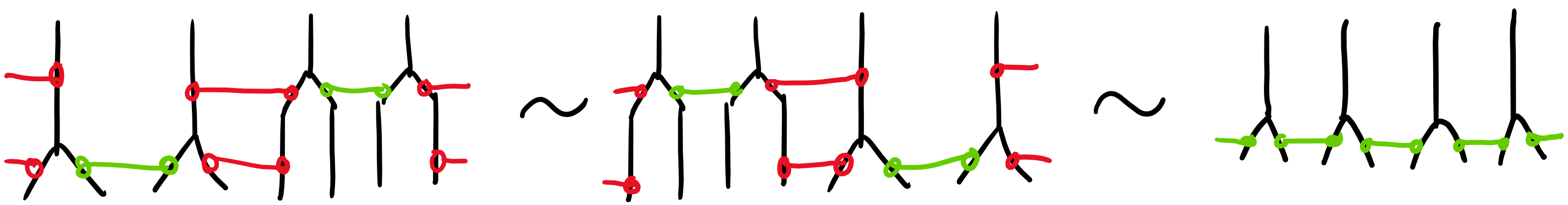
AM & G. Vidal, arXiv: 1805.12524 (2018)

(critical Ising numerics)

connection to Thompson's group(s)



unitary representation?



variational critical Ising MERA fulfills these approximately

(numerically) exact non-tree solutions exist

summary

We show how “**lattice Virasoro generators”** can be used to...

1. ...systematically identify low-energy eigenstates of critical spin chains with energy eigenstates in the CFT,
2. find “lattice primary field operators” that approximately obey the correct operator algebra,
3. identify emergent conformal transformations in Tensor Networks (e.g. MERA) that describe critical systems.

“detailed emergence of conformal symmetry in lattice systems”

extension to richer symmetry

we assumed only conformal symmetry:

$$[L_n^{CFT}, L_m^{CFT}] = (n-m)L_{n+m}^{CFT} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}$$

if there is **more** symmetry (e.g. SUSY, U(1), SU(N)…),
we can extract even more data!