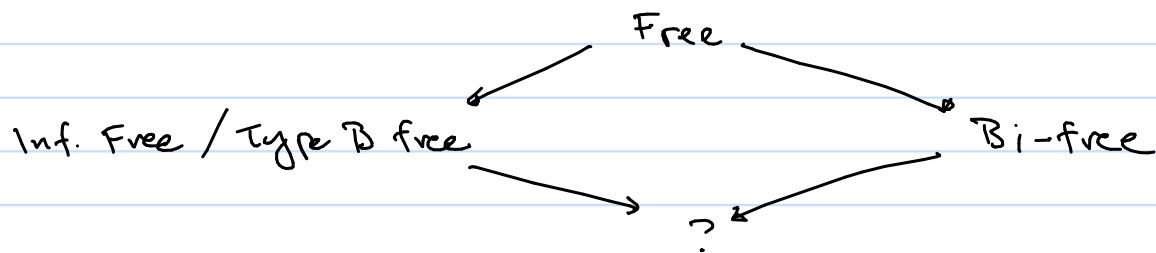


Joint w/ Zhiheng Li, Kyle Meyer, Drew Nguyen, Jennifer Pi, Anna Raichen



Def: A Coxeter system is a pair (G, S) where G is a group, S is a generating set $(e \notin S)$ s.t.

$$G = \langle S \mid R \rangle$$

where R is a set of rels of the form $(s_i s_j)^{m_{ij}} = e$ s.t.
 $m_{ij} = 1 \iff i = j$, and $m_{ij} = m_{ji}$.

Thm (Coxeter)

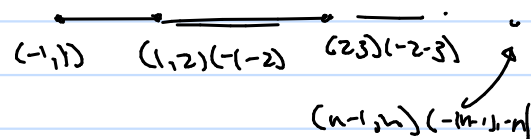
The finite indecomposable Coxeter systems are classified by Coxeter diagrams:



$$A_n \cong S_{n+1}$$



$$B_n \cong S_{\pm n} = \{ \sigma \in S_{\{-n, \dots, n\}} \text{ s.t. } \sigma(-i) = \sigma(i) \}$$



Def: The absolute order on G (coming from (G, S)) is the following

let $T = \{ g s g^{-1} \mid g \in G, s \in S \}$

Say $g = h$ if g is on a geodesic from e to h in the Cayley graph for G w/ generating set T .

A Coxeter element in (G, S) is an elt of the form $\prod_{s \in S} s$ (in any order)

$$N C(G, S) := \{ c, cT \} \text{ for any Coxeter elt. } c \text{ (same for any } c \in C)$$

For A_n or B_n , by sending cycles to blocks, we get a subset of $\mathcal{P}(n)$ or of $\mathcal{P}(\pm n)$.

An inf. n.c.p.s is a triple (A, φ, φ') where A is unital \times -alg, $\varphi: A \rightarrow \mathbb{Q}$ unital, and $\varphi': A \rightarrow \mathbb{Q}$ s.t. $\varphi'(1) = 0$

Say $\varphi_t: A \rightarrow \mathbb{C}$ all states s.t. $t \mapsto \varphi_t(a)$ is diff'ble. $\forall a \in A$, take $\varphi' = \frac{d}{dt} \Big|_{t=0} \varphi_t$

Def/Thm: • $A_1, \dots, A_n \in (A, \varphi, \varphi')$ are inf. free

• If $a_i \in A_{j_i}$, $\varphi(a_i) = 0$, $j_i \neq j_{i+1}$

$$\varphi(a_1 \dots a_n) = 0$$

$$\varphi'(a_1 \dots a_n) = \begin{cases} \varphi(a_1 a_n) \varphi'(a_2 a_{n-1}) \dots \varphi(a_{\frac{n-1}{2}} a_{\frac{n+3}{2}}) \varphi'(a_{\frac{n+1}{2}}) & \text{if } n \text{ odd} \\ & j_1 = j_n, j_2 = j_{n-1}, \dots \\ 0 & \text{otherwise} \end{cases}$$

• The mixed type-B and type-A cumulants vanish

• If $a_i \in A_{j_i}$, $j_i \neq j_{i+1}$, and $\varphi_t = \varphi + t\varphi'$

$$\varphi_t((a_1 - \varphi_t(a_1)) \cdot (a_2 - \varphi_t(a_2)) \dots (a_n - \varphi_t(a_n))) = o(t)$$

Using relation $\text{BNC}(x) = S_2^{-1} \cdot \text{NC}(n)$, can obtain analogous version of above for type-B-bitfree probability.