Can Brazilian butterfly flaps destroy the universe? How fundamental limits on knowledge and computation force Laplace's demon to become a scientist

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Abstract

What questions can ever be answered by a physical entity? In this essay, we examine this question by studying the predictive powers of Laplace's demon. We find that the powers of a physical Laplace's demon are considerably diminished by limits on what is knowable due to quantum mechanics, (classical and quantum) chaos, incompleteness, and physical limits on computation. Because the helplessness of Laplace's demon naively conflicts with the shocking comprehensibility of the world around us, we imagine a new demon intended to model what questions can ever be answered. The approach and capabilities of this demon are found to strongly resemble those of scientists. We touch on some consequences of our arguments, including for free will and predicting versus understanding.

As the sun sets on the grand vineyards of Brazil's *Vale dos Vinhedos*, bat-like spectres descend upon unguarded fruit. Among the interlopers is *Ascalapha odorata*—the black witch moth—which straightens out its proboscis tip to pierce the pulp of damaged grapes.[1] With a wingspan of up to seven inches, it braves long seasonal odysseys, sometimes even over open water; specimens have been found as far north as Canada's Hudson Bay,[2] and in locales as strange as oil platforms off the Louisiana coast.

Mexicans, and their Aztec predecessors, call it *mariposa de la muerte*: the butterfly of death. Its appearance has been linked to impending death and witchcraft. Perhaps, one legend goes, you will lose your hair if one flies over your head.[3]

A flap of the black witch moth's wings is not particularly powerful. Given that hawkmoth flaps produce on the order of tens of millinewtons worth of force,[4] our butterfly of death's flaps might produce on the order of hundreds of millinewtons at most—enough to lift itself up, but not even as strong as the force we might feel due to a light breeze.

It is difficult to imagine that one of these flaps has serious consequences for anything except the moth itself. But the Reeh-Schlieder theorem,[5] a remarkable and counterintuitive result from quantum field theory, suggests the following scenario may be plausible. One day a forgetful physicist, while on a tour of the orchard, mistakenly leaves a box on the ground. Suppose this box contains a reasonably well-isolated particle in a superposition of two possible states, and that opening the box constitutes a measurement of its state. On a windless evening, a black witch moth flies by, and one of its flaps happens to knock the lid off of the box. This local measurement of the state of the particle perturbs the quantum state of the universe in just such a way that the entire universe is annihilated, and there remains nothing but vacuum.

Of course such a thing cannot happen. But how do we *know?*

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If anyone knew the answer, it would be *Laplace's demon*, the hypothetical vast intelligence first described by the eminent scholar Pierre-Simon Laplace in an 1814 essay.[6] If the universe really *did* consist of nothing but a large number of particles obeying Newton's laws of motion, Laplace reasoned, then an intellect that (i) knew the positions and momenta of all particles at one time and (ii) could accurately solve Newton's equations of motion given that data, could know any future or past state of the universe with absolute certainty. In the Newtonian-Laplacian paradigm, there is no room in the universe for free will, teleology, or anything else; there exist nothing but particles obeying differential equations for all eternity.

But the trauma inflicted by twentieth century revelations in physics, mathematics, and computer science (among others) seems to obliterate this view, muddying the waters somewhat. Let us consider a few such advances, and contemplate their effect on the predictive powers of Laplace's demon one at a time.

Suppose for the moment that Newtonian physics offers the correct description of the universe, and that Laplace's demon knows the current positions and momenta of all particles in the universe to only *finite* precision instead of infinite precision; in other words, suppose that there is a small amount of error in its knowledge of the current state of the universe. How much does this initial inaccuracy affect its ability to predict the future, or retrodict the past? It turns out that most deterministic dynamical systems—like collections of interacting particles obeying Newton's laws—exhibit *chaos*, a feature which in practice means that predictions and retrodictions are effectively impossible beyond a certain characteristic timescale. In particular, beyond the timescale set by the deterministic universe's (maximal) Lyapunov exponent, Laplace's demon cannot possibly be confident in its projections. It can see only the recent past and near future clearly.

Edward Lorenz, the mathematician and meteorologist considered the father of modern chaos theory, was among the first to appreciate the implications of this mathematical fact for weather forecasting. Because the models of atmospheric dynamics used to make forecasts are chaotic dynamical systems, it is thought that even in principle (i.e. given the best possible weather monitoring instruments and most appropriate model of the atmosphere) predicting the weather beyond a few weeks into the future is impossible.[7]

Of course, the universe is not classical, but quantum mechanical. Supposing quantum mechanics offers the correct description of the universe, and assuming for our philosophical convenience that Everett's austere[8–10] 'many-worlds' interpretation is valid, the fundamental ontology of the universe includes only the wave function, which evolves smoothly in time according to the Schrödinger equation. Although measurement outcomes are probabilistic in quantum mechanics, from the point of view of the wave function, Laplace's clockwork universe remains intact: if Laplace's demon knows the wave function of the universe at one time (and hence has complete information about the state of the universe at that time), and it can accurately solve the Schrödinger equation, then it can obtain complete information about the wave function of the universe at any other time.

Interestingly, the universe being quantum mechanical naively offers an antidote to the problem of deterministic chaos. Because the Schrödinger equation is linear, imperfect knowledge of the current state of the universe (i.e. the wave function) yields predictions and retrodictions with the same amount of error[a].

The situation becomes more interesting if we assume Laplace's demon is itself part of the universe. If the demon solves the Schrödinger equation to predict the state of the universe at some future time, it cannot possibly *verify* its prediction as correct without making a measurement[b], the act of which disturbs the wave function in such a way that its prediction of the wave function is no longer accurate.

The predictive powers of Laplace's demon deteriorate significantly more if we assume it faces the same epistemological constraints we do—i.e. that it cannot access the wave function, which is unobservable, and instead must make measurements of observable quantities to interrogate the state of the universe. Though the classical paradigm for chaos (particle trajectories initially close in phase space separating exponentially quickly) does not quite make sense in a quantum mechanical context, there are many features of quantum systems that have an analogous impact on one's ability to make predictions, some of which are labeled "quantum chaos".

For example, suppose Laplace's demon measures some observable. How much does *that* measurement affect the results of *future* measurements of *other* observables? A flurry of recent theoretical studies[11–13] have established out-of-time-ordered correlators (OTOCs) as one tool for answering this question, and as a useful proxy for chaos in quantum mechanical settings. The growth characteristics of the OTOCs for many of the model systems investi-

[a] See endnote A.

[b] This fact was pointed out to me by Joshua Deutsch.

gated so far suggest that the influence of the first measurement on the outcome of the second can in general be quite large.

Even if chaos-related obstacles to making predictions could be overcome, a flesh and blood Laplace's demon still faces serious problems due to fundamental constraints on computation[c]. Lloyd used physical heuristics to estimate the largest possible memory capacity and fastest possible information processing rate achievable by any finite size physical system;[15] the salient consequence of his arguments for us is that both are *finite*. This essentially means that Laplace's demon requires time to think before it can make a prediction, and that it can only take so much information into account at one time. If predicting the state of the universe at some future time takes longer than just *waiting* for that future time, that prediction is clearly worthless.

Given the humongous number of logical operations required to simulate the time evolution of the universe since the big bang (roughly 10^{120} operations on 10^{120} bits, according to Lloyd[16]), and given that Laplace's demon is constrained by the aforementioned limits on finite size information processors, it lacks the computational power to see into the distant past and future, even in principle.

Does Laplace's demon have any alternative to brute-force computation? One possibility is that it takes a mathematical approach, and assumes the universe is completely characterized by some finite list of axioms. However, even if this assumption is valid, it faces at least two serious obstacles: this reframing of the problem does not save time over brute-force computation in almost all relevant cases (e.g. to prove that the future state of the universe will be some particular state, in general one must just do the computation), and not all mathematical truths about a system are derivable from a finite list of axioms characterizing that system. The latter fact is roughly the content of Gödel's first incompleteness theorem.[17]

The net effect of all of this seems to be that Laplace's demon cannot use imprecise knowledge of the present to determine the past and future, and that it cannot even carry out the calculations necessary to show that this is so.

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Because even Laplace's demon cannot fully foresee the consequences of a fundamental description of the universe, it seems the project of reductionist physics—in one view, of understanding and inferring the future and past of the universe in terms of its fundamental building blocks—is doomed to fail. But to a non-physicist, this may seem strange. We can predict (quite reliably) that the Sun will rise in the morning. We know how a ball will fall when we drop it. Cosmologists have managed to develop a compelling account of (almost all of) the first three minutes of the universe,[18] an era completely alien to our everyday experience. Why is a world mired in so many forms of fundamental uncertainty so shockingly predictable?

This is a deep and open philosophical question. Some 'trivial' answers might invoke an anthropic argument (e.g. the world must be somewhat predictable in order for us to

[c] Surprisingly, this may be more important here than limitations on obtaining knowledge due to the computational complexity-related constraints discussed by Aaronson.[14] For example, if the Hilbert space of the universe has dimension N, predicting the future quantum state of the universe given complete knowledge of its current state amounts to exponentiating a large but finite dimensional diagonalizable matrix. Because this can be done in $\mathcal{O}(N^3)$ time, the problem is the hugeness of N, not the inefficiency of available algorithms.

come into existence, survive, and be able to ask the question) or selection effect (e.g. the world is mostly incomprehensible, but our mental machinery is such that we focus on the comprehensible parts). But the former is unsatisfying, and the latter just sidesteps the issue by arguing semantics. The fact is, we know enough about the universe to journey to the Moon, treat and cure serious diseases, communicate information between distant locales almost instantaneously, and have a working framework for understanding literally everything. Even non-scientists can trust in the very many regularities of everyday life, like 'rocks are solid' and 'objects do not move unless something pushes them'. The laws of physics did not have to be this way.

Three factors contributing to the world's intelligibility are core principles of quantum field theory: locality, causality, and unitarity. Locality roughly says that distant experiments do not affect each other's outcomes[d]. Causality is closely related to locality in relativistic field theories, and seems to help prevent undesirable possibilities like faster than light communication and time travel. In the words of Schwartz,[19] unitarity roughly means that "something cannot be created from nothing, nor can something just disappear." While these postulates may or may not be true in the ultimate theory of everything (locality, for example, may be violated in quantum gravity[20]), they are certainly features of the world at all currently accessible energy scales.

The fact that the universe is well described by *renormalizable* quantum field theories means that many facts about the dynamics of the world at low energies (e.g. the behavior of rocks, balls, and people) do not sensitively depend upon the details of how the world works at high energies (e.g. quark-gluon dynamics). This is one of the reasons why there is some hope of understanding rocks without knowing a full theory of quantum gravity.

The *emergence*[21,22] of qualitatively new features from the underlying physical laws—like people, tables, and chairs—allows us to sensibly talk about the world in terms of them[e], and affords us luxuries like being able to distinguish the past from the future. The related idea of *thermalization*,[24] which refers to many physical systems tending to relax towards some kind of equilibrium, allows us to come to meaningful physical conclusions in otherwise extremely complicated situations. For example, we can successfully characterize many gases at equilibrium in terms of thermodynamic quantities like temperature and pressure, even though describing them in terms of the chaotic motion of their constituent molecules would be hopelessly difficult.

Finally, we should acknowledge that the way our brains work is partly responsible for allowing us to make sense of the world. Neuroscience research suggests that perception (receiving and interpreting sensory input, like sights and sounds) is not a passive process; instead, the brain is thought to constantly create probabilistic models of the world around it, and to try to anticipate what kind of sensory input it expects to receive next.[25] In response to new sensory input, it adjusts its models to reduce the extent to which the new data disagrees with its prediction. If we were not equipped with a way to rapidly adjust and adapt the way we think about the world in response to new sensory input, scientific inquiry—to say nothing of basic survival—would be impossible.

[d] I say *roughly* because of results like the Reeh-Schlieder theorem.

[e] The idea that there is just one physical world, but many sensible ways of talking about it, is Carroll's Poetic Naturalism.[23]

Whatever the reason that the world is comprehensible, that it is so remains an empirical fact. This fact permits us to think in terms of cause and effect; perform careful experiments on isolated systems, without worrying about the rest of the universe; construct phenomenological models of the world that are not derived from its most fundamental description, and that do not depend on it in any important way; introduce language for talking about the world that does not appear in the fundamental ontology; and change our thinking when confronted with new and possibly conflicting information. In short, the comprehensibility of the world allows us to do science to understand it.

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With Laplace's demon as it is usually imagined effectively incapacitated, one might wonder: is there some way to leverage the comprehensibility of the world to partially replenish its powers? Let us introduce a variant of the usual scenario that will help us grapple with this question.

Imagine that you stand before an entity with staggering intelligence and computational power. You can ask it a question about anything you like. You can also specify stipulations on your answer; for example, if the answer to your question is a number, you can specify the desired accuracy. What sort of powers should we endow this demon with, so that it can successfully answer as many questions as possible?

As long as this demon is itself part of the universe, it faces the same physical limitations we discussed earlier in the context of Laplace's demon: namely, it can only learn about the state of the universe by measuring observables, and its computational power and memory capacity are limited. Moreover, as a physical entity, its measurements have only finite precision, and the act of measurement must take a nonzero amount of time. It is localized in space, and requires a large amount of time to measure distant observables (e.g. it must take over four years to directly measure the position of a particle in Alpha Centauri, if it begins on Earth).

It cannot possibly know everything about the universe, according to our prior arguments. But it can learn approximate truths about limited parts of the universe by constructing models of it—imperfect representations of the universe that suffice for answering specific questions to within a desired accuracy, and whose consequences can be computed within a reasonable amount of time given limited computational power. Hence, we should endow our demon with the ability to construct models.

For it to be able to construct models efficiently, it requires several things: prior knowledge (even if one takes the position that this is not required, which is debatable, it can certainly help), the ability to obtain new knowledge, a language with which to express quantitative relationships, the ability to deduce and compute consequences of a model, and a way to check whether its model or answer is correct. In other words, it should know as much as possible about math, science, and computation; be able to conduct experiments throughout the model building process; and have some way (statistical or otherwise) of estimating error. It should also be comfortable thinking, talking, and expressing conclusions in terms of emergent concepts; this way, it can understand and answer questions involving people or chairs much more easily.

We will equip it with the following:

1. an exhaustive current knowledge of science, math, and computation, along with the ability to apply that knowledge (e.g. by carrying out abstract mathematical calculations or using specific algorithms);
2. the ability to imagine and simulate scenarios in its head (i.e. do *in silico* experiments);
3. the ability to interrogate or measure observable aspects of the universe (i.e. do real experiments);
4. the ability to parse coarse-grained language and model the world at different levels of resolution;
5. a scheme for updating its degree of belief in various propositions when provided new information.

Because a human is asking the question, we must also enforce the following constraint: the question must be asked and answered, in a format intelligible to the human, before the human dies. While this point may seem trivial, it is some amusing consequences. For example, even if the demon *could* determine the complete wave function of the universe at some time, it could not *report* all of that information in an answer within the lifetime of the questioner—without compressing it somehow, anyway.

Just as Laplace's demon is a model for what can in principle be known about a classical universe, this demon is a model for what kind of questions—about the universe or otherwise—can in principle ever be answered.

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What kind of questions might our demon be able to answer? It is clear that it cannot answer *every* possible question. If you ask it a nonsense question (for example,[26] "What is the marital status of the number five?"), it cannot answer. If you ask it a mathematical question which is provably undecidable (for example, "Given Zermelo–Fraenkel set theory plus the axiom of choice (ZFC), is the continuum hypothesis true or false?"), it cannot answer, unless its answer is that the question is provably undecidable. If you ask it to compute an uncomputable number (like a value of the busy beaver function, $\Sigma(n)$, for sufficiently large n), it cannot answer, unless its answer is that the number is provably uncomputable. If you ask it a physical question requiring an unreasonable measurement ("How many protons are in Galaxy X, two hundred light years away?"), it cannot answer exactly, but it may be able to provide an approximate answer (e.g. by making reasonable Fermi estimates). If you ask it a subjective question ("Was Newton a greater scientist than Einstein?"), then it cannot answer, except subjectively. If your question is not amenable to deduction or experiment ("What am I thinking right now?"), it cannot answer.

There are some questions that our demon cannot answer for slightly less trivial reasons. For example, asking it to exactly compute the value of the busy beaver function $\Sigma(n)$ for an n which is large, but not so large that the question is mathematically undecidable, may take longer than the lifetime of the universe—and certainly longer than the lifetime of the questioner.

On the other hand, there are clearly very many questions that it *can* answer. It can prove almost all provable mathematical statements (with the ones it cannot prove having proofs that take more than a human lifetime to search for or write down), a list which almost certainly includes famous open problems like the Riemann hypothesis and the P versus NP problem. It can answer any scientific question that can be resolved via a finite number of reasonable experiments ("Is the existence of a fourth, sterile neutrino[27] disallowed by data from past and present short baseline nuclear reactor experiments?"). It can answer any theoretical scientific question whose answer can be mathematically proved or computed within a reasonable amount of time ("What are the critical exponents of the three-dimensional Ising model?"). It can answer nonscientific questions amenable the same kind of approach ("Is the law of supply and demand statistically consistent with the behavior of United States consumers in 2020?"). Although many of our example questions need to be clarified before they can properly be asked and answered, there is in principle nothing preventing our demon from answering them.

In fact, this demon can solve all of the questions that *we* will eventually be able to solve. Its process is not so different from ours, with its main advantage being that its computational power is the largest possible. It collects information via real and *in silico* experiments, organizes and synthesizes that information using models, and evaluates the validity of its conclusions using something like Bayesian statistics. It also, by necessity, talks and thinks about the world using emergent concepts.

Perhaps it is comforting that this demon is so much like us. We may not have the power to see the past or future with perfect clarity, but what we *can* do is the next best thing.

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In some sense, claims about what this demon can do are claims about the limits of what we will ever be able to do. What do these limits mean for us? In this section, we consider a few simple consequences.

First, even if the state of the universe at every future time is completely determined by the present state of the universe, that future is effectively opaque to our demon, making a deterministic description of the world (if one exists) not particularly useful. If part of the world can be modeled as random, and that model both agrees with existing data to within a desired tolerance, and is constructed at a resolution appropriate for the specific question it intends to answer, then that part of the world may as well be considered random. The movement of pollen grains suspended in room temperature water is random. The concentration of Oct4 proteins inside a single mammalian cell is random.

Because even our demon cannot completely simulate a physical system as complicated as a person, the behavior of people may as well also be considered random. No powerful demon will ever be able to exactly predict our actions before they happen, and neither will we. We are too complicated (and therefore vulnerable to chaos in its various guises), too hard to experiment on (the experiment most relevant to predicting someone's next action can only be done once), and too computationally demanding (if it can be done at all, it is probably hard to predict someone's next action within their lifetime). For all practical purposes, humans have agency, and free will exists.

By the same argument, an artificial intelligence whose future actions are opaque to our demon may as well also be considered to have free will[f]. If there is no experiment or computation that can determine its actions within our lifetimes, even in principle, how is it useful to claim otherwise?

Second, there are likely provable theorems that humans will never be able to prove, and fundamental laws of physics that may never be discovered. Some true and provable mathematical statements might take significantly longer than a human lifetime to prove (exactly computing values of the busy beaver function again comes to mind). There may be nontrivial features (e.g. new particles) of the ultimate description of the universe that are impossible or utterly impractical to test experimentally: even a quick and powerful demon would have a hard time constructing a galaxy-sized collider within a few human lifetimes. Whether these things should be considered true, if they can never be verified, is a matter of philosophical taste.

Lastly, there is a meaningful distinction between computing a result and satisfactorily answering a question—essentially, the difference between prediction and understanding. As previously noted, even if our demon could exactly predict the wave function of the universe (or whatever else) at some future time, it could not report that information to us within our lifetimes; even if it could, we would not understand it. Thinking in terms of emergent concepts and using approximate descriptions of the world is not just important for easing the computational burden. Rather, it is essential for achieving true understanding of any phenomenon.[28] The behavior of a falling ball can be understood by treating it like a point particle in a constant gravitational field, and including superfluous complexity (by, for example, modeling every atom in the ball or describing the many-body wave function of the entire system) does not sharpen this understanding. Human understanding requires significant data compression.

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What does all of this mean for our initial question? Can a flap of some insect's wings destroy the universe as we know it? Unfortunately, Laplace's demon cannot answer this question, because a brute-force computation of the universe's future state is simply too hard. Our demon, despite its powers, cannot do it either.

The result responsible for this possibility is sometimes also called the Taj Mahal theorem, because of a dramatic interpretation due to Reinhard Werner:[29] "By acting on the vacuum with suitable operations in a terrestrial laboratory, an experimenter can create the Taj Mahal on (or even behind) the Moon!" But there are reasons that the Reeh-Schlieder theorem should not cause us to panic. For one, it only applies to a certain model of quantum field theory (although the assumptions involved are minor, as far as these things go); since we do not yet know the fundamental description of our universe, or whether that description is a quantum field theory at all, it is not clear if it even applies to our universe. Furthermore, a careful mathematical analysis of the result suggests that the local operations required to do something like create a Taj Mahal on the moon are not physically possible.[30]

[f] Clearly, random number generators do not have free will. We are implicitly assuming that it really *is* intelligent in some objective sense, e.g. it has passed some kind of Turing test.

Trusting our current best models of how the universe works, and taking seriously even their most ridiculous-sounding consequences, is one way to learn about the world. It is the only way to learn about realms of existence not susceptible to direct experimental inquiry, like the very early moments of the universe. Still, though a coarse reading of quantum field theory makes a butterfly destroying the universe sound plausible, there are simple reasons we are confident that such a thing cannot happen (or that if it is possible, the likelihood of us observing it is so small that it may safely be ignored). Just like the Sun rising and setting every day increases our confidence that it will happen again and again, every day that no butterflies destroy the universe, we update our belief to be biased towards it never happening. This is the same reason we know most anything: through the process of induction, according to which we assume that robustly observed patterns probably continue. Even if we cannot ever completely know the fundamental laws, we can still observe the world, test hypotheses about how it works, and draw careful conclusions about what is likely to happen next.

Regardless of the answer, it is remarkable that we can reasonably contemplate what we can and cannot ever know. We are fortunate to live in a universe vast and complicated enough to hide complete knowledge of the future from even the most enterprising demons, yet comprehensible enough to predict the weather—butterflies and all.

Endnotes

A. Suppose the true state of the universe at time t_0 is $|\psi(t_0)\rangle$, and Laplace's demon wrongly thinks that the state of the universe is instead $|\psi'(t_0)\rangle$. If the Hamiltonian of the universe is $\hat{H}$, then the true state of the universe after an amount of time Δt will be

$$|\psi(t_0 + \Delta t)\rangle = e^{-i\hat{H}\Delta t/\hbar} |\psi(t_0)\rangle \ ,$$

whereas Laplace's demon will predict

$$|\psi'(t_0 + \Delta t)\rangle = e^{-i\hat{H}\Delta t/\hbar} |\psi'(t_0)\rangle \ .$$

One way to measure the 'difference' between quantum states is via the magnitude of their overlap, i.e. we could define

$$\delta := |\langle\psi'(t_0)|\psi(t_0)\rangle|^2$$

to quantify the initial difference between them. Notice that

$$|\langle\psi'(t_0 + \Delta t)|\psi(t_0 + \Delta t)\rangle|^2 = \left| \langle\psi'(t_0)|e^{i\hat{H}\Delta t/\hbar} \ e^{-i\hat{H}\Delta t/\hbar}|\psi(t_0)\rangle \right|^2 = \delta \ .$$

Hence, if the magnitude of the overlap is initially close to one (as it would be if the demon's knowledge of the universe's current state was reasonably close to the truth), it remains close to one forever; initial error does not balloon out of control in the same way that it does in classical chaotic dynamical systems, in this sense.

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