Lab 11 (Photoelectric Effect) Practice Problem Sheet

Here are some problems to prepare you for the quiz on Lab 11 (photoelectric effect). If you are comfortable with these problems, you should easily do well on the quiz. Some solutions you can use to check your work are at the end of this document.

These problems are a little bit wordy, but that's mainly because I'm using them to try and help you learn stuff rather than test your understanding.

1. **Basics of photoelectric effect.**

You shine red light on a piece of metal, and no electrons get knocked off. You shine blue light on a piece of metal, and electrons *do* get knocked off.

(a) If you keep increasing the intensity of the red light on the metal (so that more photons hit it per second), will you eventually be able to knock off an electron? Why?

(b) If you keep decreasing the intensity of the blue light on the metal (so that less photons hit it per second), will you eventually stop knocking off electrons? Why? (Ignore the effects of current leakage here.)

(c) If 5 million blue photons hit the piece of metal, how many electrons get knocked off? Assume that an electron that gets knocked off doesn't itself knock off electrons.

(d) Is it plausible to believe that green light will knock off electrons? What about purple light? Why?

(e) Four red photons (each with energy E_r) and five blue photons (each with energy E_b) hit the piece of metal (which has work function W_0). How many electrons get knocked off? What are their energies, in terms of E_r and E_b?

2. **Deriving equation for stopping voltage.**

We used the equation

$$V_f = \frac{h}{e}f - \frac{W_0}{e}$$

to fit our data and measure h/e in the lab. Let's see, step by step, where this equation comes from.

(a) What is the energy of a photon with frequency f?

(b) A photon with frequency f hits a metal whose work function is W_0. If the energy of this photon is bigger than W_0, the threshold energy, what is the energy of the electron that gets knocked off of the metal? This is the electron's *initial kinetic energy* K_0.

(c) After getting kicked off of the metal, the electron is moving from a plate with a net positive charge to a plate with a net negative charge (where it will get collected so we can measure what's happening). To overcome the potential difference V between the plates, it has to expend some energy. What is the energy of the electron when it hits the negatively charged metal plate? This is the electron's *final kinetic energy* K_f.

(d) In the photoelectric effect, lots of electrons get kicked off of the positively charged plate and migrate to the negatively charged plate. What happens to V when too many electrons have migrated to the negatively charged plate? What is the smallest K_f an electron that successfully migrated is allowed to have?

(e) The highest voltage difference between the plates that allows an electron to migrate is V_f, the stopping voltage. Using the condition you derived in the previous part, plus the expressions from (b) and (c), derive the expression at the top of the page.

3. **Do we have enough energy to knock off an electron?**

Imagine shooting various colors of light at different kinds of metals. Depending on the work function of each metal, the light will or will not have enough energy to knock off an electron.

In the table below, write a check mark where light has enough energy to knock an electron out of a given metal.

Metal	**red** (685 nm)	**green** (532 nm)	**blue** (473 nm)	**UV** (250 nm)
Rubidium ($W_0 = 2.3$ eV)				
Titanium ($W_0 = 4.33$ eV)				
Silver ($W_0 = 4.5$ eV)				
Gold ($W_0 = 5.2$ eV)				

Remember that the energy of a photon is

$$E = hf = \frac{hc}{\lambda} \ ,$$

where $hc \approx 1240 \text{ eV} \cdot \text{nm}$.

4. **Graphing the photoelectric effect.**

Consider the equation

$$V_f = \frac{h}{e} f - \frac{W_0}{e} ,$$

which relates the stopping voltage V_f to the frequency f of light used, Planck's constant h, the charge of an electron e, and the metal's work function W_0.

(a) When you take f as the independent variable x, and V_f as the dependent variable y, plotting the equation above gives you a line. What is the slope and y-intercept of this line?

(b) In the real experiment, you don't see the entire line; remember that there is an energy threshold! Before the threshold, $V_f = 0$. What value of f corresponds to the frequency/energy threshold?

(c) Suppose you use a metal with a small W_0, and then a metal with a large W_0. How do the slopes, y-intercepts, and thresholds compare in each case? (i.e. are they bigger/smaller with higher W_0, or is there no change?)

Bonus: Quantitative model of leakage current.

We discussed in class how attaching a voltmeter to the two plates you're using can affect your result. The basic problem is that, since real voltmeters have *finite* resistance instead of *infinite* resistance, some of the electrons can 'leak out' of the negatively charged plate into the voltmeter. Electrons will now both constantly travel to and leave the negatively charged plate, and the system will reach an equilibrium voltage when those two processes balance, *instead of* when the two plates have the maximum possible voltage between them. Let's make a simple quantitative model of this.

(a) Let V denote the potential difference between our two plates. Suppose that the rate it's *increasing* due to electron migration is $k_{migrate}$, and that the rate it's *decreasing* due to electron leakage through the voltmeter is k_{leak}. Write a differential equation for $\frac{dV}{dt}$.

(b) $k_{migrate}$ = (how many photons hit the metal per second) $\times$ (voltage change per successful electron migration). What is the voltage change due to one electron migrating? It should be the final kinetic energy of one successfully migrated electron (in terms of V, h, f, and W_0) divided by the electron charge e.

(c) Given your answer to (b), if I is the number of photons hitting the metal per second, write an expression for $k_{migrate}$.

(d) k_{leak} = (number of electrons leaking per second) $\times$ (voltage change per leak). If we think about our two plates as a capacitor with capacitance C, the number of electrons leaking per second is given by $1/RC$, the inverse of the time constant. What is the voltage change per electron that leaks out?[1]

(e) Given your answer to (d), write an expression for k_{leak}.

(f) Given your answers to (c) and (e), rewrite the differential equation you found in part (a).

(g) The voltage will stop changing when voltage *increases* due to migration balance out voltage *decreases* due to leakage. Find the stopping voltage V_f by setting $dV/dt = 0$ in your differential equation from the previous part.

(h) How does V_f depend on intensity? Does this make sense, based on our argument from class?

(i) What happens if you take $R \to \infty$? Does your answer look familiar?

[1] *Hint:* How do changes in voltage across a capacitor relate to changes in the charge on a plate?

Partial solutions.

1. (a) No—intensity doesn't matter. Electrons will only get knocked off if an individual photon has enough energy to knock them off. If a single red photon can't knock off an electron, ten billion red photons can't either.

(b) No—theoretically (i.e. ignoring things like leakage), at least. Just one blue photon has enough energy to knock off an electron, so any number of them will still knock off electrons.

(c) 5 million blue photons knock off 5 million electrons. One photon will knock off one electron, assuming electrons don't knock off electrons.

(d) Green light has energy between red and blue light, so it may or may not knock off electrons. Purple light will definitely knock off electrons, since it has more energy than blue light, which is already past the energy threshold.

(e) The red photons won't knock off any electrons. Each blue photon will knock of an electron, so there will be five electrons, each with energy $E_b - W_0$.

2. (a) $E = hf$

(b) initial KE of electron = (energy of photon) - (energy cost of knocking off electron)

so $K_0 = hf - W_0$

(c) final KE of electron = (initial KE of electron) - (energy cost of overcoming potential difference)

so $K_f = K_0 - eV$

(d) Every time an electron moves from the positively charged plate to the negatively charged plate, V increases. When many electrons have moved, V becomes too high for electrons that get knocked off to overcome, so they can no longer travel from the positively charged plate to the negatively charged plate. The final kinetic energy of the last electron to migrate will be $K_f = 0$, so that $K_0 = eV_f$; it had exactly enough energy to overcome the potential difference, and no more.

(e) $K_f = 0 \implies K_0 = eV_f \implies hf - W_0 = eV_f \implies V_f = (h/e)f - (W_0/e)$

3. Just divide 1240 by each wavelength and see if it's bigger than W_0 in each case.

Bonus. (a) $\frac{dV}{dt} = k_{migrate} - k_{leak}$

(b) $K_f = (hf - W_0) - eV$ so $\Delta V_{migrate} = \frac{hf - W_0 - eV}{e}$

(c) $k_{migrate} = I\left(\frac{hf - W_0 - eV}{e}\right)$

(d) $\Delta V_{leak} = \frac{\Delta Q}{C} = \frac{e}{C}$

(e) $k_{leak} = \frac{1}{RC}\frac{e}{C} = \frac{e}{RC^2}$

(f) $\frac{dV}{dt} = I\left(\frac{hf - W_0 - eV}{e}\right) - \frac{e}{RC^2}$

(g) $0 = I\left(\frac{hf - W_0 - eV_f}{e}\right) - \frac{e}{RC^2} \implies V_f = \frac{hf - W_0}{e} - \frac{e}{RC^2 I}$

(h) As intensity I decreases, V_f decreases. This is just what we expected from our in-class discussion. In the limit as $I \to \infty$ (where we have a really intense light source), the last term in our expression for V_f vanishes, meaning leakage doesn't matter at all anymore.

(i) Taking $R \to \infty$ corresponds to having an ideal voltmeter with infinite resistance; in this limit, the last term in our expression for V_f goes away, and we get the same answer for V_f that we did in problem 2.