Lecture One
Introduction and Math Review

1 Introduction

Quesition: *What is Economics? What is the difference between Microeconomics and Macroeconomics? What is the common ground, linkage, connection?*

1.1 *What is Economics?*

Economics is a **social science** that studies how an **economic agent** chooses to allocate its **scarce resources**, which have **alternative uses**, to achieve a **specific objective**.

- Social science: study human behavior;
  - People are complicated: love, hatred, mistakes, irrational behavior;
  - Economics studies stylized people – rational people;
- Economic agent: consumers, firms, government etc.
- Specific objective: decision criteria are behavioral assumptions made by economists.
  - consumer: utility maximization;
  - firm: profit maximization;
  - government: complex policy objectives: low unemployment, low inflation, high economic growth, equal income distribution (not necessarily to be absolutely equal) etc.
- Scarce resources:
  - resources are limited v.s. human wants are unlimited by nature (e.g. utility maximization);
  - land, time/labor, ever clear air;
  - Basic economic problem: unlimited wants (maximization) compete for limited resources.
- Alternative uses
  - opportunity cost is the value of the **best forgone alternative** — what you gave up when you got something.
- **So, Economics studies the rational decision of economic agents when facing resource constraints.**

1.2 *Difference and connection between Microeconomics and Macroeconomics*

- Microeconomics is primarily focused on the actions of individual agents, such as firms and consumers, and how their behavior determines prices and quantities in specific markets.
- Macroeconomics studies aggregated indicators such as GDP, unemployment rates, and price indices to understand how the whole economy functions.
  - Aggregate behavior of individual agents.

1.3 Economics is both HARD and EASY

- People are complicated, but economics studies only rational people
  - A deviation: Behavioral Economics, bounds of rationality
- The World is a complex system (everything is related), but predictable to some degree;
  - **Example: Japan tsunami** (Complex but predictable)
    → Car parts factory shut down their production;
    → Toyota, Honda and Nissan all suspended production at a few of their plants in Japan;
    → Car export to the US decreases;
    → Supply of car decrease in the US;
    → Average Price of new cars goes up;
    → Demand for used car goes up;
    → Average Price of used car goes up;
    → Demand for alternative transportation increases (flight, coach)
    → Airline price goes up;
    → ······
- We can simply the complex world into economic models by making some assumptions. Some assumptions are manifestly false and perhaps even ridiculous, but we can frequently end up making very good predictions about the real world based on these assumptions. The usefulness of an assumption depends on the problem of interest.
  - **Example** (Milton Friedman, 1953, *Essays in Positive Economics, Part I The Methodology of Positive Economics*):
    Consider the density of leaves around a tree. I suggest the hypothesis that the leaves are positioned as if each leaf deliberately sought to **maximize the amount of sunlight it receives**, given the position of its neighbors, as if it knew the physical laws determining the amount of sunlight that would be received in various

positions and could move rapidly or instantaneously from any one position to any other desired and unoccupied position....Despite the apparent falsity of the "assumptions" of the hypothesis, it has great plausibility because of the conformity of its implications with observation.

The clever tree assumption is a useful assumption because it **helps to predict** the distribution of leaves around a tree. But if we concerned about how the tree feels about losing leaves in the winter, the assumption will lead us badly astray.

- **Another example:**

  We want to know how long it would take a brick dropped off of Kirkland hall to hit the ground. We model the brick as a perfect sphere of uniform density falling in a vacuum towards a planet of known mass and uniform density.

  This is all false. Should we take into account wind resistance, or gravitational anomalies, or the odds of hitting a butterfly on the way down? Our predictions would be more accurate, but we do not gain too much, so it would not be worth the effort.

- Information is imperfect: car dealers have more information about their cars;
- Information is incomplete: no one can say for sure whether it will be raining tomorrow.

1.4 Methodology of economic studies

- The above examples shows
  - On one hand, by simplifying the real world, models can make complicated problems tractable;
  - on the other hand, sometimes models are too simple to capture the main points, or assumptions are too unrealistic to be relevant to the real world.

  That is why Economics is both an art and a science.

- Art: you have to have the artistic eyes to simplify the real world in order to make resulting model tractable. Meanwhile, the model could not be so simple as to be completely irrelevant;
- Science: we can use scientific method, such as mathematics, to analyze the model we produce.

1.5 General Framework

In this course, we will focus on the microeconomic models. So we will use mathematical reasoning to study the optimizing behavior of consumers and firms. And then we will see that under very general conditions, when collections of agents act under optimization, we get an equilibrium. And this equilibrium is optimal for the whole society. This is the "invisible *hand" introduced by Adam Smith. The idea is that even though all agents are working in their own narrow interest, they are lead "as if by an invisible hand" to seek a kind of collective good.*

- Consumer Optimization
  - indifference curve, marginal utility
  - demand
  - normal good, inferior good, Giffon good)

$$\max_{x_1,x_2} U(x_1,x_2) \;\; s.t. \; p_1x_1 + p_2x_2 = I$$

- Firm Optimization
  - fixed cost, variable cost, marginal cost (long run v.s. short run)
  - Increase/constant/decreasing return to scale

$$\max_{x_1,x_2} py - p_1x_1 - p_2x_2 \; s.t. \; y = f(x_1,x_2)$$

Or

$$\min_{x_1,x_2} p_1x_1 + p_2x_2 \; s.t. \; f(x_1,x_2) = \bar{y}$$

- Market Structure / Market Power
  - Perfect Competition: price taker, supply
  - Monopoly
  - Oligopoly
  - Monopolistic Competition
- Welfare Economics (under perfect competition):
  - consumer surplus, producer surplus, social gain
  - tax, subsidy, price ceiling, price floor, tariffs, deadweight loss
  - general equilibrium, Edgeworth box, Pareto optimal, contract curve
  - 1^{st} welfare and 2^{nd} theorems, coalition, core

2 Math Review

See file math_review.

Lecture Two (Chapter 1) Demand, Supply and Equilibrium

In a sense, this chapter is logically out of place. We won't know where demand curves come from until we get to chapter 3. The point of this chapter is to give a sort of a road map to where the first seven chapters will take us.

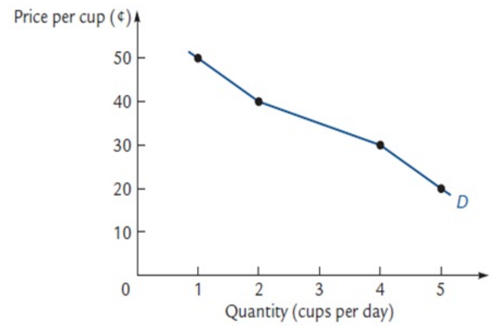
Supply and demand are one of the most powerful tools that economists have.

1 Demand

- What do you know about demand from Econ 101? **The law of demand**
  - When the price of a good goes up, the quantity demanded goes down;
  - It is equivalent to say that demand curves are downward sloping;
  - This is an *empirical law without theoretical foundation*;
  - We will see in Chapter 4 that the law of demand follows logically from certain more fundamental assumptions about human behavior.
- Elements of Demand?
  - What commodity (quality, when, where?)
  - Over what unit of time
  - By whom
  - What unit of consumption
  - What unit of exchange
  - Other conditions: e.g. weather.
- Demand Curve
  - Note that quantity is on the horizontal axis and price on the vertical axis.

EXHIBIT 1.1 **The Demand Curve**

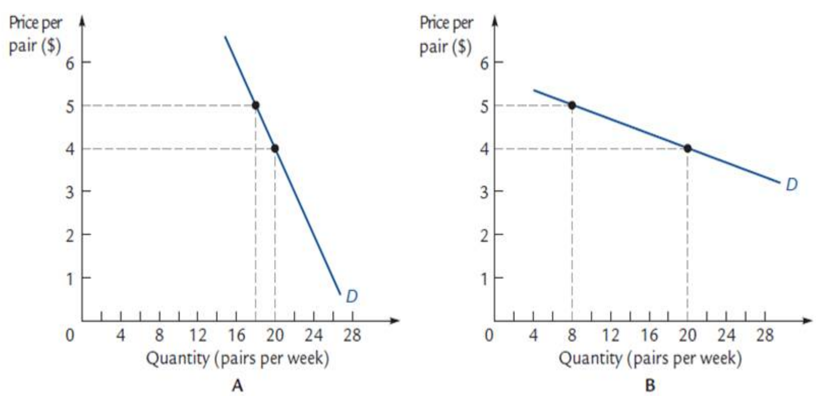
Price	Quantity
20¢/cup	5 cups/day
30¢	4
40¢	2
50¢	1



The demand table shows how many cups of coffee you would buy per day at each of several prices. The black points in the graph correspond precisely to the information in the table. The curve connecting the points is your demand curve for coffee. It conveys more information than the table because it shows how many cups of coffee you would buy at intermediate prices like 22¢ or 33½¢ per cup. If the table were enlarged to include enough intermediate prices, then the table and the graph would convey exactly the same information.

- - Slope of the curve: steeper, change of price has less effect on the quantity demanded.

EXHIBIT 1.4 **The Shape of the Demand Curve**



The two panels depict two possible demand curves for shoes. In panel A a given change in price (say from $4 per pair to $5 per pair) leads to a small change in quantity demanded (from 20 pairs of shoes per week to 18 pairs per week). In panel B the same change in price leads to a large change in quantity demanded (from 20 pairs per week to 8 pairs per week).

- In economics, the idea of demand can apply to many things that might not be conventional.
  - There is a demand curve for anything that bring a positive utility to you. e.g. an airplane, an inland

Example: Demand for reckless driving.

- Positive utility: save time, excitement, etc.

- Demand v.s. Quantity demanded

  People might say that they love to have a private inland in the tropic, but they do not have a demand for it. Not precise, we need to distinguish demand and quantity demanded.

  Question: Do you want to buy an inland given the price is \$10. Of course.

  - Quantity demanded: the amount of a good that a given consumer will choose to consumer **at a given price**.
    - A point in demand curve.
    - e.g. The quantity demanded for coffee in May by Vanderbilt students is 1000 cups at the price of $\$1/cup$.
  - Demand: Specify a relationship between two variables, price and quantity demanded. Given any possible price, the corresponding quantity demanded.
    - demand curve itself.
    - e.g. The demand for coffee in May by Vanderbilt students is 1000 cups at the price of $\$1/cup$, 2000 cups at the price of $\$\frac{1}{2}/cup$ and so on.
  - Therefore, we have a demand for an inland, but my quantity demanded of an inland is **0 given the price is very high**.

- Changes in demand

  Question: Suppose the price of coffee changes due to a revolution in Brazil. What happens to the demand for coffee?

  - Nothing! The quantity demanded changes, but not the demand curve itself. This is the first question I was ever asked on an exam and also the first I got wrong.
  - A change in demand is a shift in the curve. So for a given price, the quantity demanded changes.
  - A change in quantity demanded is a shift along a given curve. So given a price, the quantity demanded is the same.

  Question: What changes the demand?

  - Change in the price of Complements (Chapter 4)
    - When the prices of coffee mate increase, your demand of coffee might decrease.
    - Given the same price of coffee, the quantity demanded for it declines.
  - Change in the price of Substitutes (Chapter 4)
    - When the prices of tea increase, your demand of coffee might increase.
    - Given the same price of coffee, the quantity demanded for it increases.

– Change in the your taste (Chapter 4)

  * Change in income: When people become rich, the demand for McDonald might decrease.
  * Change in other conditions: If I move this class to 7am, it is very possible that for most of you, the demand for this class would decrease.

– Distortion of price (Chapter 7)

  * (Sale) Taxes: Suppose a tax of $.25 per cup is placed on the sale of coffee, your demand for it might decrease.
  * Subsidy etc.

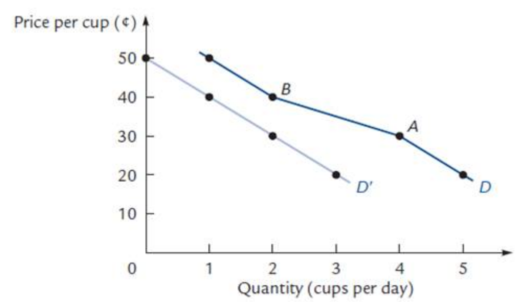
EXHIBIT **1.2** **Shifting the Demand Curve**

TABLE A. Your Original Demand for Coffee

Price	Quantity
20¢/cup	5 cups/day
30¢	4
40¢	2
50¢	1

TABLE B. Your New Demand for Coffee after Medical Advice to Cut Back

Price	Quantity
20¢/cup	3 cups/day
30¢	2
40¢	1
50¢	0



Your original demand curve for coffee is the curve labeled *D*. A change in price, say from 30¢ per cup to 40¢ per cup, would cause a movement along the curve from point *A* to point *B*. A change in something other than price, such as a doctor's suggestion that caffeine is bad for your health, can lead to a change in demand, represented by a shift to an entirely new demand curve. In this case the doctor's advice leads to a fall in demand, which is represented by a leftward shift of the curve.

2 Supply

- What do you know about supply from Econ 101? **The law of supply**

  – When the price of a good goes up, the quantity supplied goes up;
  – It is equivalent to say that supply curves are upward sloping;
  – This is also an *empirical law without theoretical foundation*;
  – We will see in Chapter 6 that the law of supply follows logically from firms' profit-maximization behavior

- Elements of Supply?

  – What commodity (quality, when, where?)
  – Over what unit of time

– By whom
– What unit of consumption
– What unit of exchange

- Supply Curve
  - The same as the demand curve, quantity is on the horizontal axis and price on the vertical axis.
  - Slope of the curve: steeper, change of price has less effect on the quantity supplied.

EXHIBIT **1.5** **The Supply of Coffee**

TABLE A. Supply of Coffee to Your City

Price	Quantity
20¢/cup	100 cups/day
30¢	300
40¢	400
50¢	500

TABLE B. Supply of Coffee to Your City Following the Development of Better Farming Methods

Price	Quantity
20¢/cup	200 cups/day
30¢	400
40¢	600
50¢	700

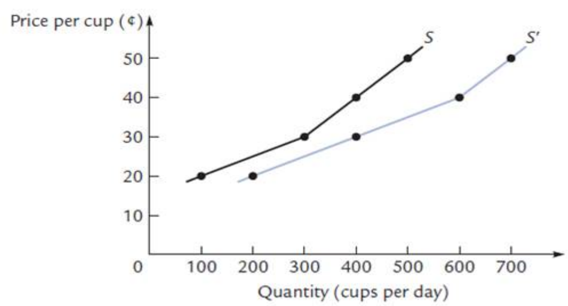


Table A shows, for each price, how much coffee would be supplied to your city. The same information is illustrated by the points in the graph. The curve labeled *S* is the corresponding supply curve. It conveys more information than the table by displaying the quantities supplied at intermediate prices. The law of supply is illustrated by the upward slope of the supply curve.

The invention of a cheaper way to produce coffee increases the willingness of suppliers to provide coffee at any given price. The new supply is shown in Table B and is illustrated by the curve *S′*. Although a change in price leads to a movement along the supply curve, a change in something other than price causes the entire curve to shift.

The curve *S′* lies to the right of *S*, indicating that the supply has increased.

- Supply v.s. Quantity supplied

  Question: Do you want to buy an inland given the price is \$10. Of course.
  - Quantity supplied: the amount of a good that a given supplier will provide **at a given price**.
    - A point in supply curve.
    - e.g. The quantity supply of coffee in May by the Starbucks on 21^{st} avenue is 2000 cups at the price of $\$1/cup$.
  - Supply: Specify a relationship between two variables, price and quantity supplied. Given any possible price, the corresponding quantity supplied.
    - supply curve itself.
    - e.g. The supply of coffee in May by the Starbucks on 21^{st} avenue is 2000 cups at the price of $\$1/cup$, 1000 cups at the price of $\$1.5/cup$.

- Changes in supply

  Question: Suppose the price of coffee decrease due to a new discover that coffee is much worse for health than what we expected. What happens to the supply of coffee?

  - Nothing! The quantity supplied changes, but not the supply curve itself.
  - A change in supply is a shift in the curve. So for a given price, the quantity supplied changes.
  - A change in quantity supplied is a shift along a given curve. So given a price, the quantity supplied is the same.

  Question: What changes the supply?

  - Change in the price of inputs (Chapter 7)
    * When the prices of coffee beans increase, the supply of coffee would decrease.
    * Given the same price of coffee, the quantity supplied for it declines.
  - Change in technology (Chapter 7)
    * When a more efficient technology is introduce to produce coffee, the supply of coffee would increase.
    * Given the same price of coffee, the quantity supplied increases.
  - Distortion of price (Chapter 7)
    * (Excise) Taxes: Suppose a tax of $.25 per cup is placed on the production of coffee, the supply of coffee would decrease.
    * Subsidy etc.

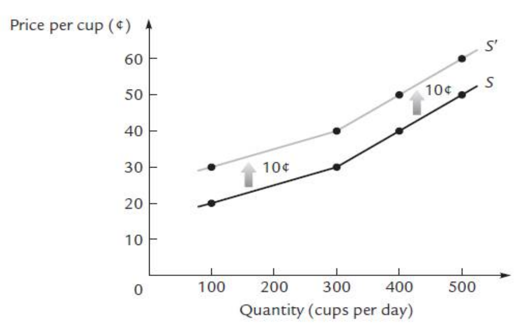
EXHIBIT **1.6** **Effect of an Excise Tax**

TABLE A. Supply of Coffee without Tax

Price	Quantity
20¢/cup	100 cups/day
30¢	300
40¢	400
50¢	500

TABLE B. Supply of Coffee with Excise Tax of 10¢ per Cup

Price	Quantity
30¢/cup	100 cups/day
40¢	300
50¢	400
60¢	500



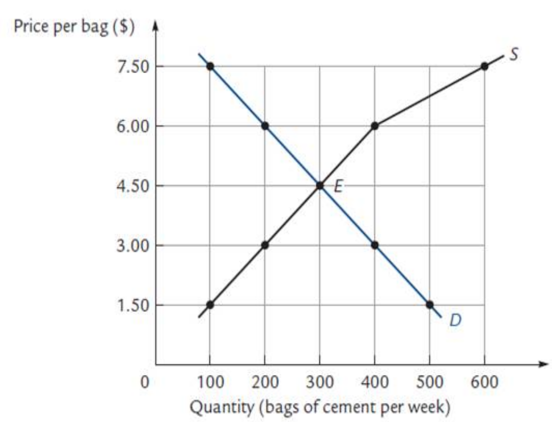
If the price of coffee is 30¢ per cup and there is an excise tax of 10¢, then a seller of coffee will actually get to keep 20¢ per cup sold. The original supply schedule (Table A) shows that under these circumstances suppliers would provide 100 cups per day. This is recorded in the first row of Table B. The other rows in that table are generated in a similar manner.

The rows of Table B contain the same quantities as the rows of Table A, but the corresponding prices are all 10¢ higher. Thus, each point on the new supply curve *S′* lies exactly 10¢ above a corresponding point on the old supply curve *S*. Therefore, *S′* lies exactly 10¢ above *S* in vertical distance. The excise tax causes the supply curve to shift upward parallel to itself a distance of 10¢.

3 Equilibrium

- **Equilibrium** is the condition of a system in which competing influences are balanced.
  - Mechanical equilibrium: the state in which the sum of the forces, and torque, on each particle of the system is zero;
  - Economic equilibrium is a state of the world where economic forces are balanced
    * quantity demanded and quantity supplied are equal;
    * the point where the supply and demand curves intersect;
    * equilibrium price and equilibrium quantity
    * given the price, buyers can buy exactly amount they want to buy, and sellers can sell exactly amount they want to sell.
  - Adjustment of equilibrium: Suppose the price to too low or two high.

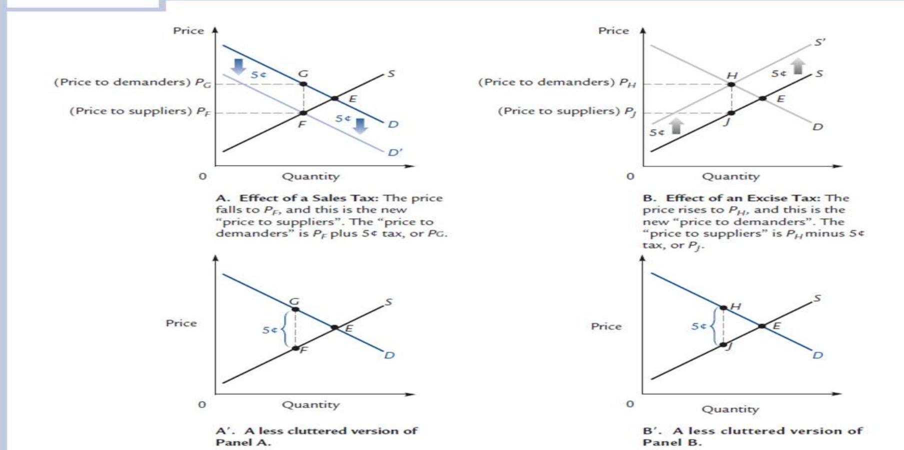
EXHIBIT **1.7** **Equilibrium in the Market for Cement**



The graph shows the supply and demand curves for cement. The equilibrium point, *E*, is located at the intersection of the two curves. The equilibrium price, $4.50 per bag, is the only price at which quantity supplied and quantity demanded are equal.

- The economic incidence of a tax is independent of its legal incidence.
  - Sales tax v.s. excise tax.
  - Consider the examples of sales and excise taxes in a graph. What happens to quantity and price?

EXHIBIT **1.10** **A Sales Tax versus an Excise Tax**



Panel A reproduces the graph from Exhibit 1.9, illustrating the effect of a 5¢ sales tax. Panel B illustrates the effect of a 5¢ excise tax: The supply curve shifts upward a vertical distance 5¢, leading to a new market equilibrium at point *H*. The corresponding price, P_H, is what demanders have to pay; the amount that suppliers get to keep is P_H minus 5¢, which is P_J. (Because *H* is on the curve *S′*, *J* must be on the curve *S*.)

Panels A′ and B′ are less cluttered versions of panels A and B. In each of these panels, we see two darkened points, one on the original demand curve *D* and one on the original supply curve *S*, separated by a vertical distance 5¢. There is only one possible location for such a pair of points.

It follows that points *G* and *F* in panel A′ (or panel A) are identical with points *H* and *J* in panel B′ (or panel B). In other words, the effects of the excise tax are identical to the effects of the sales tax, from the viewpoint of either demanders or suppliers.

- A mathematical example:
  - No tax case: Demand: $Q^d = 100 - P$; Supply : $Q^s = 3P$

$$\begin{aligned} Q^s &= Q^d \\ &\Rightarrow 3P = 100 - P \\ &\Rightarrow Q^s = Q^d = 75, P = \$25 \end{aligned}$$

  - With sales tax: $P_c = P_p + T$ and $P_p = P_n$
    Demand: $Q^d = 100 - P_c$; Supply : $Q^s = 3P_p$; Sales tax: $T = \$8$

$$\begin{aligned} Q^s &= Q^d \\ &\Rightarrow 3P_p = 100 - P_c = 100 - (P_p + \$8) \\ &\Rightarrow Q^s = Q^d = 69, P_n = P_p = \$23, P_c = \$31 \end{aligned}$$

  - With excise tax: $P_p = P_c - T$ and $P_c = P_n$
    Demand: $Q^d = 100 - P_c$; Supply : $Q^s = 3P_p$; Excise tax: $T = \$8$

$$\begin{aligned} Q^s &= Q^d \\ &\Rightarrow 3(P_c - \$8) = 3P_p = 100 - P_c \\ &\Rightarrow Q^s = Q^d = 69, P_p = \$23, P_n = P_c = \$31 \end{aligned}$$

Lecture Three (Chapter 2)
Price, Cost and Gains from Trade

1 Price

- **Questions:**
  - What do we mean when we say "the oil price is high"?
  - Are you impressed when your grandmother says "everything is so expensive these days, a movie only used to cost a quarter."

  In 1932, a movie cost \$.25; while in 2011, a movie cost \$10.

  Further Question:

  - Which world would you rather live in?
  - Although things you consume cost more **in money terms**, things you sell, like labor, also cost more.

	Movie	Hour of labor
1932	\$.25	\$.10
2011	\$10.00	\$10.00

Clearly, you would rather be alive today. Why? You have to work less to afford the same number of movies even though movies cost much more.

- Absolute Price v.s. Relative Price
  - Absolute Price: the number of dollars that can be exchanged for a unit of a given good.
  - Relative Price: the number of units of another good that can be exchanged to a unit of a given good.

  We see the absolute prices of all given in the table above. The relative prices are as follows:

  - 1932: movies cost 2.5 hours of labor
  - 2011: movies cost 1 hour of labor

Thus, movies are **absolutely more expensive, but relatively cheaper.**

- *In economics a single word "price" always refers to a relative price. So back to our original questions:*

  - When we say ”the oil price is high”, we might mean that the oil price is higher than the price last month. It is an absolute increase, so we are talking about absolute price.
  - When your grandmother says “everything is so expensive these days, a movie only used to cost a quarter.” She also means that absolute price of a movie is higher.

- Relative price change v.s. Inflation

  - Relative price change: A relative price exists for each good with respect to each other good, so usually when we say the price of a good increases, we refer to the price relative to the average of absolute prices, or to the price of a basket of goods, e.g. CPI, PPI.
  - Inflation: An ongoing rise in the average of absolute prices. E.g. Increasing of CPI.
  - Doubling all the absolute price leads to inflation, but the relative price is the same.
  - In economics, decisions are made according to the relative prices. Therefore, inflation has no effect on our economic decision. Then inflation has no effect on the real economy, this idea is called neutrality of money. This was used to argue against government intervention in political economy as a waste of time.

- Some Applications:

  - Example 1: Two kinds of orange are grown in Florida, and consumed both in Florida and Japan. Where do you think consumes better orange on average. Hint: transportation costs, say $3.

	Price of good	Price of bad	Relative price of good
Florida	$2	$1	2
Japan	$5	$4	1.25

    Relative price of good orange is cheaper in Japan, so Japanese consumes more good orange.
  - Example 2: Interest rates are relative prices between money today and money tomorrow.
  - Example 3: Exchange rates are relative prices between different currency.

2 Cost

When we discussed price, we said ”A movie costs $10 in 2011”. It sounds that the $10 is the **COST** of the movie, so price and cost are equivalent. However, in economics, it is not true. When an economist use the word ”cost”, he/she really mean ”**Opportunity cost**”.

- **Opportunity cost** is the cost of any activity measured in terms of the best alternative forgone.
  - Total value of all aspects of the best foregone opportunity necessary to obtain an object;
  - Second best choice.
- Example 1: Suppose you want to go to the movie, say *"Kung Fu Panda II"*, You have to pay \$10 of course, but this is just a direct cost. You also have to drive to the cinema, so you have to pay for the gasoline. Meanwhile, it takes about 2 hours for you to drive there and finish the movie. That time could have been spent doing something else, say hanging out with friends, or preparing the quiz tomorrow. The highest value of these alternatives is your indirect opportunity cost.
  - Ticket: \$10
  - Gasoline: \$3
  - hanging out with friends: \$30
  - preparing the quiz: \$20

  Question: What is your cost of going to the movie "Kung Fu Panda II"? \$43. (Be careful: don't double count)
- Example 2: Cost of college education a year
  - Tuition: $\$40,000$
  - Not working: $\$40,000$
  - Leaving home: $+/-\$10,000$

3 Gains from Trade

Question: *What is the source of gains from trade? Who benefits from trade?*

As long as the trade is voluntary, the second question is easy. If one party did not benefit from the trade, he/she can simply choose not to trade. Intuitively, everyone should benefit from trade.

But what is the source of benefits from trade? The following examples helps us reveal the source from different aspect.

- **Example 1:** Trade between the US and Saudi Arabia.
  - Saudi Arabia sends us oil and we send them wheat;
  - Oil in Saudi Arabia is abundant, while arable land in the US is abundant;
  - Trade based on **differences in endowment.**

- **Example 2:** Trade between the US and Japan.
  - Japan sends us SUV's and we send them Yellowfin tuna;
  - Japan should have more ocean resource, and the US should not been worse in producing SUV's;
  - Simply because Japanese likes Yellowfin tuna more and American needs/likes more SUV's;
  - Trade based on **differences in taste.**
- **Example 3:** Trade between the US and Canada.
  - Canada sends us papers and we send them computers;
  - Canada and the US have very similar endowment and taste;
  - Compared with the US, the paper industry is more efficient than the IT industry in Canada;
  - Trade based on **differences in (relative) abilities / comparative advantage.**
  - **Numerical Example:**
    Both US and Canada need 1000 papers and 1 computer.

	Papers (1000)	Computer (1)
US	\$2000	\$1000
Canada	\$1000	\$2000

    * Self-production: cost \$3000 each, so \$6000 in total;
    * US produces 2 computers and Canada produces 2000 papers, then trade: cost \$4000 in total. They can split the benefits (\$2000), which depends on the trading prices.
- **Example 4:** Trade between the US and Germany.
  - Germany sends us Mercedes-Benz and we send them Fords;
  - Both of us are good at car production;
  - **Comparative advantage** explains a small part. Within industry. Where is the source of comparative advantage?
  - Still trade based on **differences in (relative) abilities / comparative advantage,** but the comparative advantage comes from **Specialization / increasing return to scale / economy of scale.**
  - **Numerical Example:**
    Both US and Germany need 1 Benz and 1 Fords.

	Benz (1)	Fords (1)
US	\$20,000	\$10,000
Germany	\$20,000	\$10,000

By specializing in one industry, their production are both more efficient.

	Benz (2)	Fords (2)
US	$30,000	$15,000
Germany	$30,000	$15,000

* Self-production: cost $30,000 each;
* US produces 2 Fords and Germany produces 2 Benz, then trade: cost $45,000 in total. They can split the benefits ($15,000), which depends on the trading prices.

EXHIBIT **2.2** **The Electrician and the Carpenter**

Table A

	Electrician	Carpenter
Rewiring	10 hours	20 hours
Paneling	15 hours	18 hours

Table B

	Electrician	Carpenter
Rewiring	2/3 paneling	10/9 panelings
Paneling	3/2 rewirings	9/10 rewiring

Table A shows the amount of time needed for the electrician and the carpenter to rewire and to panel. Notice that the electrician can complete either job in less time than the carpenter can. We express this by saying that the electrician has an absolute advantage at each task.

Table B shows the costs of rewiring and paneling jobs performed by each individual. The costs are measured in terms of forgone opportunities; thus the cost of a rewiring job must be measured in terms of paneling jobs and vice versa. All of the information in Table B can be derived from the information in Table A.

Notice that the electrician can rewire at a lower cost than the carpenter, but that the carpenter can panel at a lower cost than the electrician. We express this by saying that the electrician has a comparative advantage at rewiring, whereas the carpenter has a comparative advantage at paneling.

Suppose that each individual wants his house rewired and his den paneled. Table C below shows the total amount of time that each will have to work in order to accomplish both jobs. In the first column we assume that each does all of the work on his own house. For example, the electrician spends 10 hours rewiring and 15 hours paneling, for a total of 25 hours. In the second column, we assume that each specializes in the area of his comparative advantage: The electrician rewires both houses and the carpenter panels both dens.

It is apparent from Table C that trade makes both parties better off. In particular, the electrician can gain from trade with the carpenter, despite his absolute advantages in both areas. This illustrates the general fact that everyone can be made better off whenever each concentrates in his area of comparative advantage and then trades for the goods he wants to have.

Table C

	Without Trade	With Trade
Electrician	25 hours	20 hours
Carpenter	38 hours	36 hours

Lecture Four (Chapter 3)
Preferences and Utility Functions

Question: Economics is about decision making. So can you make a decision for JOHN about what to order in a restaurant?

We need some information to make the decision:

- Preference / Tastes
  - Represented by utility $U(\cdot)$
  - Implicit assumption: always choose the best if he can, so max $U(\cdot)$.
  - We don't discuss why he likes steak over shrimp. We take his preference as given
- Constraint
  - Menu (price p_1, p_2, and consumption opportunity x_1, x_2)
  - Budget (how much JOHN want to spend? I)

After we know all these information, we can formalize the question as

$$\max_{x_1,x_2} U(x_1,x_2) \;\; s.t. \; p_1x_1 + p_2x_2 = I$$

1 Preference

- Preference over what?
  - Typically, we consider a finite space of goods. For example, we might think of a world with three goods: Food, clothing and shelter. We would represent a typical consumption choice here as a vector:

$$x \equiv (x_f, x_c, x_s) = (2,3,9) \in \Re^3$$

  - Thus, a consumption bundle is an element of a Euclidian space and lists a level of consumption of each good.
  - Usually, the goods space is an n-dimensional Euclidian space in this course, $\Re^n$
  - The goods space could be an infinite dimensional Euclidian space: usually deal with uncertainty
    - e.g. financial market: how much insurance to buy.
- Refine the consumption space:
  - We don't allow consumers to choose negative consumption, so we would want to restrict the choices to the **positive orthant,** $\Re^n_+$.

- Sometimes, we need to restrict the consumption bundle to permit survival.
- In general, we will restrict choices to a consumption set denoted as follows (draw this):

$$X \subseteq \Re^n_+$$

- Ordering or Ranking over the consumption set (**binary relation**)
  - $x \succ y$ means x is strictly preferred to y;
  - $x \succeq y$ means x is strictly preferred or indifferent to (at least as good as) y;
  - $x \sim y$ means x is indifferent to y.

- Assumptions on preferences:

  Rationality requirements:

  - **Completeness:** $x \succeq y$ or $y \succeq x$ or both ($\sim$), $\forall x, y \in X$, $x \neq y$;

    This means that all consumption bundles in the consumption set can be ranked against one another. You can never get to case where you can't choose.
  - **Reflexivity:** $x \succeq x$, $\forall x \in X$

    Any consumption bundle is at least as good as itself.
  - **Transitivity:** If $x \succeq y$ and $y \succeq z$, then $x \succeq z$, $\forall x, y, z \in X$.

  Technical assumption:

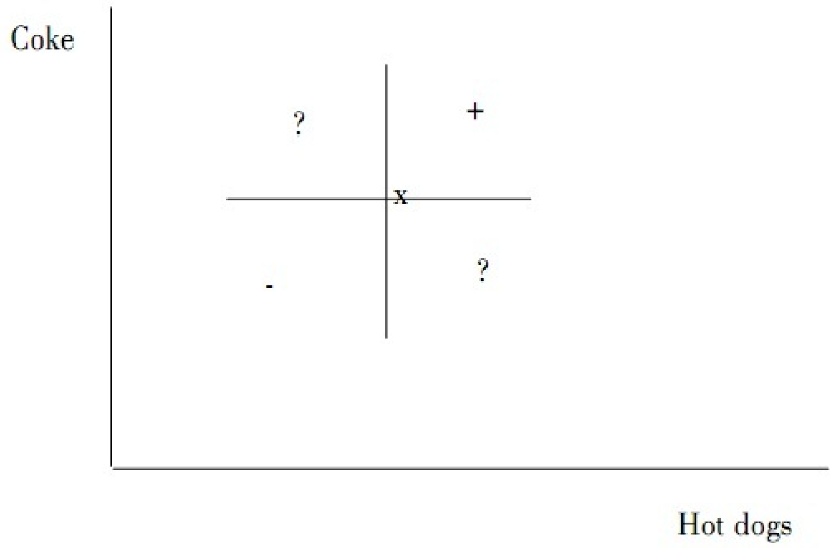
  - Continuity: $\{x \in X | x \succeq y\}$ and $\{x \in X | y \succeq x\}$ are closed, $\forall y \in X$.

- Indifference curve: A collection of consumption bundles, all of which the consumer considers equally desirable.
  - Indifference curve through a bundle $\bar{x}$ is the set of consumption bundles that are exactly as good as $\bar{x}$.

$$I(\bar{x}) \equiv \{x \in X | x \sim \bar{x}\}$$

Question: What do we know about Indifference curve by the assumptions? **Not much!! (Draw)** We need **more (behavioral) assumptions.**

- Monotonicity
  - Consider a consumption point $\bar{x}$. Which points are better or worse than this?
  - In general, we assume more is better. If an agents has the attitude toward more goods we say his preferences are monotonic.



- – **Weak Monotonicity:** A preference relation $\succeq$ is weakly monotone if

$$x_i \geq y_i, \forall i = 1, \cdots, N \text{ and } \forall x, y \in X, \text{ then } x \succeq y$$

  – **Strong Monotonicity:** A preference relation $\succeq$ is Strongly monotone if

$$\begin{aligned} x_i &\geq y_i, \forall i = 1, \cdots, N \text{ and } \forall x, y \in X, \text{ then } x \succeq y \\ \text{and } x_i &> y_i \text{ for some } i = 1, \cdots, N, \text{ then } x \succ y \end{aligned}$$

  – Examples for $N = 2$, $x = (2, 2)$ and $y = (2, 1)$, from **Weak Monotonicity**, we know $x \succeq y$; while from **Strong Monotonicity**, we know more: $x \succ y$.

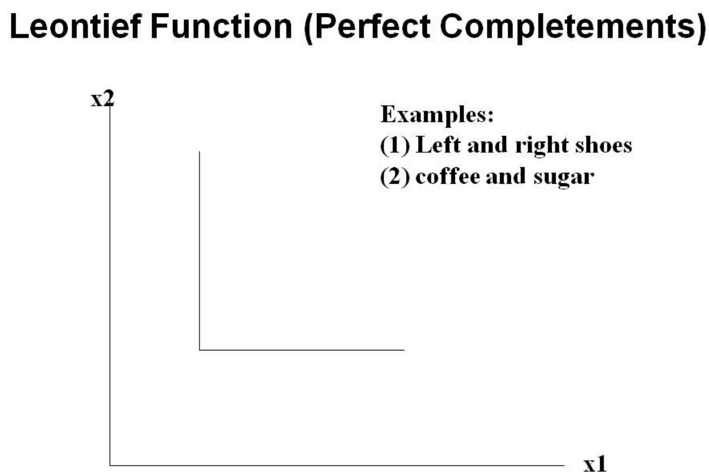
  – **Weak monotonicity** allows for the **possibility that consumers become satiated in some goods (but not all goods at once)**. For example. Once you drink 12 cokes at a football game, additional cokes do not increase your utility. You would simply ignore them if they were given to you, and would be neither better nor worse off. (Draw Leontief)

  – Similarly if you go to an *all you can eat* pizza buffet, in a formal sense you have access to an infinite quantity of pizza. You choose to consume it to point that the marginal benefit of another slice is zero. That is, you are satiated. The extra pizza you have access to neither improves nor harms your welfare.

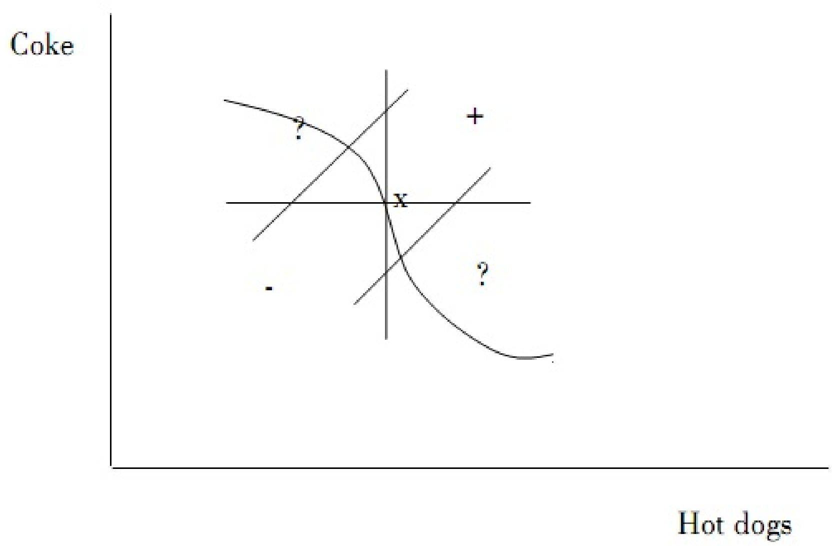
  – **Strong monotonicity** means that you are **never satiated in anything**. From any consumption point, if I increase your allocation of any good, you are made **strictly better off**.

  – Example:

$$\min(x_1, x_2) \text{ satisfies Weak Monotonicity, but not Strong Monotonicity}$$



- – If you are **satiated in all good at any point,** your preferences are **not monotonic**.
  – An example of this is a **bliss point.** At such a point changing your allocation in any direction makes you worse off.
  – Monotonicity rules out thick and upward sloping/ backward ICs. We get



- **Convexity**
  Question: What do we know about the shape of IC?
  – Definitions of the strong and weak preferred sets

$$\begin{aligned} Wpref(x) &\equiv \{z \in X | z \succeq \bar{x}\} \\ Spref(x) &\equiv \{z \in X | z \succ \bar{x}\} \end{aligned}$$

- It is equivalent to ask what we know about the shape of $Wpref(x)$?

Question: Which would you rather have, two pairs of pants or two shirts, or one of each?

Question: 4 slices of Pizza or 4 cans of Coke, or two of each?

Question: How much are you willing to pay for your 1^{st} Pizza? What about your 2^{nd}?

-
  * Average are preferred to extremes and
  * The marginal value of a good declines as you have more of it, including money
    Example: St. Petersburg Paradox
    You toss a coin until it comes up heads. If the first heads shows on the n^{th} toss, you win 2^{n-1} dollars. (Thus the payoff doubles with each coin toss that isn't heads.) How much are you willing to pay to enter this game?

$$\text{Expect Win} = \frac{1}{2} \cdot 1 + (\frac{1}{2})^2 \cdot 2 + +(\frac{1}{2})^3 \cdot 2^2 + \cdots = \infty$$

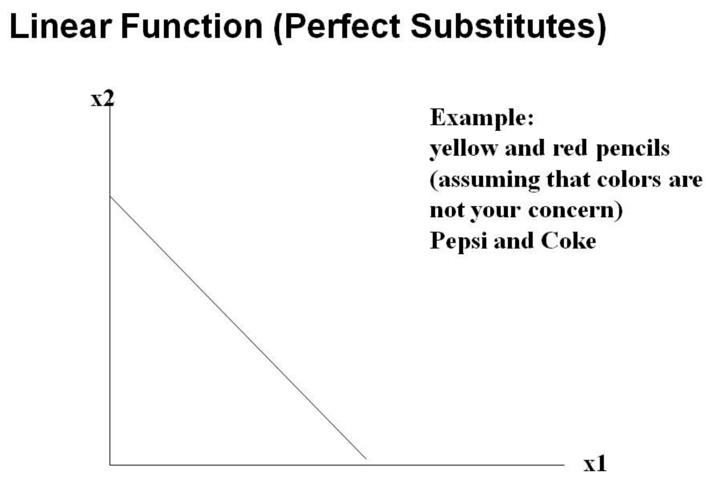
- Formally, we assume convexity: We say the $Wpref(x)$ is
  * **Weak Convex** if

$$\forall \lambda \in [0, 1], \forall x \in X \text{ and } \forall y, z \in Wpref(x), \lambda y + (1 - \lambda)z \in Wpref(x).$$

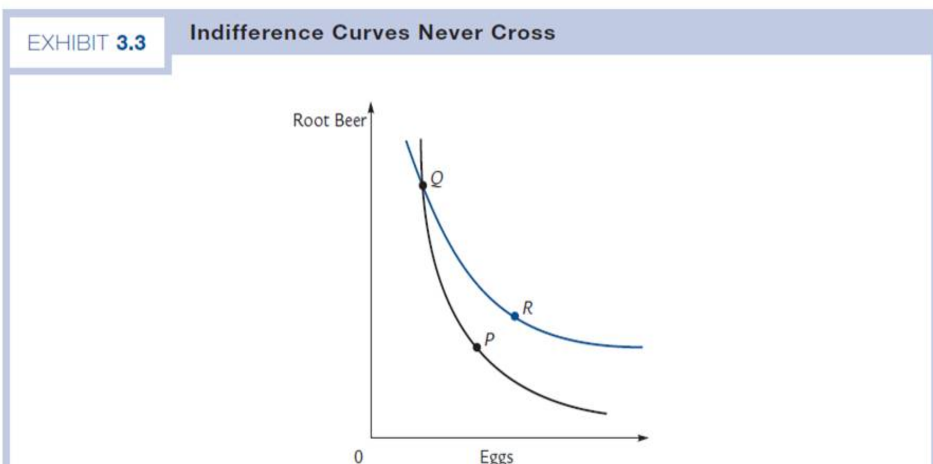
    · A set is **weakly convex** if any mixture between points in the sets is also in the set. By "mixture" we mean more formally a "linear combination" of the points. Graphically, a point is a linear combination of two points if it is somewhere on the line between the points.
    · Another way to think of this is that set is weakly convex if any lines drawn between two points in a set remain entirely within the set.
  * **Strong Convex** if

$$\forall \lambda \in (0, 1), \forall x \in X \text{ and } \forall y, z \in Wpref(x) \text{ and } y \neq z, \lambda y+(1-\lambda)z \in Spref(x).$$

    · A set is **strongly convex** if any **strict mixture** between points in the set is the in the interior of the set (and not on the boundary). A strict linear combination between two point simply excludes the degenerate case of choosing one or the other end of the line ($\lambda \neq 0$ or 1).
- **Strong convexity** means that nondegenerate averages of different consumption bundles are strictly preferred to extremes. **Indifference curves can not be flat** anywhere in this case. Note that **strongly convex preferences are also weakly convex, but weakly convex preferences may not be strongly convex.**



- Combining all the rationality assumption (completeness, reflexivity, transitivity) and behavioral assumptions (monotonicity and convexity), we get an IC with standard shape (Draw), What about the relationships among ICs? **Never Cross!! (Hint: Violating monotonicity)**



Crossing indifference curves, such as those shown in the graph, cannot occur. The consumer likes *P* and *Q* equally well because they are both on the same (black) indifference curve. He also likes *R* and *Q* equally well because they are both on the same (colored) indifference curve. We may infer that he likes *P* and *R* equally well, which we know to be false (in fact, *R* is preferred to *P*). Thus, the graph cannot be correct.

- Marginal Rate of Substitution (MRS)

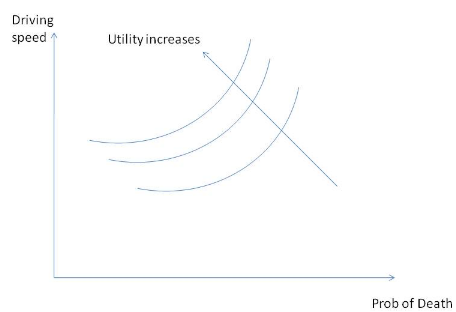
  We know from last section that the ICs slope downward, but what is the meaning of the slope of the ICs?

  – A key idea related to indifference curves is the **marginal rate of substitution between x and y**

- Formally MRS of **x** and **y** at point (x_0, y_0)

$$MRS_{x,y}|_{(x_0,y_0)} = -\frac{\Delta y}{\Delta x}$$

- In the limit, it is the minus the slope of the IC: $MRS_{x,y}|_{(x_0,y_0)} = \lim_{\Delta x \to \infty} -\frac{\Delta y}{\Delta x} = -\frac{dy}{dx}$
- Steeper slope, higher MRS;
- Strict convexity of IC implies decreasing MRS; weak convexity of IC implies non-increasing MRS;
- Let's practice with **ICs** for different cases
  - Example 1: Right shoes and left shoes, perfect complements (one example of Kinky preference);
  - Example 2: Coke and Pepsi, perfect substitutes;
  - Example 3: Reckless driving and Probability of death? (Goods and Bads)



2 From Preferences to Utility

In the last section, we said a lot about preferences. Unfortunately, **all of that stuff is not very useful** in analyzing consumer behavior, unless you want to do it one bundle at a time. However, if we could somehow **describe preferences using mathematical formulas**, we could use math techniques to analyze preferences. The tool we will use to do this is called utility functions.

- A utility function is a function $U(x)$ that assigns a number to every consumption bundle $x \in X$

$$U(\cdot) : \Re^n_+ \to \Re$$

  - Utility function $U(\cdot)$ represents preference relation $\succeq$ if for any x and y, $U(x) \geq U(y)$ if and only if $x \succeq y$.

- Preferences represented by a **binary relation** $\succeq$, by **indifference curves** and by **utility functions** are all equivalent as long as **the preference satisfies rationality assumptions**.
- E.g. The following statements are equivalent: Considering a bundle $(x, y) \in X$ and $(x', y') \in X$
  * (x, y) and (x', y') are on the same IC;
  * $(x, y) \sim (x', y')$;
  * $U(x, y) = U(x', y')$

• Since ICs and utility functions are equivalent, what is the equivalence of MRS using a utility function?

- $MRS_{x,y}|_{(x_0,y_0)} = \lim_{\Delta x \to \infty} -\frac{\Delta y}{\Delta x} = -\frac{dy}{dx}$
- **Indifference** curve $\to U(x, y) = \bar{U}$ (a constant)

$$dU(x,y) = \frac{\partial U(x,y)}{\partial x}|_{(x_0,y_0)}dx + \frac{\partial U(x,y)}{\partial y}|_{(x_0,y_0)}dy = 0$$

$\Longrightarrow$

$$MRS_{x,y}|_{(x_0,y_0)} = -\frac{dy}{dx} = \frac{\frac{\partial U(x,y)}{\partial x}}{\frac{\partial U(x,y)}{\partial y}}|_{(x_0,y_0)} = \frac{MU_x}{MU_y}|_{(x_0,y_0)}$$

• Examples of Utility functions (Draw ICs):

- Cobb-Douglass Utility function $U(x, y) = x^{1/2}y^{1/2}$
- Leontief Utility function $U(x, y) = \min(x, y)$
  * perfect complements
- Linear Utility function $U(x, y) = x + y$ or $x + ay$,where a is a constant.
  * perfect substitutes

• What if we multiplied utility functions by two? What if we added one? It would be the same.

- Utility functions are **ordinal**. We can transform them provided we don't change the shape of the IC's they generate or alter the order of the numbering system;
- Graphically, just reassign numbers to IC's;
- Mathematically: Any monotone transformations $F(U(x, y))$, where $F(\cdot)$ is a monotone function of a utility function, is equivalent in terms or the preferences it represents.
- Economists say:
  * Utility functions are ordinal.
  * Utility functions are unique up to a monotone transformation.

Lecture Five (Chapter 3)
Budget Constraints and Optimal Choice

1 Budget Constraints

- Consider a simple case of two goods, x and y. Suppose we know the prices: p_x and p_y, and the income is I.

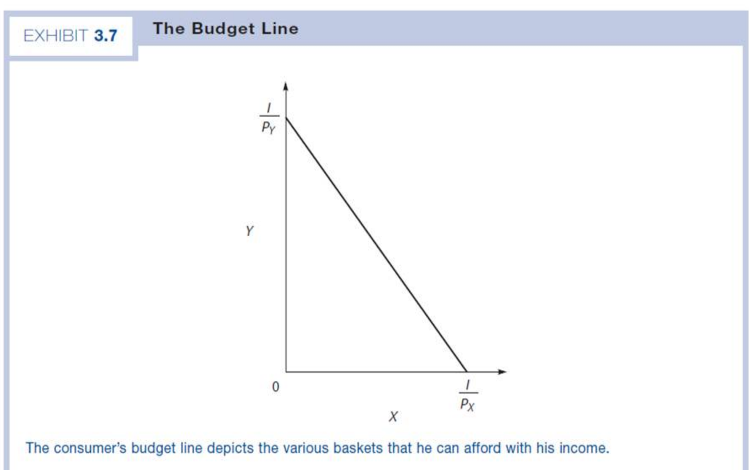
Then our choice must satisfy the following budget equation

$$p_x \cdot x + p_y \cdot y \leq I$$

- **Budget set** $B(p, I) \equiv \{(x, y) \in \Re^2_+ | p_x \cdot x + p_y \cdot y \leq I\}$
- Furthermore, due to monotonicity assumption (more is better), consumers always want to spend all their income. (Draw)

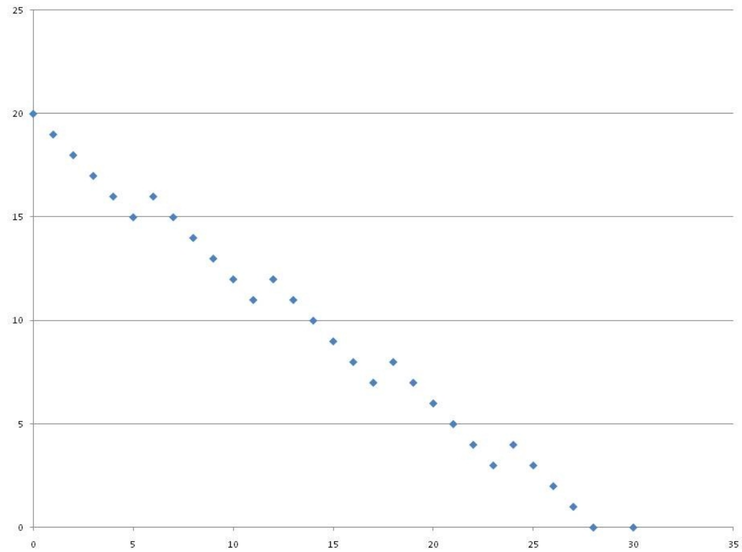
$$p_x \cdot x + p_y \cdot y = I$$

  - This is also called **budget line**, with slope $-\frac{p_x}{p_y}$.



- **Draw the following cases**
  - What happens if p_x changes?
  - What happens if p_y changes?
  - What happens if both p_x and p_y change?
  - What happens if I changes?

- Budget line might not be linear.
  - Example: A can of coke costs \$1, but six pack cost \$4 with income of \$20 and other goods cost \$1 each;



2 Optimal Choice (Graphical Analysis)

- Find the optimal consumption bundle, which gives the highest utility.

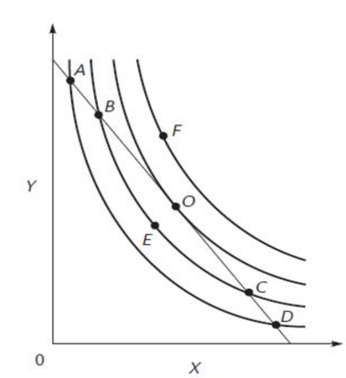


EXHIBIT **3.8** **The Consumer's Optimum**

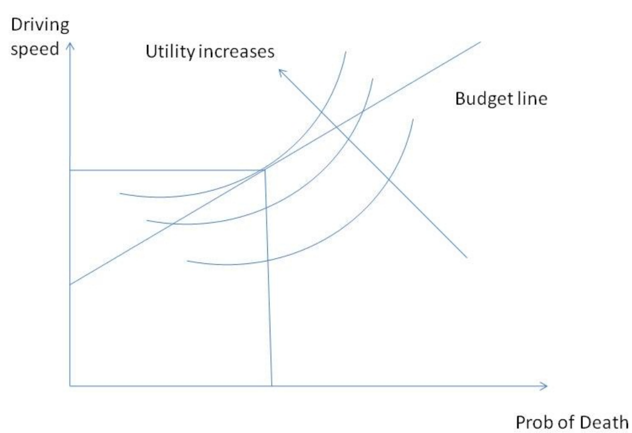
The consumer must choose one of the baskets that is on his budget line, such as *A*, *B*, *O*, *C*, or *D*. Of these, he will choose the one that is on the highest indifference curve, namely, *O*. Thus, the consumer is led to choose the basket at the point where his budget line is tangent to an indifference curve. This point is called the consumer's optimum.

At the consumer's optimum, the relative price of *X* in terms of *Y* (given by the slope of the budget line) and the marginal value of *X* in terms of *Y* (given by the slope of the tangent line to the indifference curve) are equal. The geometric reason for this is that the budget line is the tangent line to the indifference curve. The economic reason for it is that whenever the relative price is different from the marginal value, the consumer will continue to make exchanges until the two become equal.

- Intuition: Given a consumption bundle x, if you can afford another consumption bundle that gives you a higher utility, then x is not your optimal consumption bundle.

  - If a point is optimal, there will be no intersection between the feasible set and the Strongly preferred set.
  - **For the normal cases**, the optimal choice will be where the budget set and IC are tangent.
  - **For the normal cases**, It is equivalent to say that the slopes of the IC and budget line are the same

- Let's find the optimal choice with different budget lines and ICs
  - Example 1 (IC): Right shoes and left shoes, perfect complements (also kinky);
  - Example 2 (BL & IC): Reckless driving and Probability of death? (Goods and Bads)



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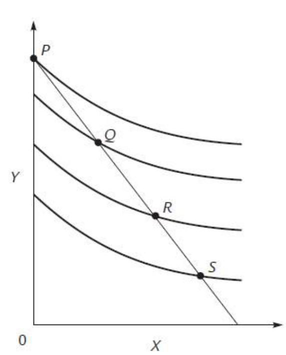
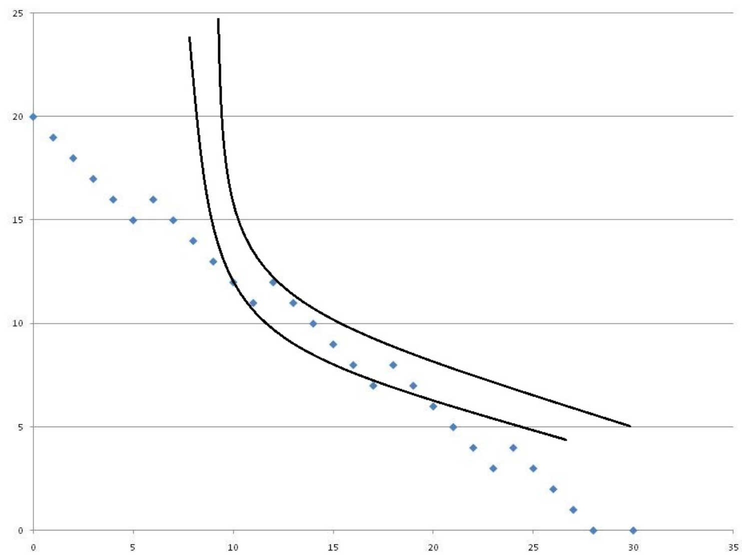
  - Example 3 (BL & IC): Case of corner solution e.g 1. normal IC 2. Coke and Pepsi when prices, perfect substitutes;

EXHIBIT 3.9 **A Corner Solution**



If the consumer's indifference curves look like those pictured, there is no tangency between his budget line and any of his indifference curves. Of all the points on the budget line, the consumer will choose the most desirable, namely, *P*. At any other point on the budget line the marginal value of *X* in terms of *Y* is less than the relative price, so the consumer can sell *Xs* for more than they are worth to him and will continue to do so until he has sold all of his *Xs*, ending up in the corner at *P*.

-
  - Example 4 (nonlinear BL): A can of coke costs \$1, but six pack cost \$4 with income of \$20 and other goods cost \$1 each;
  - Could have more than one optimal solution



3 Optimal Choice (Mathematical Analysis)

- **Question:** What do we mean mathematically when we say "*for normal cases, It is equivalent to say that the slopes of the IC and budget line are the same*"?

$$\frac{dy}{dx} = -\frac{p_x}{p_y} \Rightarrow MRS_{x,y}|_{(x^*,y^*)} = \frac{MU_x}{MU_y}|_{(x^*,y^*)} = -\frac{dy}{dx} = \frac{p_x}{p_y}$$

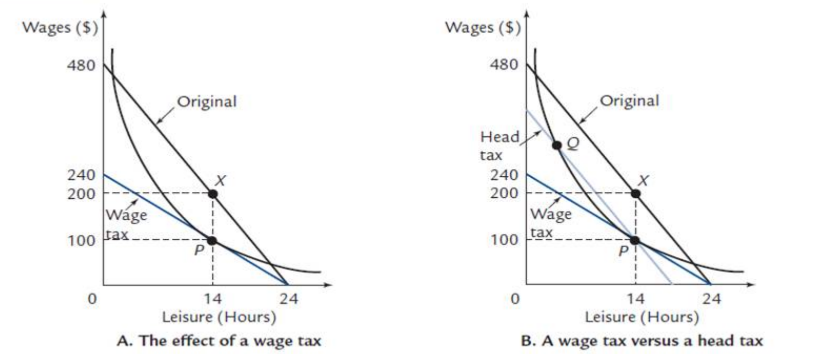
- Given the prices $p_x = 1, p_y = 2$, and income $I = \$10$, solve the following examples:
  - Example 1: Cobb-Douglas Utility function $U(x,y) = x^{1/2}y^{1/2}$
    * $x = \frac{I}{2p_x} = 5; y = \frac{I}{2p_y} = 2.5;$
  - Example 2: Leontief Utility function $U(x,y) = \min(x,y)$;
    * **CANNOT** use FOC: $x = y = \frac{10}{3}$;
  - Example 3: Linear Utility function $U(x,y) = x + y$;
    * **CANNOT** use FOC: $x = 10, y = 0$.

4 Price Indices & Cost of Living

- Let's find new optimal choice when budget line moves. Whether the change is good or bad for consumers?

– Example 1: Income decreases, prices not changes; Worse. e.g, Case of a Head Tax.

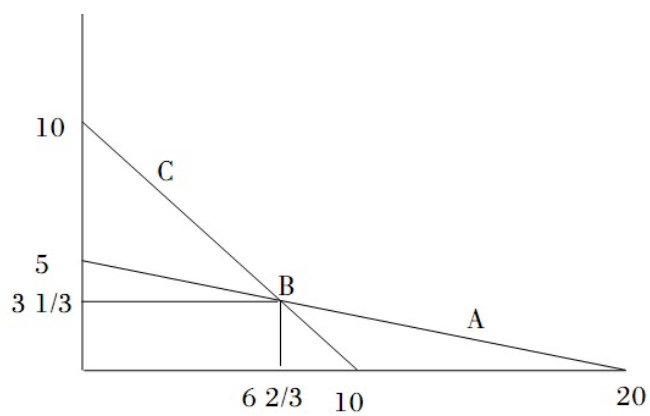
EXHIBIT **3.14** **A Wage Tax versus a Head Tax**



You have 24 hours a day to divide between leisure and working at a wage of $20 an hour; this yields the *Original* budget line. When your wages are taxed at 50%, the budget line pivots to the *Wage Tax* line in panel A, and you choose point *P*.

If the wage tax is replaced by a head tax that collects the same number of dollars, the new budget line must be parallel to the *Original* (because it represents a head tax) and must pass through point *P* (because it raises the same number of dollars as the wage tax). This yields the *Head Tax* line in panel B, which enables you to reach a higher indifference curve (with a tangency somewhere between *P* and *Q*). Thus the head tax is preferable to the wage tax.

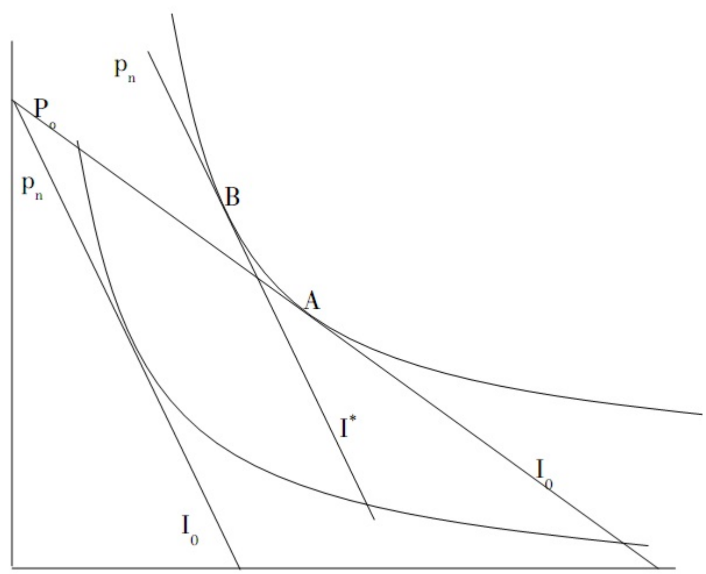
- – Example 2: Income not change, one price changes; Case of a Wage Tax.
  – Compare Example 1&2. Why do we use wage tax rather than head tax?
    * Head tax requires that the rich and the poor pay the same amount of tax. Contradict to the principle of tax, **"ability to pay"** based tax;
  – Example 3: Income not change, both prices change
    * Both increase, worse; both decrease, better;
    * One increases, one decrease, not sure.
      e.g Suppose income is 10 and prices go from $(1,1)$ to $(1/2,2)$
      · If original optimal point is at B, new prices better;
      · If original optimal point is at A or C parts, new prices uncertain;



- From the above example, we know usually we cannot say for sure **how** people are affected by price changes and **by how much**. A natural measure is the **change of the cost of living.** When the cost of living increases for a person, he/she is worse off.
  - Cost of Living: the cost of a given basket, $I = PQ = \sum_{i=1}^{n} p_i q_i$;
  - Increase of the cost of living: **price index** $\frac{I'}{I_0} > 1$;
  - A perfect price index would take the level of income required to get back to the **old IC under new prices,** and divide that by the original income:

$$PPI = \frac{I^*}{I_0}$$

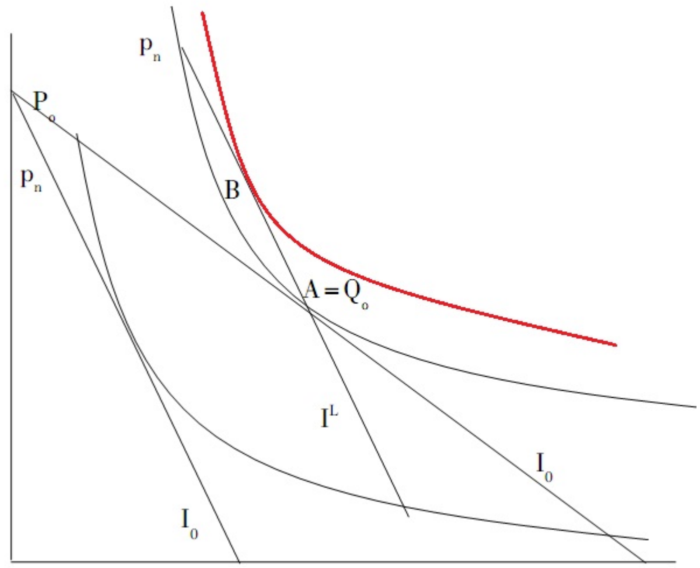
  - The cost of the new bundle B under new prices divided by the cost of the old bundle A under old prices.



- **Problem:** the new bundle B is not observable in the real life. In other words, we don't have any data about B.
  - Alternative: **Laspeyre price index & Paashe Price index**.
- **Laspeyre price index:**

$$LPI = \frac{P_n Q_0}{P_0 Q_0}$$

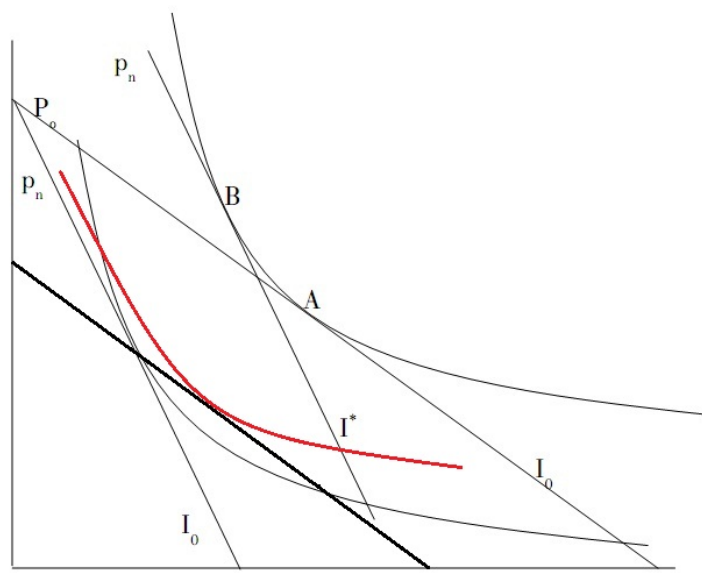
  - The idea is to find how much income it would take to afford the **old choice (rather than old IC)** of consumption bundle under **new prices** compared to old prices



- - Notice that LPI over compensates for the price change since better points are always available under this Laspeyre income.
  - This is how the BLS calculate the **CPI (Consumer Price Index).**
- **Paashe price index:**

$$PaPI = \frac{P_n Q_n}{P_0 Qn}$$

  - The only difference is the reference bundle. We use **new bundle** (chose under the new price) rather than the old choice.



- - Notice that $PaPI$ under compensates for the price change since we compare the new IC with an IC with higher utility than the original one.

Lecture Six (Chapter 4)
Consumers in the Marketplace

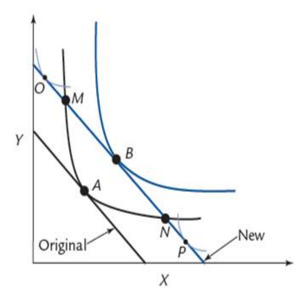
In this lecture, we will see how the change of income and the change of prices affect our optimal decision on the quantity demanded. After that, we can derive the demand function from the consumers' optimal decision.

We will concentrate on one good, denoted by X. And we use Y to denote "all other goods" with price \$1, and you can think of it as money.

1 Change in Income I

- **A parallel shift of the budget line**
  - Prices are the same, so the slope of budget line does not change.
  - Rise in income, move outward;
  - Fall in income, move inward;
  - Move of the optimal point (Income Expansion Path).

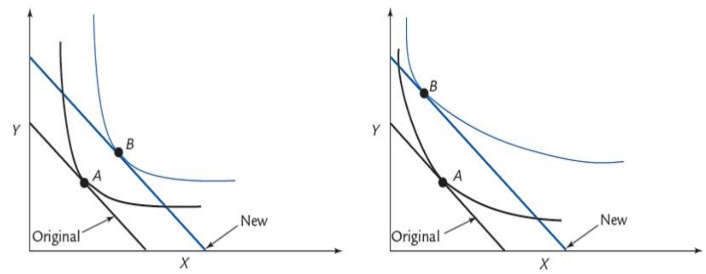
EXHIBIT **4.2** **A Rise in Income**



An increase in income causes the budget line to shift outward. If the original tangency is at *A*, then the new tangency cannot be at *O* or *P*, as either possibility would require two indifference curves to cross. (The curves that are shown tangent at these points cannot be indifference curves because they must cross the original black indifference curve.) Instead, the new tangency is at a point like *B*.

- Two possible positions of the new optimal point (tangent point)
  - normal good: a good that you consume more of when your income rises, $\frac{dQ^d}{dI} > 0$;
    - electricity, food, jewelry;
  - inferior good: a good that you consume less of when your income rises, $\frac{dQ^d}{dI} < 0$,
    - McDonald.

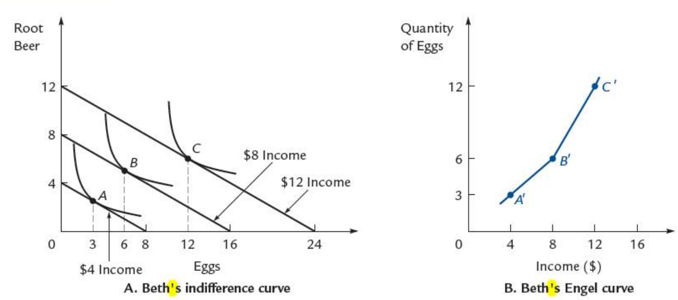
EXHIBIT 4.3 **Normal and Inferior Goods**



Suppose your original tangency is at *A* and your income increases. Then your new tangency *B* could be either to the right of *A* (as in the first panel) or to the left of *A* (as in the second panel). In the first case, a rise in income leads you to consume more *X* and we call *X* a normal good. In the second case, a rise in income leads you to consume less *X* and we call *X* an inferior good.

- The **Engel curve**
  - Given prices, the **relationship between income and the quantity** of good consumers decided to consume.
  - Rational decision
  - Normal good: upward sloping, $\frac{dQ^d}{dI} > 0$;
  - Inferior good: downward sloping, $\frac{dQ^d}{dI} < 0$.

EXHIBIT 4.4 **Constructing the Engel Curve**



Points *A*, *B*, and *C* in the first panel show Beth's optima at a variety of incomes. (The prices of eggs and root beer are held fixed at 50¢ and $1, respectively.) Points *A′*, *B′*, and *C′* in the second panel record the quantity of eggs that Beth consumes for each of three incomes; these quantities are the horizontal coordinates of points *A*, *B*, and *C*. The curve through *A′*, *B′*, and *C′* is Beth's Engel curve for eggs.

2 Changes in Price of X

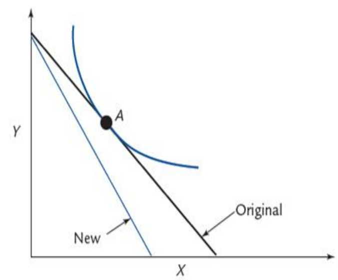
2.1 Total Effect

- **Budge line pivot around its $Y-$intercept.**
  - Price of Y does not change, so the intercept on Y axis is the same;
  - Rise in price of X, budget line pivots inward;

– Fall in price of X, budget line pivots outward;

– Move of the optimal point (Price Expansion Path).

EXHIBIT **4.6** **A Price Increase**



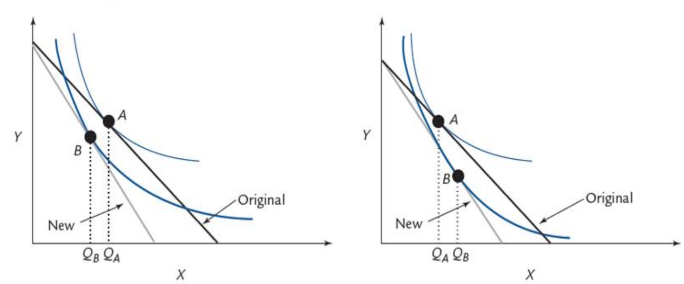
A rise in price causes the budget line to pivot inward. The original optimum is at *A*, and the new optimum could be anywhere at all on the new (brown) budget line.

- Two possible positions of the new optimal point (tangent point)

  – Ordinary good: a good that **obeys the Law of Demand**, $\frac{dQ^d}{dp} < 0$;

  – Giffen good: a good that **violates the Law of Demand**, $\frac{dQ^d}{dp} > 0$.

EXHIBIT **4.7** **Ordinary Goods and Giffen Goods**



When the price of *X* goes up, the budget line pivots inward. The optimum moves from point *A* to point *B* and the quantity of *X* that you demand changes from Q_A to Q_B. In the first panel, Q_B is less than Q_A; in other words "when the price goes up the quantity demanded goes down," as required by the law of demand. In the second panel, Q_B is greater than Q_A, so that "when the price goes up the quantity demanded goes *up*," in violation of the law of demand. When the law of demand is violated, *X* is called a Giffen good.

- The **demand curve**

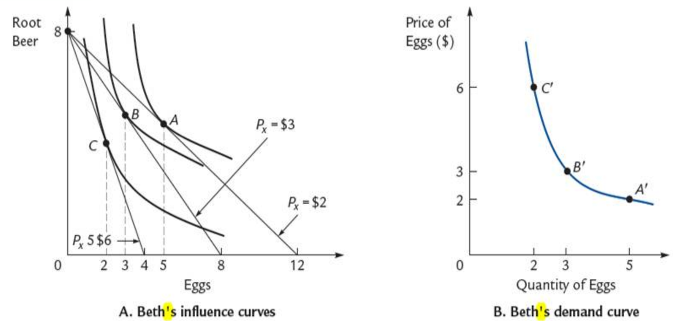
  – Given income and the prices of all other goods, the **relationship between price and the quantity** of good consumers decided to consume.

  – Rational decision

  – Ordinary good: upward sloping, $\frac{dQ^d}{dp} < 0$;

  – Giffen good: downward sloping, $\frac{dQ^d}{dp} > 0$.

EXHIBIT **4.8** **Constructing the Demand Curve**



When the price of eggs is $2 apiece, Beth chooses basket *A*, with 5 eggs. This information is recorded by point *A′* in the second panel. Points *B′* and *C′* are derived similarly. The curve through *A′*, *B′*, and *C′* is Beth's demand curve for eggs.

2.2 Separate Income Effect and Substitution Effect

Let's explore more about how an agent responds to price changes. For example, you have an annual income of $\$70,000$ but no house, when the price of houses declines, say from $\$100,000$ to $\$50,000$, you might say "I can afford a house now", that is

- Income effect: you have a larger budget set. In other words, you are wealthier.

  You might also say "Considering the relative prices of a car and a house, I decide to buy a house this year instead of a car", that is

- Substitution effect: A good is getting cheaper. Naturally we want to consume more of it.

Therefore, the observed effect of a change in the price of X is a combination of two effects:

- 1. Substitution effect: how the change of relative price alone impacts the quantity demanded of a good.
  2. Income effect: how the change of wealth alone impacts the quantity demanded of a good.

  – **Different from the direct change of income in section 1;**

- **Question: how can we separate the two effects?**

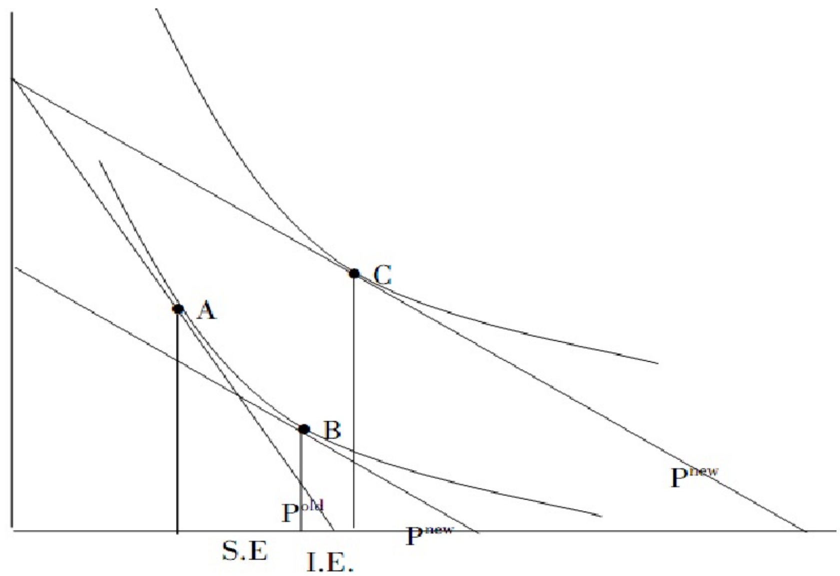
  – The existence of income effect is **because you are wealthier** after prices decline, **or poorer** after prices increase.

  – If I can **keep you feel the same** after the change of price, we actually get rid of income effect.

  – Intuitively, by saying "**keep you feel the same**", I force you to stay on the original IC, but with new relative price.

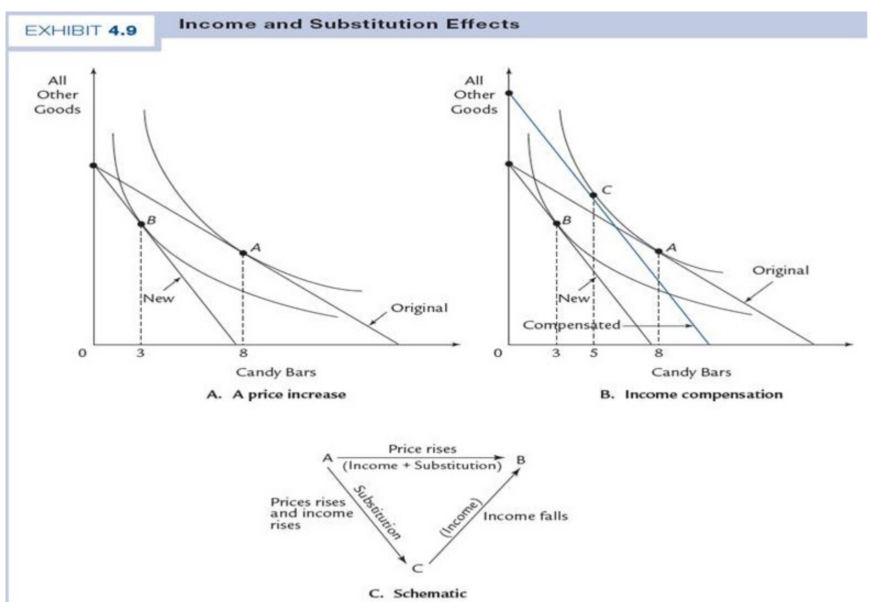
– The idea is an artificial thought experiment. We can add back the income effect by shifting the budget line in parallel later.

1. – **Example 1: Price goes down;**



- 1. – **Example 2: Price goes up;**

  – **Question:** Same substitution effect and same income effect? No, also stay on the original IC first with new price, then move to the new IC in parallel.

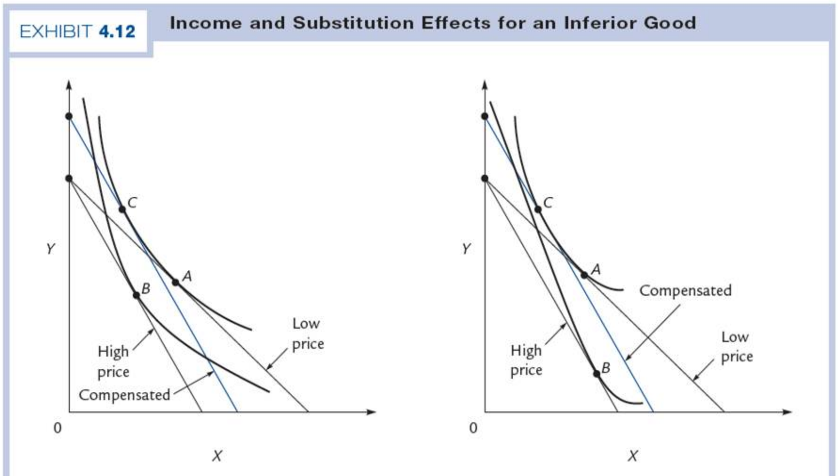


- When price falls ↓, **substitution effect always leads to increase** of consumption; income effect is ambiguous. Three possible combined effect:

– **substitution** $\uparrow$ and **income** $\uparrow$ (normal good) $\Longrightarrow$ final effect $\uparrow$;
– **substitution** $\uparrow$ and **income** $\downarrow$ (inferior good), and $sub > income \Longrightarrow$ final effect $\uparrow \Rightarrow \frac{dQ^d}{dp} < 0$ (Ordinary good);
– **substitution** $\uparrow$ and **income** $\downarrow$ (inferior good), and $sub < income \Longrightarrow$ final effect $\downarrow \Rightarrow \frac{dQ^d}{dp} > 0$ (Giffen good);

- **Observations:**
  – Giffen good has to be an inferior good;
  – An inferior good could be an ordinary good;
  – So, the demand curve for an inferior good could be upward sloping or downward sloping.

- Draw three cases



In both panels, X is an inferior good; that is, B is to the right of C. In the first panel, X is ordinary (i.e., not Giffen); that is, B is to the left of A. In the second panel, X is Giffen; that is, B is to the right of A.

- Close investigation of Giffen goods.

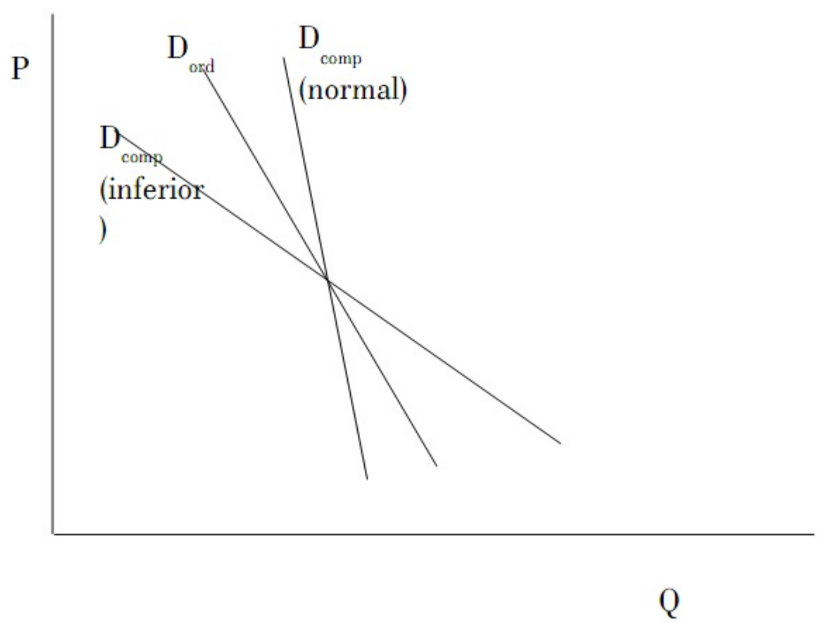
  Question: Is there any example in the real life that justifies the existence of Giffen goods.

  – **Snob effect:** the price of designer clothes is high and this makes them exclusive. In fact, could it take place that as the price increases they get more exclusive and so more people buy them? No! If this happened then they would be less exclusive since more people had them, this would mean that fewer people would want them at this price. **Designers might make more money at higher prices, but they don't sell more clothes.**
  – **Price as a signal of quality (computer prices in Bestbuy)**: people go to a high end audio store, but can't tell what is quality. They view the price of a

component as indicting its quality. Thus, a high price creates its own demand in some sense. Can this happen? Maybe, but it turns out that more people are discouraged by the high price than are encouraged by any quality signal it might convey in the real world.

- **Strong negative income effect**
  - **negative**: the good must be inferior;
  - **Strong income effect:** represent a large fraction of your spending. For example, if the cost of housing, or food or tuition went up, you might be significantly worse off. Note, however, that these items are almost always normal goods. Thus, they can't be Giffen.
  - The one case where economists thought they found a Giffen good was the case on potatoes in nineteenth century Ireland. This turned out to wrong on closer investigation.

- We have studied the substitution effect and income effect together here. In the first section, we have studied the income effect alone. **Can we study substitution effect in isolation? YES!**
  - When we do this, we get something called **compensated demand curve**.
  - Compensate for the change in welfare in order to **stay on the original indifference curve**.
  - **demand curve** v.s **compensated demand curve**
    - **demand curve: income fixed, utility variable;**
    - **compensated demand curve: utility fixed, income variable;**
  - Draw cases of inferior and normal goods.



3 Mathematical Summary

3.1 Consumer's Optimization Problem

Consider a standard consumers' problem:

$$\max_{x_1,x_2} U(x,y) \quad s.t.\ p_x x + p_y y = I$$

$$\begin{aligned} x &= \frac{I - p_y y}{p_x} \text{ and } FOC: \frac{dU}{dx} = MU_x(-\frac{p_y}{px}) + MU_y = 0 \\ MRS &= \frac{MU_x}{MU_y} = \frac{p_x}{p_y} \end{aligned}$$

- Assuming an explicit function form of utility $U(x_1, x_2) = x^{\frac{1}{2}} y^{\frac{1}{2}}$

$$\max_{x_1,x_2} x^{\frac{1}{2}} y^{\frac{1}{2}} \ p_x x + p_y y = I$$

$$\max_{x_1,x_2} (\frac{I - p_y y}{p_x})^{\frac{1}{2}} y^{\frac{1}{2}}$$

  FOC gives $y = \frac{I}{2p_y}$ and $x = \frac{I}{2p_x}$

- **Question: What is this? All of the following:**
  - a demand curve;
  - an Engle curve;
  - A PEP;
  - An IEP.
- We know

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{p_x}{p_y} = -\text{slope of IC} = -\text{slope of budget line}$$

3.2 Sensitivity Analysis

- Sensitivity of changes.
  - Sensitivity: What is the effect of the change of a variable on other variables;
  - e.g. \$1 increase of price leads to a decrease of quantity demanded by 100;
  - different implication for bread and cars.
  - A better measure is to ask **what percentage change in quantity is associated with what percentage change in price**.

- Price elasticity:
  - Own price elasticity:

$$\varepsilon_{xx} = |\frac{\Delta Q_x/Q_x}{\Delta p_x/p_x}| = |\frac{\Delta Q_x}{\Delta p_x} \cdot \frac{p_x}{Q_x}|$$

    In the limit:

$$\varepsilon_{xx} = |\frac{dQ_x}{dp_x} \cdot \frac{p_x}{Q_x}| = |\frac{d\ln Q_x}{d\ln p_x}|$$

    * perfect elasticity (∞) and perfect inelasticity (0).
  - Cross price elasticity:

$$\varepsilon_{xy} = \frac{\Delta Q_x/Q_x}{\Delta p_y/p_y} = \frac{\Delta Q_x}{\Delta p_y} \cdot \frac{p_y}{Q_x}$$

    In the limit:

$$\varepsilon_{xy} = \frac{dQ_x}{dp_y} \cdot \frac{p_y}{Q_x} = \frac{d\ln Q_x}{d\ln p_y}$$

    * $\varepsilon_{xy} > 0$ means x and y are substitutes;
    * $\varepsilon_{xy} < 0$ means x and y are complements;
- Income elasticity:

$$\eta_{xI} = \frac{\Delta Q_x/Q_x}{\Delta I/I} = \frac{\Delta Q_x}{\Delta I} \cdot \frac{I}{Q_x}$$

  In the limit:

$$\eta_{xI} = \frac{dQ_x}{dI} \cdot \frac{I}{Q_x} = \frac{d\ln Q_x}{d\ln I}$$

  - $\eta_{xI} < 0$ means the good is inferior;
  - $\eta_{xI} > 0$ means the good is normal;
  - $\eta_{xI} > 1$ means the good is luxury;

Lecture Seven (Chapter 5 & 6) Theory of the Firm

From Chapters 3 and 4, we have already figured out where demand curves come from. Now let us figure out where supply curves come from.

From an individual firm's standpoint, goods are divided between **inputs (also called factors of production)** and **outputs (also called consumption goods)**.

- Input: Capital, Labor, Land etc.
- Output: All the consumption goods we discussed before.

First, let us compare the problems of consumers and firms and discuss the similarity and differences between them

1 Consumer v.s Firm

1.1 Similarities

First of all, keep in mind that consumers and firms are both economic agents. So we study the **rational behavior** of them.

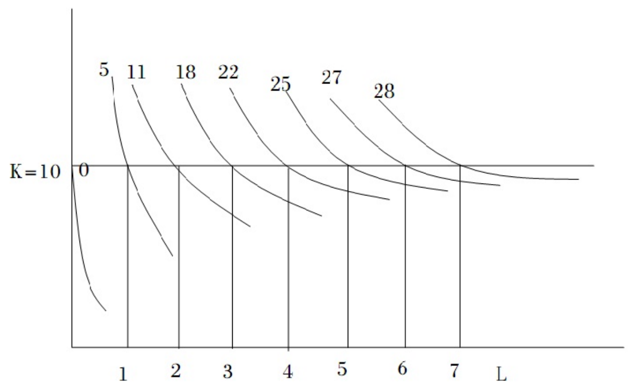
- People are represented by a set of preferences.
    - summarized by utility function: $U(x, y)$
        * Function form
        * MU_x: marginal utility gained from x;
        * diminishing marginal utility
    - drawn in graph as indifference curves (also isoutility curves): $U(x, y) = \bar{U}$
    - indifference curve is the set of bundles of consumption goods that gives you a given level of utility
    - Slope of IC: $MRS_{xy} = -\frac{dy}{dx} = \frac{MU_y}{MUx}$
        * the amount of y that can be substituted for one unit of x in order to hold utility constant.
        * diminishing MRS: equivalent to the assumption of convexity of preferences
    - preferences are monotonic:
        * utility function is increasing in every of its arguments: $U'(\cdot) > 0$;
        * more is better: more consumption good could not hurt you. You can throw it away if you don't like it, e.g. 5th pizza.
- Firms are represented by a set of technological possibilities.

- summarized by production function: $f(L, K)$
  - Function form
  - MPL: Marginal products of labor: $\frac{\partial f(L,K)}{\partial L}$
  - diminishing marginal products
- drawn in graph as isoquant curves: $f(L, K) = \bar{y}$
- isoquant is the set of bundles of inputs that produce a given level of output.
- Slope of isoquant curve: $MRTS_{LK} = -\frac{dK}{dL} = \frac{MPL}{MPK}$
  - the amount of K that can be substituted for one unit of L in order to hold output constant.
  - diminishing $MRTS$: equivalent to the assumption of convex Isoquants (Isoquants are also complete and transitive)
- A technology is monotonic
  - the production function is increasing in every of its arguments.
  - if the firm can costlessly dispose of any inputs, having extra inputs around cannot hurt the firm

1.2 Differences

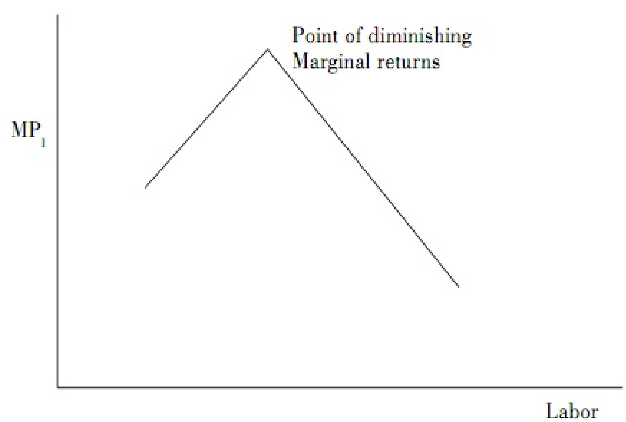
- Preference or utility function
  - Ordinal utility
  - Objective: utility maximization, there is no expenditure minimization because the expenditure will equal to the income level I.
  - Budget constraint
  - Do not discuss time horizon
    - It is interesting to discuss short run and long run decisions for a consumer.
    - Decision on durable goods e.g. House v.s Food.
- Technology or production function
  - Cardinal production
  - Objective: profit maximization or cost minimization
  - No budget constraint
  - Essential to consider different time horizon
    - Short run: a period of time over which at least some inputs are fixed;
    - Long run: a period of time over which all inputs are variable.
      - "In the long run, we're all dead." John Maynard Keynes.

* NOTICE **that there are many potential short runs (some input is fixed for one short run, but variable for another one), but only one long run.**
  E.g. $f(L, \bar{K})$. In the short run, Capital is fixed at 10 units, while labor is a variable factor. It would take 10 units of capital and 6 units of labor to produce 27 Frisbees.



- One thing we can learn from this is **change of the marginal product of labor**. We can see **the incremental effect** on output of one more unit of labor for a fixed capital level.
  - We can see the typical pattern of gains from specialization at first;
  - followed by diminishing marginal products.

Q_1	TP_1	MP_1
1	5	5
2	11	6
3	18	7
4	22	4
5	25	3
6	27	2
7	28	1



2 Objectives of Firms

From the comparison above, we know one of the main differences between consumers and firms is the objective they want to achieve. For firms, the objective is to maximize their profit given the technology constraints $f(L, K)$ or to minimize their cost given production level $\bar{y}$ and technology constraints $f(L, K)$.

2.1 Profit Maximization

- Profits are defined as total revenues minus total cost.
- Total Revenue of firms: Money received from selling output goods;
- Total Costs of firms
  - Cost of hiring Capital: rental price;
  - Cost of hiring Labor: wage.
  - **Question: Is it harder to break into an industry when the incumbents already own their capital?**

For example, suppose that some guy owns a block on Miami beach. This land is worth $\$100,000$. Does he have an advantage over you in starting a hotel there? Will his hotel be able to charge lower prices and make more profits because he does not have to pay for land?

NO! Everybody still have the same opportunity costs.

- Profit Maximization

$$\begin{aligned}\pi &= \text{Total Revenue} - \text{Total Cost} \\ &= pf(L, K) - wL - rK\end{aligned}$$

where p is the price of the consumption goods the firm produces; w is the price of labor, i.e. wage; r is the price of capital, i.e., rental price or interest.

- Long-Run Profit Maximization

$$\max_{(L,K)} pf(L, K) - wL - rK$$

- Short-Run Profit Maximization

$$\max_{L} pf(L, \bar{K}) - wL - r\bar{K}$$

2.2 Cost Minimization

- Long-Run Cost Minimization

$$\begin{aligned} & \min_{(L,K)} wL + rK \\ s.t.\ f(L,K) &= \bar{y} \end{aligned}$$

- Short-Run Cost Minimization

$$\begin{aligned} & \min_{L} wL + r\bar{K} \\ s.t.\ f(L,\bar{K}) &= \bar{y} \end{aligned}$$

As we can see, the level of labor used to produce $\bar{y}$ is uniquely determined by the technology constraints given the level of capital used $\bar{K}$. Therefore, in effect, there is not a short-run cost minimization problem.

3 Costs of Firms

We have talked about **physical production**, now let us turn to the **costs of production**.

3.1 Fixed Cost v.s. Variable Cost

Given the discussion about the difference between decisions made by consumers and by firms, we know one of the essential difference is the consideration of time horizon in firm's decision. There we need to distinguish two kinds of cost:

- Fixed cost FC: Costs that do not vary with the quantity of output.
  - Cost of fixed input, e.g Capital
  - This are no fixed inputs in the long run, then there is no fixed cost in the long run;
- Variable cost VC: Costs that vary with the quantity of output.
  - Cost of variable input, e.g. Labor
  - Every input is variable in the long run, then there is only variable cost in the long run;
- **Question:** suppose we have spent \$100 billion of research and development to produce the new B-2 stealth bomber. We only have 4 prototypes, but all the technology has been developed and the production line set up. Given this, we could have another 100 B-2's for only \$4 billion more.

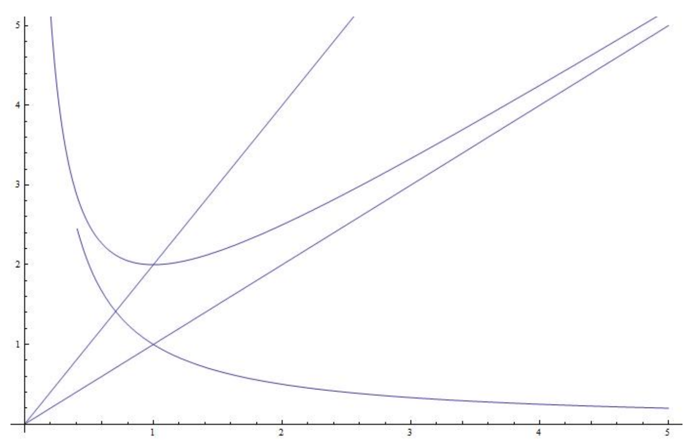
- What is the fixed cost? \$100 billion of research and development.
  * Sunk cost: a realized fixed cost.
- What is the variable cost? \$40 million per piece of B-2.

3.2 Other Notions of Costs

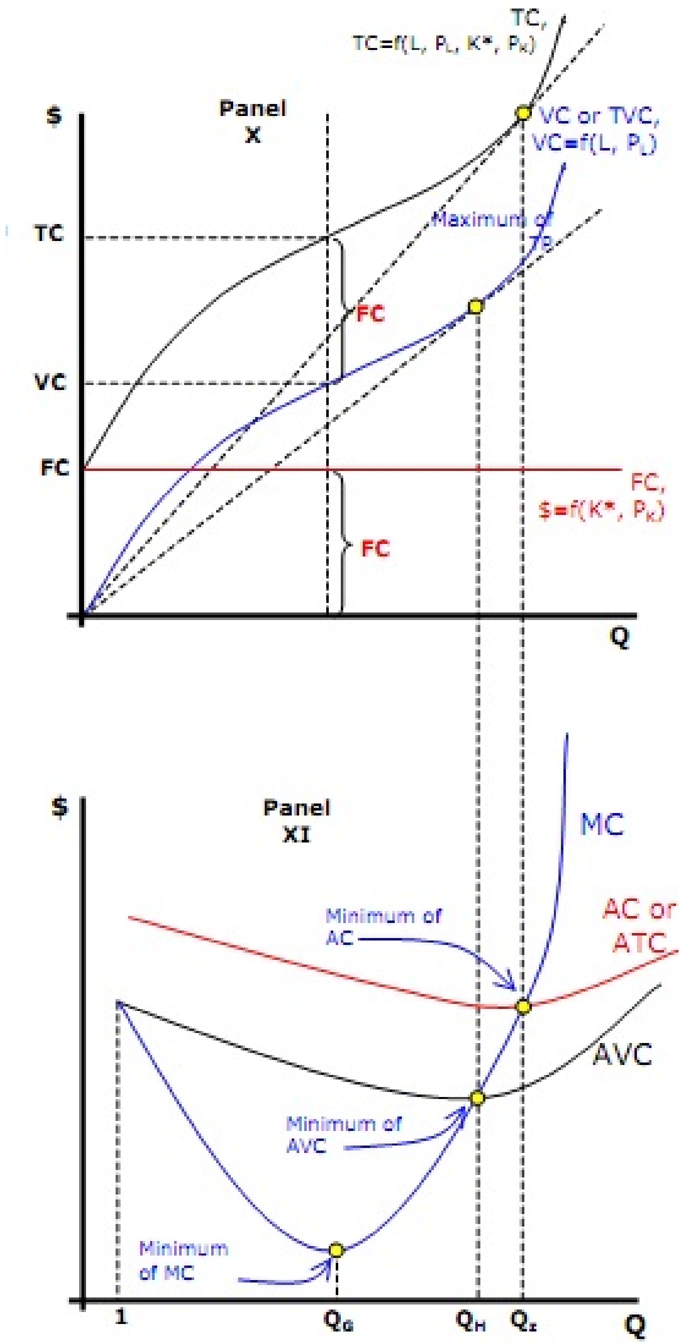
- Total Cost: $TC = FC + VC$;
- Average Cost: $AC = \frac{FC+VC}{Q} = AFC + AVC$;
- Average Fixed Cost: $AFC = \frac{FC}{Q}$;
- Average Variable Cost: $AVC = \frac{VC}{Q}$;
- Marginal Cost: $MC = \frac{dTC}{dQ} = \frac{dVC}{dQ}$ since $\frac{dFC}{dQ} = 0$

3.3 The Shape of the Cost Curves

- Example $TC(Q) = Q^2 + 1$



- Relationship between TC, AC, AFC, AVC *and* MC.



- Relationship between MC and AC: AC is the area under MC.

4 Profit Maximization by Choosing Output Q

$$\max_{Q} \pi = TR - TC$$

4.1 Graphical Solution

- From TC and TR graphs
  - Profit is the gap between TC and TR, so maximizing profit is to look for the largest gap.
  - When the slopes of TC and TR are the same. It is equivalent to say $MC = MR$.
- From MC and MR graphs
  - The area under MC is AC; the area under MR is TR
  - Profit= $TR - FC - AC$. Since FC is and constant, so maximizing profit is to look for the largest difference of the areas under MC and MR.

4.2 Algebraical Solution

$$\max_{Q} \pi = TR - TC$$

FOC:

$$\frac{d\pi}{dQ} = \frac{dTR}{dQ} - \frac{dTC}{dQ} = MR - MC = 0$$

5 Production Functions $f(L, K)$

- Examples:
  - Fixed Proportions: $f(L, K) = A \min\{L, K\}$;
  - Perfect Substitutes $f(L, K) = A(L + aK)$;
  - Cobb-Douglas $f(L, K) = AL^a K^b$;
  - Constant Elasticity of Substitution $f(L, K) = A(L^a + K^a)^b$, $0 < a < 1$

  Where A is the technology parameter, a and b are parameters with different meanings for different production function.
- Unlike utility function, a monotonic transformation of a production function does not mean anything. A production function is determined by the current technology, while a utility function is just an index.
- When we fix the output Q, the relationship between L and K is depicted by Isoquant curves.
- Marginal Product: how much more output can be produced by adding one (small) unit of factor, say labor. It is the partial derivative of the production function w.r.t the factor input.

- Diminishing marginal product, check the example

$$MPL = \frac{\Delta Q}{\Delta L} = \frac{\partial f(L,K)}{\partial L} \text{ in the limit}$$

- Example: CES production function $f(L,K) = A(L^a + K^a)^b$

$$\begin{aligned} MPL &= Ab(L^a + K^a)^{b-1} a L^{a-1} = abA(L^a + K^a)^{b-1} L^{a-1} \\ MPK &= abA(L^a + K^a)^{b-1} K^{a-1} \end{aligned}$$

- Example: Cobb-Douglas $f(L,K) = AL^a K^b$

$$\begin{aligned} MPL &= AaL^{a-1}K^b \\ MPK &= AbL^a K^{b-1} \end{aligned}$$

- Technical rate of substitution (TRS) or marginal rate of technical substitution (MRTS)
  - The counterpart of TRS/MRTS is MRS in consumer theory;
  - How much factor 2 can be replaced with one unit of factor 1 in order to produce the same amount of output.
  - Diminishing MRTS, check the example
  - MRTS is the slope of isoquant, similar to MRS, we have

$$\begin{aligned} \Delta Q &= MPL \cdot \Delta L + MPK \cdot \Delta K = 0, \\ \Rightarrow & \; MRTS(L,K) = -\frac{\Delta K}{\Delta L} = \frac{MPL}{MPK} \end{aligned}$$

  - Example: CES production function $f(L,K) = A(L^a + K^a)^b$

$$MRTS(L,K) = \frac{MPL}{MPK} = (\frac{L}{K})^{a-1}$$

  - Example: Cobb-Douglas $f(L,K) = AL^a K^b$

$$MRTS(L,K) = \frac{MPL}{MPK} = \frac{aK}{bL}$$

  - Elasticity of Substitution: percentage change of the ratio of two inputs to a production function with respect to percentage change of the ratio of their marginal products:

$$ES = \frac{\frac{\Delta(K/L)}{(K/L)}}{\frac{\Delta MRTS(L,K)}{MRTS(L,K)}}$$

- **Returns to Scale** measures the change to the output if we double all the (variable) inputs. There are three possibilities.

- Increasing returns to scale (IRS) if

$$f(tK,tL) > tf(K,L) \text{ for all } t > 1$$

- Constant returns to scale (CRS) if

$$f(tK,tL) = tf(K,L) \text{ for all } t > 1$$

- Decreasing returns to scale (DRS) if

$$f(tK,tL) < tf(K,L) \text{ for all } t > 1$$

Examples:

- Example 1: CES production function $f(L,K) = A(L^a + K^a)^b$

$$f(tK,tL) = A(t^a L^a + t^a K^a)^b = t^{ab} A(L^a + K^a)^b = t^{ab} f(K,L)$$

  * IRS if $ab > 1$
  * CRS if $ab = 1$
  * DRS if $ab < 1$

- Example 2: Cobb-Douglas $f(L,K) = AL^a K^b$

$$f(tK,tL) = At^a L^a t^b K^b = t^{a+b} AL^a K^b = t^{a+b} f(K,L)$$

  * IRS if $a + b > 1$
  * CRS if $a + b = 1$
  * DRS if $a + b < 1$

- Notice that it is related to the slope of MC or the shape of TC

  * IRS: downward sloping MC or concave TC;
  * CRS: perfect flat MC or linear TC;
  * DRS: upward sloping MC or convex TC;

6 Profit Maximization by Choosing Inputs K and/or L

6.1 Profit Maximization in the Short Run

In the short run, assume K is the fixed factor. Therefore, the profit maximization problem is about choosing L to maximize the profit:

$$\max_L pf(L,\bar{K}) - wL - r\bar{K}$$

- FOC:

$$p \cdot \frac{df(L,\bar{K})}{dL} - w = 0 \Leftrightarrow p \cdot MPL = w$$

- Economic meaning of the FOC: **market value of the marginal product** of a variable factor must equal to the **per unit cost of the variable factor**.
  - If $p \cdot MPL < w$, then the "last unit" of labor costs more (w) than it makes ($p \cdot MPL$), so profit will be higher if the firm fires a unit of labor;
  - If $p \cdot MPL > w$, then the "last unit" of labor costs less (w) than it makes ($p \cdot MPL$), so profit will be higher if the firm hires more labor;

- **Question:** Is this optimal condition different from what we have learned, $MR = MC$?

  NO! They are equivalent!

$$\begin{aligned} MR &= MC \\ &\Leftrightarrow \frac{dpf(L,\bar{K})}{dQ} = \frac{d(wL + r\bar{K})}{dQ} \\ &\Leftrightarrow p \cdot \frac{df(L,\bar{K})}{dQ} = w \cdot \frac{dL}{dQ} \\ &\Leftrightarrow p \cdot \frac{df(L,\bar{K})}{dL} = w \end{aligned}$$

6.2 Profit Maximization in the Long Run

In the long run, assume K is a variable factor. Therefore, the profit maximization problem is about choosing L and K to maximize the profit:

$$\max_{L,K} pf(L,K) - wL - rK$$

- FOC:

$$\begin{aligned} p \cdot \frac{\partial f(L,K)}{\partial L} - w &= 0 \Leftrightarrow p \cdot MPL = w \\ \text{and} \quad p \cdot \frac{\partial f(L,K)}{\partial K} - r &= 0 \Leftrightarrow p \cdot MPL = r \end{aligned}$$

  Therefore

$$MRTS(L,K) = \frac{MPL}{MPK} = \frac{w}{r}$$

- Economic meaning of the FOC: **market value of the marginal product** of a variable factor must equal to the **per unit cost of the variable factor**. Similar explanation as the case in the short run.

- **Question:** Is this optimal condition different from what we have learned, $MR = MC$?

NO! They are equivalent!

$$
\begin{aligned}
MR &= MC \\
&\Leftrightarrow \frac{dpf(L,K)}{dQ} = \frac{d(wL+rK)}{dQ} \\
&\Leftrightarrow p \cdot [\frac{\partial f(L,K)}{\partial L}\frac{dL}{dQ} + \frac{\partial f(L,K)}{\partial K}\frac{dK}{dQ}] = w \cdot \frac{dL}{dQ} + r \cdot \frac{dK}{dQ}
\end{aligned}
$$

6.3 Profit Maximization and Returns to Scale

It is important to notice that if a firm has either IRS or CRS production function, there would be no maximized profit for the firm. Intuitively, it the firm doubles its scale it will double its profit. This means that the firm is always able to increase its profit by increasing its scale. Therefore, only with a DRS production function, we can study the profit maximization problem.

- That is also why a firm never stop at the part of downward sloping MC.
- Example: Cobb-Douglas $f(L,K) = AL^aK^b$, assuming the FOC satisfied, then

$$
\begin{aligned}
p \cdot MPL &= w \text{ and } p \cdot MPK = r \\
\text{so } p \cdot AaL^{a-1}K^b &= w \text{ and } p \cdot AbL^aK^{b-1} = r
\end{aligned}
$$

  Then

$$\frac{aK}{bL} = \frac{w}{r}$$

  Thus

$$p \cdot Aa(\frac{b}{a})^b(\frac{w}{r})^bL^{a+b-1} = w$$

  If $a + b > 1$, then we can always increase L to make the value of marginal product higher than the cost w. In that case, the firm can achieve infinite amount of profit by increasing L. Therefore, there is no optimal solution in this scenario.

7 Cost Minimization by Choosing Inputs K and/or L

7.1 Cost Minimization in the Short Run

In the short run, assume K is the fixed factor. Therefore, the cost minimization problem is about choosing L to minimize the profit:

$$
\begin{aligned}
& \min_L wL + r\bar{K} \\
s.t.\ f(L,\bar{K}) &= \bar{y}
\end{aligned}
$$

- In the short run, L is uniquely determined by $f(L,\bar{K}) = \bar{y}$, so this is not an legitimate question.

7.2 Cost Minimization in the Long Run

In the long run, assume K is a variable factor. Therefore,the cost minimization problem is about choosing L and K to minimize the profit:

$$\begin{aligned} &\min_{(L,K)} wL + rK \\ s.t.\ f(L,K) &= \bar{y} \end{aligned}$$

- FOC:

$$\begin{aligned} \lambda \cdot \frac{\partial f(L,K)}{\partial L} - w &= 0 \Leftrightarrow \lambda \cdot MPL = w \\ \text{and} \quad \lambda \cdot \frac{\partial f(L,K)}{\partial K} - r &= 0 \Leftrightarrow \lambda \cdot MPL = r \end{aligned}$$

  Therefore

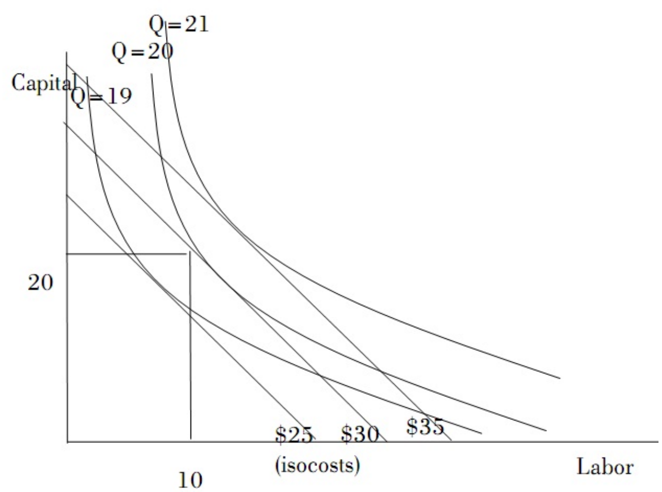
$$MRTS(L,K) = \frac{MPL}{MPK} = \frac{w}{r}$$

- Same as profit maximization!

7.3 Graphical Analysis

- Isoquant and Isocost curves

  In this example, the cheapest way to produce 20 units is to spend \$30 and use 10 units of labor and 20 of capital, each costing \$1.

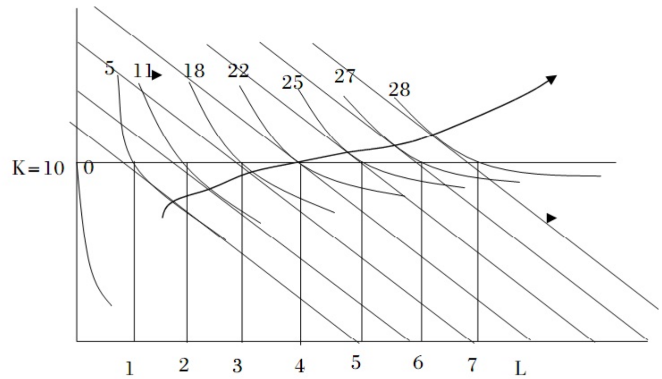


- Under the given prices, we can trace out the least cost ways of producing different levels of output. This is called an **(LR) expansion path**. If the prices changes, we get another expansion path.

- **Each expansion path is represented by a LRTC curve in LRTC/Q graph.**

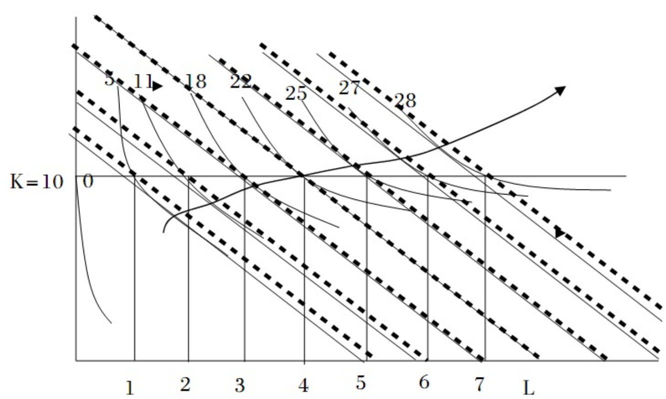
7.4 Relations Between the Short Run and Long Run

7.4.1 LRTC and SRTC

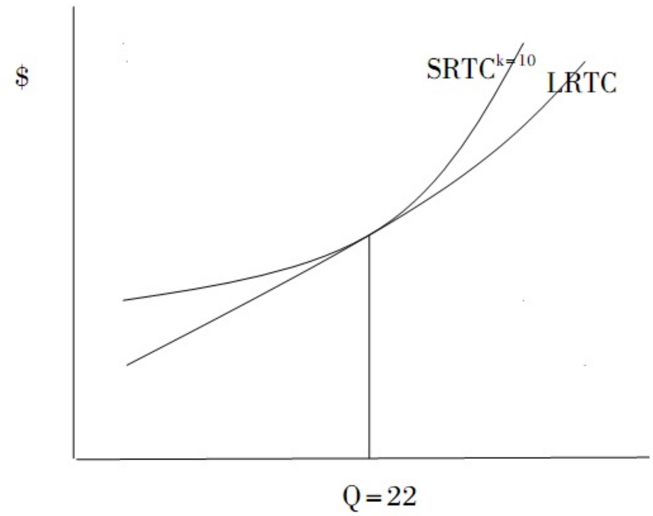


- For production levels below $Q = 22$, the cost minimizing level of capital is less than 10;
- For production levels $Q = 22$, the cost minimizing level of capital is equal to 10;
- For production levels above $Q = 22$, the cost minimizing level of capital is more than 10;

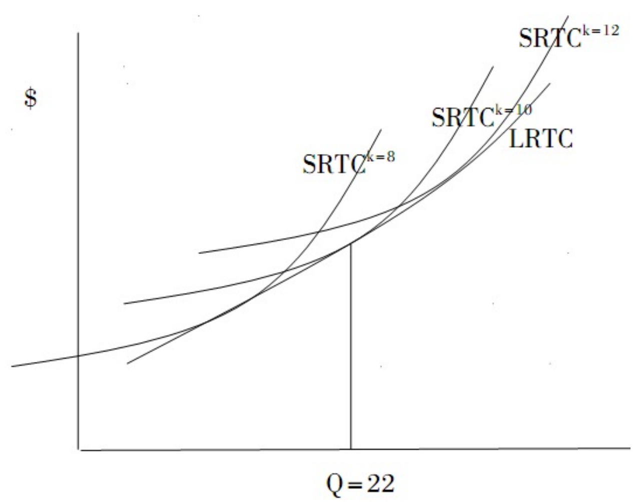
Observation: The firm will find itself over or under capitalized, therefore will adjust to the cost minimizing levels in the long run.



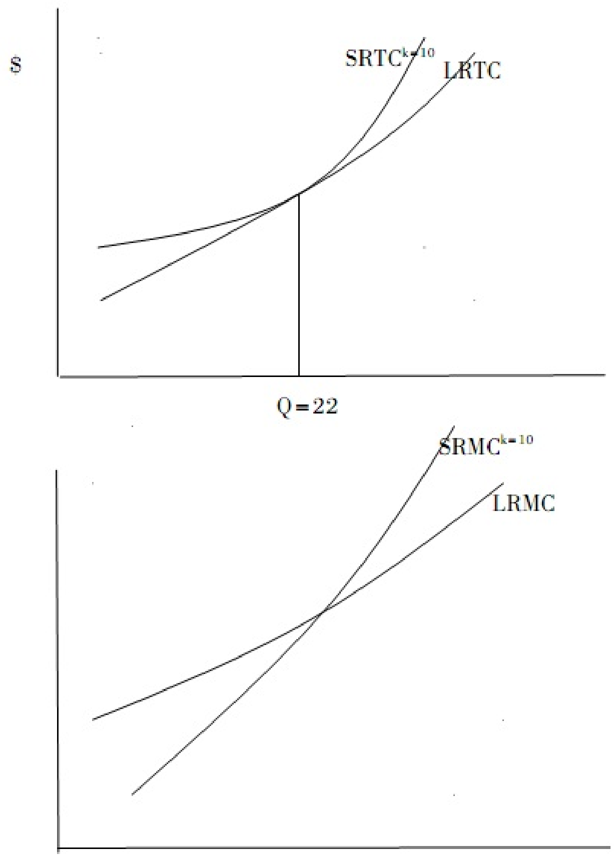
- The dotted lines are isocost lines through the SR ways to produce different levels of output. The solid line are the cost minimizing ways to produce the same levels of output in the long run.
- The LRTC is lower than SRTC when $Q \neq 22$, and LRTC=SRTC when $Q = 22$. Therefore we have



- If we draw for different levels of output. We would notice that the LRTC was an envelope that contains all the SRTC with each SRTC tangent at only one point.



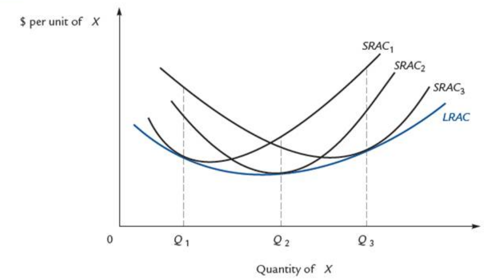
7.4.2 LRMC and SRMC



- When $Q < 22$, the slope of the SRTC is smaller than the slope of LRTC;
- When $Q = 22$, the slope of the SRTC is eqaul to the slope of LRTC;
- When $Q > 22$, the slope of the SRTC is larger than the slope of LRTC;

7.4.3 LRAC and SRAC

EXHIBIT 6.15 **Many Short-Run Average Cost Curves**



The curves $SRAC_1$, $SRAC_2$, and $SRAC_3$ show short-run average cost for a small, a medium-size, and a large plant. To produce Q_1 units, the firm finds that the small plant minimizes average cost, and so chooses that size plant in the long run. Thus, $LRAC = SRAC_1$ when quantity is Q_1. If only three plant sizes are available, the *LRAC* curve consists of the black portions of the *SRAC* curves shown. If a continuous range of plant sizes is available, there are many other *SRAC* curves, and the *LRAC* curve is the color curve shown.

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Lecture Eight (Chapter 7) Short-run and Long-run Supply of Competitive Firms

In chapters 3 and 4, we discuss how consumers make decisions given their preferences and budget constraints. In chapters 5 and 6, we learn how firms make decision given their technology and input prices. From chapter 7, we will start to discuss the economic environments in which consumers and firms interacts.

Market structure describes the state of a market with respect to competition, measured by number and distribution of firms, indicating the competitivity of the market. There are different types of market structure.

- **Perfect competition**: a theoretical market structure that features **unlimited contestability (or no barriers to entry), an unlimited number of producers and consumers, and a perfectly elastic demand curve**
  - e.g. Corner grocery store
- **Monopoly**: there is **only one provider of a product or service**
  - e.g. Microsoft
- **Oligopoly:** a market is **dominated by a small number of firms** that together **control the majority of the market share.**
  - e.g. OPEC
- **Monopolistic competition**: where there are **a large number of firms**, each having a **small proportion of the market share** and **slightly differentiated products**.
  - e.g. different brands of toothpaste
- **Monopsony**: there is **only one buyer** in a market.
  - e.g Walmart.

In chapter 7, we consider firms in a perfectly competitive market, an extreme type of market structure.

1 The Competitive Firm

Informally, markets are competitive if there are many suppliers and many consumers. This implies

- No agent has any power over the market
- All agent accept the conditions of the market and the actions of others as given

In most of the markets, we can observe many consumers. Therefore, usually when we study the market structure, we discuss the behavior of firms in different types of markets.

- A firm is competitive
  - The firm has no power to determine the market price alone.
  - The firm is a price taker.
  - The firm serves a small part of the market.
  - The firm faces a horizontal (perfect elastic) demand curve for its product.
    * Since an individual firm in the competitive market is small compared to the whole market, one firm cannot support the entire market.
    * Note that the industry demand curve still slopes downward, it is just that the part of the total output that one firm could ever supply is so small compared to total output, that its production decisions have no effect on its own price.
  - The firm can sell any quantity it wants to at the (given) market price.
  - No barriers for other firms to enter the industry.
- For all the firms, in perfect competitive market or not, total revenue is $TR = PQ$. In general, the price P is a function of quantity Q (since demand curve is downward sloping), so $TR = P(Q) \cdot Q$.
- The total revenue function summarizes the market constraints, which are determined by the **market environment**.
- For the special case of competitive firms, however, price is constant, therefore
  - $TR = \bar{P}Q$
  - $MR = \bar{P}$: flat at the level of the market price.

2 Supply Decision of A Competitive Firm

- In the long run, firms can exit.
  - Exit: leave the industry entirely, so no longer incur any costs
- In the short run, firms can shut down but cannot exit;
  - Shutdown: stop producing output, but stay in the industry, so continue to incur fixed costs.

2.1 Supply Curve in the Long Run

1. *Step 1:* **Supply curve** $= LRMC$**.**

   - Put $MR = \bar{P}$ with what we know about profit maximization $MR = LRMC$, we get the following profit maximizing solution for competitive firms

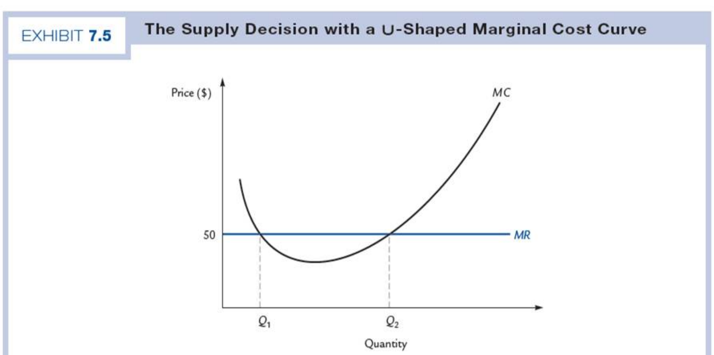
   $$LRMC(Q) = MR = \bar{P}$$

     There is to say, to maximize profit, competitive firms should choose a quantity that equates MC and the competitive price $\bar{P}$.

   - Since $LRMC$ is a function of Q, by varying $\bar{P}$, we get a relationship between Q^s by the firm and the market price P. That is the **supply curve for the firm!**
   - We conclude that the $LRMC$ **is exactly the same as the supply curve for competitive firms**.
   - Note that the presence of fixed costs do not affect this since fixed cost is unavoidable.

2. *Step 2:* **Supply curve** $=$ upward sloping part of $LRMC$**.**

   - This is too hasty. Recall the shape of MC curves:



At a market price of $50 the firm produces Q_2 items (assuming it produces at all). It takes losses on the first Q_1 of these, all of which are produced at a marginal cost of more than $50, and it earns positive profits on the others. If those positive profits fail to outweigh the losses on the first Q_1 items, the firm will shut down.

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- **Question:** The price equals the MC at two points, Q_1 and Q_2. Which quantity should the firm supply?
- Notice that from 0 to Q_1, the $LRMC$ is always above the price. Thus each time a unit is produced, it costs more than the firm gets in revenue.
- Remind that the firm would never produce on the downward sloping part of the $LRMC$ since this is the IRS part.

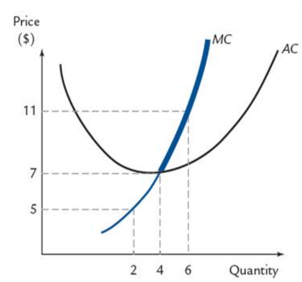
3. *Step 3:* **Supply curve** = upward sloping part of $LRMC$ **with** $P > AC$.

- We are still being too hasty. We know at Q_2, the firm makes **maximized profits**, but is the firm making **positive profits** at Q_2? If it is not, the firm would be better off leaving the industry. After all, zero profit is better than negative profits.
- How can we tell?
  - If the gains from the "smile part" are larger than loss in the triangle part, then the firm makes positive profits.
  - If the gains are smaller than the loss, the firms makes negative profits and it should exit.
- Therefore, only part of the upward sloping part of the $LRMC$ yields positive profits and is therefore the supply curve.
- Notice that $\Pi = TR - TC > 0$. Then

$$\frac{TR - TC}{Q} > 0 \Rightarrow AR = P > AC$$

- This means that in the LONG RUN, as long as the price is above the minimum of the average cost curve, they should supply along the $LRMC$, otherwise it should exit from the industry.

EXHIBIT **7.12** **The Firm's Long-Run Supply Curve**



At any given price, Sam's House of Widgets sells the quantity where price equals marginal cost. Therefore, as long as Sam wants to be in business at all, his (long-run) supply curve coincides with his (long-run) marginal cost curve.

But at sufficiently low prices (like \$5), Sam prefers to exit the industry. Therefore, the point corresponding to \$5 is excluded from the supply curve. The supply curve is the boldfaced portion of the marginal cost curve—the part that lies above average cost.

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- **Conclusion:** LR supply curve is the LRMC curve with $P > AC$.

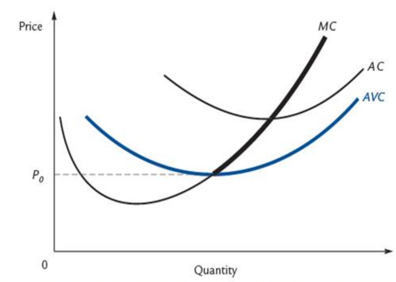
2.2 Supply Curve in the Short Run

The above is true for the long run, but in the short run, the firm cannot avoid fixed cost. Thus, if a firm's revenues at least cover its variable costs, the firm at least partly **offsets its fixed costs by staying in the business**. If it shuts down, the losses are even greater. Thus,

- Operation condition in the SHORT RUN is $TR > VC$, (or $TR - TC > -FC$). That is

$$\frac{TR}{Q} > \frac{VC}{Q} \Rightarrow AR = P > AVC$$

EXHIBIT 7.6 The Competitive Firm's Short-Run Supply Curve



As long as the price exceeds P_0, the firm's supply curve coincides with its marginal cost curve. At prices below P_0, the firm produces nothing. Therefore, the firm's supply curve is equal to the boldfaced portion of the marginal cost curve.

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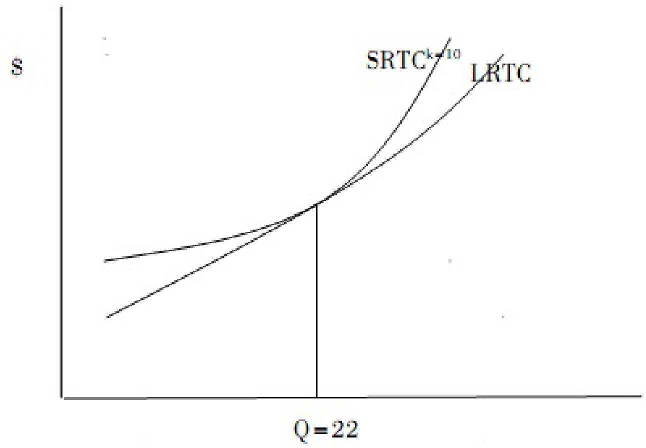
- **Observation:**
  - In the SR, the firm shuts down (without leaving the industry) if price falls below AVC;
  - In the LR, the firm exits if price falls below AC.
  - Firms can accept negative profit (so low price) in the SR in order to cover fixed cost, but cannot tolerate negative profit in the LR, so the critical exiting price is higher than the critical shutdown price.

- **Conclusion:** SR supply curve is the SRMC curve with $P > AVC$.

$$Q^s_{SR}(P) = \begin{cases} y^*(P) \text{ if } P > AVC(y^*(P)) \text{ and } MC(y^*(P)) = P \\ 0 \quad\ \text{ if } P < AVC(y^*(P)) \text{ and } MC(y^*(P)) = P \end{cases}$$

2.3 Relationship Between SR supply curve and LR supply curve

That is the relationship between SRMC curve and LRMC curve



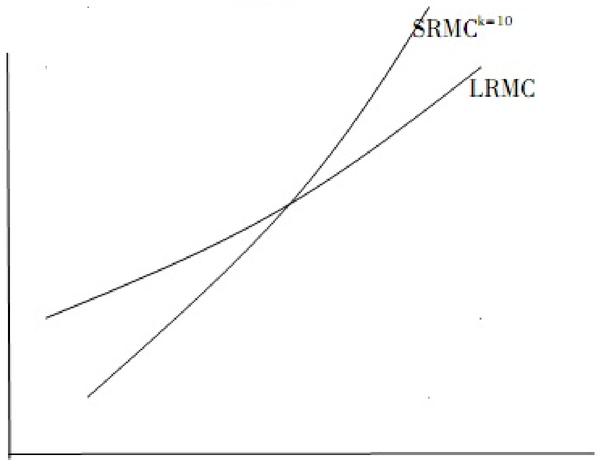
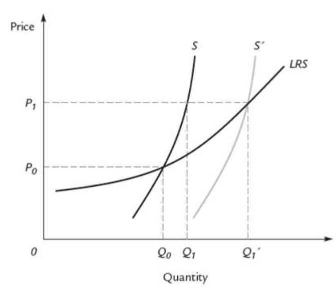


EXHIBIT 7.11 **Long-Run and Short-Run Supply Responses**



In long-run equilibrium at P_0, the firm is on both its long-run and short-run supply curves. A change in price, to P_1, has the immediate effect of causing the firm to move along its short-run supply curve S to the quantity Q_1. In the long run, the firm can vary its plant capacity (for example, a hamburger stand can install more grills) and move along its long-run supply curve, *LRS*, to Q_1'. With the new plant capacity, the firm has a new short-run supply curve S'. In the new equilibrium at price P_1 and quantity Q_1', the firm is again on both its long-run and short-run supply curves.

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2.4 The (Price) Elasticity of Supply

$$\begin{aligned} E_s &= \frac{\text{Percentage change in quantity supplied}}{\text{Percentage change in price}} \\ &= \frac{\Delta Q^s / Q^s}{\Delta P / P} \\ &= \frac{dQ^s}{dP} \cdot \frac{P}{Q^s} \text{ as } \Delta P \to 0 \end{aligned}$$

3 A Practice Example

Consider the production function $f(L,K) = L^{1/2}K^{1/2}$. Fix input prices at $w = r = 1$, and suppose that in the short run $K = 100$.

1. The short-run conditional factor L demand as a function of output y;

$$y = L^{1/2}100^{1/2} \Rightarrow L = \frac{y^2}{100}$$

2. The short-run total cost function;

$$SRTC = wL + r\bar{K} = \frac{y^2}{100} + 100$$

3. The short-run marginal cost

$$SRMC = \frac{dSRTC}{dy} = \frac{y}{50}$$

4. The short-run average cost

$$SRAC = \frac{SRTC}{y} = \frac{y}{100} + \frac{100}{y}$$

5. The short-run average variable cost

$$SRAVC = \frac{SRVC}{y} = \frac{y}{100}$$

6. The long-run total cost function

$$\frac{MPL}{MPK} = \frac{w}{r} = 1 \Rightarrow \frac{\frac{1}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}}{\frac{1}{2}L^{\frac{1}{2}}K^{-\frac{1}{2}}} = 1 \Rightarrow K = L$$

Substitute this into the production function, we find

$$y = L = K$$

Then LRTC

$$LRTC = wL + rK = 2y$$

7. The long-run marginal cost

$$LRMC = \frac{dLRTC}{dy} = 2$$

8. The long-run average cost

$$LRAC = \frac{LRTC}{y} = 2$$

9. The short-run function

$$SRMC = p \Leftrightarrow \frac{Q^s_{SR}}{50} = p \Leftrightarrow Q^s_{SR} = 50p$$

Check the shutdown condition: $AVC(y^*(P)) = \frac{Q^s_{SR}}{100} = \frac{50p}{100} = \frac{1}{2}p > p$.

10. The long-run supply function

$$p = LRMC = 2$$

Check the not-exiting condition ($P > AC$):

$$LRAC = 2 = p$$

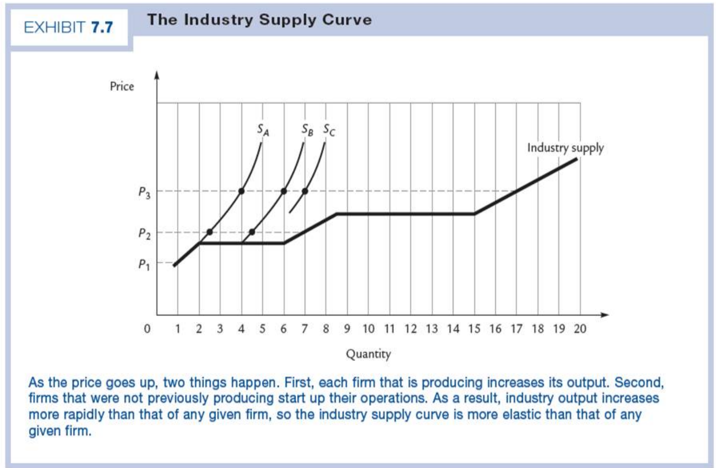
Lecture Night (Chapter 7)
Supply Curves of the Competitive Industry

In order to study the supply curves of an competitive industry, we need to distinguish the long run and short run behavior of the industry, as we study the supply curves of a competitive firm.

- In the short run,
  - Firm (shutdown): Stop producing output, but stay in the industry, so continue to incur fixed costs;
  - Industry: No firm can enter or exit the industry. So number of firms in the industry is constant.
- In the long run,
  - Firm (exit): Leave the industry entirely, so no longer incur any costs;
  - Industry: Any firms that wants to can enter or leave the industry. So number of firms in the industry is changing.

1 Industry Supply Curve in the Short Run

- Aggregation: we add up the short-run supply of each firm to get the industry supply.
- As the price goes up, two things happen,
  - Each firm that is producing **increases its output**;
  - Firms that were not previously producing **start up their operations**.
- Therefore, **the industry supply curve**
  - is **upward sloping** as the supply curve of competitive firms;
  - **is flatter but less smooth than the supply curves of competitive firms.**



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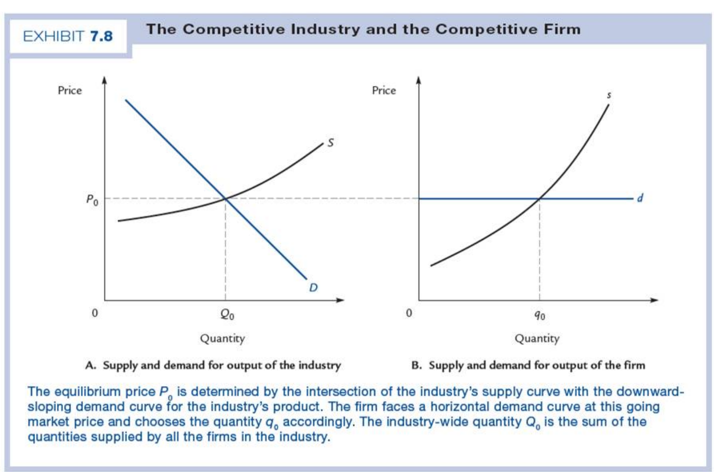
1.1 Relationship Between the competitive industry and the competitive firm in the short run

- In a competitive market, **market price is determined by the market demand and market supply** (supply from the whole industry)
- The **firm faces a horizontal demand curve at the going market price**, and chooses the quantity q_0 accordingly.
- The **industry supply Q_0 is the sum of the quantities supplied by all the firms** in the industry.



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1.2 Effects of changes in costs and demand

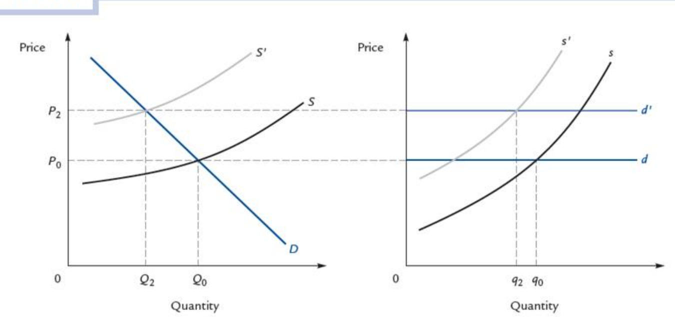
1.2.1 Changes in fixed costs

- In the short run, fixed cost is always incurred,
- The changes in fixed costs do not change the marginal cost,
- Therefore, the supply curve of a competitive firm does not shift,
- The industry supply curve, as the sum of supply curves of all the firms, does not shift,
- Conclusion: *NOTHING'S CHANGED.*

1.2.2 Changes in variable costs (a rise in variable cost as an example)

- For a competitive firm, given the quantity produced, when variable cost rise, marginal costs rise as well;
- The firm supply shifts upward;
- The industry supply shifts upward;
- The new equilibrium is determined by the intersection of the industry demand curve and the new industry supply curve;
- Equilibrium market price rises, and equilibrium market supply declines;
- *The quantity supplied by each firm is ambiguous: decreases (as the figure below) or increases (**some firms shutdown their production, so the remaining firm needs to produce more to satisfy the market demand.**)*

EXHIBIT 7.9 A Rise in Variable Costs



A. Supply and demand for output of the industry

B. Supply and demand for output of the firm

A rise in variable costs causes the firm's supply curve to shift left from s to s' in panel B. The industry supply curve shifts left from S to S' in panel A, both because each firm's supply curve does and because some firms may shut down. The new market price is P_2. The firm operates at the intersection of s' with its new horizontal demand curve at P_2. Depending on how the curves are drawn, the firm could end up producing either more or less than it did before the rise in costs. (That is, q_2 could be either to the left or to the right of q_0.)

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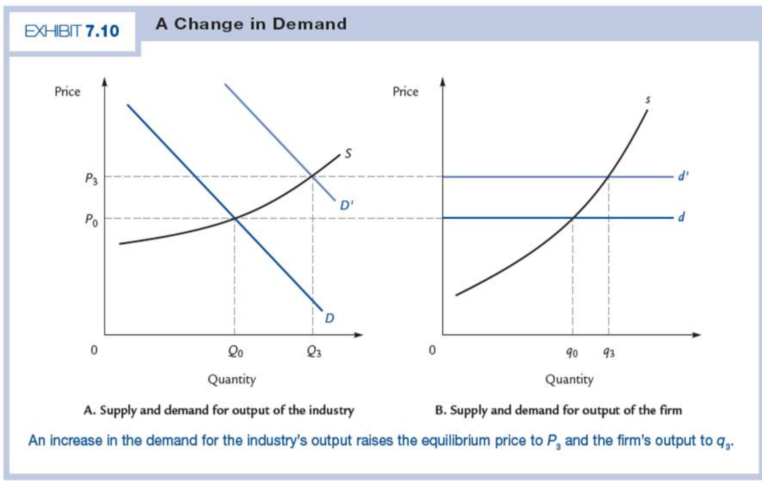
1.2.3 Changes in demand (a rise in demand as an example)

- Demand curve shifts to the right;
- Equilibrium market price rises, and equilibrium market supply rises as well;
- *The quantity supplied by each firm increases as well.*



An increase in the demand for the industry's output raises the equilibrium price to P_3 and the firm's output to q_3.

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2 Industry Supply Curve in the Long Run

- **Question**: Can we add up the long-run supply of each firm to get the industry supply?
  - Firms can enter or exit in the long run. **We don't even know which firms to count in!**
- **The long-run economic profit of a competitive industry should be**
  - **nonpositive for all the potential entrants;**
    - If the long-run economic profit is positive for potential entrants, those potential entrants would enter the industry since this decision is profitable for them. This profit-pursuing behavior drives the market price down until the long-run economic profit is nonpositive for new entrants.
  - **nonnegative for all the existing firms.**
    - If the long-run economic profit is negative for some existing firms, they would exit the industry since they are losing money in the long run by staying in the industry. Given a market price, this exit decreases the quantity supplied by the market. In turn, the decline of the market supply bids up the market price until the economic profit is nonnegative for all the existing firms.

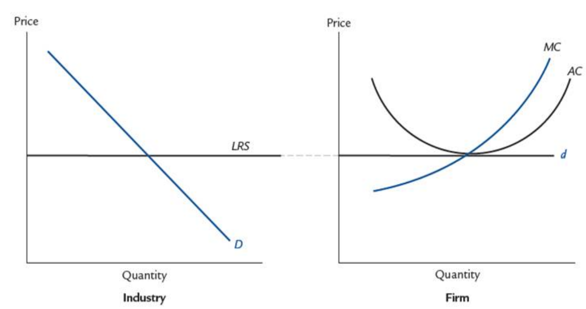
- **Question:** How can we use this principle to get the long-run industry supply curve?

  We need to have more information on the characteristics of the industry.

2.1 A benchmark type of industry (Constant-Cost Industry)

- **Constant-cost industry (CCI):** all firms have identical cost curves, and the cost curves do not change as the industry expands or contracts.
  - **The potential entrants and existing firms are identical, therefore the long-run economic profits for them should be zero (both nonpositive and nonnegative);**
  - **Their cost curves do not change as the industry expands or contracts, therefore the price satisfying the zero-profit condition is the same as the industry expands or contracts.**
- Therefore, for an CCI, the **long-run market price** should satisfy the **zero-profit condition at all the quantity**.
  - Break-even price: the price at which a seller earns zero profits.
  - Long-run industry supply curve is flat at a price where all firms earn zero economic profit.

EXHIBIT 7.14 Long-Run Competitive Equilibrium



The industry-wide equilibrium occurs at the break-even price. Firms face a demand curve that is flat at that price.

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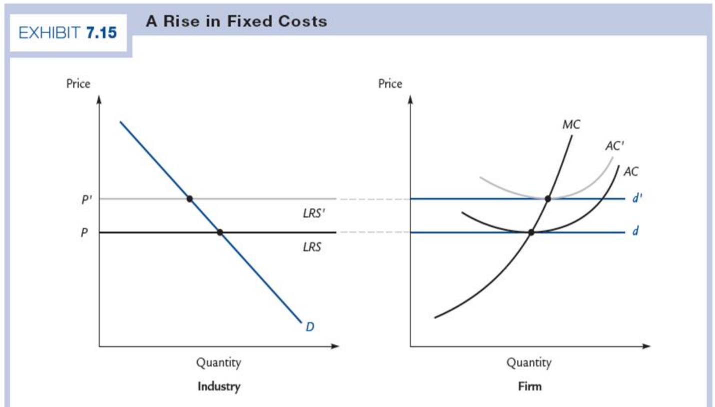
2.1.1 Changes in fixed costs (a rise in fixed cost as an example)

- A rise in fixed cost increases break-even price ($\pi = P_{break}Q - FC - VC = 0$);
- The long-run industry supply curve shifts upwards. For a firm, the AC curve shifts upwards, while the MC curve is unchanged;

- Equilibrium market price is higher, equilibrium market supply is lower. However, each firm supplies more to the market since less firms stay in the industry.



If fixed costs rise, the break-even price rises also, so the long-run industry supply curve rises from *LRS* to *LRS'*. Quantity increases at each individual firm and decreases in the industry. The firm's average cost curve rises from *AC* to *AC'*, indicating that profit is zero in the new equilibrium.

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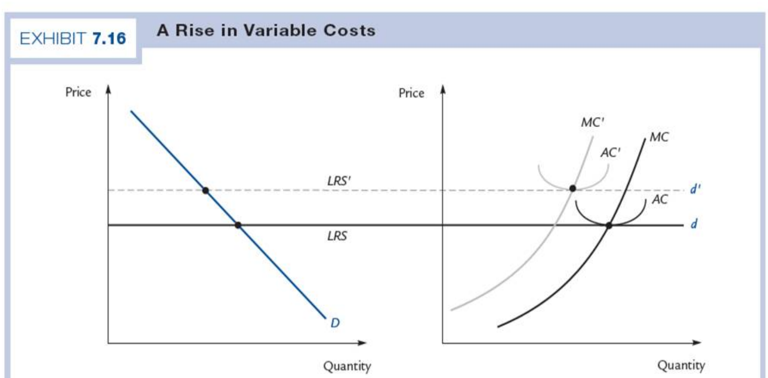
2.1.2 Changes in variable costs (a rise in variable cost as an example)

- A rise in variable cost increases break-even price ($\pi = P_{break}Q - FC - VC = 0$);
- The long-run industry supply curve shifts upwards. For a firm, the AC curve shifts upwards, and the MC curve shifts upwards as well;
- Equilibrium market price is higher, equilibrium market supply is lower. However, *The quantity supplied by each firm is ambiguous: decreases (as the figure below) or increases (**some firms exit from the industry, so the remaining firm needs to produce more to satisfy the market demand.**)*



If variable costs rise, the firm's marginal cost curve rises from *MC* to *MC'*. The break-even price rises, so the long-run industry supply curve rises from *LRS* to *LRS'*. The industry quantity falls; the firm quantity can either fall or rise. The average cost curve shifts from *AC* to *AC'*, and the firm earns zero profit at the new equilibrium.

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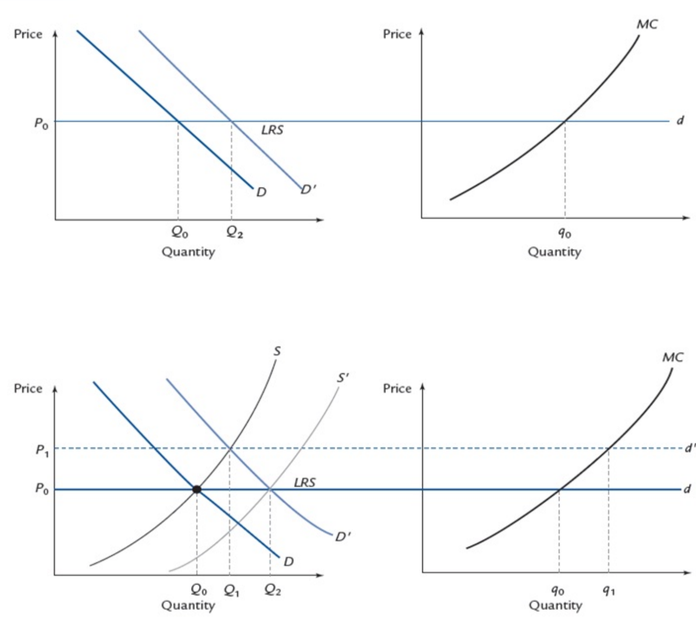
2.1.3 Changes in demand (a rise in demand as an example)

- Demand curve shifts rightwards;
- Equilibrium market supply increases, price is unchanged. Each firm produce the same. More firms enter the industry to satisfy the excess demand.

EXHIBIT **7.17** **A Rise in Demand**

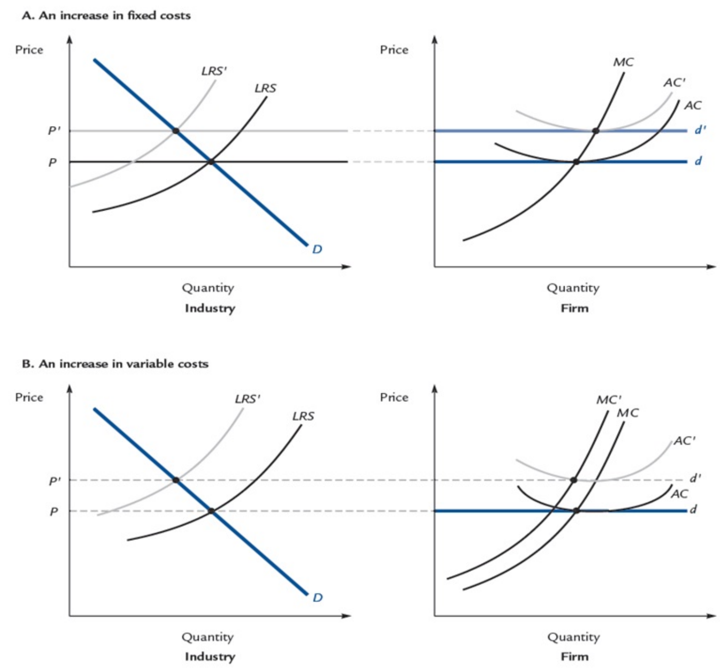


The top of this exhibit shows the long-run effect of an increase in demand for the product of a constant-cost industry. The industry demand curve shifts from D to D'. There is no change in price and hence no change in the "firm" part of the picture. Firms produce exactly as before, but the industry quantity increases. The bottom of the exhibit contrasts the short-run and long-run responses. The industry is initially in both short-run and long-run equilibrium at price P_0. When demand shifts from D to D', the price is bid up to P_1. Firms increase their output from q_0 to q_1, and the industry output rises to Q_1. Now firms earn positive profits, so in the long run there is entry. Entry continues until the price is bid back down to P_0. At this point, firms return to producing quantity q_0, and the industry produces quantity Q_2. Entry causes the short-run supply curve to shift rightward to S'. The short-run supply curve shifts in the long run, not in the short run.

2.2 Increasing-Cost Industry

- **Increasing-cost industry (ICI):** A competitive industry where the break-even price for new entrants increases as the industry expands.
  - Some firms might be less efficient, so have higher break-even prices. Given a market price lower than their break-even price, they will not enter the industry. They will enter the market when the market price is increased to be higher or equal to their break-even price. Therefore, we get a upward sloping industry supply curve.
  - Factor Price Effect (FPE): an expansion of an industry might bid up the price of some factor of production and thereby bids up the break-even price for every firm.
- Changes in fixed costs and changes in variable costs in an ICI

EXHIBIT 7.20 An Increase in Costs in an Increasing-Cost Industry



The top panels show an increase in fixed costs and the bottom panels show an increase in marginal costs. In both cases, the break-even price increases, so the long-run industry supply curve shifts. The firm's marginal cost curve shifts only in the second of the two examples. In both examples, the price rises and the industry supplies a smaller quantity. In the first example, the firm's quantity surely increases; in the second, the firm's quantity could increase or decrease.

2.3 Decreasing-Cost Industry

- **Decreasing-cost industry (DCI):** A competitive industry where the break-even price for new entrants falls as the industry expands.
  - If there is an economy of scale in the factor market, the cost of inputs can go down.
  - When the production rises, more inputs are required. With more inputs produced, the unit cost/price is lower. This leads to the decrease of MC and AC.
  - Therefore, we get a downward sloping industry supply curve.

3 Applications

General Question: What is the effect of the change in (fixed cost, variable cost, demand, supply) on the SR/LR (price change, quantity supplied by the firm/industry) for a LR (ICI, CCI, DCI)?

Lecture Ten (Chapter 8) Efficiency Analysis of Economic Policies

We have learned from Chapter 2 that there are gains from trade for both buyers and sellers as long as the trade is voluntary. We discussed the source of gains (difference in endowment, taste, ability etc.), but we never studied how we can measure the magnitude of the gains.

In this chapter, we will talk about how to gauge the gains from trade for consumers, firms and the whole society. After that, we can evaluate some economic policies, such as tax, subsidy, price ceiling, minimum wage and so on, by comparing the social gains or loss before and after the policies. In chapters 10 and 11, we will the same analysis to evaluate different types of market structure.

1 Measuring the Gains from Trade

When you buy a car from a producer directly. Both parties gains from this trade. Our first task is to measure the magnitude of the gains for both sides.

1.1 Gains for Consumers (Consumer's Surplus)

- **Question: If you decide to buy a Lexus SUV at $\$40,000$, what is your gains from the purchase? Or why are you willing to pay for the car?**

  Economically, we might say "my utility gains from having the Lexus are higher than the utility loss from losing $\$40,000$"

- It seems that the gap between your utility gains and utility loss is a potential measure of your net gains from the trade.

- However, utility is ordinal. The magnitude has no absolute economic meanings. Do we have any alternative?

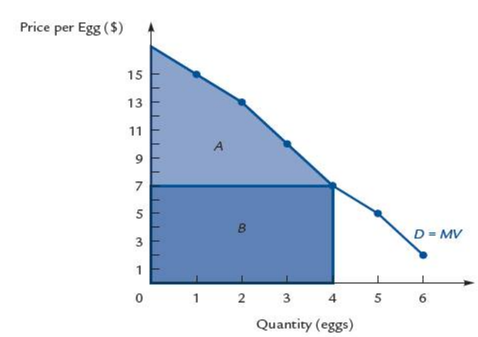
- Monetary measure is meaningful.

  - Gap between "**the maximum amount you are willing to pay**" and "**the amount you actually pay**".
  - E.g., you are willing to pay $\$60,000$ for your 1^{st} Lexus SUV, but the market price is $\$40,000$.
    * The **marginal value** of the 1^{st} Lexus SUV for you is $\$60,000$. That is also the total gross gains from trade;
    * The marginal cost of the 1^{st} Lexus SUV is $\$40,000$. That is the total cost from trade;

    * The net gains is $\$20,000$.
  - **Question: what about you buying 2^{nd} Lexus SUV?**
    You don't value the 2^{nd} Lexus SUV as high as the 1^{st} one since you have got one. So, Say, you are willing to pay $\$50,000$ for it. In this case,
    * The **marginal value** of the 2^{st} Lexus SUV for you is $\$50,000$.
    * The **total value** from buying two Lexus SUVs is $\$110,000$.
    * The marginal cost of the 2^{st} Lexus SUV is still $\$40,000$.
    * The total cost from buying two Lexus SUVs is $\$80,000$.;
    * The net gains from buying two Lexus SUVs is $\$30,000$.
  - **Question: what about you buying 3^{rd} Lexus SUV?**
    If your marginal value of the 3^{rd} **Lexus SUV** is higher than the marginal cost of it ($\$40,000$), you will still buy it, and receives a positive net gains.

- The total value of the consumer's purchases is equal to the area under the demand curve out to the quantity demanded.
- The total cost of the purchases is equal to the area under the market price out to the quantity demanded.
- **Consumer's Surplus: The gap between the total value and the total cost of the purchases.**
  - The amount by which the value of the purchase exceeds what a consumer actually pays for them.

EXHIBIT **8.3** **The Consumer's Surplus**



In order to acquire 4 eggs, the consumer would be willing to pay up to the entire shaded area, *A* + *B*. At a price of $7 per egg, his actual expenditure for 4 eggs is $28, which is area *B*. The difference, area *A*, is his consumer's surplus.

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- **Consumer's surplus of the whole market:** We are deal with individual demand so far. We can easily get the consumer's surplus of the whole market by replacing the individual demand curve with a market demand since market demand is simply an aggregation of all the individual consumers' demand.

1.2 Gains for Producers (Producer's Surplus)

Question: What is the gains from the sale for the producer? Or why do you think the producer is willing to sell you the car?

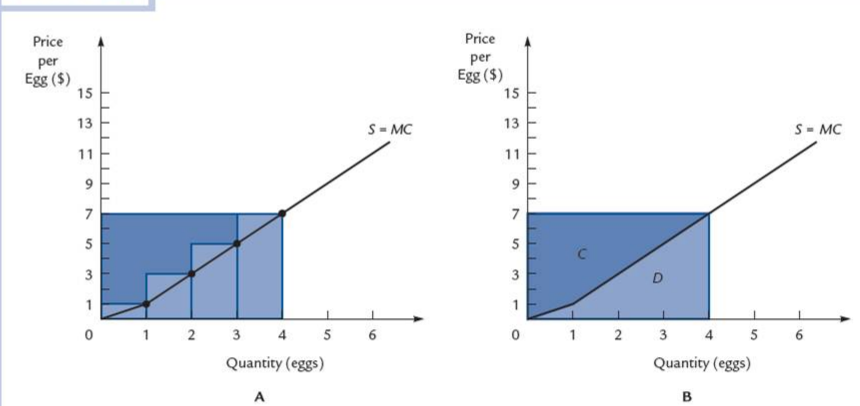
Economically, the deal is profitable, which means the price is higher than the MC.

- We are considering only how the producer is affected by trade. Since the producer would incur the fixed costs even without trading, this makes the fixed cost irrelevant to the discussion.

- The gap between "the sale price" and "the marginal cost" serves as a natural measure of the gains from **selling one additional car.**

- **Producer's Surplus:** The amount by which the producer's revenue exceeds his/her variable production costs.
  - The area above the supply curve (MC curve) up to the price received and out to the quantity supplied.
  - The area under the MC curve is the VC.

EXHIBIT 8.4 The Producer's Surplus



If the producer supplies 4 eggs, his cost is the sum of the 4 marginal costs, which are represented by the rectangles in panel A. If we measure eggs in very small units, we find that an exact measure of his cost is area *D* in panel B. At a market price of $7, revenue is $7 × 4 = $28, which is the area of rectangle *C* + *D*. Thus, the producer's surplus is area *C*.

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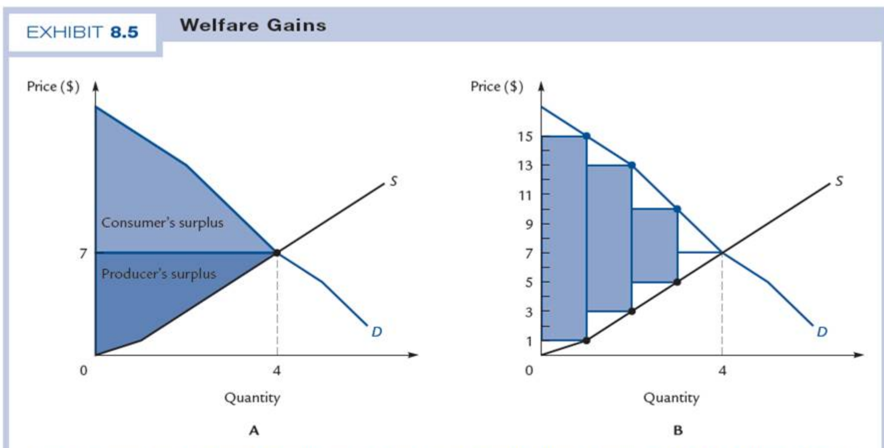
- **Producer's Surplus of the whole market:** We are deal with individual firm so far. We can easily get the producer's surplus of the whole market by replacing the supply curve of individual firm with a market supply.

1.3 Social Gain from Trade

- Social gain (welfare gain): the sum of the gains from trade to all participants.
  - $SG = CS + PS$
  - market demand and market supply
  - Think of the market as a whole: **Total Value – Total Cost for the whole society**



Panel A shows the consumer's surplus and the producer's surplus when 4 eggs are sold at a price of $7. The sum of these areas is the total welfare gain. The second panel shows another way to calculate the welfare gain. The first egg creates a gain equal to the area of the first rectangle, the second creates a gain equal to the area of the second rectangle, and so on. When units are taken to be small, the sum of these areas is the shaded region, which is the sum of the consumer's and producer's surpluses.

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2 The Efficiency Criterion and Policy Evaluation

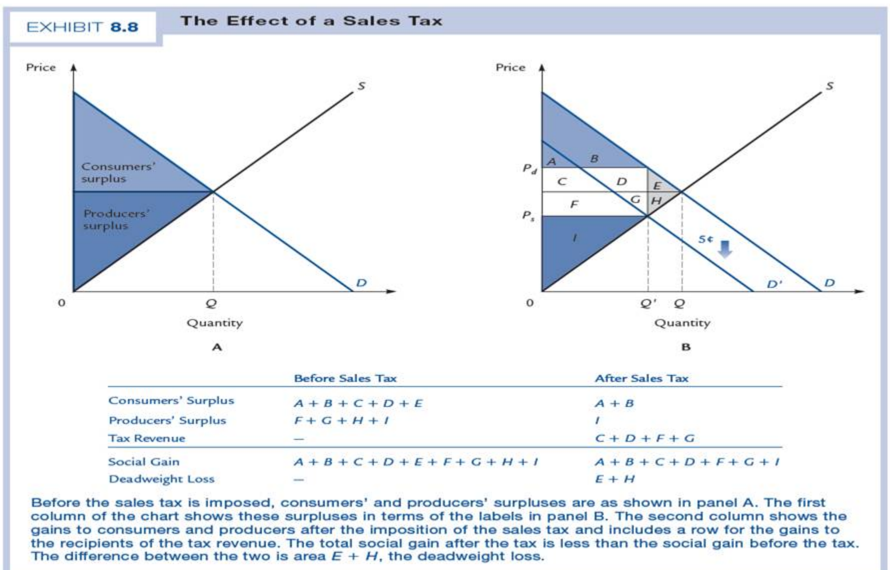
- Normative Economics
  - Study what should be accomplished
  - Begins with predetermined criteria and use them to evaluate policies
    - The policy is good or bad according to some criterion.
  - Not objective
- Positive Economics
  - Study why the economy works like that.
  - Establishes cause-and-effect relationships among economic variables.
  - Formulates "If...then" hypotheses that can be checked against facts

– Objective

- Efficiency Criterion
  - An normative criterion concerning on the SG, CS and PS defined above.
  - A policy achieving higher CS is good for consumers, higher PS is good for producers, higher SG is good for the society
  - **DO NOT CARE ABOUT FAIRNESS!**

2.1 Examples

2.1.1 Sales Tax



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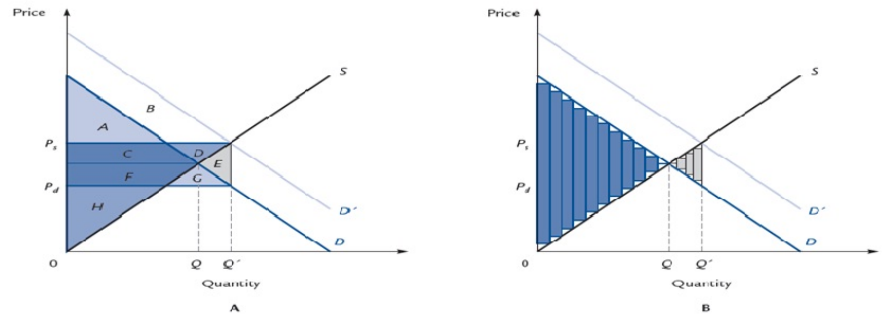
- Two Hidden Assumptions:
  - Competitive market assumption: Without tax, market price is determined by the intersection of supply and demand;
  - Equal value tax Revenue assumption: The tax revenue collected by the goverment worth the same for the individuals and the society.

2.1.2 Subsidies

EXHIBIT **8.12** **The Effect of a Subsidy**

	Before Subsidy	After Subsidy
Consumers' Surplus	$A + C$	$A + C + F + G$
Producer's Surplus	$F + H$	$C + D + F + H$
Cost to Taxpayers	—	$-(C + D + E + F + G)$
Social Gain	$A + C + F + H$	$A + C + F + H - E$
Deadweight Loss	—	E



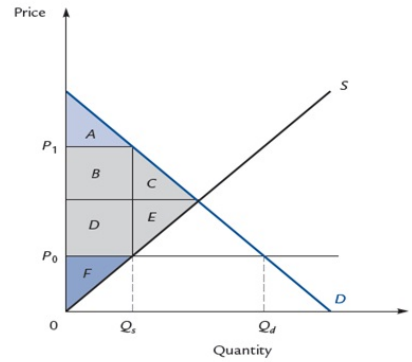
The table shows the gains to consumers and producers before and after the institution of a \$50-per-unit government subsidy to home insulation. With the subsidy in effect, there is a cost to taxpayers that must be *subtracted* when we calculate the social gain. We find that the social gain with the subsidy is lower by *E* than the social gain without the subsidy. *E* is the deadweight loss.

To check our work, we can consider the social gain created by each individual unit of insulation, shown in panel B. Each unit up to the equilibrium quantity *Q* creates a rectangle of social gain. After *Q* units have been produced, we enter a region where marginal cost exceeds marginal value. Each unit produced in this region creates a social *loss* equal to the excess of marginal cost over marginal value; these losses are represented by the gray rectangles, which stop at the quantity *Q′* that is actually produced. The social gain is equal to the sum of the colored rectangles minus the sum of the gray ones. Because the social gain without the subsidy is just the sum of the colored rectangles, the gray rectangles represent the deadweight loss.

2.1.3 Price Ceilings

- Price Ceilings: A maximum price at while a product can be legally sold
  - Effective price ceiling: a price ceiling set below the equilibrium

EXHIBIT 8.14 **A Price Ceiling**



	Before Ceiling	After Ceiling
Consumers' Surplus	$A + B + C$	A
Producers' Surplus	$D + E + F$	F
Social Gain	$A + B + C + D + E + F$	$A + F$
Deadweight Loss	—	$B + C + D + E$

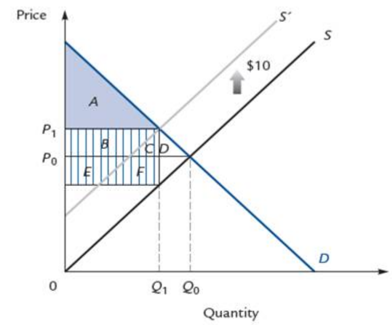
At a maximum legal price of P_0, demanders want to buy more than suppliers want to sell. Therefore, they compete against each other for the available supply, by waiting in line, advertising, and so forth. This increases the actual price to consumers. The full price to consumers must be bid all the way up to P_1 because at any lower price the quantity demanded still exceeds the quantity supplied, leading to increases in the lengths of waiting lines.

The deadweight loss comes about for two reasons. First, there is the reduction in quantity from equilibrium to Q_s. This loss is the area $C + E$. Second, there is the value of the consumers' time spent waiting in line. This is equal to $P_1 - P_0$ times the quantity of items purchased, which is the rectangle $B + D$.

2.1.4 Tariffs of a large country without domestic industry

- Large country for good A: World price of good A could be influenced by the domestic policy of the country: e.g. A special fruit bought only by the US.
- Supply curve of the product for the country is upward sloping.
- A country could be a large country for some products, but small country for some others. E.g. US for some tropic fruit.

EXHIBIT 8.15 **A Tax on Imported Cameras**



	Before Tariff	After Tariff
Consumers' Surplus	$A+B+C+D$	A
Tariff Revenue	—	$B+C+E+F$
Social Gain	$A+B+C+D$	$A+B+C+E+F$

If cameras are supplied by foreigners and purchased by Americans, then a tariff affects Americans through the consumers' surplus and through the tax revenue that it generates.

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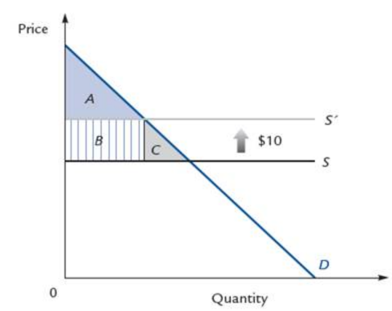
2.1.5 Tariffs of a small country without domestic industry

- Small country for good A: World price of good A could not be influenced by the domestic policy of the country.
- Supply curve of the product for the country is flat.

EXHIBIT 8.16 **A Tariff on Imported Cameras That Are Elastically Supplied**



	Before Tariff	After Tariff
Consumers' Surplus	$A+B+C$	A
Tariff Revenue	—	B
Social Gain	$A+B+C$	$A+B$
Deadweight Loss	—	C

If the United States is a small part of the market to which the Japanese sell cameras, then Americans will face a flat supply curve. In this case, a tariff always reduces the welfare of Americans.

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2.1.6 Tariffs of a small country with domestic industry

EXHIBIT **8.17** **A Tariff When There Is a Domestic Industry**

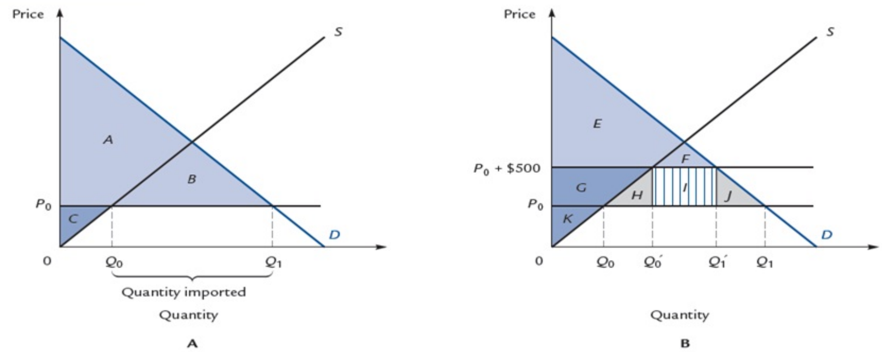


Table A

Consumers' Surplus	$A + B$
Producers' Surplus	C
Social Gain	$A + B + C$

Table B

	Before Tariff	After Tariff
Consumers' Surplus	$E + F + G + H + I + J$	$E + F$
Producers' Surplus	K	$G + K$
Tax Revenue	—	I
Social Gain	$E + F + G + H + I + J + K$	$E + F + G + K + I$
Deadweight Loss	—	$H + J$

We assume that Americans can buy any number of cars from Japan at the price P_0. The supply curve S shows how many cars American manufacturers will provide at each price. At the price P_0, American producers supply Q_0 cars and American consumers purchase Q_1 cars. The difference, $Q_1 - Q_0$, is the number of cars imported. Table A shows the gains to Americans.

In panel B we see the effect of a $500 tariff on imported cars. The price of a foreign car rises to P_0 + $500, and the number of imports falls to $Q_1' - Q_0'$. Table B compares gains before and after the tariff. Note that the first column of Table B is identical to Table A except that it uses the labels from panel B rather than panel A. The tariff revenue is computed by observing that the area of rectangle I is $(Q_1' - Q_0') \times$ $500.

Lecture Eleven (Chapter 8) General Equilibrium and The Efficiency of Perfect Competition

"It is not from the benevolence of the butcher, the brewer or the baker, that we expect our dinner, but from their regard to their own self interest. We address ourselves, not to their humanity but to their self-love, and never talk to them of our own necessities but of their advantages." – Adam Smith, "*The Wealth of Nations*"

The theory of the Invisible Hand states that *if each consumer is allowed to choose freely what to buy and each producer is allowed to choose freely what to sell and how to produce it, the market will settle on a product distribution and prices that are beneficial to all the individual members of a community, and hence to the community as a whole. All agents are lead "as if by an invisible hand" to seek a kind of collective good.*

In this lecture, we will demonstrate that the competitive market is the most efficient market structure according to the efficiency criterion discussed in the last lecture. However, we could not say it is the best market structure since the competitive market might end up with very extreme income distribution that one person owns everything, but the others have nothing.

1 General Equilibrium and Partial Equilibrium

- Economic equilibrium is a state of the world where economic forces are balanced
  - quantity demanded and quantity supplied are equal;
  - given the price, buyers can buy exactly amount they want to buy, and sellers can sell exactly amount they want to sell.
- Two Implication:
  - Market clearing;
  - Individual consumer maximizes his/her utility;
  - Individual firm maximizes its profit.

1.1 Two types of equilibria

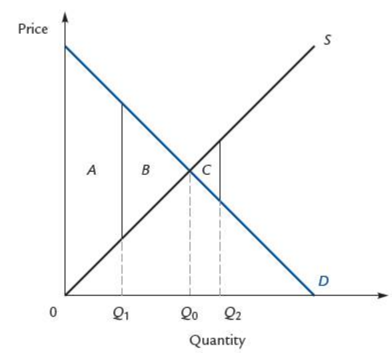
- General Equilibrium: studies the behavior of supply, demand and prices in a whole economy with several or many interacting markets.
  - Emphasize the interaction between markets;
  - The prices, quantity of all goods are determined simultaneously.

- Partial Equilibrium: studies the behavior of supply, demand and prices in an isolated market, assuming prices, quantities of other goods, as well as income levels of consumers, fixed.
  - Neglects the effect of changes in one market on other markets.
  - The prices of all other goods (substitutes and complements), as well as income levels of consumers are constant.

2 Efficiency Analysis in Partial Equilibrium

- **Question:** What is the social gain from trade in an isolated competitive market?
  - $SG = CS + PS$

EXHIBIT 8.20 The Invisible Hand



Under competition, the quantity produced is Q_0, where supply equals demand. A benevolent dictator who wanted to maximize social gain would employ the equimarginal principle and order potatoes to be produced to a quantity where marginal cost equals marginal value. This also occurs at quantity Q_0.
If the dictator ordered Q_1 potatoes produced, social gain would be area A; if he ordered Q_2, social gain would be $A + B - C$. The maximum social gain, at Q_0, is $A + B$.

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- **Question:** What is the possible maximum social gain we are able to achieve from trade?
  - Continue until marginal value is higher than marginal cost
- The point of partial equilibrium is also the point of maximum social gain.
  - The social gain from trade in a competitive market achieve the possible maximum social gain.
- **Question:** Do we prove the theory of "invisible hand"?

- **NO!** The theory of "invisible hand" is about **the whole economy**, rather than a single, isolated market. In other words, the "invisible hand" hypothesis could be proved only in the general equilibrium setting.

3 Efficiency Analysis in General Equilibrium with No Production

- **Question:** What is the social gains from trade when we study all the competitive markets as a whole?
  - Add up the social gains from individual markets
- **Question:** What is the possible maximum social gains we are able to achieve when we consider all the competitive markets as a whole?
  - All markets are related. By achieving maximum welfare gains in one market, we might impose more loss in other markets.
  - It is too hard, if not impossible, to **CALCULATE** the possible maximum social gains.

3.1 Pareto Efficiency (Pareto Optimality)

- **Pareto Criterion:** A normative criterion according to which one policy is better than another when it is preferred unanimously.
  - Different from efficiency criterion
    - No tax is better than tax according to efficiency criterion;
    - Tax recipients prefer to have tax. We cannot say which policy is better according to pareto criterion.
- **Pareto Improvement:** Given an initial allocation of goods among a set of individuals, a change to a different allocation that makes at least one individual better off without making any other individual worse off is called a Pareto improvement.
- **Pareto Efficiency (Pareto Optimality):** Any allocation where we cannot make one person better off without harming another person is Pareto efficient or Pareto optimal.

3.2 Pareto Efficiency Analysis in An Edgeworth Box

- Simplification Assumptions:
  - Exchange Economy: No production, so the numbers of goods available are constant;
  - Two agents: A and B;

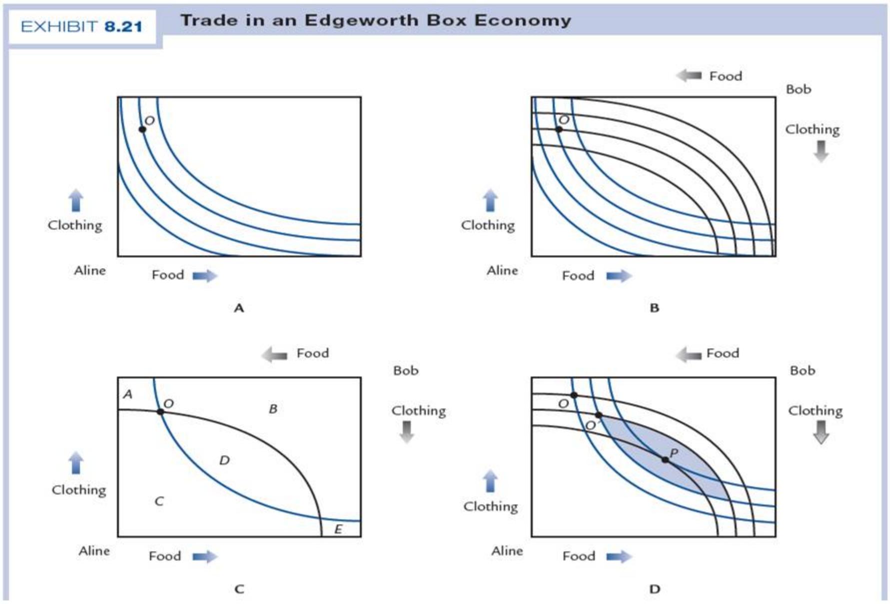
- Two goods: x and y;
- Initial Endowment:
  * For agent $A : \omega_A = (\omega_x^A, \omega_y^A)$,
  * For agent $B : \omega_B = (\omega_x^B, \omega_y^B)$,
  * The whole economy: $\omega = (\omega_x, \omega_y)$ with $\omega_x = \omega_x^A + \omega_x^B$ and $\omega_y = \omega_y^A + \omega_y^B$,

- Edgeworth Box
  - The height of the Edgeworth box is set equal to the total endowment of good y while the width is set as the total endowment of good x.
  - The initial endowment is a point in point O in the Edgeworth box.
  - The origin for agent A is in the lower left while the origin for agent B is in the upper right.
    * The amount of good x in agent A's endowment is the horizontal distance from agent A's origin to the endowment point;
    * The amount of good x in agent B's endowment is the horizontal distance from agent B's origin to the endowment point;
    * Good y is similar, but measured vertically.

- A Key thing about the Edgeworth box is that **EVERY POINT in the box represents a feasible allocation of the goods**.

$$x^A + x^B = x_y \text{ and } y^A + y^B = \omega_y$$

- Finding all the Pareto efficient points in the Edgeworth Box
  - Draw ICs for agents A and B
  - Given the endowment point O, where is the weakly preferred sets for agents A and B
  - Region of mutual advantage (area D in the graph): the joint set of the weakly preferred sets for agents A and B
    * It is a set that **Pareto Improvement could be done**.
    * The **Pareto Improvement** is done through exchange (from O to O')
  - Exchange will continue until no **Pareto Improvement** is possible.
    * Point P in the graph.
    * By definition of **Pareto Efficiency (Pareto Optimality),** point P is Pareto efficient or Pareto optimal
    * Agents A and B's ICs are tangent to each other at point P.
    * Higher utility for both agents A and B than the original endowment.



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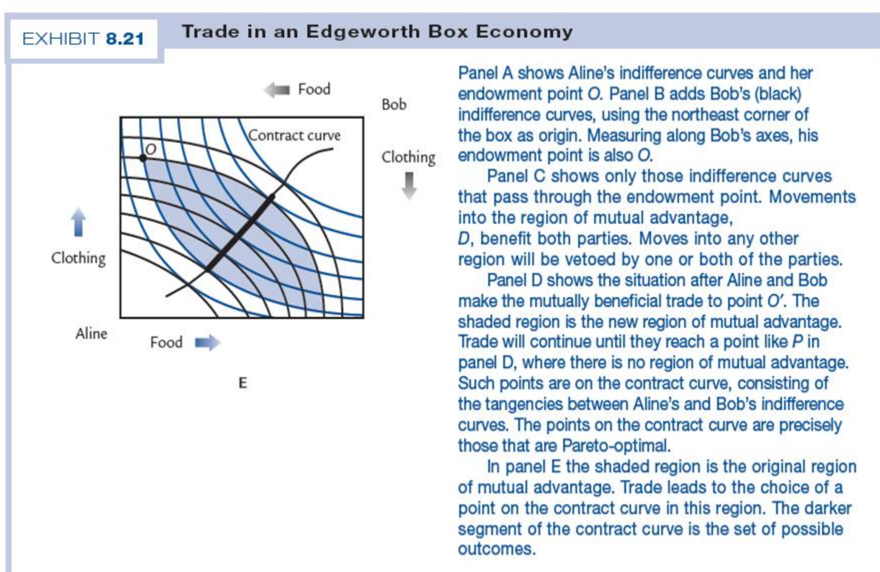
- – The collection of **all Pareto-optimal points** forms a curve, which is called the **contract curve**.
    - ∗ Final consumption points could be any point on the contract curve.



Panel A shows Aline's indifference curves and her endowment point *O*. Panel B adds Bob's (black) indifference curves, using the northeast corner of the box as origin. Measuring along Bob's axes, his endowment point is also *O*.

Panel C shows only those indifference curves that pass through the endowment point. Movements into the region of mutual advantage, *D*, benefit both parties. Moves into any other region will be vetoed by one or both of the parties.

Panel D shows the situation after Aline and Bob make the mutually beneficial trade to point *O′*. The shaded region is the new region of mutual advantage. Trade will continue until they reach a point like *P* in panel D, where there is no region of mutual advantage. Such points are on the contract curve, consisting of the tangencies between Aline's and Bob's indifference curves. The points on the contract curve are precisely those that are Pareto-optimal.

In panel E the shaded region is the original region of mutual advantage. Trade leads to the choice of a point on the contract curve in this region. The darker segment of the contract curve is the set of possible outcomes.

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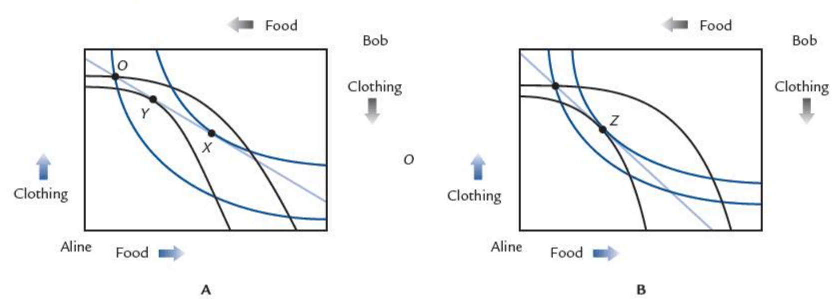
3.3 Competitive Equilibrium in The Edgeworth Box

From the last section, we know agents A and B can reach the **Pareto Efficient** points through exchange, and all the points on the contract curve are possible outcomes (final consumption). Now we restrict the exchange by agents A and B that they are required to bargain through the mechanism of a price system. That is also the mechanism of the competitive market.

EXHIBIT **8.22** **Competitive Equilibrium in an Edgeworth Box Economy**



In panel A a relative price has been suggested that leads to the budget line pictured. (This is Aline's budget line from her perspective and Bob's budget line from his.) Aline chooses point *X* and Bob chooses point *Y*. But these points are not the same; the quantities that Aline wants to buy and sell are not the same quantities that Bob wants to sell and buy.

In panel B a different relative price has been suggested. At this price Aline's desires are compatible with Bob's. Point *Z* is a competitive equilibrium.

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- Given a price, agents A and B should decide individually how much of x and y they would like to buy or sell.
  - That is what we have learned: Given the budget line, find the optimal consumption bundle.
  - Notice that the budge line has to pass through the original endowment since it is the point you decide to trade zero amount of x and y.
- If their desires are compatible under the price, they carry out the transaction.
  - The amount of x (y) agent A wants to buy (sell) is exactly the amount of x (y) agent B wants to sell (buy).
  - It is the market clearing condition.
- Competitive Equilibrium: A point (point Z in our case) that everyone (agent A and B in our case) will choose to trade to, for some appropriate market prices.
  - The appropriate market prices is called **competitive equilibrium prices**.

  - The final allocation (point Z) is called **competitive equilibrium allocation**.

- Notice that there might be mutiple competitive equilibrium points.

3.4 Invisible Hand in The Edgeworth Box (First Welfare Theorem)

- Some Implications from Competitive Equilibrium
  - Both agents A and B's ICs are tangent to the budget line, therefore agents A and B's ICs are tangent to each other.
  - So the competitive equilibrium points are on the contract curve.
  - Since all the points on the contract curve are **Pareto-optimal**, the competitive equilibrium points are **Pareto-optimal**.

- **First Fundamental Theorem of Welfare Economics:** If all markets are perfectly competitive, the allocation of resources will be Pareto efficient.

3.5 An Example

Jack Sprat could eat no fat, his wife could eat no lean. Suppose Jack is endowed with 4 pounds of fat and 8 pounds of lean, and his wife was endowed with 6 pounds of fat and 2 pounds of lean. In an Edgeworth box, show the endowment point, the Pareto optimal allocation(s), and the competitive equilibrium. Is the competitive equilibrium Pareto optimal? Assume the price of fat is \$1 per pound. What is the price of lean at the competitive equilibrium?

Lecture Thirteen (Chapter 10) Monopoly

Chapters 7 discusses one extreme type of market structure, i.e. perfectly competitive market and how consumers and firms interacts in this type of market. Chapter 8 starts with discussion about efficiency criterion, and how to measure gains for consumers, firms and the whole society in a perfectly competitive market using the criterion. Then Chapter 8 introduces the Pareto criterion to illustrate the invisible hand theorem in a general equilibrium setting. The invisible hand theorem is officially called the 1^{st} fundamental theorem of welfare economics, which states that If all markets are perfectly competitive, the allocation of resources will be Pareto efficient.

In Chapter 10, we will continue our discussion of market structure. We will learn another extreme market structure, that is, monopoly. Since perfect competition and monopoly are two extreme cases, it is easy for us to compare the definition and implication of the two extreme market structures.

- Definition
  - **Perfect competition**: a theoretical market structure that features **unlimited contestability (or no barriers to entry), an unlimited number of producers and consumers, and a perfectly elastic demand curve**
  - **Monopoly**: there is **only one provider of a product or service**
- Number of suppliers
  - PC: **unlimited number** of producers;
  - Monopoly: **one** producer;
- Market power
  - PC: no power to determine the market price alone, so a **price taker**;
  - Monopoly: have power to determine the market price alone, so a **price maker**;
- Market share
  - PC: **a small portion** of the market;
  - Monopoly: **whole** market;
- Demand curve
  - PC: face a **horizontal (perfect elastic) demand curve** for its product;
  - Monopoly: face a **downward sloping demand curve** for its product (since a monopoly firm is the whole industry and industry demand curve slopes downward);

- Quantity supplied
  - PC: can sell **any quantity it wants to** at the (given) market price;
  - Monopoly: quantity supplied and the price should be determined together;
- Barriers
  - PC: **no barriers** for other firms to enter the industry;
  - Monopoly: depends
    * High Tech (e.g. Microsoft): very low barrier;
    * High Fixed Cost Industry (e.g. telephone company, electricity company): high barrier;

1 Profit Maximization Problem of A Monopolistic Firm

The objectives for all the firms are the same, that is, to maximize their profit. As a review, we know, for any firm, profit maximization problem could be specified as follows

$$\max_Q \pi = TR - TC$$

and the corresponding FOC is

$$\frac{d\pi}{dQ} = \frac{dTR}{dQ} - \frac{dTC}{dQ} = MR - MC = 0$$

For all the firms, in a perfect competitive market or not, total revenue is $TR = PQ$.

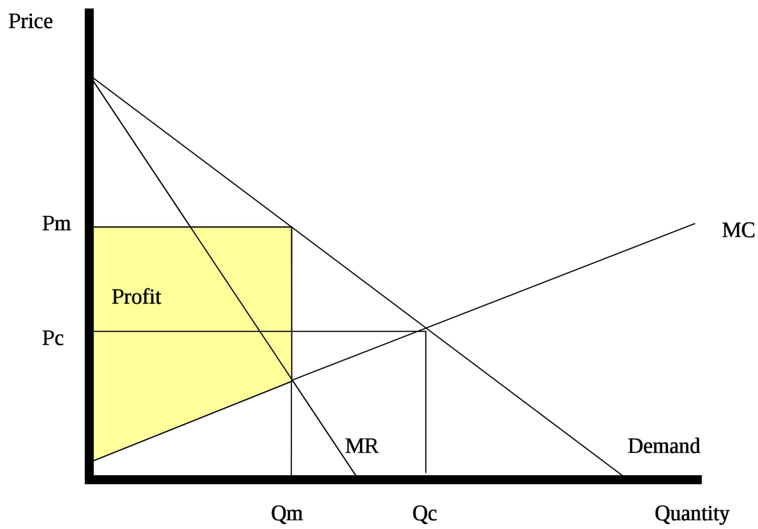
- For competitive firms, price is constant, $P = \bar{P}$. Therefore,
  - $TR = \bar{P}Q$
  - $MR = \bar{P}$: flat at the level of the market price
  - FOC implies $\bar{P} = MC$
- For monopolistic firms, since demand curve is downward sloping, the price P is a decreasing function of quantity Q, $P'(Q) < 0$. Therefore,
  - $TR = P(Q) \cdot Q$
  - $MR = \frac{dTR}{dQ} = P'(Q) \cdot Q + P(Q)$
    * Since $P'(Q) < 0$, then $MR(Q) < P(Q)$ for any Q. (MR curve lies everywhere below the demand curve.)
  - FOC implies $P'(Q) \cdot Q + P(Q) = MC$

1.1 Profit-maximizing Price Setting

- Algebraical Solution

$$P'(Q) \cdot Q + P(Q) = MC$$

- Graphical Solution



1.2 Price Elasticity of Demand and Monopoly Power Measurement

- Note that what the monopolistic firm is doing is cutting back on quantity produced in order to raise the price on all the units it sells. In other words, the monopolist is weighing the profit gains from increasing price and the profit loss from reducing sales. Therefore, by increasing price by ΔP,
  - The monopolist is able to get gain ΔP more for every unit of sale, which leads to a rise of TR by $\Delta P \cdot Q$.
  - The monopolist has to reduce his production by ΔQ, which leads to a reduction of TR by $P \cdot \Delta Q$.
  - That is described by total derivative

$$dTR = d(PQ) = \Delta P \cdot Q + P \cdot \Delta Q$$

- We use price elasticity of demand to study the relationship between ΔP and ΔQ,

$$\eta = \frac{\Delta Q/Q}{\Delta P/P} = \frac{dQ}{dP}\frac{P}{Q}$$

- Therefore

$$\begin{aligned} MR &= P'(Q) \cdot Q + P(Q) \\ &= \frac{P}{Q}\frac{1}{\eta} \cdot Q + P \\ &= P \cdot (1 - \frac{1}{|\eta|}) \end{aligned}$$

- Since monopolistic firms set price at $MR = MC > 0$, then $MR = P \cdot (1 - \frac{1}{|\eta|}) > 0$. Thus, $|\eta| > 1$. That is to say
  - A monopolist always operates on the elastic portion of the demand curve.
- The difference between price charged and the MC is called **markup**, $P - MC$. Sometimes we express the markup as a fraction of the price, which is called **Lerner Index**:

$$\textbf{Lerner Index} = \frac{P - MC}{P}$$

At the optimal choice point (also equilibrium point) of a monopolist, $MC = MR = P \cdot (1 - \frac{1}{|\eta|})$. Thus

$$\textbf{Lerner Index} = \frac{P - P \cdot (1 - \frac{1}{|\eta|})}{P} = \frac{1}{|\eta|} = |\frac{\Delta P/P}{\Delta Q/Q}|$$

  - Perfect competitive firm: $|\eta| \to \infty$ (**perfect elastic**), so $MR = P$ and Lerner Index =0.
  - Monopolistic firm: $|\eta| > 1$, so $MR < P$ and Lerner Index $\in (0, 1)$.
    - The higher the Lerner Index is (not possible to be higher than 1), the higher the market power is for a firm.

1.3 An Example: Linear Demand Curve

Suppose that demand function is $Q = \frac{a}{b} - \frac{1}{b}P$ and that the cost function is $C(Q) = cQ$,where $a, b, c > 0$ and $a > c$. Then the inverse demand is $P = a - bQ$. Thus

$$\begin{aligned} TR &= P(Q)Q = (a - bQ)Q = aQ - bQ^2 \\ MR &= \frac{dTR}{dQ} = a - 2bQ \\ MC &= c \end{aligned}$$

Thus,

$$\begin{aligned}
MR &= MC \\
&\Rightarrow Q^* = \frac{a-c}{2b} \text{ and } P^* = \frac{a+c}{2} \\
&\Rightarrow \pi = TR - TC = aQ^* - bQ^{*2} - cQ^* = \frac{(a-c)^2}{4b}
\end{aligned}$$

And the price elasticity of demand at the optimal point is

$$\begin{aligned}
\eta &= \frac{dQ}{dP}\frac{P}{Q}|_{P^*,Q^*} = -\frac{1}{b}\frac{P^*}{Q^*} = -\frac{1}{b}\frac{\frac{a+c}{2}}{\frac{a-c}{2b}} = -\frac{a+c}{a-c} \\
|\eta| &= \frac{a+c}{a-c} > 1 \\
Lerner\ Index &= \frac{1}{|\eta|} = \frac{a-c}{a+c}
\end{aligned}$$

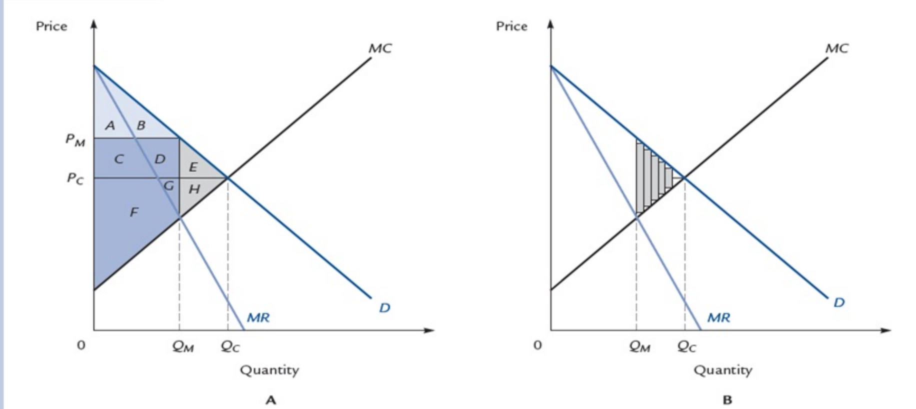
1.4 No Supply Curve for Monopolists

- When we look for a supply curve of a firm, we ask "How much would the firm produce at a given price".
- There are questions that a monopolist is never asked because the monopolistic firm is a price maker, rather than price taker.
- Therefore, there is no supply curve for a monopolistic firm.

2 Welfare Analysis and Remedies for the Efficiency Loss

2.1 Monopoly v.s. Competition

EXHIBIT **10.2** **Monopoly versus Competition**



	Competition	Monopoly
Consumers' Surplus	$A + B + C + D + E$	$A + B$
Producers' Surplus	$F + G + H$	$C + D + F + G$
Social Gain	$A + B + C + D + E + F + G + H$	$A + B + C + D + F + G$
Deadweight Loss	–	$E + H$

The table assumes that a monopoly and a competitive industry would have the same marginal cost curve. The competitive industry produces the equilibrium quantity Q_C, and the monopolist produces its profit-maximizing quantity Q_M. Because marginal value still exceeds marginal cost at Q_M, it would be efficient for additional units to be produced. The social gains from additional units after Q_M are represented by the rectangles in panel B. Because the monopolist does not produce those units, those social gains are sacrificed, giving a deadweight loss of $E + H$.

- Social Loss (DWL) $E + H$ is due to the existence of the monopoly.
  - From the society's point of view, when output is at Q_M, the social marginal value (demand curve) still exceeds social marginal cost. Therefore, it is socially beneficial to produce more.
  - From the monopolist's point of view, when output is at Q_M, the firm marginal revenue (MR curve) equals to the marginal cost. Therefore, it is an optimal output for the firm.

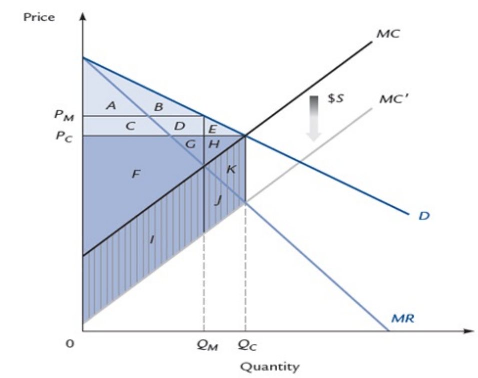
- Failure of the "Invisible Hand"?
  - **YES!** The original story of *"Invisible Hand"* states that *if each consumer is allowed to choose freely what to buy and each producer is allowed to choose freely what to sell and how to produce it, the market will settle on a product distribution and prices that are beneficial to all the individual members of a community, and hence to the community as a whole. All agents are lead "as if by an invisible hand" to seek a kind of collective good.*
  - **NO!** The modern story of *"Invisible Hand", which is the **First Fundamental Theorem of Welfare Economics*** states that *If all markets are perfectly competitive, the allocation of resources will be Pareto efficient.*

2.2 Remedies to Reduce the Efficiency Loss from Monopoly

- The efficiency loss from monopoly is because the optimal production level chosen by the monopolistic firm is lower than the socially optimal level.
- Therefore, in order to reduce or even eliminate the efficiency loss, we should find a way to make the monopolist choose the socially optimal level of production.

2.2.1 Subsidize Monopolists

EXHIBIT **10.3** **A Subsidized Monopolist**

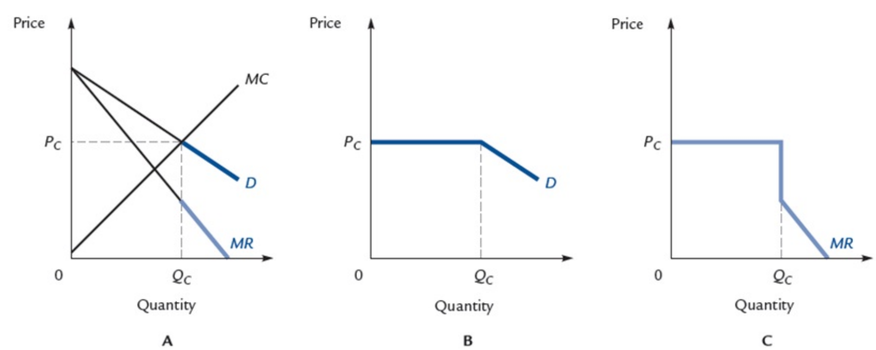


	Competition	Unsubsidized Monopoly	Subsidized Monopoly
Consumers' Surplus	$A + B + C + D + E$	$A + B$	$A + B + C + D + E$
Producers' Surplus	$F + G + H$	$C + D + F + G$	$F + G + H + I + J + K$
Cost to Taxpayers	—	—	$I + J + K$
Social Gain	$A + B + C + D + E + F + G + H$	$A + B + C + D + F + G$	$A + B + C + D + E + F + G + H$

An unsubsidized monopolist produces the quantity Q_M. The subsidy of $S per unit of output, which lowers the marginal cost curve to MC', is chosen to be of just the right size so that the monopolist will now produce the competitive quantity Q_C. Because the competitive quantity maximizes social gain, the deadweight loss is eliminated. The table confirms that social gain is the same as it would be under competition.

2.2.2 Price Ceiling

EXHIBIT **10.4** **A Price Ceiling**



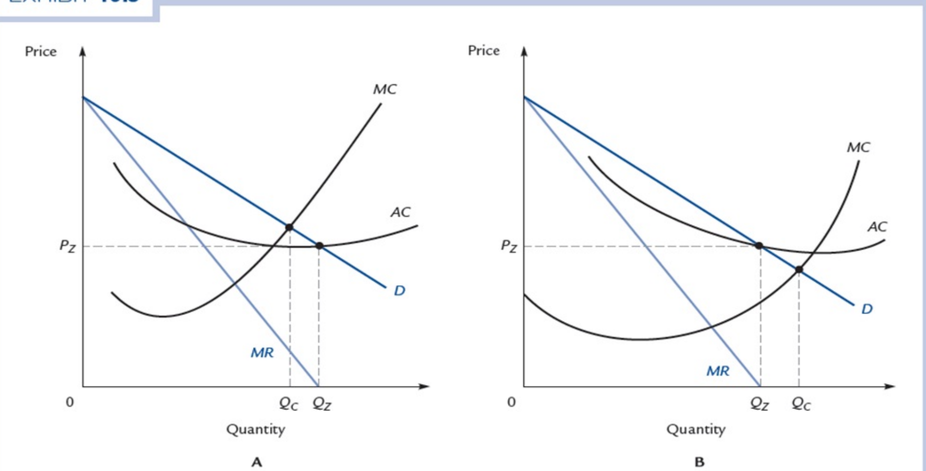
If a monopolist is required by law to charge no more than the competitive price P_c, then it effectively faces the demand and marginal revenue curves shown in panels B and C. It produces at the point Q_c, where marginal cost and marginal revenue are equal.

- The problem with both subsidizing and price ceiling is how do you know
  - how much subsidy to give?
  - what level of price ceiling to set?
- You need to know both MC and MR to figure out the socially optimal level of production. However, the monopolistic firm has no incentive to tell the regulator the true MC.
- Therefore, in practice, the regulator use an alternative way: Rate-of-Return Regulation.

2.2.3 Rate-of-Return Regulation (RRR, also called Average-cost Pricing)

- The idea is that since there are no economic profits in a free entry competitive equilibrium, but positive profits under monopoly, we can force a monopolistic firm to behave like a competitive firm by regulating their price such that $P = AC$. By doing that, the monopolistic firm must earn zero economic profits.
- **Problem: zero-profit production level is usually not the socially optimal production level under monopoly.**
  - Zero-profit level could be **either greater or less** than socially optimal level
  - The resulting DWL could even higher than the unregulated case.

EXHIBIT **10.5** **Zero-Profit Regulation of Monopoly**



The two panels show two possible configurations of demand, marginal revenue, marginal cost, and average cost curves for a monopolist. If the monopolist is required by law to earn zero profits, it will produce that quantity Q_z at which the demand price is equal to average cost. The efficient level of output is Q_c, where marginal cost equals demand. As the two panels show, Q_z could be either greater or less than Q_c.

- **Additional Problems:** RRR provides the monopolists with
  - no incentive to have technology improvement;

– an incentive to waste, which increases AC

  ∗ e.g. Sinopec: \$.24 million dollars of alcohol purchases and \$1.76 million for one pendant lamp.

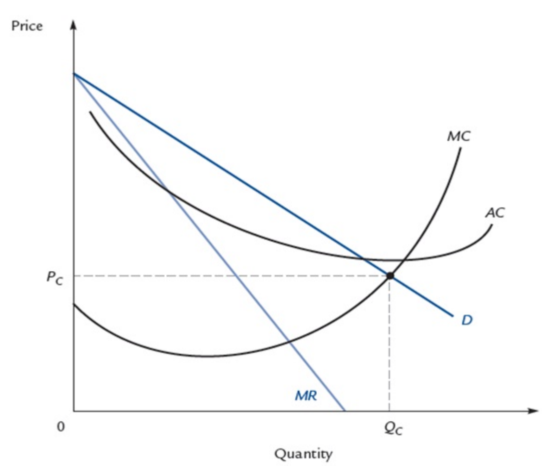
3 Sources of Monopoly Power

In this section, we ask the question of why monopolists arise in the first place.

3.1 Natural Monopoly

- Natural Monopoly: An industry in which each firm's average cost curve is decreasing at the point where it crosses market demand.
- Two possible reasons
  - An industry characterized by high FC but low MC such as railroads, water utilities
    - ∗ Draw TC curves with high and low FC, show the AC. When the FC is high, the minimum point of AC appears when quantity supplied Q is larger.
  - An industry experiencing increasing returns to scale to a high production level such as telephone company (Network externalities).
    - ∗ Draw TC curves with short-period and long-period of IRS.
- Under conditions of natural monopoly, a competitive industry cannot survive.
  - In other words, socially optimal level of production always leads to negative economic profits.

EXHIBIT **10.6** **Natural Monopoly**



A natural monopoly occurs when each firm's average cost curve is downward sloping at the point where it crosses industry demand. Because marginal cost crosses average cost at the bottom of the U, marginal cost must cross demand at a point where price is below average cost. Thus, if the firm priced competitively, it would earn negative profits.

- In practice, two ways to deal with natural monopoly
  - **Rate-of-Return Regulation**: resulting production level is better than monopoly pricing, but is still less than the socially optimal level.
  - **Public Ownership**: produce at the socially optimal level, and then get lump-sum subsidy.
- For some industry, it is possible to decouple the natural monopoly part from the competitive part. Take electricity company for example, power transmission lines could be owned by public or by a regulated monopolistic firm, but electricity supplying can be provided by competitive firms.

3.2 Patents

These are legally enforced monopolies. They are awarded for a 17 year period in order to encourage innovation and compensate firms for putting money into research and development. On the other hand, the cost to society is that for the 17 years, there are DWL's from monopoly patent holders.

It is hard to set a patent's length and breadth correctly. Some innovations are easy and would take place with little or no reward. Other innovations are very difficult and so certain areas of research may be too speculative to payoff under the 17 year system. One size does not fit all! On the other hand, if I invented the blow dryer, show I get a patent on all hot air drying technologies, or just the specific blow-drying machine I developed. Too broad a patent, and you foreclose innovation and over compensate inventors. Too narrow and you discourage innovation (my competitor just puts out the same machine as me but in blue plastic instead of green.) It is also difficult for the patent office to determine what was the "prior art", something that existed already or was obvious to anyone in the industry and so is not by law patentable.

If patent protection is too strong then too many people try to innovate. This means that some of the best people are not going into economics, medicine or politics, and this could be costly to society.

3.3 Legal Barriers to Entry

Sometimes the state awards a monopoly or makes it illegal to enter an industry.

- Examples:
  - Highway oasis;
  - Local cable service;
  - Hudson's Bay Company
  - Philippine beer monopoly
- These generate rents that people try to get. How do we see rent seeking in real life?

- Lobbying
- Campaigning
- Marry into the royal family
- Bribery
- Offer something of value to society in return

- Question: What are the welfare implications?

3.4 Natural Resource Monopoly

when someone owns all of a particular item that exists in the world. This is most common with natural resources.

- Examples: Alcoa and aluminum, OPEC and Oil, DeBeers and diamonds, Picasso and his paintings.
- This is the classic bad monopoly. There is no offsetting social gain. It is good if you are a seller of natural resources, but bad if you are a consumer.

4 Price Discrimination

So far we have considered a very simple use of monopoly power. We have assumed that the monopolist sets a **single price** and sells at these prices **to everyone**.

However, if the monopolist use his monopoly power in a more complicated way, he might **earn more profits.**

- For example, a dealer has the only two cars in town worth \$500 on the outside market and he wants to sell them to two people, one who was willing to pay \$2000, and one who would only pay at most \$1500.
  - If the dealer could only set a single price and sells car to everyone at the price. Then he is comparing
    - Set $P = \$2000$, only one buyer. Profits $\pi = \$2000 - \$500 = \$1500$;
    - Set $P = \$1500$, two buyers. Profits $\pi = (\$1500 - \$500) \times 2 = \$2000$;
  - If the dealer is able to charge different buyers at different prices. Then
    - Charge $P_1 = \$2000$ to the first buyer and $P_1 = \$1500$ to the second buyer. Profits $\pi = (\$2000 - \$500) + (\$1500 - \$500) = \$2500$;
- Sometimes a monopolist can increase its profits by charging different prices for identical items. This practice is know as **Price Discrimination**. There are three type of **Price Discrimination:**

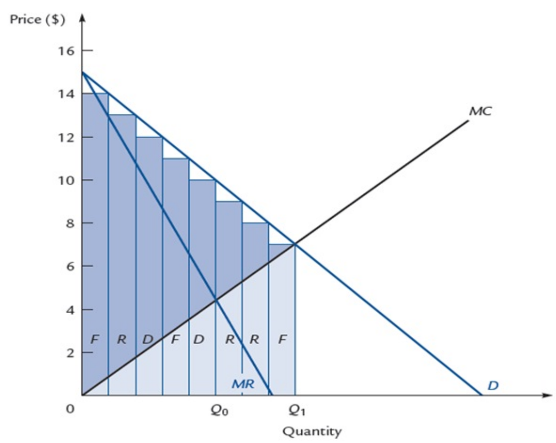
– **First-degree price discrimination:** Charging each customer the most that he/she would be willing to pay for each item that he/she buys; in other words, every customer is charged exactly his/her **reservation price**.

– **Second-degree price discrimination:** Charging the same customer different prices for identical items; different units of the good are priced differently.

– **Third-degree price discrimination:** Charging different prices for identical items in different markets;

4.1 First-degree Price Discrimination

EXHIBIT **10.8** First-Degree Price Discrimination



The rectangles show the marginal values of pies to Mrs. Lovett's customers, with each labeled by the initial of the corresponding customer. If she charges each customer the maximum amount that she is willing to pay for a pie, Flicka will have to pay $14 for her first pie, Ricka will pay $13 for her first pie, and so on. Because each customer pays her marginal value for each pie, there is no customers' surplus. All of the surplus is earned by Mrs. Lovett, who gains the entire shaded area.

- Welfare Analysis:
  – **Pareto Efficient**
  – **increase in production**
  – **all the surplus is earned by producers**
  – **consumers are worse off**

- Example One: In 1990, IBM introduced the LaserPrinter E. The difference is that it printed 5 ppm rather than 10 ppm. They did so by **ADDING** 5 **chips** in the E model. The purpose of the chips was to make the printer WAIT. The price of the new laserprinter E was 60% of the old one. Why did **IBM pay for** a reduction in the speed?
  – Suppose Jim values the faster printer at $1000 and the slower printer at $700, and Sean values the faster printer at $700 and the slower printer at $600.

- It costs \$450 to make the faster printer and \$475 to make the slower printer. What should IBM charge for either printer?
- If IBM only sells the fast printer, what should it charge? \$700. $Profits = (\$700 - \$450) \times 2 = \$500$.
- If IBM wants to sell the fast printer to Jim and the slow printer to Sean, what should it charge? \$1000 for faster printer and \$600 for slower printer. $Profits = (\$1000 - \$450) + (\$600 - \$475) = \$675$.

- Example Two: Fast delivery service may hold back packages that are 2^{nd} day rather than overnight.

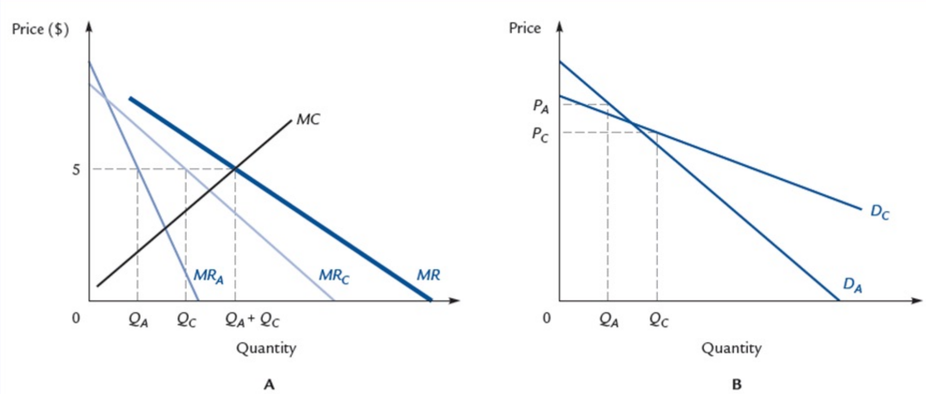
4.2 Second-degree Price Discrimination

- Welfare Analysis
  - **Pareto Improvement**
  - **increase in production**
  - **benefits the producers**
  - **consumers might be worse off or better off**
- Example One: Quantity discount, Coke \$1 each, but \$4 for a pack of six.
- Example Two: Dental Plaster

4.3 Third-degree Price Discrimination

- Monopoly in two markets with different demands

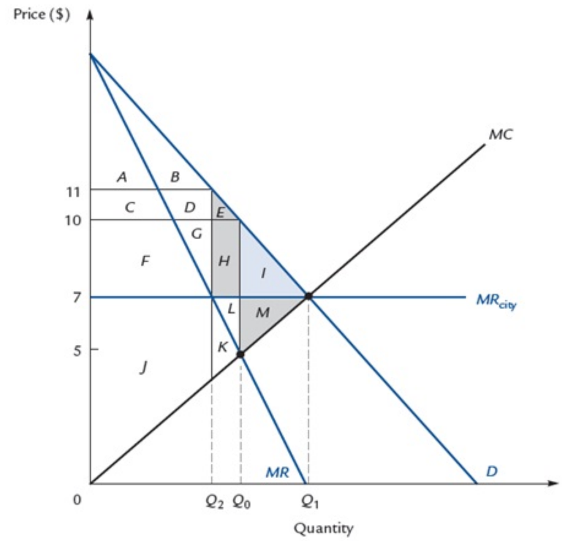
EXHIBIT **10.10** **Third-Degree Price Discrimination by a Monopolist in Two Markets**



Benjamin Barker sells haircuts to adults and children. The two groups have different marginal revenue curves, labeled MR_A and MR_C in panel A. The heavier curve, MR, is obtained by horizontally summing the curves MR_A and MR_C. Benjamin produces the quantity $Q_A + Q_C$ where MC crosses MR, selling Q_A haircuts to adults and Q_C to children. He chooses the corresponding prices off the adults' and children's demand curves, which are shown in panel B.

- Monopoly in one market and competition in another

EXHIBIT 10.9 **Third-Degree Price Discrimination with Monopoly in One Market and Competition in Another**



	Competition	Ordinary Monopoly	Price-Discriminating Monopoly
Consumers' Surplus	$A+B+C+D+E+F+G+H+I$	$A+B+C+D+E$	$A+B$
Producers' Surplus (Local)	$J+K+L+M$	$F+G+H+J+K+L$	$C+D+F+G+J$
Producer's Surplus (City)	–	–	$K+L+M$
Social Gain	$A+B+C+D+E+F+G+H+I+J+K+L+M$	$A+B+C+D+E+F+G+H+J+K+L$	$A+B+C+D+F+G+J+K+L+M$
Deadweight Loss	–	$I+M$	$E+H+I$

The demand and marginal revenue curves are from Mrs. Lovett's hometown market. In the distant city she can sell all of the pies she wants to at the competitive price of \$7. In that case, she will sell only Q_2 pies at home, as opposed to the ordinary monopoly quantity Q_0. The reason is that she can always earn \$7 marginal revenue by selling pies in the city, so that she will not sell pies at home when her marginal revenue there falls below \$7. When she sells Q_2 pies at home, she sets a price of \$11, higher than the ordinary monopoly price of \$10. The table shows what social gains would be if the pie industry were competitive, if Mrs. Lovett were an ordinary monopolist, and if Mrs. Lovett were able to sell pies in both markets at different prices. In each case, the consumers' surplus comes entirely from the local market. There is no consumers' surplus in the city market, because the demand curve there for Mrs. Lovett's pies is flat.

- Welfare Analysis
  - **Not necessarily to be Pareto Improvement**
  - **increase or decrease in production**
  - **benefits the producers**
  - **consumers might be worse off or better off**
- Example One: Book publisher having a cheap international edition of a book, even cheaper edition for some countries.
- Example Two: Publisher charging a higher rate to libraries than to individuals.
- Example Three: Airline Tickets

4.3.1 Elasticities and Third-degree Price Discrimination

Suppose there are two agents, A and B with separate demand curves. Profits are maximized at $MR_A = MR_B = MC$. Otherwise, the producer should keep output the same and sell more in the high MR market and less in the low MR market.

We have learned

$$MR = P \cdot (1 - \frac{1}{|\eta|})$$

By $MR_A = MR_B$, we get

$$P_A \cdot (1 - \frac{1}{|\eta_A|}) = P_B \cdot (1 - \frac{1}{|\eta_B|})$$

Therefore, if $|\eta_A| > |\eta_B|$, then $P_A < P_B$.

- Conclusion: The group with the more elastic demand is charged the lower price.
  - In other words, a price-discriminating monopolist offers the lowest prices to the most price-sensitive customers.

4.4 Conditions for Price Discrimination

Since price discrimination is usually profitable, why don't we observe price discrimination in all monopolistic industries?

- There are several conditions necessary for an industry to make profits by employing price discrimination:
  - **Monopoly Power**: if I try to charge above MC and there are other competitive firms that can enter the market, they will undercut my price and still make profits. Thus, I must have market power if I hope to charge any price higher than MC.
  - **Identification:** If I cannot tell who the high demand guy is, both will claim to be the low demand guy. Thus, I can either sell one car at \$2000 or two cars at \$1500. If I can tell who is who, on the other hand, I can just set the price at their reservation level and tell them take it or leave it.
  - **No Resale:** Even if I can identify the type of buyers if the low demand guy can walk in and get the low price and then turn around and sell to the high demand guy, then I will not be able to price discriminate.

Lecture Thirteen (Chapter 11) More on Market Power (Merge and Oligopoly)

Chapters 7 discusses perfect competition and Chapter 10 talks about monopoly. These are two extreme cases of market structure. In Chapter 11, we are going to learn another type of market structure, i.e. oligopoly.

1 Acquiring Market Power

We start by exploring some methods that firms used to acquire and exploit market power.

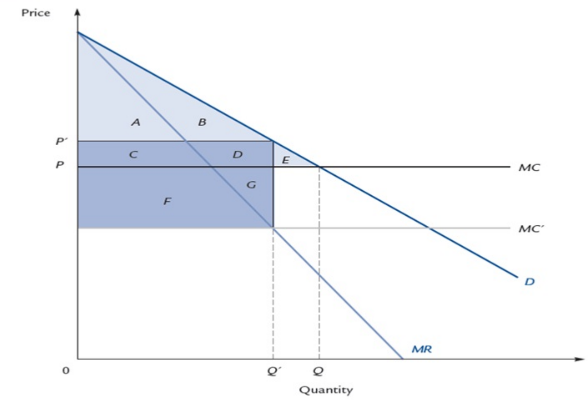
1.1 Mergers

One common observation in business is that two or more firms merge to form a larger firm. It is one practice for firms to acquire market power since the market share of the new firm is larger. Mergers can be roughly classified into two types: **Horizontal Integration and Vertical Integration.**

1.1.1 Horizontal Integration

- Horizontal Integration: A merger of firms that produce the same product.
- Purposes:
  - Increase efficiency (lower MC, explore economies of scale)
  - Gain market power (power to set price)
- Welfare Analysis:
  - **in one hand, low MC leads to more production and low price**
  - **in the other hand, market power leads to less production and higher price.** Therefore,
    - Good for firms: increasing PS;
    - CS: uncertain (depends on the new price);
    - Society: uncertain.

EXHIBIT **11.1** **A Horizontal Merger**

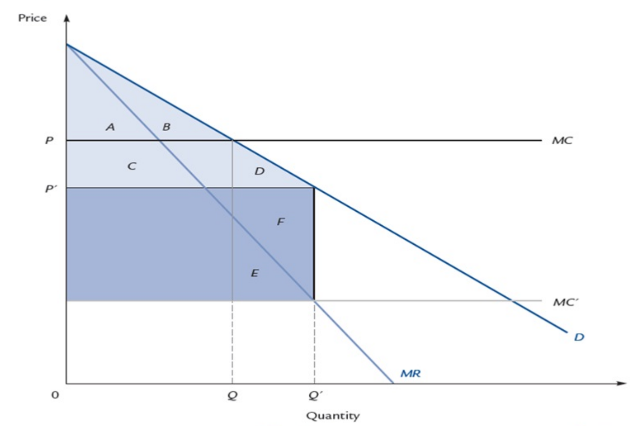


	Before Merger	After Merger
Consumers' Surplus	$A + B + C + D + E$	$A + B$
Producers' Surplus	—	$C + D + F + G$
Social Gain	$A + B + C + D + E$	$A + B + C + D + F + G$

Initially, the industry's marginal cost (= supply) curve is *MC*. If the industry is competitive, it produces the equilibrium output *Q* at the price *P*. Because the *MC* curve is horizontal, there is no producers' surplus.

Following a merger, marginal cost is reduced to *MC′*, but the newly created firm has monopoly power and so produces the quantity *Q′*, where *MC′* crosses the marginal revenue curve *MR*. The monopoly price is *P′*. The table above computes welfare before and after the merger.

EXHIBIT **11.2** **A Horizontal Merger Leading to a Large Cost Reduction**



	Before Merger	After Merger
Consumers' Surplus	$A + B$	$A + B + C + D$
Producers' Surplus	—	$E + F$
Social Gain	$A + B$	$A + B + C + D + E + F$

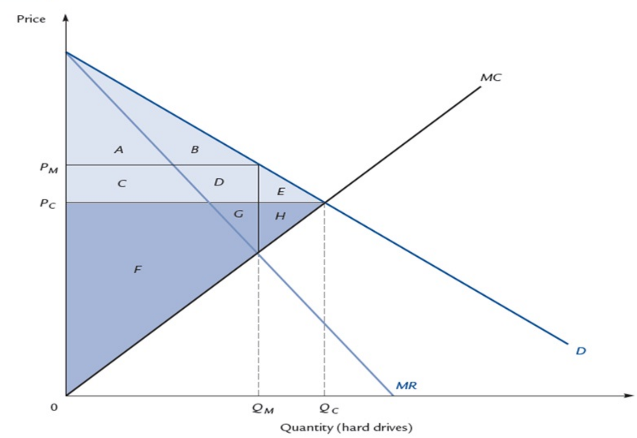
If the competitive industry's marginal cost curve is *MC*, and if a merger converts the industry into a monopoly with the much lower marginal cost curve *MC′*, then price will fall from *P* to *P′*, benefiting both consumers and producers.

1.1.2 Vertical Integration

- Vertical Integration: A merger between a firm that produces an input and a firm that uses that input.
- Purposes:
  - Internalize input costs (lower MC)
  - Gain market power (power to set price)
- Welfare Analysis: **Very COMPLICATED**

- **Depends on the specifics of the two firms (Monopoly or Competitive)**
- One simple example: two monopolists merges (Dell and Seagate)
  * Input costs are internalized, so the new firm is maximizing the sum of CS and PS in the input market;
  * Lower input costs
  * More final production
  * Good for the new firm: increasing PS;
  * Good for consumers: increasing CS;
  * Society: SG rises.

EXHIBIT **11.3** **Vertical Integration**



A monopoly hard drive manufacturer (Seagate) produces Q_M hard drives for sale to a monopoly computer manufacturer (Dell). This maximizes producer's surplus at $C + D + F + G$ while restricting consumers' surplus to $A + B$.

If Dell acquires ownership of Seagate, it will earn both the producer's and the consumers' surpluses and will therefore want to maximize the sum of the two. This is accomplished by producing the quantity Q_C of hard drives, creating a gain equal to the sum of all the lettered areas. Social gain is increased by $E + H$. More hard drives are produced, more computers are produced, and the price of computers goes down.

1.1.3 Predatory Pricing

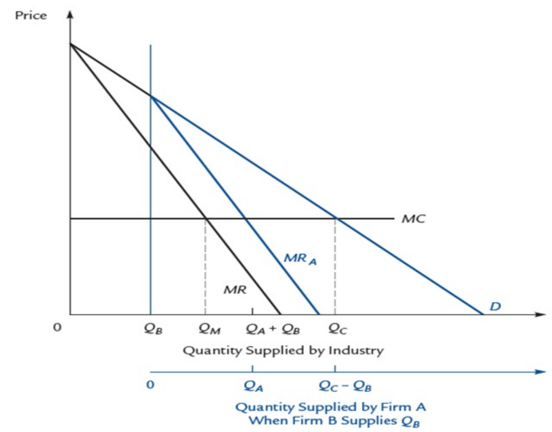
1.1.4 Resale Price Maintenance

2 Oligopoly

- Oligopoly: a market structure in which a market is dominated by a small number of sellers (oligopolists).
- Duopoly: a specific type of oligopoly where only two producers exist in one market.
- Collusion: an agreement among firms to set prices and outputs.
- We introduce two models in modeling oligopoly.
  - Common assumptions: **Two firms with no collusion**;
  - Cournot Model: Firms take their rivals' **output as given**;
  - Bertrand Model: Firms take their rivals' **price as given**.

2.1 Cournot Model

EXHIBIT 11.9 The Cournot Model of Oligopoly



We assume that two identical firms have the flat marginal cost curve *MC* and face a market demand curve *D*. A competitive industry would produce the quantity Q_C. A monopolist would produce the quantity $Q_M = \frac{1}{2}Q_C$, where *MC* crosses the marginal revenue curve *MR*.

If Firm A assumes that Firm B will always produce quantity Q_B, then Firm A views itself as a monopolist in the market for the remaining quantity. The demand curve in that market is the colored part of the market demand curve, measured along the colored axis. The marginal revenue curve is the colored curve MR_A. Firm A produces the monopoly quantity Q_A, which is half the competitive quantity $(Q_C - Q_B)$. Combining this fact with the equation $Q_A = Q_B$ (which follows from the fact that the firms are identical), we compute that $Q_A = Q_B = \frac{1}{3}Q_C$. Thus, the industry output is $\frac{2}{3}Q_C$, less than the competitive output but more than the monopoly output.

2.1.1 Symmetric Example:

Assume two identical firms with cost function $TC(Q) = 30Q$ and market demand function $Q = 120 - p$.

- Take firm 2's output Q_2 as given, firm 1 maximize its profit

$$\max_{Q_1} \pi = pQ_1 - TC(Q_1) = (120 - Q_1 - Q_2)Q_1 - 30Q_1$$

  FOC:

$$Q_1 = \frac{90 - Q_2}{2}$$

- Similarly, firm 2 take firm 1's output Q_1 as given and maximize its profit, we have

$$Q_2 = \frac{90 - Q_1}{2}$$

- Equilibrium

$$\begin{aligned} Q_1 &= \frac{90 - Q_2}{2} = \frac{90 - \frac{90 - Q_1}{2}}{2} \\ &\Rightarrow Q_1 = Q_2 = 30 \end{aligned}$$

2.2 Bertrand Model

The assumptions are the same as in the Cournot model except that firms decide on prices rather than quantities. However, the small change leads to totally different conclusion. Since in the Bertand model, firms compete with price as instrument. As long as price exceeds marginal cost, any firm in the Bertrand model will always want to undercut its rivals by offering a slightly lower price. This tiniest of price cuts leads to a sizable increase in sales, so the price cutting procedure will go on until price equals to marginal cost.

- Conclusion: Bertand competition leads to $P = MC$. This is the same condition we get under perfect competition.

2.3 Bertrand Competition v.s. Cournot Competition

- Bertrand predicts a duopoly is enough to push prices down to marginal cost level; a duopoly will result in an outcome exactly equivalent to what prevails under perfect competition.
- Neither model is necessarily "better." The accuracy of the predictions of each model will vary from industry to industry, depending on the closeness of each model to the **industry situation**.
- If capacity and output can be easily changed, Bertrand is generally a better model of duopoly competition. Or, if output and capacity are difficult to adjust, then Cournot is generally a better model.
- Under some conditions the Cournot model can be recast as a two stage model, where **in the first stage firms choose capacities, and in the second they compete in Bertrand fashion.**

Lecture Fourteen (Chapter 18) Decision Making Under Risk and Uncertainty

In our first class of introduction, we know that **Economics studies the rational decision of economic agents when facing resource constraints**. Until now, we have been concerned with the rational decisions made by consumers and producers in a world of absolute certainty. For example, consumers know the prices of all commodities and knows that any feasible consumption bundle can be obtained with certainty. Clearly, economic agents in the real world cannot always operate under such pleasant conditions. Many economic decisions are made under partial or even total ignorance. Take purchasing decision of an automobile as an example, the consumer might consider the future price of gasoline, expenditure on repairs (expensive repairment fee of German car in US), and the resale value of the car several year later (Japanese car depreciates less). None of these factors is known with certainty at the time of the decision. Decisions like this involve uncertainty about the outcome of the choice that is made. In this lecture, we explore how the theory we learned before can be modified to take uncertainty into consideration.

More examples of decisions involving uncertainty for both consumers and producers:

- buying car / home insurance;
- saving for retirement;
- deciding on a production plan for your firm;
- participating in a joint venture;
- investing in R&D;
- hiring a manager.

1 Consumption Bundles and Budget Line Revisit

1.1 Consumption Bundles

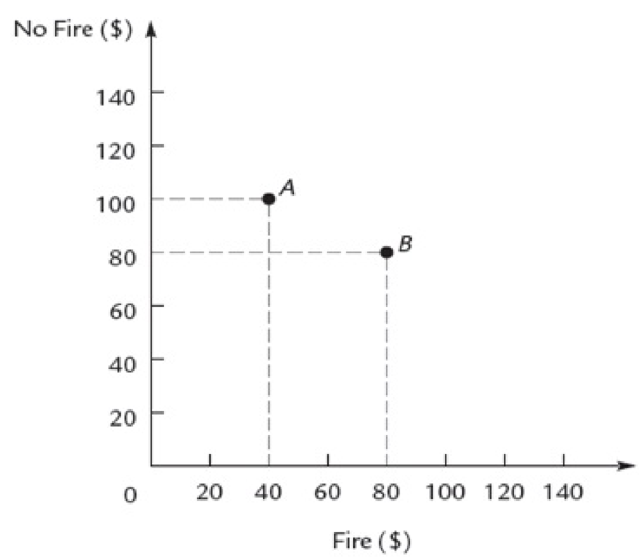
In the early classes, the consumer was assumed to have a preference over all consumption bundle x in the consumption set X.

- Previously, we distinguish consumption good in a bundle x by their physical characteristics.
  - An apple is not the same consumption good as a haircut or a car.
- Under uncertainty, we should also distinguish consumption good in a bundle x by the conditionality of commodities.

- An umbrella at a rainy day is not the same consumption good as an umbrella at a shining day.
- **State of the world: a potential set of conditions**, **potential outcomes of an uncertain situation**.
- The usefulness/value of an commodity depends on the state of the world.
- The price could be different in different state of the world.
- People have preference over consumption bundles like (# of umbrella when it rains, # of umbrella when the sun is shining)

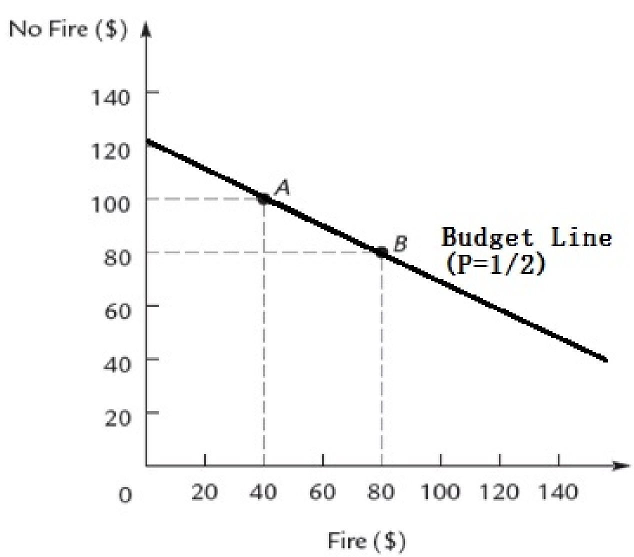
• In general, we can modify the consumption bundle $x = (x_1, x_2)$ to be (consumption of x in state 1, and consumption of x in state 2).

- Example: your wealth depends on whether there is a fire (state of the world).
- Your total wealth is \$100 if there is no fire and is \$40 if there is a fire, so the original consumption bundle (also endowment bundle) is (\$100, \$40).
- By buying an insurance of \$20, the insurance company guarantees to pay you \$60 when there is a fire and \$0 when there is no fire. So the consumption bundle is (\$80, \$80).
  - ∗ No fire: $\$100 - \$20 = \$80$;
  - ∗ Fire: $\$40 - \$20 + \$60 = \$80$.
- Your preference over two consumption bundles (\$100, \$40) and (\$80, \$80) determines whether you want to buy the insurance.
  - ∗ Ex ante preference: before the state of the world is known.



1.2 Budget Line (Opportunity)

- Budget line is all the consumption bundles that you have the opportunity to reach given all the prices.
- Insurance Example:
  - Full insurance: cover all possible loss, buy an insurance to claim all the losses when bad things happen.
    * Buy an insurance to get \$60 from insurance company when there is a fire.
  - Partial insurance: cover part of possible loss, buy an insurance to claim part of the losses when bad things happen. (of course you pay less for partial insurance than full insurance)
    * Buy an insurance to get \$10 from insurance company when there is a fire.
  - Insurance plan, policy rate (price of insurance).
    * policy rate$= \frac{\text{coverage when fires}}{\text{payment to buy an insurance}} = \frac{C}{I} = \frac{\$60}{\$20} = \frac{3}{1}$.
  - Budget line
    * Pass the endowment point (\$100, \$40);
    * Slope of the budget line equals to $\frac{\text{Gain if no fire}}{\text{Gain if fire}} = \frac{-\$20}{\$40} = -\frac{1}{2}$.
    * **Notice that some points on the budget line corresponding to your sales, rather than purchase of insurance.**



2 Preferences and Optimal Choice Revisit

Once we get all the consumption bundles, we can use ICs to represent an individual's preferences among the bundles. The discussion of the shape of the ICs can be easily discussed after we study iso-expected value line.

2.0.1 Iso-expected Value Line

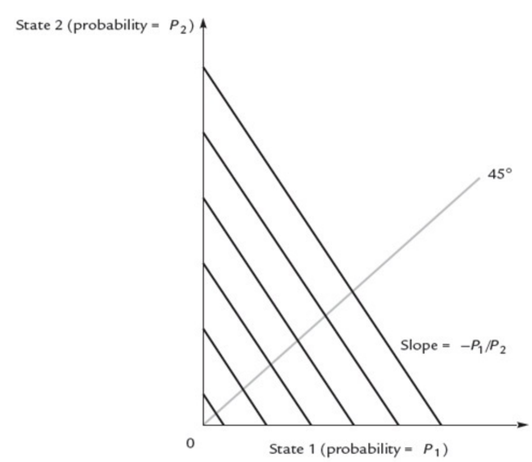
- Expected value: the average value over all states of the world, with each state weighted by its probability.

$$
\begin{aligned}
E(\text{wealth}) &= \text{Prob(fire)} \times (\text{wealth when there is a fire}) \\
&\quad + \text{Prob(no fire)} \times (\text{wealth when there is no fire})
\end{aligned}
$$

- For example, Prob(fire)$= .25$, Prob(no fire)$= .75$, then
  - No insurance
  $$E(\text{wealth}) = .25 \times \$40 + .75 \times \$100 = \$85$$
  - With insurance
  $$E(\text{wealth}) = .25 \times \$80 + .75 \times \$80 = \$80$$

- Bundles with the same Expected Value (Iso-expected Value Line)
  - $P_1 \cdot w_1 + P_2 \cdot w_2 = \bar{E}(w)$
  - $w_2 = \frac{\bar{E}(w) - P_1 \cdot w_1}{P_2}$
  - $Slope = -\frac{P_1}{P_2}$
  - Riskiness: variation in potential outcomes.
  - Riskfree: Same value in any state of the world
    - Bundles on the 45° line.

EXHIBIT **18.2** **Baskets with the Same Expected Value**



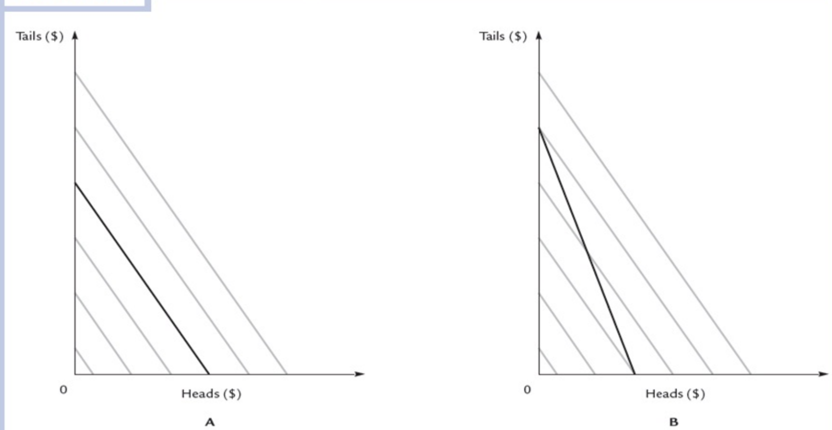
If the probability of state 1 is P_1 and the probability of state 2 is P_2 (so that $P_1 + P_2 = 1$), then all of the baskets along a line of slope $-P_1/P_2$ have the same expected value. The graph shows a family of such lines.
The baskets along the 45° line are risk-free, because a person holding such a basket will have the same wealth in either state of the world. Moving along an iso-expected value line away from the 45° line in either direction, the baskets become successively riskier.

- Fair Odds: Odds that reflect the true probabilities of various states of the world.
  - When an individual is offered fair odds, his budget line coincides with an iso-expected value curve.
  - Insurance example: fair odds should be $.75 : .25 = 3 : 1$.

2.0.2 Preferences and optimal choices (Indifference Curves)

- **Risk-neutral: caring only about expected value.**
  - indifferent among alternatives with the same expected value
  - risk does not matter
  - ICs are straight lines, parallel to the fair odds line (remember fair odds line gives the same expected value)
  - Optimal Choice
    - ∗ At fair odds (BL coincides with an iso-expected value curve), indifferent as to how much he bets;
    - ∗ At unfair odds, bet everything he owns on one or the other outcome.

EXHIBIT 18.4 Risk Neutrality

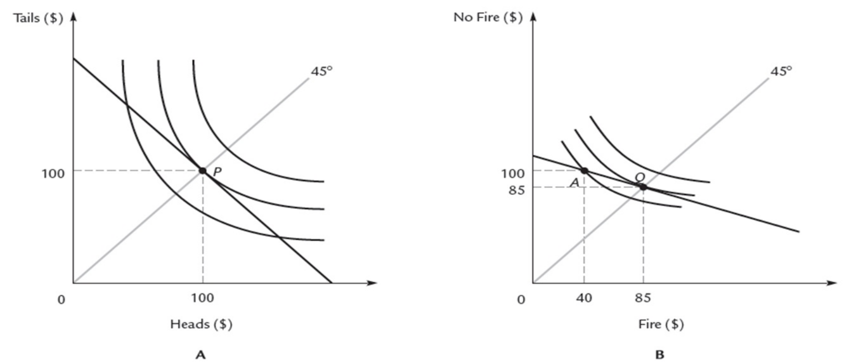


A risk-neutral individual has indifference curves that coincide with the iso-expected value lines, shown in gray in both panels. When he is offered fair odds, his budget line coincides with one of the indifference curves, as in panel A. In that case the individual is indifferent among all of the options available to him. When he is offered any odds other than fair odds, his budget line has a different slope than his indifference curves, like the black budget line in panel B. In that case, he will always choose a corner and bet everything he has on one outcome or the other.

- **Risk-averse: always preferring the least risky among bundles with the same expected value.**
  - prefer the riskfree bundle among alternatives with the same expected value
  - risk matters
  - ICs are convex, tangent to the fair odds line (remember fair odds line gives the same expected value)
  - Optimal Choice

* At fair odds, always choose the riskfree bundle;
* At unfair odds, depends.

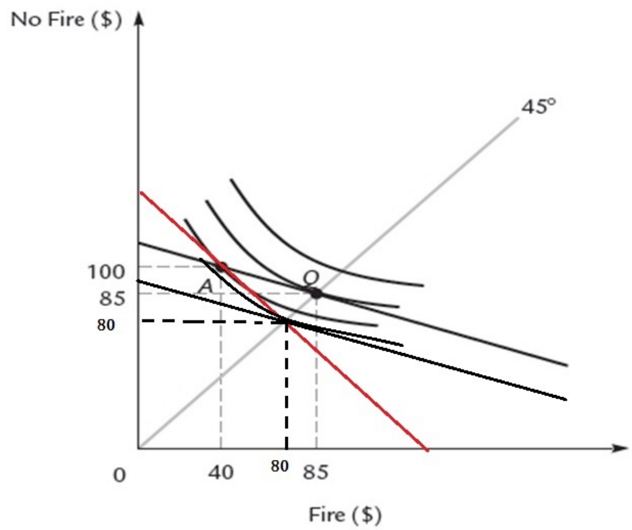
EXHIBIT **18.5** **Risk Aversion**



The two panels illustrate the indifference curves of individuals facing fair odds. In panel A the individual has initial wealth of $100 and is offered the opportunity to bet at even odds on the toss of a fair coin. His endowment is at point *P*, which is already on the 45° line. This is also his optimum, so he places no wager.

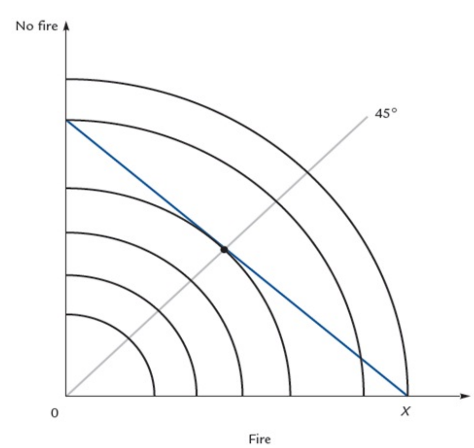
In panel B the individual has initial wealth of $100, which will be reduced to $40 in the event of a fire. His endowment is at point *A*. We assume that the probability of "no fire" is 3 times as great as the probability of "fire." Thus, the fair odds for an insurance policy are 3 to 1, and we assume that such a policy is available. This gives the illustrated budget line, which crosses the 45° line at (85, 85). Because he is risk-averse, his optimum is at *Q*. He achieves this point by purchasing $15 worth of insurance.

- Insurance Example:
  - Fair odds is $\frac{1}{3}$, and slope of BL is $\frac{1}{2}$. BL is steeper.
  - If BL is $\frac{1}{3}$ as graph, you will choose full insurance.
    * Insurance company earns no profit.
  - If BL is still $\frac{1}{2}$ and the you are risk averse, you will not choose full-insurance, but partial insurance or even buy negative amount of insurance (i.e. sell insurance to others at the rate 1 : 2).
    * In reality, not everyone can sell insurance, so at this unfair odds, people will choose to have partial insurance.
    * In this case, insurance companies earn positive expected profits. Since insurance companies are risk-neutral, they care only about expect profit. **That is why insurance is a win-win game under an unfair odds.**
    * You are willing to bear some risk by yourself (partial insurance) or even bear more risk (sell insurance).



- **Risk-preferring (Risk-love): always preferring the most risky among bundles with the same expected value.**
  - the riskfree bundle among alternatives with the same expected value is the worst
  - risk matters
  - ICs are concave, tangent to the fair odds line (remember fair odds line gives the same expected value)
  - Optimal Choice
    * At fair odds, always avoid the riskfree bundle;
    * At unfair odds, depends.

EXHIBIT **18.6** **Risk Preference**



The risk-preferring individual always chooses a corner solution, regardless of the odds he faces. This individual chooses point *X*, where his wealth becomes zero if there is no fire. He can accomplish this by spending all of his income on fire insurance, hoping for a fire that will make him rich.

3 Utility Function Revisit (Expected Utility)

3.1 Expected Utility Functions

As we have learned in Chapter 3, we can use utility functions to represent preferences. The fact that we are considering decision making under uncertainty does add a special structure to the utility functions. In general, a person's utility depends not only on the consumption levels in different states of the world, but also on the probability that the states in question will actually occur.

- Under uncertainty, a utility function depends on
  - probabilities;
  - consumption levels.
- Consider a simplified example with only two mutually exclusive states.
  - c_1 and c_2 are consumptions in states 1 and 2.
  - π_1 and π_2 are probabilities that state 1 or state 2 actually occurs.
- Given the notation, we can write the utility function for consumption in states 1 and 2 as $U(c_1, c_2; \pi_1, \pi_2)$.
  - Example 1: $U(c_1, c_2; \pi_1, \pi_2) = \pi_1 \cdot c_1 + \pi_2 \cdot c_2$.
  - Example 2: $U(c_1, c_2; \pi_1, \pi_2) = c_1^{\pi_1} c_2^{\pi_2}$. Monotonic transformation $\ln U(c_1, c_2; \pi_1, \pi_2) = \pi_1 \cdot \ln c_1 + \pi_2 \cdot \ln c_2$.
- Under some assumptions, we have a particularly convenient form that the utility function might take
$$U(c_1, c_2; \pi_1, \pi_2) = \pi_1 \cdot u(c_1) + \pi_2 \cdot u(c_2)$$
  - This says that utility can be written as a weighted sum of some function of consumption in each state, $u(c_1)$ and $u(c_2)$, where the weights are given by the probabilities π_1 and π_2.
  - This utility function form is called as an **expected utility function**, or **von Neumann-Morgenstern utility function.**

3.2 Expected Utility and Risk Attitudes

As we know an indifference curve is just all the consumption bundle with a constant level of utility. From the last section, we have learn three types of ICs, which corresponds to three types of risk attitudes, i.e., risk-neutral, risk-averse and risk-love.

- **Question:** How can we distinguish the three types of risk attitudes in the expected utility functions?

- Let's apply the expected utility functions to our insurance example. A general form of expected utility for this example is

$$U(w_1, w_2; \pi_1, \pi_2) = E(u(w)) \equiv \pi_1 \cdot u(w_1) + \pi_2 \cdot u(w_2) = .75 \cdot u(w_1) + .25 \cdot u(w_2)$$

  - (w_1, w_2) is the consumption bundle we can choose, e.g. endowment ($100, $40) and wealth with insurance ($80, $80).
  - The only thing unspecified is the function form of $u(\cdot)$.

- **Risk-neutral: caring only about expected value.**
  - linear function: $u(w) = w$, so $U(w_1, w_2; \pi_1, \pi_2) = .75 \cdot w_1 + .25 \cdot w_2$ =expected value.
  - $\max U(w_1, w_2; \pi_1, \pi_2)$ is to maximize expected value.
  - Optimal choice: highest expected value.

- **Risk-averse: always preferring the least risky among bundles with the same expected value.**
  - strictly concave function: for example $u(w) = \ln w$, so $U(w_1, w_2; \pi_1, \pi_2) = .75 \cdot \ln w_1 + .25 \cdot \ln w_2$
  - Compare $E(u(w))$ and $u(E(w))$.
  - Optimal choice depends on the function form.

- **Risk-preferring (Risk-love): always preferring the most risky among bundles with the same expected value.**
  - strictly convex function: for example $u(w) = w^2$, so $U(w_1, w_2; \pi_1, \pi_2) = .75 \cdot w_1^2 + .25 \cdot w_2^2$.
  - Compare $E(u(w))$ and $u(E(w))$.
  - Optimal choice depends on the function form.

3.2.1 Insurance Example:

- Insurance policy as before: if you pay $20, the insurance company will pay you $60 when there is a fire.

- Assume you can choose a sequence of insurance plan. Therefore, if you pay $\$20\gamma$, the insurance company will pay you $\$60\gamma$ when there is a fire.
  - γ is any real number, which represents the amount of policy you buy.

- We have

$$w_1 = 100 - 20\gamma \text{ and } w_2 = 40 - 20\gamma + 60\gamma = 40 + 40\gamma$$

– That is consistent with our budget line $w_1 = 120 - \frac{1}{2}w_2$.

- By choosing the optimal amount of insurance, you want to maximize the expected utility as follows,

$$\begin{aligned}\max_{\gamma} E(u(w)) &= .75 \cdot \ln w_1 + .25 \cdot \ln w_2 \\ &= .75 \cdot \ln(100 - 20\gamma) + .25 \cdot \ln(40 + 40\gamma)\end{aligned}$$

- FOC:

$$\frac{3}{4} \cdot \frac{1}{100 - 20\gamma} \cdot (-20) + \frac{1}{4} \cdot \frac{1}{40 + 40\gamma} \cdot 40 = 0$$

$\Rightarrow$

$$\gamma = \frac{1}{2}$$

- Confirm our hypothesis form last section that under unfair odds, partial insurance is optimal for a risk averse person.

4 Extension: Limit of Expected Utility Theory

4.1 Example: Ellsberg paradox

Suppose you have an urn containing 30 red balls and 60 other balls that are either black or yellow. You don't know how many black or yellow balls there are, but that the total number of black balls plus the total number of yellow equals 60. The balls are well mixed so that each individual ball is as likely to be drawn as any other. You are now given a choice between two gambles:

- Gamble A: You receive \$100 if you draw a red ball
- Gamble B: You receive \$100 if you draw a black ball

Also you are given the choice between another two gambles (about a different draw from the same urn):

- Gamble C: You receive \$100 if you draw a red or yellow ball
- Gamble D: You receive \$100 if you draw a black or yellow ball

4.2 Mathematical Interpretation

Mathematically, your estimated probabilities of each color ball can be represented as: R, Y, and B. **If you strictly prefer Gamble A to Gamble B,** by utility theory, it is presumed this preference is reflected by the expected utilities of the two gambles: specifically, it must be the case that

$$R \cdot U(\$100) + (1 - R) \cdot U(\$0) > B \cdot U(\$100) + (1 - B) \cdot U(\$0)$$

where $U(\cdot)$ is your utility function. If $U(\$100) > U(\$0)$ (you strictly prefer \$100 to nothing), this simplifies to:

$$R \cdot [U(\$100) - U(\$0) > B \cdot [U(\$100) - U(\$0)$$

Then

$$R > B$$

If you also strictly prefer Gamble D to Gamble C, the following inequality is similarly obtained:

$$B \cdot U(\$100) + Y \cdot U(\$100) + R \cdot U(\$0) > R \cdot U(\$100) + Y \cdot U(\$100) + B \cdot U(\$0)$$

This simplifies to

$$B \cdot [U(\$100) - U(\$0) > R \cdot [U(\$100) - U(\$0)$$

Then

$$B > R$$

- This contradiction indicates that your preferences are inconsistent with expected-utility theory.