How Costly to Sell A Company? A Structural Analysis of Takeover Auctions*

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Abstract

To explain why sellers in takeover auctions limit bidders' entry, we structurally measure two negative effects of inviting an additional bidder on the seller's revenue: *information cost* and *operation cost.* In particular, our auction model allows for bidders to discount their synergy values when their rivals obtain the confidential information – we refer to the induced revenue loss as the information cost. We establish the identification of the model primitives with unobserved heterogeneity, naturally arising from the confidential information. From a sample with 287 M&A deals of U.S. public companies, we find that the unobserved heterogeneity explains 75.3% of the variation of the value and bidders lower their values by 7.6% for each rival. We quantify the information cost and operation cost using counterfactual analysis, and discuss policy implications for the seller as well as regulatory and judiciary authorities.

Keywords: Mergers and Acquisitions, Takeover Auctions, Structural Identification and Estimation, Information Cost, Operation Cost

JEL Classifications: C57, D22, D44, G34

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The literature of the mergers and acquisitions (M&A) has reported that sellers of takeover auctions routinely restrict bidders' entry. In particular, Boone and Mulherin (2007) document that about a half of the company sales in their M&A sample invite one acquiror and even when they do more than one, they allow only a few. This observation seems to contradict the received theory: competition raises the seller's (expected) revenue.[1] We explain the entry regulation by formalizing a takeover auction with costs incurred by the seller and empirically measuring the model primitives.

We consider two well-documented costs: *operation cost* and *information cost*. The operation cost includes advisory fees and other economic costs explicitly or implicitly incurred by the seller to conduct a company sale.[2] The operation cost varies across target companies, reflecting sellers' private information on their own business, e.g., the loss of potential business opportunities, and therefore is mostly unknown to the acquirors. We bound the operation cost by contrasting the theoretical prediction on the seller's revenue against the number of invited bidders observed in the sample. While the operation cost is one of key determinants in the seller's choice, our methodology focuses more on the information cost because it is implied by bidders' evaluation process and bidding strategy. Specifically, a bidder would discount her value on the target whenever other bidders, who are mostly industry rivals, gain the confidential information on the target, which then can be strategically exploited. Hence, bidders would lower their bids, and this causes a premium loss for the seller, which we call the information cost. Notice that the information cost is a unique feature of the takeover auction where the confidential information plays a crucial role in and even after the auction.

[1] Bulow and Klemperer (1996) show, for example, that no amount of bargaining power is as valuable to the seller as attracting one extra bona fide bidder. There is a range of situations where this claim can fail, e.g., auctions with almost common value in Klemperer (1998) and auctions with voluntary entry in Li and Zheng (2009). The takeover auction, however, does not fit into those frameworks.

[2] Rosenbaum and Pearl (2009) and DePamphilis (2014) discuss in detail about various economic costs associated with disruption of business, negative impacts on the employee morale, and foregoing of business opportunities in a takeover process.

Hansen (2001) first introduces the notion of information cost in takeover auctions, which has been, since then, discussed in a number of articles in corporate finance, including Boone and Mulherin (2007); Rogo (2014); Schlingemann and Wu (2015). Legal studies of corporate takeovers also echo the *"legitimate proprietary concerns"* on sensitive information being disseminated to their industry rivals. Although the fiduciary duty requires that the boards of the target company act as auctioneers to maximize shareholders' benefits, the Delaware Court of Chancery indicated in many cases that a full-blown auction may not be desirable, as the cost could outweigh the benefits. Among those cases, the potential loss of competitive information is one of the major concerns. For example, in the case of *Lear Corp. Shareholder Litigation*, the target company rejects a potential acquiror on the ground that there could be a risk of losing an initial bidder or the initial bidder would pay less if the target engaged with additional bidders.[3]

In order to quantify the seller's information cost, we first develop a takeover auction model, accounting for the synergy value discount, which we term the information disclosure discount. Specifically, we approximate the takeover transaction by the first-price sealed-bid auction where bidders are uncertain about the number of opponents, reflecting the practice that the highest bidder takes over the target at the price that she bids and the seller keeps bidders uninformed of their rivals. Assuming that bidders are ex-ante identical yet privately learn their synergy values, we derive a symmetric Bayesian Nash equilibrium with a strictly increasing bidding strategy for the game induced by the auction.

We then establish the identification of the auction model with unobserved heterogeneity, which naturally arises from the confidential information that is latent by nature. In particular, we use the deconvolution method of Kotlarski (1966) to identify the distribution of unobserved heterogeneity from the within-auction bid variation.[4] After separating out the

[3] See *In Re Lear Corp. Shareholder Litigation, 926 A.2d 94, 119.* More examples can be found in Sautter (2013).

[4] Li and Vuong (1998) first introduce Kotlarski (1966) to identify models with measurement error. Using this technique, Li et al. (2000) identify a class of auctions with conditionally independent private values and

unobserved heterogeneity, we identify the distribution of synergy values and the information disclosure discount by the cross-auction bid variation and the exogenous variation of the number of potential bidders.

Integrating out the unobserved heterogeneity in the Bayesian framework, we analyze a sample of 287 takeover auctions of U.S. public companies that took place between 2000 and 2008. We find that the bidders lower their synergy value by 7.6 percent (posterior mean) for each additional opponent joining the bidding competition. In addition, after controlling for the observed characteristics, the unobserved auction heterogeneity accounts for 75.3 percent of the bid variation, corroborating the importance of the unobserved heterogeneity in empirical investigation of takeover auctions.

Our structural approach allows us to quantify the competition effect and the information cost via counterfactual analysis. The result indicates a substantial information cost; for example, the predictive takeover premium decreases by 2.6%, which amounts to $23 million, for the median target company in the consumer industry when the seller chooses 2 final bidders from 4 potential bidders. Furthermore, we use the observed shortlisting pattern to bound the seller-specific operation cost. In the same example, we find that the operation cost is at least 4.4% of the transaction value. This economic cost is much larger than observable advisory fees: Hunter and Jagtiani (2003) report advisory fees of 0.84% of the transaction value.

The main contribution of this paper is to employ a structural method to measure the information cost and operation cost incurred by the seller in a sale of company. Many studies in corporate finance use the existence of the information cost to explain some stylized features in M&A transactions. For example, Boone and Mulherin (2007) find that negotiations and auctions accrue similar transaction premiums and suggest that the presence of information cost may explain the choice of sale mechanism. Schlingemann and Wu (2015) find that the likelihood of choosing auction over negotiation decreases in R&D intensity, which may

in a different context, Krasnokutskaya (2011) identifies auction models with unobserved heterogeneity.

proxy the cost of disclosing proprietary information. These studies, however, take a reduced form approach that cannot measure the magnitude of the information cost or disentangle the information cost from the competition effect on the revenue, as the information cost is essentially structural.

There are a few articles that use a structural approach to study other aspects of takeover competitions. Taking the pool of acquirors as given, Gorbenko and Malenko (2014) estimate an auction model to study how strategic and financial bidders evaluate target companies and find that different targets appeal to different groups of bidders. Gentry and Stroup (2015) model potential acquirors' entry decision, i.e., a decision on whether to sign confidentiality agreements, and study how the pre-entry uncertainty on the target value affects the bidders' entry and bidding behavior. Unlike those previous papers, we focus on the seller's active role in determining the pool of final competitors and study how the seller's shortlisting affects bidding behavior, from which we can recover the costs incurred by the seller.

Moreover, none of aforementioned empirical papers formally consider the unobserved target heterogeneity. We highlight the importance of the confidential information in takeover transactions, which is latent by nature. We then formally consider the unobserved target heterogeneity in the econometric identification and document its economic significance for the first time in the M&A literature.

This paper also contributes to theoretical studies on takeover auctions. The extant research largely resides on the idea that information acquisition is costly for the bidders; for example, Ye (2007), Quint and Hendricks (2015), and Lu and Ye (2016). Ye (2007) finds that costly entry cannot explain why the seller of a company uses an indicative bidding stage to shortlist bidders into the final bidding competition. Quint and Hendricks (2015) show that in a special scenario, where the indicative bids sort bidders into a finite number of groups, the seller can use indicative bids to optimally shortlist bidders. These studies acknowledge the active role of the seller in the takeover process, but do not consider the costs incurred by the seller. Deviating from this strand of studies, we build a novel model where information disclosure is costly to the seller to provide a new insight on why the seller restricts bidders' entry in takeover auctions.

The paper proceeds as follows. Section I provides the institutional background and outlines our data. Section II develops our auction model and section III establishes the identification of the model. Section IV proposes a method to estimate the structural parameters and section V reports the empirical results with counterfactual analyses. Section VI concludes by discussing policy implications. Appendices collect proofs and technical details for computation.

I. Sale of Companies

Before modeling the takeover process, we describe a typical sale of a company in a private takeover and describe our data.[5] Then we conduct preliminary reduced-form analyses to gain some intuition useful for modeling in the next section.

A. Institutional Background

A corporate takeover is initiated either by a target company considering the sale of its enterprise as a strategic alternative or by an acquiring company launching an unsolicited inquiry. Once the takeover begins, the target hires an investment bank for financial advice and retains a law firm for legal counsel – we call the target and its advisors as the seller in this paper. The seller contacts prospective buyers with the delivery of a teaser and confidentiality and standstill agreements.[6] For public companies, Regulation Fair Disclosure concerns govern the content of the teaser; at this stage no material nonpublic information is revealed to the

[5] A public takeover could follow the announcement of a private takeover. For the purposes of this paper, we describe only the private takeover process. See Boone and Mulherin (2007) and Hansen (2001) for a complete description.

[6] The confidentiality agreement governs how buyers can use the information obtained and the standstill agreement precludes the prospective buyer from making unsolicited offers, or purchasing of the target's shares etc., in a specified period of time.

buyers.[7] Among the prospective buyers, those who are interested sign the confidentiality and standstill agreements and obtain the Confidential Information Memorandum (CIM), which contains detailed non-public information on the target company, including divisional data, order backlogs, proprietary contracts, and the R&D status. After the preliminary due diligence based on CIM, if further interested, a bidder submits a non-binding indication of interest that specifies a number of elements: (a) indicative purchase price (typically presented in a range) and form of consideration (cash vs. stock mix); (b) key assumptions to arrive at the stated indicative purchase price; (c) information on financing sources; (d) treatment of management and employees; (e) key conditions to signing and closing. For the research purpose of this paper, we call bidders indicating their interest the *potential bidders*.

After reviewing potential bidders' indications of interest, the seller selects a subset of the potential bidders for the final round of comprehensive due diligence, where the invited *final bidders* are granted access to the most confidential materials, on-site visits, consultations with target management, etc. This shortlisting decision depends on various considerations, such as speed to the transaction closing, fulfillment of fiduciary duties, disruption of business, and confidentiality concern. Note that the concerns on dissemination of confidential information is particular to takeover auctions. For example, in the sale of Spinnaker Exploration Company, the seller's financial advisor Randall & Dewey suggested "*limited marketing approach as it minimized the exposure of Spinnaker's sensitive confidential information to a smaller group of competitors*" because "*the likely buyers for Spinnaker were all competitors.*"[8]

Following the final due diligence, the final bidders submit their binding bids by the due date. The seller then analyzes the final bids and selects the winner to work on the final

[7] The Regulation Fair Disclosure mandates that all publicly traded companies must disclose material information to all investors at the same time. According to this rule, the bidders may not even know which company is on sale before signing the confidentiality agreement because it may constitute selective disclosure of material information (i.e., that the company is for sale) to the contacted bidders. See discussion in Hansen (2001).

[8] The *italic* texts are extracted directly from the *DEFM14A* filing by Spinnaker to SEC on November 10, 2005.

definitive agreement, which includes the purchase price, the method of payment, various deal protection devices, and fiduciary-out provisions. Upon signing the M&A agreement, the seller and buyer publicly announce the deal, which ends the takeover process.[9] Throughout the process, the seller keeps bidders uninformed about the participation of other bidders as well as their identities and bids for legal and competitive reasons; see Gorbenko and Malenko (2014) and Gentry and Stroup (2015).

B. Takeover Auction Data[10]

Our sample comes from the Mergers and Acquisitions database of the Securities Data Corporation (SDC) and consists of 287 takeover deals that were announced and completed between January 1, 2000 and September 6, 2008 and meet the following criteria:

(i) The target is a publicly traded non-financial (SIC codes 6000 - 6999 excluded) U.S. company.

(ii) The winning bid is made in cash only.

(iii) The winning bidder obtains 100% of target shares after transaction.

(iv) The deal is not a spin-off, recap, self-tender, exchange offer, repurchase, minority stake purchase, acquisition of remaining interest, or privatization.

(v) The deal is fulfilled by an auction, which we define as a deal with at least two buyers showing indications of interest.

[9] The announcement could be followed by a public takeover battle for the target. In practice, less than 4 percent of deals are subject to public competition (e.g., Moeller et al. (2007)). Betton et al. (2009) show that the winner from the private competition usually wins the public contest; 70.7% in their sample of 7,076 takeover contests. That is, the new bidder takes over the target only with 1.2% of chance. Moreover, when it happens, the seller compensates the original winner following the final agreement. Note that Boone and Mulherin (2007), Gorbenko and Malenko (2014), and Gentry and Stroup (2015) study only the private takeover stage.

[10] We thank Alexander Gorbenko and Andrey Malenko for sharing their data used in Gorbenko and Malenko (2014). To serve our research purpose, we enlarge their dataset by hand-collecting additional structural variables from SEC filings.

(vi) The deal background is available in SDC, or EDGAR filings on the SEC, or MergerMetrics.

(vii) Quarterly financial data on the target is available on the Compustat database.

Conditions (i)-(iv) are imposed to make takeover deals and bids comparable. First, M&A deals from the financial sector are excluded because its valuation is fundamentally different from that for non-financial firms. For example, a high leverage normal to financial firms is an indicator of great distress to firms in other industries.[11] Second, the winning payment is cash-only, in which case the deal value is known with certainty. This excludes other payment arrangements whose values could depend on unobserved factors. For instance, in a deal payment with securities exchanges, the deal value depends on the unobserved winner's characteristics, and thus cannot be reliably compared to a cash-only deal or other bids involving security payments. Third, the winner eventually owns 100% share of the target so that it is a full-scale merger rather than an equity investment. Finally, we exclude deals with motivations other than a business combination.[12]

Structural Variables from SEC Filings

For each takeover in the sample, we collect additional information from the background sections in filings of the US Securities and Exchange Commission (SEC).[13] In particular, we observe the bidder participation in each stage and the competing bids for each takeover auction.[14]

[11] Therefore, it is a common practice in corporate finance research to separate financial firms from others; e.g., Fama and French (1992).

[12] It is arguable that the true value of a bid depends on all terms of the final merger proposals, and thus is different from the cash value of the bid. We argue that, in the context of our structural model, this deviation reflects measurement error, which is captured as bidders' heterogeneity.

[13] Boone and Mulherin (2007) first use the hand-collected data from SEC filings to study the takeover competition. Deal backgrounds are contained in SC-TOT, 14D-9, PREM14C, DEFM14C, DEFS 14A, and S-4 filings, available at `http://www.sec.gov/edgar/searchedgar/companysearch.html`.

[14] SDC provides the final deal value, i.e., winning bid, for each auction; while SEC filings often document other (losing) bids.

First, we identify the two measures of bidders' participation: (i) N: the number of potential bidders, i.e., the ones showing indications of interest; (ii) n: the number of final bidders, i.e., the ones shortlisted for the final due diligence. Note that our theoretical model is built upon N and n. Second, we complement the bid data by collecting all the reported losing cash-only bids from the SEC documents.[15] Following other empirical studies on M&A, we normalize each bid by the corresponding target stock price four weeks before the announcement date, and use the resulting (raw) bid premiums in our empirical study.[16]

Explanatory Variables from Compustat and CRSP

Following Gorbenko and Malenko (2014), we construct obtain accounting data of the target companies from the Compustat database and construct eight target-specific variables: (i) *Size* is the firm size defined as the book value of the total assets (in millions); (ii) *Leverage* is the market leverage defined as the ratio of the book value of debt to the sum of the market value of equity; (iii) *Q-ratio* is defined as the ratio of the sum of the market value of equity and the book value of debt to the book value of the target; (iv) *CashFlow* is the cash flow over the last four quarters; (v) *Cash* is the cash balance, i.e., the sum of cash, short-term investments, and marketable securities; (vi) *R&D* is the R&D expenses; and (vii) *Intangibles* represents accounting measure for the intangible assets; (viii) *Industry* is the industry category as defined by Fama and French (1997).[17] In addition, we scale the *CashFlow*, *Cash*, *R&D* and *Intangibles* by *Size*. Standard filters are applied to exclude unreasonable values of these covariates that are likely to be mistakes. Specifically, we exclude observations with market leverage below zero and above 100%; Q-ratio in excess of 10; cash flow in excess of 10; and negative cash.

[15] We exclude some anomalous losing cash-only bids from the sample because of its incomparability to other bids for various reasons: (1) bids are subject to conditions; (2) bidders cannot provide sufficient financing proofs; (3) bidders quit after winning the auction; etc. In some of these cases, the losing cash bids could be higher than the winning cash bids.

[16] See Eckbo and Thorburn (2009) for a discussion on the choice of base prices.

[17] The Fama-French five industry classification is used because the twelve industry classification results in few observations for some industries.

We also obtain two economy-wide variables from the Center for Research in Security Prices (CRSP) to control for the general economic environment: (ii) *MarketReturn* is the market return defined as the cumulative return on the S&P 500 index over the 12 months prior to the announcement date; and (iii) *CreditSpread* is the credit spread defined as the rate on Moody's Baa bonds preceding the announcement date minus the rate on 10-year Treasury bonds on the announcement date.

Finally, our sample consists of 287 takeover auctions with a total of 372 observed bids and 193 auctions with winning bid only. Table I reports some descriptive statistics of the auction and target characteristics for the full sample and across the Fama-French five industries. Panel A first confirms the common practice of entry restriction in all industries; the average invitation rate ranges from 62% to 74% across the five industries. In addition, we observe substantial takeover premiums across the industries, especially in the HighTech, Health, and Others. Panel B investigates the accounting features of the target companies. The target firms in HighTech and Health industries appear to be growth companies, as indicated by their high Q-ratio, low leverage, and large cash balance. These companies also stand out in their R&D spending and percentage of intangible assets.

Determinants of Shortlisting Decisions

As mentioned in the institutional background, many factors affect sellers' shortlisting decisions. To better understand the determinants of the decision, we consider a Probit regression (Model I) where the dependent variable is a dummy variable equal to 1 if a potential bidders is shortlisted by the seller and 0, otherwise. Model I includes a set of observed target characteristics and economic conditions described above. To see whether the result of Model I is driven by industry effects, Model II expands Model I to include industry dummies following the five-industry classification of Fama and French (1997). Given the panel structure of our data, we further run a target-specific Random-Effect Probit regression (Model III) to control

Table I
Descriptive Statistics of Auction and Target Characteristics

This table reports descriptive statistics (mean, standard deviation and median) of the auction and target characteristics for the full sample and across five industries, as classified by Fama and French (1997). Within the five-industry classification, *Consumer* includes consumer durables, nondurables, wholesale, retail, and some services (laundries, repair shops); *Manufacturing* includes manufacturing and energy; *HighTech* includes business equipment, telephone and television transmission; *Health* includes healthcare, medical equipment, and drugs; and *Others* includes mines, construction, construction materials, transportation, hotels, business services, and entertainment. Panel A reports the means and medians (in brackets) of N (the number of potential bidders), n (the number of final bidders), and n/N (the invitation rate), and the means and standard deviations (in parentheses) of the takeover premium (winning bid scaled by the target size). Panel B reports the means and standard deviations (in parentheses) of the target characteristics. The sample contains 287 takeover auctions took place between January 1, 2000 to September 6, 2008.

	All	Consumer	Manufacturing	HighTech	Health	Others
Panel A						
N (Potential)	4.7	5.1	6.2	4.3	3.5	4.8
	[3]	[4]	[6]	[3]	[3]	[4]
n (Final)	2.7	2.8	3.1	2.6	2.5	2.7
	[2]	[2]	[2.5]	[2]	[2]	[2]
n/N (Invitation Rate)	0.68	0.62	0.62	0.70	0.74	0.70
	[0.67]	[0.62]	[0.65]	[0.71]	[0.88]	[0.75]
Premium	1.34	1.27	1.22	1.38	1.37	1.40
	(0.30)	(0.30)	(0.21)	(0.31)	(0.30)	(0.28)
Panel B						
Size (in millions)	659.0	719.6	731.6	569.5	435.8	975.5
	(2345.0)	(1400.8)	(1898.6)	(3103.2)	(528.5)	(2449.3)
Leverage	0.15	0.23	0.19	0.07	0.13	0.27
	(0.22)	(0.26)	(0.21)	(0.13)	(0.22)	(0.26)
Q-ratio	1.53	1.14	1.28	1.53	2.69	1.17
	(1.17)	(0.62)	(0.50)	(0.99)	(1.99)	(0.81)
CashFlow	0.02	0.07	0.11	-0.03	-0.02	0.05
	(0.26)	(0.10)	(0.08)	(0.35)	(0.24)	(0.21)
Cash	0.25	0.10	0.09	0.37	0.31	0.18
	(0.23)	(0.09)	(0.13)	(0.22)	(0.25)	(0.24)
R&D	0.02	0.00	0.00	0.03	0.03	0.00
	(0.03)	(0.00)	(0.01)	(0.04)	(0.04)	(0.00)
Intangibles	0.15	0.09	0.09	0.19	0.17	0.13
	(0.19)	(0.15)	(0.14)	(0.19)	(0.22)	(0.20)
# of obs.	287	57	36	113	40	41

for the unobserved heterogeneity.[18]

The regression results in Table II show that several target characteristics help to predict sellers' shortlisting decisions. Among those explanatory variables, we are particularly interested in the impact of intangible assets, representing accounting measure of many confidential items such as patents and trade secrets. A higher value of intangible assets suggests that the confidential information is more important to the target value.[19] All three regressions show significantly negative effects of intangible assets on the shortlisting probability, which supports our premise that the seller restricts bidders' participation concerning the dissemination of the confidential information.

II. Takeover Auction Model

Based on the institutional background described above, we develop an auction model to approximate bidders' behavior in a takeover process. We aim to quantify the seller's revenue loss arising from bidders' value discount due to the release of confidential information. Since the bids in the final stage determine the revenue, we model the bidding behavior only in that stage, which is sufficient to achieve our goal.[20]

To develop an auction model that suits our objective, moreover, it is necessary to understand how bidders perceive the competition intensity in the final stage of auction. As mentioned above, the seller's shortlisting decision depends on many factors, some of which

[18] For a given auction, a repeated sample of the shortlisting decision on each potential bidder is observed. The unobserved target-specific heterogeneity reflects the seller's private knowledge on the target and private consideration for the auction, e.g., the speed of sale and the disruption of the business. For a given auction, we treat the auction as an individual and regard the seller's shortlisting decision on each potential bidder as a response in different time.

[19] For example, it is well known that Coca-Cola has its own formula for their products, which is confidential. Its market value is recorded in intangible assets.

[20] Given our objective we can abstract away bidders' entry decision, especially, the decision on whether to sign the confidentiality agreement. Note also that Gentry and Stroup (2015) explicitly model this entry decision in their model where bidders who enter choose optimal bids using the value distribution conditional on their signal being larger than the cut-off point for entry. This conditional distribution amounts to the distribution of synergy value in this paper.

Table II
Determinants of Shortlisting Decisions

This table reports estimation results for three Probit regressions on shortlisting decisions. Model I includes a set of target characteristics and market conditions. Model II expands Model I by including industry dummies following the five-industry classification by Fama and French (1997). Model III runs a target-specific Random-Effect Probit regression. t statistics are in parentheses beneath the coefficient estimates and statistical significance is indicated by ***, **, and * for the 0.01, 0.05, and 0.10 levels. The sample size (total numbers of potential bidders from 287 takeover auctions) is $1,339$. The sample covers January 1, 2000 to September 6, 2008.

	Model I Probit	Model II Probit (Industry Effect)	Model III Random-Effect Probit
Constant	-0.811***	-0.645**	-0.518
	(-2.69)	(-2.01)	(-1.01)
Log(Size)	0.121***	0.119***	0.133***
	(4.12)	(4.00)	(2.95)
Leverage	0.204	0.197	0.188
	(1.10)	(1.05)	(0.60)
Q-ratio	0.211***	0.195***	0.206***
	(4.19)	(3.80)	(3.14)
CashFlow	-0.371*	-0.412*	-0.596
	(-1.70)	(-1.78)	(-1.62)
Cash	0.103	-0.235	-0.040
	(0.52)	(-1.05)	(-0.11)
R&D	3.068	2.057	1.235
	(1.57)	(1.12)	(0.61)
Intangibles	-0.580***	-0.883***	-0.827**
	(-3.02)	(-4.16)	(-2.52)
MarketReturn	-0.091	0.110	0.115
	(-0.23)	(0.27)	(0.19)
CreditSpread	5.064	5.676	4.536
	(0.60)	(0.66)	(0.34)
FF-Industry-1		-0.195	-0.263
		(-1.61)	(-1.37)
FF-Industry-2		-0.229*	-0.223
		(-1.78)	(-1.00)
FF-Industry-3		0.120	0.047
		(1.01)	(0.23)
FF-Industry-4		0.114	-0.002
		(0.73)	(-0.01)

can be inferred from the observed target characteristics, but others are private to the seller. Therefore, we assume that the final bidders regard the shortlisting rule as a random process and perceive it through learning the number of potential bidders N and the shortlisting probability

$$p_n := \Pr(n \text{ bidders are invited out of } N \text{ potential bidders})$$

with aids from their financial advisors. Moreover, the financial advisors employ similar databases and analysis methods to obtain this information and, therefore, we further assume that N and p_n are common knowledge among the final bidders.[21] Note that we suppress the dependence on N in this section for notational simplicity.

Now, consider bidder i in the final bidding stage. After the comprehensive due diligence, she privately discovers her synergy value and discounts it to reflect the potential damage of competitive information revelation. To this end, we express bidder i's net utility when she wins the auction with n final bidders as

$$u_i = v_i - D(n, v_i), \tag{1}$$

where v_i is her *synergy value* upon merging with the target and $D(n, v_i)$ denotes her *information disclosure discount*. We consider the synergy value as private information to the bidder because it reflects the bidder's opportunity costs of merging with the target and business compatibility that is particular to each bidder-target combination. We also assume that $(v_1, \ldots, v_n)$ are independent draws from the valuation distribution F_v with density f_v supported on $[\underline{v}, \overline{v}] \subset \mathcal{R}_+$, which is common knowledge among the final bidders. Note that the bidder symmetry can be justified because bidders do not know the identities of their opponents. Moreover, we maintain the following assumption regarding the structure of in-

[21] A major method adopted by the investment bank is the precedent transactions analysis, where the information of the previous M&A transactions are collected through SEC filings and business databases such as SDC Platinum. See Chapter 2 of Rosenbaum and Pearl (2009) for more details.

formation disclosure discount.

ASSUMPTION 1. $D(n, v)$ satisfies

i. $D(n, v)$ is increasing in n;

ii. $D(n, v)$ is increasing in v, but not faster than the identity;

iii. $D(n, \underline{v}) = 0$.

Assumption 1 items (i) and (ii) reflect that a greater discount arises either as more competitors gain the confidential information or as a larger synergy value is at stake; item (ii) preserves the order of the bidders' valuations in the presence of the information disclosure discount, i.e., $v_i > v_j$ implies $u_i > u_j$; and item (iii) normalizes the discount on the lower boundary to be zero.

In order to derive the Bayesian equilibrium for the game induced by the auction, we consider bidder i's optimal bidding. Suppose that all other bidders follow a strictly increasing bidding strategy $\beta(\cdot)$. Since $\{F_v, D, N, p_n\}$ is common knowledge in the bidding game, we write bidder i's expected payoff of bidding b as

$$\begin{aligned}\pi(b, v) &= \sum_{n=1}^{N} p_n \left[\prod_{j=1, j\neq i}^{n} Pr(\beta(v_j) \leq b)(v - D(n, v) - b) \right] \\ &= \sum_{n=1}^{N} p_n F_v(\beta^{-1}(b))^{n-1}(v - D(n, v) - b).\end{aligned}$$

By taking a derivative of this payoff function with respect to b, we obtain the first-order necessary condition for the bidder's optimization problem as

$$\sum_{n=1}^{N} p_n F_v(v)^{n-1} \left(\frac{\partial}{\partial v} \beta(v) \right) = \sum_{n=2}^{N} p_n (n-1) F_v(v)^{n-2} f_v(v) \left(v - D(n, v) - \beta(v) \right). \tag{2}$$

Solving the differential equation (2), we obtain an analytical expression for $\beta(v)$. In order to simplify the expression, we denote bidder i's equilibrium winning probability by $H(v) := \sum_{m=1}^{N} p_m F_v(v)^{m-1}$ and its derivative by $h(v) := dH(v)/dv = \sum_{m=2}^{N} p_m(m-1)F_v(v)^{m-2} f_v(v)$. We then find the solution to (2) as

$$\begin{aligned}\beta(v) &= \frac{H(\underline{v})}{H(v)}\beta(\underline{v}) + \sum_{n=2}^{N} \frac{p_n}{H(v)} \int_{\underline{v}}^{v} (t - D(n,t))dF_v(t)^{n-1} \\ &= \frac{p_1}{H(v)}\underline{v} + \sum_{n=2}^{N} \frac{p_n F_v(v)^{n-1}}{H(v)} \int_{\underline{v}}^{v} (t - D(n,t))d\left[\frac{F_v(t)}{F_v(v)}\right]^{n-1},\end{aligned}$$

for which we use the boundary condition, $\beta(\underline{v}) = \underline{v}$: the bidder with $\underline{v}$ always bids $\underline{v}$, because any bid lower than $\underline{v}$ is not accepted and any bid higher than $\underline{v}$ exceeds her value. We may write $\beta(v)$ in a more intuitive form. To do so, we define the contingent bidding function

$$\widetilde{\beta}_n^I(v) := \begin{cases} \underline{v}, & \textbf{for } n = 1, \\ \int_{\underline{v}}^{v} (t - D(n,t))d\left[\frac{F_v(t)}{F_v(v)}\right]^{n-1}, & \textbf{for } n = 2, \dots, N. \end{cases}$$

Notice that when there is no information disclosure discount, i.e., $D(v, n) = 0$, $\widetilde{\beta}_n^I(v)$ for $n \geq 2$ is the optimal bidding strategy in the first price auction where the bidders know n. Hence, we consider $\widetilde{\beta}_n^I(v)$ as the bidding strategy that incorporates the value discount for each contingent n. Using this expression, then, we can re-write $\beta(v)$ as a weighted average of the contingent bidding function;

$$\beta(v) = \sum_{n=1}^{N} w_n(v)\widetilde{\beta}_n^I(v). \tag{3}$$

where $w_n(v) := p_n F_v(v)^{n-1}/H(v)$ is a conditional winning probability, i.e., $w_n(v) \geq 0$ for all n and v and $\sum_{n=1}^{N} w_n(v) = 1$ for all v by construction.

Now we establish the existence of a symmetric Bayesian Nash Equilibrium (BNE) characterized by the bidding strategy (3) by showing no bidder can profitably deviate from it when all rival bidders follow it.

PROPOSITION 1. Suppose that the bidding strategy (3) is strictly increasing in v, then $\beta(v; N)$ characterizes a symmetric monotone BNE.

Proof. See Appendix A.1. $\square$

The existence of BNE relies on the presumption of the strict monotonicity of $\beta(v)$, which does not follow the strict monotonicity of $\widetilde{\beta}_n^I(v)$ because the weights $w_n(v)$ may not be monotone and some of $w_n(v)$ even may decrease. We do not establish a sufficient and necessary condition for the monotonicity at the level of model primitives. Instead, we present a sufficient condition that only involves a reduced form parameter p_n to verify that there exists a set of structural parameters under which $\beta(v)$ is strictly increasing. This condition generally restricts p_n but it always holds for $N = 2$ and 3.

PROPOSITION 2. The bidding strategy (3) is strictly increasing if for any N and $m \leq N$,

$$p_1 \geq \mathbb{1}(N \geq 3) \times \sum_{m=3}^{N} (m-2) p_m. \tag{4}$$

Proof. See Appendix A.2. $\square$

Note that we do not impose the sufficient condition (4) for the identification and estimation of the structural model. Instead, in what follows we establish identification of (F_v, D) assuming that bidding monotonicity is satisfied and then we develop an empirical method that rules out (F_v, D) under which bidding monotonicity is violated.

III. Identification of the Structural Model

In this section, we study the identification of the takeover model, in which the observed bids are generated from the BNE described in Proposition 1. As noted earlier, the final bidders observe the confidential information through the due diligence, but since it is confidential, the econometrician cannot observe it. In order to handle the problem of unobserved heterogeneity, we let $Z := (X, \tau)$ be the vector of auction characteristics that the bidders observe in the final stage, where $X \in \mathcal{R}^k$ is observed by the econometrician but $\tau \in \mathcal{R}_+$ is not, i.e., unobserved heterogeneity. Since all the identification arguments are conditional on observed heterogeneity, we suppress the notational dependence on X in this section.

We consider a two stage valuation updating process: in the first stage, bidder i observes public information X and obtains an initial assessment ν_i on her synergy value v_i; upon being shortlisted to the final stage, she examines the confidential information that updates her value to $v_i = \tau \cdot \nu_i$. This structure then corresponds to the specification of the unobserved heterogeneity of Krasnokutskaya (2011). Since the synergy value is unique to each bidder-target pair, we consider ν_i as private information for bidder i. Moreover, since τ represents the confidential information, we assume that it is orthogonal to $\{\nu_1, \ldots, \nu_n\}$. Furthermore, we assume that the number of potential bidders N is exogenous to $(\tau, \{\nu_1, \ldots, \nu_n\})$ because an acquiring company's decision to make indication of interest, i.e., becoming a potential bidder, largely depends on her private opportunity costs, e.g., alternative investment opportunities. Formally, we make the following assumption,

ASSUMPTION 2. For each auction ,

(i) The joint distribution of $(\tau, \nu_1, \ldots, \nu_n)$ has the structure of

$$F(\tau, \nu_1, \ldots, \nu_n | N) = F_\tau(\tau) \prod_{i=1}^{n} F_\nu(\nu_i),$$

where $F_\tau(\cdot)$ is the marginal distribution of τ with $\tau \in [\underline{\tau}, \overline{\tau}]$ and $\underline{\tau} > 0$; and $F_\nu(\cdot|N)$ is the marginal distribution of ν_i with $[\underline{\nu}, \overline{\nu}]$ and $\underline{\nu} > 0$;

(ii) $D(n, v_i) = D(n, \tau \cdot \nu_i) = \tau \cdot D(n, \nu_i)$ for all $n \in \{2, \ldots, N\}$;

(iii) $p_{n|N} = p_{n|N,\tau}$, for all $n = \{1, \cdots, N\}$.

Assumption 2 item (i) combines the discussion above and our model assumptions in section II, item (ii) imposes the multiplicative structure on the information disclosure discount in the same way as the value, and item (iii) states that τ does not affect bidders' estimation of the shortlisting probability. As described in section II, bidders obtain the common knowledge of the shortlisting probability $p_{n|N}$ by studying precedent transactions using their public information. Therefore, the estimation of the shortlisting probability does not depend on the confidential information τ.

Our objective is to examine if we can learn (F_ν, F_τ, D) from the data $(b_1, \ldots, b_n, N, n)$. Since the identification is a large sample statement, we assume that we have the bid distribution $G(b|N)$ with density $g(b|N)$ for all N in the sample. The identification consists of two steps: we first identify $F_\tau(\cdot)$ by exploring the within-auction bid variation and then identify $F_\nu(\cdot)$ and $\{D(n,\cdot)\}_{n=2}^N$ by the cross-auction bid variation and the exogenous variation of N.

First, we identify $F_\tau(\cdot)$ by deconvoluting the joint bid distribution following Krasnokutskaya (2011). Since a model with (τ, ν) and a model with $(\tau/2, 2\nu)$ generate the same bid distribution, a location normalization is introduced.

ASSUMPTION 3. $\log(\tau)$ has a non-vanishing characteristic function with $E[\log \tau] = 0$.

This assumption implies that $\log \nu_i$ is an unbiased first-stage estimate of the true value $\log v_i$, i.e., $\log \nu_i = E(\log v_i)$. Under Assumption 2, we have $b_i = \beta(v_i; N) = \tau \cdot \beta_1(\nu_i; N)$, where $\beta_1(\nu_i; N)$ is a hypothetical bid that bidder i would have submitted if she observed $\tau = 1$. Let $\mathtt{b}_i := (b_i/\tau) = \beta_1(\nu_i; N)$ with an associated density $g_{\mathtt{b}}(\cdot|N)$. The following Proposition provides conditions for identifying $f_\tau(\cdot)$ and $g_{\mathtt{b}}(\cdot|N)$.

PROPOSITION 3. Under Assumptions 2 and 3, $F_\tau(\cdot)$ and $G_{\mathtt{b}}(\cdot|N)$ are nonparametrically identified.

Proof. Under the hypothesis, the joint distribution of $(\log b_1, \log b_2)$ is directly identified. Since $\log \mathtt{b}_i$ has a compact support $[\log \underline{\nu}, \log \beta(\overline{\nu}; N)]$, its characteristic function is non-vanishing ; see Lemma $A1$ of Krasnokutskaya (2011). Under Assumption 2, the characteristic function of $\log \tau$ is also non-vanishing. Since $\log b_i = \log \tau + \log \mathtt{b}_i$, the joint distribution of $(\log b_1, \log b_2)$ determines the joint distribution of $(\log \tau, \log \mathtt{b}_1, \log \mathtt{b}_2)$ up to a change of location; see Kotlarski (1966). We fix the location at $E[\log \tau] = 0$ (Assumption 2), concluding the proof. □

Based on Proposition 3 we regard $\{G_{\mathtt{b}}(\cdot|N)\}_N$ as known and we identify F_ν and $D(n, \cdot)$. Since we use $G_{\mathtt{b}}(\cdot|N)$, we consider an auction with $\tau = 1$, for which we can rewrite the first order necessary condition (2) in terms of the bid distribution functions by the change of variable via the equilibrium bidding strategy, $\beta_1(v; N)$;

$$\sum_{n=2}^{N} (n-1) p_{n|N} G_{\mathtt{b}}(b|N)^{n-2} g_{\mathtt{b}}(b|N)(v - D(n, v) - b) = \sum_{n=1}^{N} p_{n|N} G_{\mathtt{b}}(b|N)^{n-1}. \tag{5}$$

Note that we express the dependence of the shortlisting probability $p_{n|N}$ on the number of potential bidders, N, which is useful for identification. In order to simplify the equilibrium relationship between the value and the bid in (5), we define

$$\lambda_N(b) := \frac{\sum_{n=1}^{N} p_{n|N} G_{\mathtt{b}}(b|N)^{n-1}}{\sum_{m=2}^{N} (n-1) p_{m|N} G_{\mathtt{b}}(b|N)^{m-2} g_{\mathtt{b}}(b|N)},$$

which is the ratio of the overall winning probability to its derivative, and

$$\eta_{n,N}(b) := \frac{(n-1) p_{n|N} G_{\mathtt{b}}(b|N)^{n-2}}{\sum_{m=2}^{N} (m-1) p_{m|N} G_{\mathtt{b}}(b|N)^{m-2}},$$

which forms a probability distribution, as it is positive and sum to 1 over n. Using the notations, we rewrite (5) as

$$v = b + \lambda_N(b) + \sum_{n=2}^{N} \eta_{n,N}(b) D(n, v), \tag{6}$$

where the second term on the right hand side is the markdown due to competition intensity and the third term is the weighted expected value deduction due to information disclosure discount. Note that $\lambda_N(\cdot)$ and $\eta_{n,N}(\cdot)$ are directly identified because they are determined by $G_{\mathsf{b}}(b|N)$. We can construct $\overline{N}-1$ equations from (6), one for each $N \in \{2, \ldots, \overline{N}\}$ with $\overline{N} \geq 2$. Using the equations, we hope to identify $F_\nu(\cdot)$ and $D(n, \cdot)$ for all $n \in \{2, \ldots, \overline{N}\}$, which gives $\overline{N}$ unknown functions. Since we have more unknowns than knowns, however, the model primitives would not be identified without a further structure. To this end, we consider the following specification.

ASSUMPTION 4. For any $v \in [\underline{v}, \overline{v}]$ and each $n = 2, \ldots, N$, the information disclosure discount has the form of $D(n, v) = (n-1) \cdot D(v)$ with $(n-1)dD(v)/dv < 1$.

Assumption 4 has an intuitive interpretation: since bidders do not know the identities of their competitors, they naturally treat each rival bidder as a representative competitor, and discount their synergy values by the same expected loss for each potential competitor. Accordingly, the discount factor $D(v)$ represents this expected loss per rival bidder. Besides, this specification alleviates the data requirement for identification and the empirical application later. The identification problem is now boiled down to identifying $D(v)$ and $F_\nu(\cdot)$.

PROPOSITION 4. Under Assumptions 2 and 4, $D(\cdot)$ and $F_\nu(\cdot)$ are identified by $G_{\mathsf{b}}(\cdot|N_1)$ and $G_{\mathsf{b}}(\cdot|N_2)$ with $N_1 \neq N_2$ when $p_{N_1|N_1} > 0$ and $p_{N_2|N_2} > 0$.

Proof. See Appendix A.3. □

IV. Estimation Method

A. Estimation Strategy

Now we bring the theoretical model and the data together to recover the model primitives. We observe a sample of T independent takeover auctions, $\{b_{1t}, \ldots, b_{n't}, N_t, n_t, X_t\}_{t=1}^T$ where

$n'_t \le n_t$ denotes the number of observed cash bids, i.e., winning cash bids and some losing cash bids.[22] We postulate that the omitted losing bids are lower than the winning bid because the target board of directors has the fiduciary duty to accept the superior offer; and therefore the cash values of the losing bids, reported or not, should be lower than the observed cash-only winning bid. Given the symmetric IPV paradigm, then, the joint density of the bids for auction t is given by

$$\prod_{i=1}^{n'_t} g(b_{it}|N_t, X_t; \tau_t, \theta)\left[G(b_t^w|N_t, X_t; \tau_t, \theta)\right]^{n_t - n'_t},$$

where b_t^w denotes the winning bid, τ_t is the unobserved heterogeneity, and θ collects all the model parameters, which will be specified later. Let $\boldsymbol{y} := \{b_{1t}, \ldots, b_{n't}\}_{t=1}^T$, $\boldsymbol{\tau} := \{\tau_t\}_{t=1}^T$, $\boldsymbol{n} := \{n_t\}_{t=1}^T$, and $\boldsymbol{X} := \{N_t, X_t\}_{t=1}^T$. Conditional on $(\boldsymbol{\tau}, \boldsymbol{X}, \theta)$, then, the joint density of the observed bids $\boldsymbol{y}$ and the number of final bidders $\boldsymbol{n}$ is given by

$$L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}, \boldsymbol{X}, \theta) = \prod_{t=1}^{T}\left[p(n_t|N_t, X_t; \theta)\prod_{i=1}^{n'_t} g(b_{it}|N_t, X_t; \tau_t, \theta)\left[G(b_t^w|N_t, X_t; \tau_t, \theta)\right]^{n_t - n'_t}\right], \quad (7)$$

where $p(n_t|N_t, X_t; \theta)$ denotes the conditional shortlisting probability.

Now, we specify each element in (7) on the right hand side: the shortlisting probability and the bid distribution functions. We model the shortlisting probability by the binomial distribution, i.e.,

$$p(n = n_t|N_t, X_t; \delta) := \begin{pmatrix} N_t - 1 \\ n_t - 1 \end{pmatrix} \Phi(X_t'\delta)^{n_t - 1}\left[1 - \Phi(X_t'\delta)\right]^{N_t - n_t}, \quad (8)$$

where $\delta \in \mathcal{R}^k$. In order to construct the bid density, we need to specify several factors. First,

[22] Losing bids are omitted mainly for the following reasons: (a) a bidder is invited, but did not submit a final bid; (b) a submitted bid is not comparable to the winning bid, e.g, not a cash-only bid; and (c) a losing bid is simply not reported in the SEC filings.

we consider the valuation premium on target t in the form of

$$v_{it} := \frac{V_{it}}{M_t} = \tau_t \exp(X_t'\gamma)\varepsilon_{it} \tag{9}$$

where V_{it} is bidder i's absolute synergy value and M_t is the observed standalone value of the target.[23] Here, $\tau_t \exp(X_t'\gamma)$ with $\gamma \in \mathcal{R}^k$ refers to the common synergy premium and ε_{it} denotes the idiosyncratic synergy premium, specific to each bidder-target combination. We specify that $\log v_{it} = X_t'\gamma + \log \tau_t + \log \varepsilon_{it}$ and

$$\log \varepsilon_{it} \sim \mathcal{N}(0, h_\varepsilon^{-1})\mathbb{1}(\log \varepsilon_{it} \in [-dh_\varepsilon^{-1/2}, dh_\varepsilon^{-1/2}]), \tag{10}$$

$$\log \tau_t \sim \mathcal{N}(0, h_\tau^{-1}), \tag{11}$$

where h_ε and h_τ are the precision parameters of the normal distributions and $d := \Phi^{-1}(99.5\%) = 2.576$ with $\Phi(\cdot)$ being the CDF of $\mathcal{N}(0,1)$. We use $F_\varepsilon(\cdot|h_\varepsilon)$ and $f_\varepsilon(\cdot|h_\varepsilon)$ to denote the CDF and PDF of ε_{it} and $F_\tau(\cdot|h_\tau)$ and $f_\tau(\cdot|h_\tau)$ similarly for τ_t.

Second, we specify the discount factor $D(v_{it})$ as a linear function of v_{it} which is zero at the lower bound, i.e., $D(v_{it}|\eta) = \eta[v_{it} - \underline{v}_t]\mathbb{1}(\eta > 0)$, where the event that $\eta \leq 0$ which corresponds to no information discount. Then, together with Assumptions 2 and 4, the information disclosure discount is written as

$$D(n_t, v_{it}|X_t; \eta) = (n_t - 1)\tau_t \exp(X_t'\gamma)\eta[\varepsilon_{it} - \underline{\varepsilon}_{it}]\mathbb{1}(\eta > 0),$$

where $(n_t - 1)\eta < 1$ for any given n_t to be consistent with Assumption (1) that valuation order is preserved in the presence of information disclosure discounts.

Now, we use $\theta := (\gamma, \delta, \eta, h_\varepsilon, h_\tau)$ to index the model primitives. We interpret observed bids as equilibrium outcomes, i.e., the latent synergy values are linked to the observed

[23] Gorbenko and Malenko (2014) and Gentry and Stroup (2015) use a similar synergy value structure but without the unobserved heterogeneity τ_t.

bids through the strictly increasing bidding function. Then, we derive the bid distribution functions in (7) by change of variables under each parameter θ.[24] Finally, given the data $\boldsymbol{z} := (\boldsymbol{y}, \boldsymbol{n}, \boldsymbol{X})$ and a prior $\pi(\theta)$, the posterior density is

$$\pi(\boldsymbol{\tau}, \theta|\boldsymbol{z}) \propto \pi(\theta) \times \prod_{t=1}^{T} f_\tau(\tau_t|h_\tau) \times L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}, \boldsymbol{X}, \theta),$$

and the posterior predictive mean is given as

$$E[c(\boldsymbol{\tau}, \theta)|\boldsymbol{z}] = \iint c(\boldsymbol{\tau}, \theta)\pi(\boldsymbol{\tau}, \theta|\boldsymbol{z})d\boldsymbol{\tau}d\theta. \tag{12}$$

for a measurable function of interest, $c(\boldsymbol{\tau}, \theta)$.

B. Implementation

The posterior predictive mean (12) requires the integration of a high-dimensional function. To overcome the computational difficulty, we draw a sample $\{\boldsymbol{\tau}^{(s)}, \theta^{(s)}\}_{s=1}^{S}$ from the posterior distribution $\pi(\boldsymbol{\tau}, \theta|\boldsymbol{z})$ using Metropolis-within-Gibbs algorithm, then approximate the equation (12) by

$$\frac{1}{S}\sum_{s=1}^{S} c(\boldsymbol{\tau}^{(s)}, \theta^{(s)}) \xrightarrow{a.s} E[c(\boldsymbol{\tau}, \theta)|\boldsymbol{z}]. \tag{13}$$

The algorithm is summarized as follows with details given in Appendix B.2.

Prior We use the following prior distributions for $\theta = (\gamma, \delta, \eta, h_\varepsilon, h_\tau)$: $h_\tau \sim \mathcal{G}a(\alpha_\tau, \lambda_\tau^{-1})$, $h_\varepsilon \sim \mathcal{G}a(\alpha_\varepsilon, \lambda_\varepsilon^{-1})$, $\gamma \sim \mathcal{N}(\mu_\gamma, \sigma_\gamma^2 I_k)$, $\delta \sim \mathcal{N}(\mu_\delta, \sigma_\delta^2 I_k)$, and $\eta = \eta^* \mathbb{1}(\eta^* > 0)$ with $\pi_{\eta^*}(\eta^*) \propto \mathbb{1}(\eta^* > \underline{\eta}^*)$ for some $\underline{\eta}^* < 0$, where I_k is a k-dimensional identity matrix and $\alpha_\tau, \lambda_\tau, \alpha_\varepsilon, \lambda_\varepsilon, \mu_\gamma, \sigma_\gamma, \mu_\delta, \sigma_\delta$ and $\underline{\eta}^*$ are known parameters. It is worth emphasizing that we set $\underline{\eta}^* < 0$ to allow the de-

[24] See Appendix B.1 for the detailed derivation of bid distribution functions.

tection of no information disclosure discounts, and thus no information cost.

Gibbs Sampling At each iteration $s = 1, \dots, S$, we split θ into scalar blocks and sample each scalar parameter separately, conditional on the most recent values of the other parameters.

Metropolis-Hastings Updating An adaptive Gaussian Metropolis-Hastings algorithm is used to update the parameters within each block sampling. We use Gaussian proposal distribution for all the parameters except $\gamma = (\gamma_1, \dots, \gamma_k)$ and h_ε, for which truncated Gaussian proposal distributions are used by incorporating theoretical parameter bounds in order to improve computational efficiency. For each iteration, we tune the variance of the proposal distribution following Haario et al. (2001, 2005). In particular, the variance is updated according to the variance of the parameter values sampled so far.

V. Estimation Results

In this section, we present the estimation results from our structural analysis. We run the sampling algorithm for $500,000$ iterations, then keep every 10th draw after an initial $100,000$ burn-in period. This procedure gives a sample of $M = 40,000$ draws from the posterior distribution.[25]

A. Parameter Estimates

The key model primitive of our interest is the information disclosure discount, which is summarized by the parameter η, the percentage synergy value discount for each additional competitor learning the confidential information. The posterior mean of η is 7.6% with

[25] The thinning procedure reduces the autocorrelation of the posterior sample and reduces computing time of the counterfactual analyses.

the 95% credible interval of $[6.1\%, 9.2\%]$, which amounts to tens of millions of dollars for a typical takeover transaction worth billions of dollars. For example, for an average firm in our sample, the 7.6% of discount translates to $67 million synergy value loss.[26]

Table III summarizes the posterior distribution of the coefficients attached to auction covariates X_t by their posterior means and 95% credible intervals. The left panel reports the parameter estimate of γ, which represents the direct association of synergy values with the target characteristics and market conditions; see our specification (9). The estimation result shows that *Log(size)* and *Q-ratio* are negatively associated with synergy value. The negative effect of target size on the synergy value is likely due to decreasing returns to scale.[27] The negative association with *Q-ratio* may reflect high takeover premiums for undervalued target firms, proxied by low Q-ratio. In addition, the positive predictive coefficients of *R&D* and *Intangibles* are intuitive: a main source of synergy value creation comes from technological enhancement in the combined entity.

The estimate of δ, reported in the right panel of Table III, demonstrates that a few observed target characteristics help to predict the shortlisting probability. This result supports our early assertion in section II that bidders can estimate the shortlisting probability by studying the precedent transactions using their public information, i.e., observed target characteristics. Moreover, we find that the company-specific characteristics, i.e., *Log(Size)*, *Q-ratio*, *CashFlow*, *Cash* and *Intangibles*, rather than the general market conditions and industry factors, are more relevant to seller's shortlisting decisions. In particular, the negative association of intangible assets with the shortlisting probability suggests that bidder's entry is more tightly regulated for target companies with more intangible assets. This result is coherent with our premise of information costs.

[26] The average firm is defined as a target company with an average size, leverage, Q-ratio, CashFlow, Cash and R&D in our sample.

[27] An alternative explanation of the negative size coefficient could be due to our focus on takeovers with cash-only bids. Cash bidders usually finance the acquisition through investment banks and the cost of cash financing increases with the size.

Table III
Posterior Estimation of Parameters

This table reports the posterior means and 95% credible intervals of the coefficients on a set of target characteristics and market conditions for valuation and shortlisting estimation; see specification (8 and 9). The results are calculated based on $40,000$ draws from the posterior distribution.

	γ's in valuation		δ's in shortlisting	
Variables	Mean	95% Credible Interval	Mean	95% Credible Interval
Constant	0.761	[0.560, 0.957]	-0.792	[-1.215, -0.530]
Log(Size)	-0.043	[-0.068, -0.015]	0.150	[0.094, 0.223]
Leverage	0.029	[-0.221, 0.243]	0.028	[-0.353, 0.408]
Q-ratio	-0.045	[-0.087, -0.001]	0.177	[0.053, 0.291]
CashFlow	0.031	[-0.150, 0.222]	-0.854	[-1.416, -0.305]
Cash	0.052	[-0.184, 0.313]	-0.506	[-0.936, -0.070]
R&D	0.712	[-0.491, 1.878]	0.673	[-0.953, 2.281]
Intangibles	0.070	[-0.190, 0.342]	-0.904	[-1.354, -0.437]
CreditSpread	0.250	[-1.618, 2.017]	0.166	[-1.740, 2.072]
MarketReturn	-0.320	[-0.655, 0.034]	-0.213	[-0.856, 0.426]
FF-Industry-1	-0.064	[-0.213, 0.083]	-0.228	[-0.487, 0.053]
FF-Industry-2	-0.114	[-0.278, 0.048]	-0.201	[-0.466, 0.065]
FF-Industry-3	-0.034	[-0.174, 0.102]	0.155	[-0.090, 0.398]
FF-Industry-4	0.031	[-0.130, 0.205]	0.082	[-0.271, 0.429]

B. Predictive Densities of τ_t and ε_{it}

Our specification (9) forms bidder i's synergy value by three components: the idiosyncratic bidder-target synergy ε_{it}, observed target heterogeneity $X_t'\gamma$, and unobserved target heterogeneity τ_t. Our specification along with the identification result allows us to measure the contribution of each of the three components on the variation of the bid data. To this end, we define the (posterior) predictive density of τ as

$$f_\tau(\tau|\boldsymbol{z}) := \int f_\tau(\tau|h_\tau)\pi(h_\tau|\boldsymbol{z})dh_\tau. \tag{14}$$

where $f_\tau(\cdot|h_\tau)$ is the density of τ in (11) and $\pi(h_\tau|\boldsymbol{z})$ is the marginal posterior density of h_τ. We approximate (14) by an ergodic sample, $\{h_\tau^{(m)}\}_{m=1}^M$ drawn from the posterior, i.e., for each τ

$$\frac{1}{M}\sum_{m=1}^{M} f_\tau(\tau|h_\tau^{(m)}) \xrightarrow{a.s} f(\tau|\boldsymbol{z}),$$

and summarize its uncertainty by a pointwise 95% credible interval $[a,b]$ such that

$$95\% = \frac{1}{M}\sum_{m=1}^{M} \mathbb{1}\left\{f_\tau(\tau|h_\tau^{(m)}) \in [a,b]\right\}.$$

which forms a credible band as we vary τ. Specifically, we use the 2.5 and 97.5 percentiles for a and b. We similarly construct the predictive density of ε_{it} denoted by $f_\varepsilon(\cdot|\boldsymbol{z})$ and its 95% credible band.

Figure 1 presents the predictive densities $f_\tau(\cdot|\boldsymbol{z})$ and $f_\varepsilon(\cdot|\boldsymbol{z})$ in solid lines with their 95% credible band in dashed lines. It shows that $f_\tau(\cdot|\boldsymbol{z})$ is more diffuse than $f_\varepsilon(\cdot|\boldsymbol{z})$, implying that a bidder can dramatically change her evaluation on the synergy value after examining the confidential information. More specifically, we find that the confidential information would raise the target value by more than half with probability of 11.2% and it would lower by more than half with probability of 2.0%. In contrast, the effect of bidder-specific synergy is relatively small: it raises the target value more than half with probability of 1.8% and lowers more than half with essential zero probability. We further consider the ratio of the variance of τ to the sum of variances of $\log(\tau)$ and $\log(\varepsilon)$;

$$\frac{1/h_\tau}{1/h_\tau + 1/h_\varepsilon},$$

which can be interpreted as the relative variation of the confidential information conditional on the observed target heterogeneity X_t. The posterior mean of the ratio is 75.3% with the 95% credible interval of $[70.6\%, 79.7\%]$. This result provides a quantitative support of our conjecture that the unobserved confidential information plays an essential role in the target

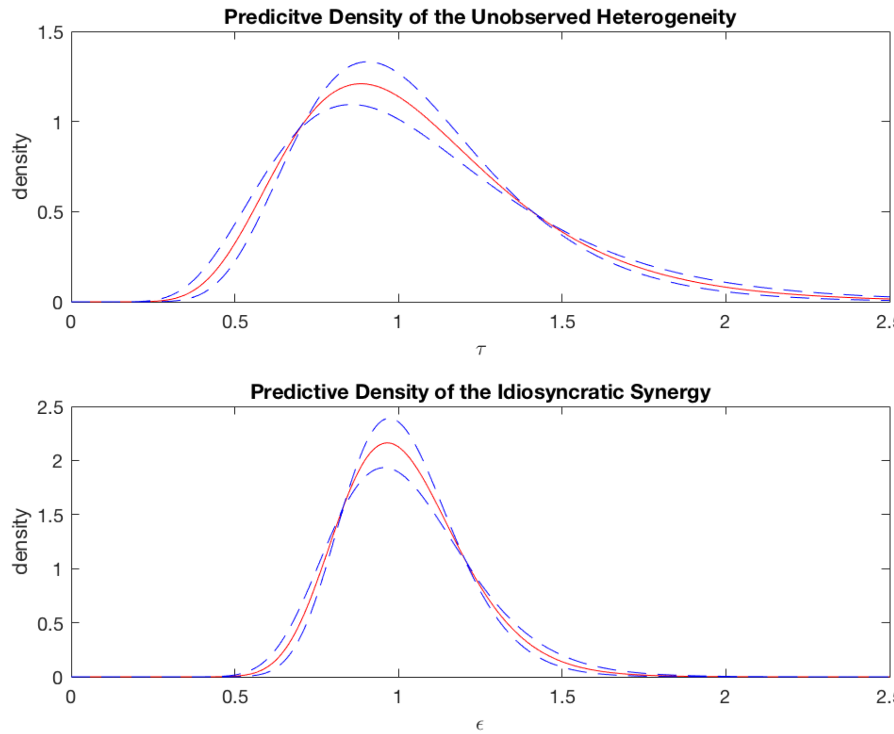

Figure 1. Predictive Densities of τ_t and ε_{it}. This Figure shows the predictive densities of the unobserved heterogeneity and the idiosyncratic synergy, based on $40,000$ draws from the posterior distribution.

valuation process and raises doubts on the models ignoring the latent factor.

C. Counterfactual Analyses

In this subsection, we conduct a range of counterfactual analyses. First, we disentangle the information cost and the competition gain. Then, we show that the English auction may reduce information cost and thus increase the seller's expected revenue. Finally, we bound the operation cost by exploring the seller's shortlisting decision.

Let $R(X, \tau, n|N, \theta)$ denote the expected revenue (i.e., takeover premium) when the seller of a target with (X, τ) invites n final bidders out of N potential bidders under the parameter

θ. The multiplicatively separable valuation structure implies that the expected revenue is also multiplicatively separable, i.e., $R(X, \tau, n|N, \theta) = \tau \cdot R(X, \tau = 1, n|N, \theta)$. For the counterfactual analyses, we consider a range of auctions in the Consumer and HighTech industries with $\tau_t = 1$ and other covariates equal to their sample medians, and suppress the dependence of $R(X, \tau, n|N, \theta)$ on (X, τ). Formally,

$$R(n|N, \theta) := \int b_w g_w(b_w|N, n, \theta) db_w, \tag{15}$$

where b_w is the winning bid from the auction and $g_w(\cdot|N, n, \theta)$ is the corresponding probability density. Moreover, we define the posterior predictive revenue as

$$\begin{aligned} R(n|N, \boldsymbol{z}) &:= \int \left\{ \left[\int b_w g_w(b_w|N, n, \theta) db_w \right] \right\} \pi(\theta|\boldsymbol{z}) d\theta \\ &= \int b_w \left\{ \int \left[g_w(b_w|N, n, \theta) \right] \pi(\theta|\boldsymbol{z}) d\theta \right\} db_w \\ &= \int b_w g_w(b_w|N, n, \boldsymbol{z}) db_w \end{aligned}$$

where the second equality applies the Fubini's theorem and the last uses the definition of the posterior predictive density of b_w. That is, $R(n|N, \boldsymbol{z})$ is the posterior mean of the winning bid. Hence, we approximate it by $M^{-1} \sum_{m=1}^{M} b_w^{(m)}$ with $b_w^{(m)} \sim g_w(b_w|N, n, \theta^{(m)})$ for each draw $\theta^{(m)}$ from the posterior, and summarize its uncertainty by the 95% credible interval based on the 2.5 and 97.5 percentiles of $\{b_w^{(m)}\}_{m=1}^{M}$.

C.1. Information Cost and Competition Effect

Our structural model allows us to disentangle two opposing effects, i.e., competition effect and information cost, on the takeover premium when one more bidder is invited. Denote the revenue in the absence of information disclosure discount by $R_0(n|N, \theta)$, which is identical to $R(n|N, \theta)$ except that η is restricted to be zero. Then we define the total information cost (TIC) incurred when n final bidders are invited as the revenue loss induced by the information disclosure discount, i.e.,

$$\mathbf{TIC}(n; N) := R_0(n|N, \theta) - R(n|N, \theta).$$

Then the (marginal) information cost (IC) by inviting an additional bidder is

$$\mathbf{IC}(n, n+1; N) := \mathbf{TIC}(n+1; N) - \mathbf{TIC}(n; N). \tag{16}$$

We also define the competition effect (CE) as the revenue difference in the absence of the information disclosure discount, i.e.,

$$\mathbf{CE}(n, n+1; N) := R_0(n+1|N, \theta) - R_0(n|N, \theta). \tag{17}$$

Then, we can decompose the revenue change into the competition effect and the information cost:

$$\begin{aligned} R(n+1|N, \theta) - R(n|N, \theta) =& R_0(n+1|N, \theta) - R_0(n|N, \theta) \\ & - [R_0(n+1|N, \theta) - R(n+1|N, \theta)] \\ & + [R_0(n|N, \theta) - R(n|N, \theta)] \\ =& \ \mathbf{CE}(n, n+1; N) - \mathbf{IC}(n, n+1; N). \end{aligned}$$

Considering a series of auctions in the Consumer and HighTech industries with $N = (2, 4, 8)$, Table IV reports the predictive revenue along with its 95% credible intervals and the change of revenue along with its decomposition into competition effect and information cost. First, column (A) shows that acquirors generally value HighTech targets more than the Consumer targets ceteris paribus. This posterior prediction is consistent with the observed premium difference in Table I; for example, on average a HighTech target receives 11% higher premium than a Consumer target, i.e., 138% v.s. 127%. Second, column (C) shows that the information cost reduces a substantial portion of the takeover premium; for example, for the median participation of $(N, n) = (4, 2)$ in Consumer industry, the predictive premium decreases by 2.6%, which amounts to \$23 millions for a firm with average size, i.e., \$719.6 millions. Finally, column (F) separates the positive competition effect by explicitly singling out the change of information cost calculated in column (E).

C.2. Operation Cost

Suppose that the sellers choose the optimal number of final bidders to maximize their expect revenue. We can bound the operation cost by contrasting the theoretical prediction on the sellers' revenue against the observed number of shortlisted bidders.

Recall that n is unknown whereas N is common knowledge among the bidders, and the bidding strategy depends on N, but not on n. However, since $R(n|N, \theta)$ is the expectation of the highest bid among submitted n bids, we have $R(n|N, \theta) > R(n-1|N, \theta)$ for all $n \leq N$. Now let OC be the operation cost for inviting an additional bidder. Then, the seller would invite the n^{th} bidder if the expected revenue increment is larger than OC, i.e., $R(n|N, \theta) - R(n-1|N, \theta) \geq OC$. Similarly, he would not invite $(n+1)^{\text{th}}$ bidder if the marginal revenue is smaller than OC, i.e., $R(n+1|N, \theta) - R(n|N, \theta) < OC$. Therefore, when the seller chooses to invite n bidders, it must be that

$$R(n+1|N, \theta) - R(n|N, \theta) < OC \leq R(n|N, \theta) - R(n-1|N, \theta). \tag{18}$$

From the inequality (18), we can bound OC by contrasting the theoretical revenue and observed n. Formally, we have $OC \in [\underline{OC}, \overline{OC}]$ where

$$\underline{OC} := R(\min\{n+1, N\}|N, \theta) - R(n|N, \theta) \tag{19}$$

Table IV
Predictive Revenue, Information Cost and Competition Effect

This table reports the posterior results for a range of hypothetical auctions with the median values of explanatory variables and the unobserved heterogeneity $\tau = 1$ for Consumer and HighTech industries. Columns (A)&(B) report the predictive revenue and its 95% credible interval; column (C) reports the predictive change of revenue when the number of final bidders increases from $n-1$ to n; columns (D)&(E) decompose the revenue change into the information cost and competition effect. The results are calculated based on 40,000 draws from the posterior distribution.

(N, n)	(A) Revenue	(B) 95% CI for (A)	(C) Δ Revenue	(D) Info. Cost	(E) Comp. Effect
		Consumer Industry			
(2,1)	1.015	[0.866, 1.191]	–	–	–
(2,2)	1.055	[0.902, 1.214]	0.040	0.001	0.041
(4,1)	1.140	[0.886, 1.448]	–	–	–
(4,2)	1.225	[0.954, 1.481]	0.085	0.006	0.091
(4,3)	1.269	[1.013, 1.498]	0.044	0.002	0.047
(4,4)	1.297	[1.058, 1.509]	0.028	0.001	0.029
(8,1)	1.240	[0.904, 1.616]	–	–	–
(8,2)	1.348	[1.011, 1.657]	0.108	0.021	0.129
(8,3)	1.402	[1.099, 1.678]	0.054	0.010	0.063
(8,4)	1.435	[1.161, 1.693]	0.033	0.005	0.038
(8,5)	1.458	[1.206, 1.706]	0.024	0.003	0.027
(8,6)	1.476	[1.237, 1.713]	0.017	0.002	0.019
(8,7)	1.489	[1.258, 1.721]	0.013	0.001	0.014
(8,8)	1.501	[1.276, 1.725]	0.011	0.001	0.012
		HighTech Industry			
(2,1)	1.094	[0.897, 1.326]	–	–	–
(2,2)	1.151	[0.949, 1.352]	0.057	0.002	0.059
(4,1)	1.242	[0.920, 1.607]	–	–	–
(4,2)	1.344	[1.018, 1.646]	0.103	0.009	0.112
(4,3)	1.396	[1.097, 1.666]	0.051	0.003	0.055
(4,4)	1.428	[1.158, 1.679]	0.032	0.001	0.033
(8,1)	1.309	[0.942, 1.691]	–	–	–
(8,2)	1.414	[1.082, 1.738]	0.106	0.034	0.139
(8,3)	1.465	[1.174, 1.763]	0.051	0.016	0.066
(8,4)	1.497	[1.231, 1.779]	0.032	0.009	0.041
(8,5)	1.519	[1.263, 1.791]	0.022	0.005	0.028
(8,6)	1.536	[1.287, 1.801]	0.017	0.003	0.020
(8,7)	1.550	[1.305, 1.808]	0.014	0.002	0.016
(8,8)	1.562	[1.322, 1.816]	0.012	0.001	0.013

$$\overline{OC} := R(n|N,\theta) - R(n-1|N,\theta) \tag{20}$$

are the implied lower and upper bounds of OC, respectively, for $n \in \{1,\ldots,N\}$. Here we assume the expected revenue is 1 under the current management, $R(0|N,\theta) = 1$, which corresponds to the scenario of inviting no bidders ($n = 0$).

Table V reports the predictive lower and upper bounds of the marginal operation costs OC, defined in (19) and (20), for a series of hypothetical auctions with $N = (2,4,8)$ in Consumer and HighTech industries. The result indicates a substantial marginal operation cost to accommodate bidders in the final due diligence. For example, for the median participation of $(N,n) = (4,2)$ in consumer industry, the marginal operation cost consumes on average 4.4% to 8.5% of the takeover premium, which amounts to \$35 to \$79 millions. This estimate is substantially higher than the reported advisory fee, e.g., 0.84% in Hunter and Jagtiani (2003), indicating the importance of other economic cost, such as foregoing of business opportunities, in the process of sales of companies. The operation cost and the information cost together explain the practice of entry regulation by the seller.

C.3. English Auction of Companies

The English auction reveals the number of final bidders to all competitors in the bidding game. By dissolving bidders' uncertainty on the number of bidders, the English auction can increase the seller's revenue when the actual pool of bidders is not large. For a series of auctions with the number of final bidders n ranging from 2 to 8, we report the predictive revenues and their 95% credible intervals in Table VI.[28]

In contrast to the predictive revenues in column (A) of Table IV, where the first-price takeover auctions are conducted, we find that the English auction increases the revenue for

[28] We omit the counterfactual comparison of the English auction with only one bidder because it would never improve the takeover revenue than the current first-price auction, as the single bidder would bid the reserve price.

Table V
Estimated Bounds on Operation Cost

This table reports the predictive lower bound ($\underline{OC}$) and upper bound ($\overline{OC}$) of operation cost for a range of hypothetical auctions with the median values of explanatory variables and the unobserved heterogeneity $\tau = 1$ for Consumer and HighTech industries. The results are calculated based on $40,000$ draws from the posterior distribution.

	(A) Consumer Industry		(B) HighTech Industry	
(N, n)	$\underline{OC}$	$\overline{OC}$	$\underline{OC}$	$\overline{OC}$
(2,1)	0.040	0.015	0.057	0.094
(2,2)	0.000	0.040	0.000	0.057
(4,1)	0.085	0.140	0.103	0.242
(4,2)	0.044	0.085	0.051	0.103
(4,3)	0.028	0.044	0.032	0.051
(4,4)	0.000	0.028	0.000	0.032
(8,1)	0.108	0.240	0.106	0.309
(8,2)	0.054	0.108	0.051	0.106
(8,3)	0.033	0.054	0.032	0.051
(8,4)	0.024	0.033	0.022	0.032
(8,5)	0.017	0.024	0.017	0.022
(8,6)	0.013	0.017	0.014	0.017
(8,7)	0.011	0.013	0.012	0.014
(8,8)	0.000	0.011	0.000	0.012

some takeovers. For example, for an auction with $(N, n) = (4, 2)$ in the Consumer industry, the first price auction would generate the revenue of 1.225. On the other hand, the seller would earn higher revenues of 1.265, i.e., extra \$35,000 on an average firm, by switching to the English auction.

VI. Concluding Remark

This study finds that the information cost and operation cost incurred by the seller of a company can be economically significant, which explains the common practice that the seller limits bidders' participation. Our findings provide a few policy implications to the

Table VI
Predictive Revenue for English Auctions

This table reports the predictive revenue and its 95% credible interval for a series of hypothetical English auctions with the median values of explanatory variables and the unobserved heterogeneity $\tau = 1$ for Consumer and HighTech industries. The results are calculated based on $40,000$ draws from the posterior distribution.

	(A) Consumer Industry		(B) HighTech Industry	
n	Revenue	95% CI	Revenue	95% CI
2	1.265	[0.912, 1.709]	1.305	[0.937, 1.764]
3	1.286	[0.975, 1.673]	1.324	[1.004, 1.723]
4	1.236	[0.958, 1.581]	1.273	[0.985, 1.626]
5	1.154	[0.905, 1.466]	1.189	[0.928, 1.511]
6	1.055	[0.825, 1.343]	1.087	[0.846, 1.383]
7	0.944	[0.721, 1.220]	0.973	[0.741, 1.255]
8	0.825	[0.601, 1.094]	0.850	[0.618, 1.125]

regulatory and judiciary authorities; and our structural method can be useful in resolving some legal disputes related to takeover transactions.

The large amount of the information cost, e.g., \$23 millions for a firm with the average size in the Consumer industry, indicates a substantial loss of social welfare due to the informational externality. It advocates the reinforcement of regulation on the use of information acquired from the takeover process.[29] Moreover, our quantitative approach can be useful in settling some takeover lawsuits. In particular, a seller's shortlisting decision is often challenged in court by its shareholders, who argue that the entry restriction discourages competition and hurts the shareholders' benefits. Given the observed target characteristics, our method provides a reasonable range of operation costs for similar transactions. These estimates may serve to guide the court judgment by checking whether the implied operation cost lies in the reasonable range.

In addition, the large seller's cost also explains why a target company is willing to sign

[29] It is beyond the scope of the paper to develop a formal decision method to choose a policy parameter because it requires additional elements outside the auction framework, e.g., the probability of detecting violation of the agreement and the socially desirable level of economic efficiency.

an exclusivity agreement with a single bidder. The exclusivity agreement sends a credible signal to the bidder that no confidential information is exposed to other competitors, and thus prevents the information disclosure discounts. In addition, although we focus on successful takeovers in the analyses, the implication of information cost on failed takeovers is straightforward. When a deal fails to consummate, the market value of the target company is expected to fall because the target firm, as the eventual owner, bears the information cost.

Appendix A: Technical Proofs

This Appendix provides the proofs of Propositions 1, 2 and 4.

A.1. Proof of Proposition 1

Proof. This proof extends Proposition 2.2 in (Krishna, 2002) to our model with uncertain number of bidders and varying valuation.

Suppose that all but bidder i follow the bidding strategy (3). We argue that it is optimal for bidder i to follow (3) as well. Assuming that the bidding strategy (3) is an increasing and continuous function. Thus, in equilibrium the bidder with the highest value submits the highest bid and wins the auction. It is not optimal for any bidder to bid over $\beta(\overline{v}; N)$.

First, with the rank preserving property assumed in Assumption 1, we are able to rewrite the $\widetilde{\beta}_n^I(\cdot)$ as

$$\widetilde{\beta}_n^I(v) = \beta_n^I(\tilde{v}) := \begin{cases} \underline{\tilde{v}}, & \textbf{for } n = 1 \\ \frac{1}{\widetilde{F}_n(\tilde{v})} \int_{\underline{\tilde{v}}}^{\tilde{v}} \tilde{t} d\widetilde{F}_n(\tilde{t}), & \textbf{for } n \in \{2, \ldots, N\}, \end{cases}$$

where $\tilde{v} := v - D(n, v)$ with a CDF $\widetilde{F}_{\tilde{v}}(\cdot)$, and $\widetilde{F}_n(\cdot) := \widetilde{F}_{\tilde{v}}(\cdot)^{n-1}$.[30] Then, let $y := \beta^{-1}(b; N)$.

[30] To see this, note that the rank preserving assumption implies that

$$F_v(t) = Pr(v \leq t) = Pr(\tilde{v} \leq \tilde{t}) = \widetilde{F}_{\tilde{v}}(\tilde{t}).$$

The expected payoff of bidder i with value $v \in (\underline{v}, \overline{v})$ if he bids an amount $b = \beta(y; N)$ is

$$\begin{aligned}
\pi(\beta(y;N), v; N) &= \sum_{n=1}^{N} p_{n|N} F_v(y)^{n-1} \left[v - D(n,v) - \beta(y;N)\right] \\
&= \sum_{n=1}^{N} p_{n|N} F_v(y)^{n-1} \tilde{v} - H(y|N)\beta(y;N) \\
&= \sum_{n=1}^{N} p_{n|N} F_v(y)^{n-1} \tilde{v} - \sum_{n=1}^{N} H(y|N) w_n(y) \beta_n^I(\tilde{y}) \\
&= \sum_{n=1}^{N} p_{n|N} F_v(y)^{n-1} \left[\tilde{v} - \beta_n^I(\tilde{y})\right] \\
&= p_{1|N}(v - \underline{v}) + \sum_{n=2}^{N} p_{n|N} \widetilde{F}_n(\tilde{y}) \left[\tilde{v} - \frac{1}{\widetilde{F}_n(\tilde{y})} \int_{\underline{\tilde{v}}}^{\tilde{y}} \tilde{t} d\widetilde{F}_n(\tilde{t})\right] \\
&= p_{1|N}(v - \underline{v}) + \sum_{n=2}^{N} p_{n|N} \left[\widetilde{F}_n(\tilde{y})\tilde{v} - \int_{\underline{\tilde{v}}}^{\tilde{y}} \tilde{t} d\widetilde{F}_n(\tilde{y})\right] \\
&= p_{1|N}(v - \underline{v}) + \sum_{n=2}^{N} p_{n|N} \left[\widetilde{F}_n(\tilde{y})\left(\tilde{v} - \tilde{y}\right) + \int_{\underline{\tilde{v}}}^{\tilde{y}} \widetilde{F}_n(t) d\tilde{t}\right]
\end{aligned}$$

Observe that $\pi(\beta(\underline{v}; N); N), \underline{v}; N) = 0$, so bidding $\beta(\underline{v}; N) = \underline{v}$ is incentive compatible for the bidder with the lowest value. Next, we exam the change of profit by deviating from the bidding strategy (3),

$$\pi(\beta(v;N), v; N) - \pi(\beta(y;N), v; N) = \sum_{n=2}^{N} p_{n|N} \left[\widetilde{F}_n(\tilde{y})(\tilde{y} - \tilde{v}) + \int_{\tilde{y}}^{\tilde{v}} \widetilde{F}_n(\tilde{t}) d\tilde{t}\right] \geq 0,$$

regardless of whether $\tilde{y} \leq \tilde{v}$ or $\tilde{z} \geq \tilde{v}$ because the term in the square brackets is non-negative for $n = 2, \ldots, N$.

We have thus shown that if all other bidders are following the strategy (3), an arbitrary bidder with a value of $v \in [\underline{v}, \overline{v}]$ cannot benefit by bidding anything other than $\beta(v; N)$.

Therefore, $\beta(\cdot; N)$ given by (3) characterizes a symmetric equilibrium bidding strategy for the model. $\square$

A.2. Proof of Proposition 2

Proof. We rewrite (3) as follows,

$$\begin{aligned}
\beta(v;N) &= \left(1-\sum_{n=2}^{N} w_{n|N}\right)\underline{v}+\sum_{n=2}^{N} w_{n|N}(v)\widetilde{\beta}_n^I(v) \\
&= \underline{v}+\sum_{n=2}^{N} w_{n|N}(v)\left(\widetilde{\beta}_n^I(v)-\underline{v}\right) \qquad \text{(A.1)}
\end{aligned}$$

Since $\widetilde{\beta}_n^I(v)-\underline{v}(\geq 0)$ is strictly increasing in v for all $n \geq 2$, then (3) is strictly increasing as long as $\omega_{n|N}(v)$ is nondecreasing in v for all $n \geq 2$. To simplify, we suppress dependence of v and N in $\omega_{n|N}(v)$.

$$\begin{aligned}
\frac{d}{dv}\omega_n &= \frac{1}{H^2}\left[(n-1)p_nF^{n-2}f\sum_{m=1}^{N} p_mF^{m-1} - p_nF^{n-2}F\sum_{m=2}^{N} p_m(m-1)F^{m-2}f\right] \\
&= \frac{1}{H^2}\left[(n-1)p_nF^{n-2}f\sum_{m=1}^{N} p_mF^{m-1} - p_nF^{n-2}f\sum_{m=2}^{N} p_m(m-1)F^{m-1}\right] \\
&= \frac{p_nF^{n-2}f}{H^2}\left[\sum_{m=1}^{N} p_m(n-1)F^{m-1} - \sum_{m=2}^{N} p_m(m-1)F^{m-1}\right] \\
&= \frac{p_nF^{n-2}f}{H^2}\left[\sum_{m=1}^{N} p_m(n-m)F^{m-1}\right].
\end{aligned}$$

Therefore, for $n=2$, we obtain

$$\frac{d}{dv}\omega_2 = \frac{p_nF^{n-2}f}{H^2}\left[p_1-\sum_{m=3}^{N} p_m(m-2)F^{m-1}\right] \geq \frac{p_nF^{n-2}f}{H^2}\left[p_1-\sum_{m=3}^{N} p_m(m-2)\right] \geq 0,$$

where the last inequality is due to (4).

Note that $p_n F^{n-2} f / H^2 \geq 0$ and $\sum_{m=1}^{N} p_m (n-m) F^{m-1}$ is increasing with respect to n for any $n \geq 2$. So, $d\omega_2/dv \geq 0$ implies $d\omega_n/dv \geq 0$ for all $n > 2$, which in turn implies the strict monotonicity of equation (A.1). □

A.3. Proof of Proposition 4

Proof. Let $v(\alpha)$ be the α-quantile of $F_v(\cdot)$. Then $b_N(\alpha) = \beta(v(\alpha); N)$ because $b_N = \beta(v; N)$ and $\beta(v; N)$ is strictly increasing in v. Therefore, from equation (6), we have,

$$v(\alpha) = b_N(\alpha) + \lambda_N(b_N(\alpha)) + \sum_{n=2}^{N} \eta_{n,N}(b_N(\alpha)) D(n, v(\alpha)) \tag{A.2}$$

for any $\alpha \in (0, 1]$ and an $N \geq 2$.

Now suppose we observe two distinct numbers, N_1 and N_2, of potential bidders and let $N_2 > N_1 \geq 2$ without loss of generality. Under the Assumption (4), we can substitute $D(n, v(\alpha)) = (n-1) D(v(\alpha))$ into the equation (A.2). Then for $N = N_1$ and N_2, we have

$$v(\alpha) = b_N(\alpha) + \lambda_N(b_N(\alpha)) + \left[\sum_{n=2}^{N} (n-1) \eta_{n,N}(b_N(\alpha)) \right] D(v(\alpha)). \tag{A.3}$$

First note that $\sum_{n=2}^{N} (n-1) \eta_{n,N}(b_N(\alpha)) \neq 0$ given $p_{N|N} \neq 0$. Then by taking a difference of the two equations, we get

$$\begin{aligned} &(b_{N_2}(\alpha) - b_{N_1}(\alpha)) + (\lambda_{N_2}(b_{N_2}(\alpha)) - \lambda_{N_1}(b_{N_1}(\alpha))) \\ &+ \sum_{n=2}^{N_2} (n-1) \eta_{n,N_2}(b_{N_2}(\alpha)) D(v(\alpha)) - \sum_{n=2}^{N_1} (n-1) \eta_{n,N_1}(b_{N_1}(\alpha)) D(v(\alpha)) = 0, \end{aligned}$$

which leads to

$$D(v(\alpha)) = \frac{(b_{N_1}(\alpha) - b_{N_2}(\alpha)) + (\lambda_{N_1}(b_{N_1}(\alpha)) - \lambda_{N_2}(b_{N_2}(\alpha)))}{\sum_{n=2}^{N_2} (n-1) \eta_{n,N_2}(b_{N_2}(\alpha)) - \sum_{n=2}^{N_1} (n-1) \eta_{n,N_1}(b_{N_1}(\alpha))}$$

$$= \frac{(b_{N_1}(\alpha) - b_{N_2}(\alpha)) + (\lambda_{N_1}(b_{N_1}(\alpha)) - \lambda_{N_2}(b_{N_2}(\alpha)))}{\sum_{n=2}^{N_2} n \cdot \eta_{n,N_2}(b_2(\alpha)) - \sum_{n=2}^{N_1} n \cdot \eta_{n,N_1}(b_{N_1}(\alpha))}.$$

The last equation is arrived by recognizing that $\sum_{n=2}^{N} \eta_{n,N}(b) = 1$ for any $b \in (\underline{b}, \overline{b}]$ and any $N \geq 2$ when $p_{1|N} < 1$. This concludes the identification of $D(v(\alpha))$ at α-quantile.[31]

With $D(v(\alpha))$ identified at each quantile $\alpha \in (0, 1]$, we can identify $v(\alpha)$ for each $\alpha \in (0, 1]$ by equation (A.3). As an immediately result, the valuation distribution $F_v(\cdot)$ is nonparametrically identified on $(\underline{v}, \overline{v}]$. And the information disclosure discount factor $D(v)$ is also nonparametrically identified on $(\underline{v}, \overline{v}]$ by equation (A.3). □

Appendix B: Estimation Methodology

B.1. Derivation of Bid Distribution Functions

In order to construct the bid density, consider an auction with characteristics $(\tau, X) = (1, 0)$ and N potential bidders. Notice that the success rate of the shortlisting probability (8) for this auction is $q := \Phi(X'\delta) = \Phi(0)$. We let (h_ε, η) index (F_ε, D), respectively. For this auction, we use $\beta(\cdot|q, N, h_\varepsilon, \eta)$ to denote the equilibrium bidding strategy. Then, interpreting observed bids in data as equilibrium outcomes, we may write

$$\begin{aligned} b_{it} &= \beta(v_{it}|\Phi(X'\delta), N, h_\varepsilon, \eta) \\ &= \beta(\tau \exp(X'\gamma)\varepsilon_{it}|\Phi(X'\delta), N, h_\varepsilon, \eta) \\ &= \tau \exp(X'\gamma)\beta(\varepsilon_{it}|\Phi(X'\delta), N, h_\varepsilon, \eta) \end{aligned}$$

[31] $\sum_{n=2}^{N_2} n \cdot \eta_{n,N_2}(b_2(\alpha)) - \sum_{n=2}^{N_1} n \cdot \eta_{n,N_1}(b_{N_1}(\alpha)) \neq 0$. If it is zero, then the bidding strategy is not strictly monotone.

for the auction with characteristics (X,τ). The bid density can then be written as The bid density can then be written as

$$g(b_{it}|N,X;\tau,\theta) = \frac{f_\varepsilon\left[\beta^{-1}\left(\frac{b_{it}}{\tau\exp(X'\gamma)}\middle|\Phi(X'\delta),N,h_\varepsilon,\eta\right)\middle|h_\varepsilon\right]}{\tau\exp(X'\gamma)\beta'\left[\beta^{-1}\left(\frac{b_{it}}{\tau\exp(X'\gamma)}\middle|\Phi(X'\delta),N,h_\varepsilon,\eta\right)\middle|\Phi(X'\delta),N,h_\varepsilon,\eta\right]}$$
$$\times\ \mathbb{1}\left\{\frac{b_{it}}{\tau\exp(X'\gamma)}\in[\underline{\varepsilon},\beta(\overline{\varepsilon}|\Phi(X'\delta),N,h_\varepsilon,\eta)]\right\},$$

where $\theta := (\gamma,\delta,\eta,h_\varepsilon,h_\tau)$. Accordingly, the distribution function of b_{it} is

$$G(b_{it}|N,X;\tau,\theta) := F_\varepsilon\left[\beta^{-1}\left(\frac{b_{it}}{\tau\exp(X'\gamma)}\middle|\Phi(X'\delta),N,h_\varepsilon,\eta\right)\middle|h_\varepsilon\right].$$

B.2. Posterior Sampling

This Appendix provides details on sampling $\{\boldsymbol{\tau}^{(s)},\theta^{(s)}\}_{s=1}^S$ from the posterior distribution $\pi(\boldsymbol{\tau},\theta|\mathbf{y})$ using Metropolis-within-Gibbs algorithm. Suppose the current parameter value is $\theta^{(s)} = (\gamma^{(s)},\delta^{(s)},\eta^{(s)},h_\varepsilon^{(s)},h_\tau^{(s)})$ and $\boldsymbol{\tau}^{(s)}$, we describe how we draw $\theta^{(s+1)}$ and $\boldsymbol{\tau}^{(s+1)}$ for $s=1,\dots,S$.

1. Draw $\gamma^{(s+1)} = (\gamma_1^{(s+1)},\dots,\gamma_k^{(s+1)})$: For each j starting from 1 to k, we update $\gamma_j^{(s+1)}$ using a slightly modified Gaussian Metropolis-Hastings algorithm. In order to improve computational efficiency, we compute the conditional support $[\underline{\gamma}_j^{(s)},\overline{\gamma}_j^{(s)}]$ and draw a candidate $\tilde{\gamma}_j$ from the truncated normal distribution.

   Let $b^{\min}$ and $b^{\max}$ be the lowest and the highest (i.e. winning) observed bids in auction t respectively. Theoretically, any bid b_{it} is bounded by

$$\underline{\varepsilon}^{(s)} = \underline{b}^{(s)} \le \frac{b_{it}}{\tau^{(s)}\exp(x\gamma^{(s)})} \le \overline{b}^{(s)} := \beta(\overline{\varepsilon}^{(s)}|\Phi(x'\delta^{(s)}),N,h_\varepsilon^{(s)},\eta^{(s)}),$$

   where $\underline{\varepsilon}^{(s)} = \exp(-dh_\varepsilon^{(s)-1/2})$ and $\overline{\varepsilon}^{(s)} = \exp(dh_\varepsilon^{(s)-1/2})$. Then if $x_{jt} > 0$, from the first inequality, we have

$$\overline{\gamma}_j^{(s)} := \min_{t\in\{1,\cdots,T\}} \left\{ \frac{1}{x_{jt}} \log \left[\frac{b^{\min}}{\underline{b}^{(s)}\tau^{(s)} \exp\left(\sum_{j'\neq j} x_{j't}\gamma_{j'}^{(s)}\right)} \right] \right\}$$

and from the second inequality,

$$\underline{\gamma}_j^{(s)} := \max_{t\in\{1,\cdots,T\}} \left\{ \frac{1}{x_{jt}} \log \left[\frac{b^{\max}}{\overline{b}^{(s)}\tau^{(s)} \exp\left(\sum_{j'\neq j} x_{j't}\gamma_{j'}^{(s)}\right)} \right] \right\}.$$

For $x_{jt} < 0$, we obtain similar support $[\underline{\gamma}_j^{(s)}, \overline{\gamma}_j^{(s)}]$, and do not repeat it here.

We draw a candidate $\tilde{\gamma}_j \sim \mathcal{N}(\gamma_j^{(s)}, \sigma_{\gamma_j}^2)\mathbb{1}(\tilde{\gamma}_j \in [[\underline{\gamma}_j^{(s)}, \overline{\gamma}_j^{(s)}])$. In particular, we draw $u \sim \mathcal{U}nif(0,1)$, then let

$$\tilde{\gamma}_j := \gamma_j^{(s)} + \sigma_{\gamma_j}\Phi^{-1}\left\{ u \cdot \left[\Phi\left(\frac{\overline{\gamma}_j^{(s)} - \gamma_j^{(s)}}{\sigma_{\gamma_j}} \right) - \Phi\left(\frac{\underline{\gamma}_j^{(s)} - \gamma_j^{(s)}}{\sigma_{\gamma_j}} \right) \right] + \left(\frac{\underline{\gamma}_j^{(s)} - \gamma_j^{(s)}}{\sigma_{\gamma_j}} \right) \right\}.$$

We then set $\gamma_j^{(s+1)} := \tilde{\gamma}_j$ with probability

$$\min\left\{ 1, \frac{\pi(\tilde{\theta})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\tilde{\theta})}{\pi(\hat{\theta}^{(s)})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\hat{\theta}^{(s)})} \times \frac{\Phi\left(\frac{\overline{\gamma}_j^{(s)}-\tilde{\gamma}_j}{\sigma_{\gamma_j}}\right) - \Phi\left(\frac{\underline{\gamma}_j^{(s)}-\tilde{\gamma}_j}{\sigma_{\gamma_j}}\right)}{\Phi\left(\frac{\overline{\gamma}_j^{(s)}-\gamma_j^{(s)}}{\sigma_{\gamma_j}}\right) - \Phi\left(\frac{\underline{\gamma}_j^{(s)}-\gamma_j^{(s)}}{\sigma_{\gamma_j}}\right)} \right\};$$

otherwise, $\gamma_j^{(s+1)} := \gamma_j^{(s)}$, where $\tilde{\theta} := (\gamma_1^{(s+1)}, \ldots, \gamma_{j-1}^{(s+1)}, \tilde{\gamma}_j, \gamma_{j+1}^{(s)} \ldots, \gamma_k^{(s)}; \delta^{(s)}, \eta^{(s)}, h_\varepsilon^{(s)}, h_\tau^{(s)})$ contains the most recent updated values for all other parameters but $\tilde{\gamma}_j$; and $\hat{\theta}^{(s)} := (\gamma_1^{(s+1)}, \ldots, \gamma_{j-1}^{(s+1)}, \gamma_j^{(s)}, \gamma_{j+1}^{(s)} \ldots, \gamma_k^{(s)}; \delta^{(s)}, \eta^{(s)}, h_\varepsilon^{(s)}, h_\tau^{(s)})$ contains the most recent updated values for all other parameters but $\gamma_j^{(s)}$.[32]

[32]Without confusion, we recycle the notations $\tilde{\theta}$ and $\tilde{\theta}^{(s)}$ for each parameter updating.

Finally, we tune σ_{γ_j} following Haario et al. (2001, 2005). In particular, for the first 10 iteration $s = 1, \ldots, 10$, we use $\sigma_{\gamma_j} = 0.01$; then for $s = 11, \ldots, S$, we use $\sigma_{\gamma_j} = 1.54 \times \widehat{\mathrm{Std}}(\gamma_j^{(1)}, \ldots, \gamma_j^{(s-1)})$ with probability 95% and $\sigma_{\gamma_j} = 0.0001$ with probability 5%.[33]

2. Draw $\delta^{(s+1)} = (\delta_1^{(s+1)}, \ldots, \delta_k^{(s+1)})$: For each j starting from 1 to k, we update $\delta_j^{(s+1)}$ by drawing a candidate $\tilde{\delta}_j \sim \mathcal{N}(\delta_j^{(s)}, \sigma_{\delta_j}^2)$ We then set $\delta_j^{(s+1)} := \tilde{\delta}_j$ with probability

$$\min\left\{1, \frac{\pi(\tilde{\theta})L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}^{(s)}, \boldsymbol{X}, \tilde{\theta})}{\pi(\hat{\theta}^{(s)})L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}^{(s)}, \boldsymbol{X}, \hat{\theta}^{(s)})}\right\};$$

   otherwise, $\delta_j^{(s+1)} := \delta_j^{(s)}$, where $\tilde{\theta}$ and $\hat{\theta}^{(s)}$ are defined similarly for parameter δ_j. As before, we tune σ_{δ_j} by setting $\sigma_{\delta_j} = 0.01$ for first ten iterations, after which we use $\sigma_{\delta_j} = 1.54 \times \widehat{\mathrm{Std}}(\delta_j^{(1)}, \ldots, \delta_j^{(s-1)})$ with probability 95% and $\sigma_{\delta_j} = 0.0001$ with probability 5%.

3. Draw $\eta^{(s+1)}$: Similarly, we draw a candidate $\tilde{\eta} \sim \mathcal{N}(\eta^{(s)}, \sigma_\eta^2)$ and update $\eta^{(s+1)} = \tilde{\eta}$ with probability

$$\min\left\{1, \frac{L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}^{(s)}, \boldsymbol{X}, \tilde{\theta})}{L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}^{(s)}, \boldsymbol{X}, \hat{\theta}^{(s)})}\right\};$$

   where $\tilde{\theta}$ and $\hat{\theta}^{(s)}$ are defined similarly for parameter η. We tune the scale parameter σ_η as before.

4. Draw $h_\varepsilon^{(s+1)}$: Similar to step 1, we use the theoretical restrictions to truncate the proposal density in order to improve computational efficiency. From the following restriction,

$$\underline{\varepsilon}^{(s)} = \exp(-dh_\varepsilon^{(s)-1/2}) \leq \frac{b}{\tau \exp(x\gamma^{(s)})} \leq \overline{b}^{(s)} := \beta(\overline{\varepsilon}^{(s)}|\Phi(x'\delta^{(s)}), N, h_\varepsilon^{(s)}, \eta^{(s)}).$$

[33] The reference proves the convergence of the algorithm and provide a recursive formula to compute $\widehat{\mathrm{Std}}(\gamma_j^{(1)}, \ldots, \gamma_j^{(s-1)})$ at each s.

we obtain

$$\overline{h}_{\varepsilon}^{(s)} = \min_{t\in\{1,\ldots,T\}} d^2 \left\{ \log\left[\frac{b^{\min}}{\tau^{(s)}\exp(x\gamma^{(s)})} \right] \right\}^{-2}.$$

Similarly, we could obtain the exact lower bound from the second inequality, but we do not because it requires us to compute the equilibrium bidding strategy for each $t \in \{1,\ldots,T\}$. Instead, we only use the fact that $h_{\varepsilon} > 0$. We draw the candidate $\tilde{h}_{\varepsilon} \sim \mathcal{N}(h_{\varepsilon}^{(s)}, \sigma_{h_{\varepsilon}}^2)\mathbb{1}(\tilde{h}_{\varepsilon} \in [0, \overline{h}_{\varepsilon}^{(s)}])$. In particular, we let

$$\tilde{h}_{\varepsilon} := h_{\varepsilon}^{(s)} + \sigma_{h_{\varepsilon}}\Phi^{-1}\left\{ u \cdot \left[\Phi\left(\frac{\overline{h}_{\varepsilon}^{(s)} - h_{\varepsilon}^{(s)}}{\sigma_{h_{\varepsilon}}} \right) - \Phi\left(\frac{0 - h_{\varepsilon}^{(s)}}{\sigma_{h_{\varepsilon}}} \right) \right] + \left(\frac{0 - h_{\varepsilon}^{(s)}}{\sigma_{h_{\varepsilon}}} \right) \right\}$$

with $u \sim \mathcal{U}nif(0,1)$. We then update $h_{\varepsilon}^{(s+1)} = \tilde{h}_{\varepsilon}$ with probability

$$\min\left\{ 1, \frac{\pi(\tilde{\theta})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\tilde{\theta})}{\pi(\hat{\theta}^{(s)})L(\boldsymbol{y},\boldsymbol{n}|\boldsymbol{\tau}^{(s)},\boldsymbol{X},\hat{\theta}^{(s)})} \times \frac{\Phi\left(\frac{\overline{h}_{\varepsilon}^{(s)} - \tilde{h}_{\varepsilon}}{\sigma_{h_{\varepsilon}}}\right) - \Phi\left(\frac{0 - \tilde{h}_{\varepsilon}}{\sigma_{h_{\varepsilon}}}\right)}{\Phi\left(\frac{\overline{h}_{\varepsilon}^{(s)} - h_{\varepsilon}^{(s)}}{\sigma_{h_{\varepsilon}}}\right) - \Phi\left(\frac{0 - h_{\varepsilon}^{(s)}}{\sigma_{h_{\varepsilon}}}\right)} \right\}$$

where $\tilde{\theta}$ and $\hat{\theta}^{(s)}$ are defined similarly for parameter h_{ε}. We tune the scale parameter $\sigma_{h_{\varepsilon}}$ as in the previous steps.

5. Draw $h_{\tau}^{(s+1)}$: h_{τ} is the hyperparameter that governs the distribution of the unobserved heterogeneity τ. Hence we draw $h_{\tau}^{(s+1)}$ from its conditional density given $\boldsymbol{\tau}^{(s)}$. Using the conjugacy of gamma prior for normal precision, the conditional density of h_{τ} is proportional to

$$\begin{aligned}\pi_{\tau}(h_{\tau})\prod_{t=1}^{T} f_{\tau}(\tau|h_{\tau}) &\propto h_{\tau}^{\alpha_{\tau}-1}\exp(-\lambda_{\tau}h_{\tau})\prod_{t=1}^{T} h_{\tau}^{\frac{1}{2}}\exp\left(-\frac{h_{\tau}}{2}(\log\tau)^2\right) \\ &\propto h_{\tau}^{\alpha_{\tau}+\frac{T}{2}-1}\exp\left[-\left(\lambda_{\tau} + \frac{1}{2}\sum_{t=1}^{T}(\log\tau)^2\right)h_{\tau}\right].\end{aligned}$$

Hence, we draw

$$h_\tau^{(s+1)} \sim \mathcal{G}a\left(\widehat{\alpha}_\tau, (\widehat{\lambda}_\tau^{(s)})^{-1}\right)$$

where $\widehat{\alpha}_\tau = \alpha_\tau + \frac{T}{2}$ and $\widehat{\lambda}_\tau^{(s)} := \lambda_\tau + \frac{1}{2}\sum_{t=1}^{T}(\log \tau^{(s)})^2$.

6. Draw $\boldsymbol{\tau}^{(s+1)}$: we draw $\tilde{\tau}$ simultaneously for each $t \in \{1, \ldots, T\}$. Since τ has to be positive, we draw $\tilde{\tau} \sim \mathcal{N}(\tau^{(s)}, \sigma_\tau^2)$ individually and collect $\tilde{\boldsymbol{\tau}} = \{\tilde{\tau}\}_{t=1}^{T}$. Then we update $\tau^{(s+1)} = \tilde{\tau}$ with probability

$$\min\left\{1, \frac{f_\tau(\tilde{\tau}|h_\tau^{(s+1)})L(\boldsymbol{y}, \boldsymbol{n}|\tilde{\boldsymbol{\tau}}, \boldsymbol{X}, \theta^{(s+1)})}{f_\tau(\tau^{(s)}|h_\tau^{(s+1)})L(\boldsymbol{y}, \boldsymbol{n}|\boldsymbol{\tau}^{(s)}, \boldsymbol{X}, \theta^{(s+1)})}\right\}.$$

We tune the scale parameter σ_τ as in the previous steps.

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