COMPLEX DIFFERENTIAL GEOMETRY
SHANKS WORKSHOP

VANDERBILT UNIVERSITY, MARCH 2 - 3, 2018

Titles and Abstracts

Tristan Collins, Harvard University

Title: The Inverse Monge-Ampère flow

Abstract: I will discuss a new parabolic Monge-Ampère equation with applications to the existence (and non-existence) of Kähler-Einstein metrics. I will discuss the long-time existence of the flow, and convergence on Kähler-Einstein manifolds. Time permitting, I will also discuss the convergence of the flow to the maximally destabilizing degeneration on toric Fano manifolds. This is joint work with T. Hisamoto, and R. Takahashi.

Ronan Conlon, Florida International University

Title: New examples of complete Calabi-Yau metrics on $\mathbb{C}^n$ for $n \geq 3$

Abstract: I will present new examples of non-flat complete Calabi-Yau metrics on $\mathbb{C}^n$ for $n \geq 3$ having Euclidean volume growth and a tangent cone at infinity with a singular cross section. This is joint work with Frédéric Rochon (UQÀM).

Claude LeBrun, Stony Brook University

Title: Anti-Self-Dual 4-Manifolds, Quasi-Fuchsian Groups, and Almost-Kaehler Geometry

Abstract: If a smooth manifold M admits a symplectic form, it also admits Riemannian metrics g that are related to the symplectic form by means of an adapted almost-complex structure. Such metrics are said to be almost-Kaehler, because they are Kaehler if and only if the almost complex structure is integrable. If M is compact and 4-dimensional, one can then show that the conformal classes of almost-Kaehler metrics sweep out an open subset in the space of the conformal classes. This provides a natural tool for exploring difficult global problems in 4-dimensional conformal geometry, leading to non-trivial results and motivating broader conjectures in the subject.

However, this technique certainly has its limitations. For example, if a 4-manifold admits scalar-flat Kaehler metrics, these can be deformed into anti-self-dual almost-Kaehler metrics, and these then sweep out an open set in the moduli space of anti-self-dual conformal structures. One might somehow hope that this subset would also turn out to be closed, and so sweep out entire connected components in the moduli space. Alas, however, this simply isn't true! In this talk, I?ll explain recent joint work with Chris Bishop that constructs a large hierarchy of counter-examples by studying the limit sets of quasi-Fuchsian groups.

Mehdi Lejmi, CUNY, Bronx Community College

Title: Chern-Yamabe problem

Abstract: On an almost Hermitian manifold the Chern connection is the unique Hermitian connection with J-anti-invariant torsion. In this talk, we compare the Chern scalar curvature with the Riemannian one. Moreover, we study an analog of Yamabe problem by looking for an almost Hermitian metric with constant Chern scalar curvature in a conformal class extending results of Angella, Calamai and Spotti to the non-integrable case. We also study the Chern-Yamabe flow and get existence of solutions when the Chern scalar curvature is small enough. These are joint works with Markus Upmeier and Ali Maalaoui.

Heather Macbeth, MIT

Title: Kähler-Ricci solitons on crepant resolutions

Abstract: By a gluing construction, we produce steady Kähler-Ricci solitons on equivariant crepant resolutions of $\mathbb{C}^n/G$, where G is a finite subgroup of $SU(n)$, generalizing Cao's construction of such a soliton on a resolution of $\mathbb{C}^n/\mathbb{Z}_n$. This is joint work with Olivier Biquard.

Yann Rollin, Université de Nantes, France

Title: Discrete geometry and isotropic surfaces

Abstract: We consider smooth isotropic immersions from the 2-dimensional torus into $\mathbb{R}^{2n}$, for $n \geq 2$. When $n = 2$ the image of such map is an immersed Lagrangian torus of $\mathbb{R}^4$. We prove that such isotropic immersions can be

approximated by arbitrarily C^0-close piecewise linear isotropic maps. The proofs are obtained using analogies with an infinite dimensional moment map geometry due to Donaldson. As a byproduct of these considerations, we introduce a numerical flow in finite dimension, whose limit provide, from an experimental perspective, many examples of piecewise linear Lagrangian tori in $\mathbb{R}^4$.

Christina W. Tønnesen-Friedman, Union College

Title: Weighted extremal Kähler metrics on admissible manifolds

Abstract: This talk, which is based on work in progress with Apostolov and Maschler, will be concerned with a newer geometric problem introduced by Apostolov, Calderbank, Gauduchon, Legendre, and Maschler and recently developed further by Lahdili. Specifically, we will study a boundary value problem arising from generalizing conformally Kähler, Einstein–Maxwell metrics, as defined by LeBrun, Apostolov, Calderbank, Gauduchon, and Maschler, in the setting of the so-called admissible manifolds.

An older (but still very active) geometric problem is the study of Calabi's extremal Kähler metrics. About a decade ago, in joint work with Apostolov, Calderbank, and Gauduchon, we derived some interesting results by studying these metrics on the admissible manifolds.

In this talk I shall exhibit just how similar the boundary problems are for the two geometric problems. This naturally leads to a general existence result and in particular a wealth of new examples of conformally Kähler, Einstein–Maxwell metrics in complex dimension higher than two.