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Spatio-Temporal Trade-Off for Initial Data Best Approximation

Roza Aceska
Ball State University
raceska@bsu.edu
Coauthors: Alessandro Arsie and Ramesh Karki

We present a mathematical framework and efficient computational schemes to solve some classes of PDEs from scarcely sampled initial data. Full knowledge of the initial conditions in an initial value problem (IVP) is essential but often impossible to attain; the way to overcome this impairing is to exploit the evolutionary nature of the problem at hand, while working with a reduced number of sensors. Our framework combines spatial samples of various temporal states of the system of interest, thus compensating for the lack of knowledge of the initial conditions.

Stable phase retrieval in infinite dimensions

Rima Alaifari
ETH Zurich, Switzerland
rima.alaifari@math.ethz.ch
Coauthors: Ingrid Daubechies (Duke University), Philipp Grohs (University of Vienna), Rujie Yin (Duke University)

Phase retrieval is the problem of reconstructing a signal from only the magnitudes of a set of complex measurements. The missing information of the phase of the measurements severely obstructs the signal reconstruction.

While the problem is stable in the finite dimensional setting, it is very ill-conditioned. This makes it necessary to study the stability and regularization properties in an infinite-dimensional setting. We show that in some sense the ill-conditioning is independent of the redundancy of the measurements. However, the instabilities observed in practice are all of a certain type. Motivated by this observation, we introduce a new paradigm for stable phase retrieval.

We demonstrate that in audio processing applications this new notion of stability is natural and meaningful and that in this new setting stability can actually be achieved for certain measurement systems.

Lipschitz extensions in inverse problems
Radu Balan
University of Maryland
rvbalan@math.umd.edu

This talk is devoted to Lipschitz analysis on several inverse problems in applied harmonic analysis. Specifically we analyze the phase retrieval problem, the low-rank quantum tomography, the compressive sampling, and sparse blind signal separation. In each case, for appropriate metric space structures, the forward nonlinear map is bi-Lipschitz. The problem is whether a globally Lipschitz left inverse reconstruction map exists, and if so, to find its smallest Lipschitz constant. Two extension principles are analyzed: the Whitney-McShaun (also known as the nonlinear Hahn-Banach) extension, and the Kirszbraun extension. In the phase retrieval problem, the Lipschitz extension is possible with low cost in the optimal constant; in the low-rank quantum tomography problem, Lipschitz extension is not possible due to topological obstructions; in the compressive sampling, Lipschitz extension is possible with dimension dependent bounds; in the sparse blind signal separation, the answer is not known.

A new approach to non square integrability for irreducible representations of semidirect products.
Davide Barbieri
Universidad Autonoma de Madrid
davide.barbieri@uam.es
Coauthors: Filippo de Mari, Ernesto de Vito

Unitary irreducible representations of semidirect products, in many cases of interest, happen to be not square integrable. A new strategy to recover reproducing formulas in the general setting will be presented, based on the structure of dual orbits.

Spectral extensions for all Radon measures via Beurling extrapolation
John J. Benedetto
Norbert Wiener Center, U. of Maryland
jjb@umd.edu
Coauthors: Weilin Li

We address the question: Given spectral data defined on a finite set of d-dimensional multi-integers; of all complex Radon measures on the d-dimensional torus, whose Fourier transform equals this data, does there exist exactly one with minimal total variation? This is a mathematical formulation in the area of spectral estimation of a class of super-resolution problems that arises in image processing, addressed by Candes and Fernandez-Granda for discrete dimensions in one or two dimensions. It generalizes some problems in compressed sensing.

We prove a theorem, based on Beurling minimal extrapolation and the idea of admissibility range, that has quantitative implications about the possibility and impossibility of constructing such a unique measure. Our method is well-suited for the construction of explicit examples.

This is a collaboration with Weilin Li.

On computing PDFs of products of random variables

Gregory Beylkin
University of Colorado at Boulder
`beylkin@colorado.edu`
Coauthors: Lucas Monzon and Ignas Satkauskas

Given the Probability Density Functions (PDFs) of two independent random variables, the seemingly simple (and basic) problem of computing the PDF of their product, turns out to be quite involved. While the PDF of the sum of two random variables can be evaluated as the convolution of their PDFs, the integral describing their product is significantly more complicated. For example, the PDF of the product of two normally distributed random variables with zero means is the modified Bessel function of the second kind which has a logarithmic singularity at the origin. However, even for normal distributions with non-zero means no analytic answer is available. The only general numerical algorithm relies on a Monte-Carlo approach, where one samples the individual PDFs, computes the products, and collects enough samples to achieve certain accuracy but, due to the slow convergence of such methods, achieving high accuracy using this approach is not feasible.

For independent random variables (taking both positive and negative values), we show how to accurately and efficiently compute the PDF of the product (or of the quotient) using an approximate multiresolution analysis (MRA), where the scaling function is a Gaussian. In contrast with a sampling method, the new algorithm is both fast and accurate. While a sampling method only provides a histogram, we directly obtain the result in a functional form that can be used in further computations. In particular, we avoid the need of a kernel density estimation of the result of sampling the product PDF and do not have any obstacles dealing with heavy-tailed distributions.

For non-negative independent random variables, for any user-selected accuracy, we represent PDFs via a product of a monomial factor and linear combinations of decaying exponentials with complex exponents. Using a fast algorithm involving Hankel matrices, we develop a general numerical method for computing the PDF of the sums, products, or quotients of non-negative independent random variables yielding the result in the same type of functional representation.

This is a joint work with Lucas Monzon and Ignas Satkauskas.

Applications of a distributional fractional derivative to Fourier analysis and its related differential equations

Evan Camrud
Iowa State University
ecamrud@iastate.edu

Fractional calculus is a burgeoning field in modern analysis. But how do the most recent results overlap with harmonic analysis? A new definition of a fractional derivative has recently been developed, making use of a fractional Dirac delta function as its integral kernel. This derivative allows for the definition of a distributional fractional derivative, and as such paves a way for application to many other areas of analysis involving distributions. This includes (but is not limited to): the fractional Fourier series (i.e. an orthonormal basis for fractional derivatives), the fractional derivative of Fourier transforms, fundamental solutions to differential equations such as the wave equation, and the inclusion of fractional initial conditions in differential equations. Each of these areas will be discussed in detail, elucidating an intersection between two popular areas of modern analysis.

Three problems on exponential bases

Laura De Carli
Florida International University
decarlil@fiu.edu
Coauthors: Alberto Mizrahi and Alex Tepper

We consider three special and significant cases of the following problem. Let D be a (possibly unbounded) set of finite Lebesgue measure in R^d. Find conditions on D for which the set of exponentials $e^{2\pi i x.n}$, with n in Z^d, is a frame, or a Riesz sequence, or a Riesz basis on D.

The Solution to the Quantum Detection Problem

Peter Casazza
University of Missouri
casazzap@missouri.edu
Coauthors: Sara Bothello-Andrade, Desai Cheng, Tin Tran

We will give a complete solution to the POVM quantum detection problem in quantum mechanics. We will solve both parts of the problem including the quantum injectivity problem and the quantum state estimation problem. We will solve both the real and convex cases of the problem and both the finite and infinite dimensional cases of the problem.

Poisson Summation, Selberg Trace, and Sampling on General Manifolds

Stephen D. Casey
American University
scasey.american@gmail.com

Sampling theory is a fundamental area of study in harmonic analysis and signal and image processing. Our talk will connect sampling theory with the geometry of the signal and its domain. It is relatively easy to demonstrate this connection in Euclidean spaces, but one quickly gets into open problems when the underlying space is not Euclidean. In particular, we discuss spherical geometry, hyperbolic geometry, and the geometry of general surfaces.

We first look at the key role the Poisson Summation Formula plays in sampling, and show its connection to the Selberg Trace Formula. We then focus on the connection of the Poisson Summation Formula to the Selberg Trace Formula in non-Euclidean settings.

There are numerous motivations for extending sampling to non-Euclidean geometries. Applications of sampling in spherical and hyperbolic geometries are showing up areas from EIT to cosmology. Sampling in spherical geometry has been analyzed by many authors, e.g., Driscoll, Healy, Keiner, Kunis, McEwen, Potts, and Wiaux, and brings up questions about tiling the sphere. Irregular sampling of band-limited functions by iteration in hyperbolic space is possible, as shown by Feichtinger and Pesenson. In Euclidean space, the minimal sampling rate for Paley-Wiener functions on R^d, the Nyquist rate, is a function of the band-width. No such rate has yet been determined for hyperbolic or spherical spaces. We look to develop a structure for the tiling of frequency spaces in both Euclidean and non-Euclidean domains. In particular, we develop an approach to determine Nyquist tiles and sampling groups for spherical and hyperbolic space. We then connect this to arbitrary orientable analytic surfaces using Uniformization.

On landmark-based large scale spectral clustering: recent advances and a unified view

Guangliang Chen
San Jose State University
guangliang.chen@sjsu.edu

Spectral clustering has emerged as a very effective clustering approach, due to its capability of separating nonconvex, non-intersecting manifolds, however, it is computationally very expensive. As a result, there has been considerable effort in the machine learning community to develop fast, approximate spectral clustering algorithms that are scalable to large data, most of which use a small set of landmark points selected from the given data. In this talk we present two new scalable spectral clustering algorithms that are also landmark based but through novel document-term and bipartite graph models. We demonstrate the superior performance of our proposed algorithms by comparing them with the state-of-the-art methods on benchmark data. Finally, we provide a unified view of all the old and new landmark-based spectral clustering methods.

Recovery of dictionary-sparse signals with random measurements
Xuemei Chen
University of San Francisco
xchen@math.usfca.edu
Coauthors: Pete Casazza, Rick Lynch

We study the problem of recovering signals, sparse in a frame or dictionary of n atoms, under subgaussian measurements. We first show that the null space property is preserved under a subgaussian map with high probability. We can therefore prove that, through the null space property, the l1 synthesis method is very efficient with subgaussian measurements as long as the number of measurements is on the order of slog(n). This generalization from basis case to dictionary case is not trivial and main techniques used are Mendelsons small ball method and Gaussian width.

Stable Phaseless Sampling and Reconstruction
Cheng Cheng
Duke and SAMSI
cheng87@math.duke.edu
Coauthors: Qiyu Sun

In this talk, we consider the stable reconstruction of real-valued signals with finite rate of innovations (FRI), up to a sign, from their magnitude measurements on the whole domain or their phaseless samples on a discrete subset. FRI signals appear in many engineering applications such as magnetic resonance spectrum, ultra wide-band communication and electrocardiogram. For an FRI signal, we introduce an undirected graph to describe its topological structure. We establish the equivalence between the graph connectivity and phase retrievability of FRI signals, and we apply the graph connected component decomposition to find all FRI signals that have the same magnitude measurements as the original FRI signal has. We also propose a stable algorithm with linear complexity to reconstruct FRI signals from their phaseless samples on the above phaseless sampling set.

Frames and dynamical sampling

Ole Christensen
Technical University of Denmark
ochr@dtu.dk
Coauthors: Marzieh Hasannasab

One of the central questions in dynamical sampling is to identify when and how a given frame can be represented via iterated actions of a bounded operator on a single element in the underlying Hilbert space. The talk will provide various characterizations of the frames for which this can be done. The talk presents joint work with Marzieh Hasannasab.

Efficient Methods for Large and Dynamic Inverse Problems

Julianne Chung
Virginia Tech
jmchung@vt.edu

In many physical systems, measurements can only be obtained on the exterior of an object (e.g., the human body or the earth's crust), and the goal is to estimate the internal structures. In other systems, signals measured from machines (e.g., cameras) are distorted, and the aim is to recover the original input signal. These are natural examples of inverse problems that arise in fields such as medical imaging, astronomy, geophysics, and molecular biology.

In this talk, we describe efficient methods to compute solutions to large, dynamic inverse problems. We focus on addressing two main challenges. First, since most inverse problems are ill-posed, small errors in the data may result in significant errors in the computed solutions. Second, in many realistic scenarios such as in passive seismic tomography or dynamic photoacoustic tomography, the underlying parameters of interest may change during the measurement procedure. To address these challenges, we describe efficient, iterative, matrix-free methods based on the generalized Golub-Kahan bidiagonalization, and we demonstrate these methods on a range of applications.

On the reduction of the interferences in the Born-Jordan distribution

Elena Cordero
University of Torino (Italy)
elena.cordero@unito.it
Coauthors: Maurice de Gosson (University of Vienna, Austria), Fabio Nicola (Politecnico di Torino, Italy)

One of the most popular time-frequency representation is certainly the Wigner distribution. To reduce the interferences coming from its quadratic nature, several related distributions have been proposed, among which the so-called Born-Jordan distribution. It is well known that in the Born-Jordan distribution the ghost frequencies are in fact damped quite well, and the noise is in general reduced. However, the horizontal and vertical directions escape from this general smoothing effect, so that the interferences arranged along these directions are in general kept. Whereas these features are graphically evident on examples and heuristically well understood in the engineering community, there is not at present a mathematical explanation of these phenomena, valid for general signals in L^2 and, more in general, in the space $\mathcal{S}'$ of temperate distributions. In the present note we provide such a rigorous study using the notion of wave-front set of a distribution. We use techniques from Time-frequency Analysis, such as the modulation and Wiener amalgam spaces, and also results of microlocal regularity of linear partial differential operators.

An abstract Calderon condition for wavelets on noncommutative domains

Bradley Currey
Saint Louis University
bradley.currey@slu.edu
Coauthors: Brent Wessel

In 1964 Calderón discovered a necessary and sufficient condition that a square-integrable function on $\mathbb{R}$ be a continuous wavelet. More recently a generalized Calderón condition proved by H. Führ, where the domain $\mathbb{R}$ is replaced by a locally compact separable commutative group and dilations are group automorphisms. In this talk a generalized Calderón condition is presented for noncommutative domains satisfying certain representation-theoretic conditions. We apply this condition to various examples in order to construct wavelets with rapid decay.

On Smooth Whitney Extensions of almost isometries with small Distortion, Interpolation and Alignment in d-dimensional Euclidean space.

Steven Damelin
AMS
damelin@umich.edu
Coauthors: Charles Fefferman

We study the following problem: Let $D \geq 2$ and let $E \subset R^D$ be a finite set. Suppose that we are given a map $\phi : E \to R^D$ with ϕ a small distortion on E. How can one decide whether ϕ extends to a smooth small distortion $\Phi : R^D \to toR^D$ which agrees with ϕ on E. We also ask how to decide if in addition Φ can be approximated well by certain rigid and non-rigid motions from $R^D \to R^D$. Since E is a finite set, this question is basic to interpolation and alignment of data in R^D. We also show connections of this problem to the BMO space. (Functions of Bounded Mean Oscillation on R^D).

Quasi-Tight framelets with minimum number of generators and generalized matrix spectral factorization

Chenzhe Diao
University of Alberta
diao@ualberta.ca
Coauthors: Bin Han

As a generalization of orthonormal wavelets in $L_2(\mathbb{R})$, tight framelets (also called tight wavelet frames) are of importance in wavelet analysis and applied sciences due to their many desirable properties in applications such as image processing and numerical algorithms. Tight framelets are often derived from particular refinable functions satisfying certain stringent conditions. Consequently, a large family of refinable functions cannot be used to construct tight framelets. This motivates us to introduce the notion of a quasi-tight framelet, which is a dual framelet but behaves almost like a tight framelet. It turns out that the study of quasi-tight framelets is intrinsically linked to the problem of the generalized matrix spectral factorization for matrices of Laurent polynomials. In this talk, we provide a systematic investigation on the generalized matrix spectral factorization problem and compactly supported quasi-tight framelets. As an application of our results on generalized matrix spectral factorization for matrices of Laurent polynomials, we prove in this paper that from any arbitrary compactly supported refinable function in $L_2(\mathbb{R})$, we can always construct a compactly supported quasi-tight framelet having the minimum number of generators and the highest possible order of vanishing moments.

Anisotropic diffuse interface functionals based on sparse representations

Julia Dobrosotskaya
Case Western Reserve University
julia.dobrosotskaya@case.edu
Coauthors: W. Czaja (Norbert Wiener Center, University of Maryland College Park)

We consider Laplacian-like diffusive operators based on sparse representation systems such as shearlets and composite dilation wavelets in 2D. The directional sensitivity of these systems allow for adaptive design of anisotropic behavior in the diffuse interface set-ups analogous to the Ginzburg Landau/Mumford Shah functionals. The associated energies approximate weighted perimeter functionals (regular or anisotropic TV) in the variational sense, with minimizers exhibiting sharp phase transitions and little interface blur. They can be effectively utilized in creating adaptive variational techniques for multipurpose image processing. We illustrate the theoretical findings with examples of image inpainting and supperresolution.

Orthonormal bases generated by Cuntz algebras

Dorin Dutkay
University of Central Florida
ddutkay@gmail.com
Coauthors: Gabriel Picioroaga

We show how wavelet bases, Fourier bases, Walsh bases and Fourier bases on fractal measures can be generated using a class of representations of the Cuntz algebra.

Sparsity of Lvy processes

Julien Fageot
EPFL
julien.fageot@epfl.ch
Coauthors: Michael Unser, Shayan Aziznejad, Alireza Fallah, John Paul Ward

As is well-known, Gaussian random processes are inadequate to model the sparsity observed in many naturally-occurring signals. The recent theory of sparse stochastic processes has proved that this limitation can be overcome by considering the more general class of Lvy processes. In this talk, we analyse the compressibility of Lvy processes in terms of their best N-term approximation in wavelet bases. The result is achieved thanks to a careful analysis of the Besov regularity of the underlying Lvy white noise. The main outcome is a quantified characterisation of the compressibility of continuous probabilistic models.

Distributed Learning with Manifold Regularization

Zhiying Fang
City University of HongKong
zyfang4-c@my.cityu.edu.hk

We consider a distributed learning algorithm with the least squares regularization and an extension with manifold regularization that enforces smoothness with respect to the structure of input data. This scheme is studied under a framework of semi-supervised learning in a reproducing kernel Hilbert space. We show error bounds in the L^2-metric by using a novel second order decomposition of operator differences and the global output function of distributed learning is a good approximation of the regression function.

Statistical Learning Approach to Modal Regression

Yunlong Feng
The State University of New York at Albany
ylfeng@albany.edu
Coauthors: Jun Fan, and Johan Suykens

In this presentation, I will talk about the modal regression problem from a statistical learning point of view. We will show that modal regression can be approached by means of empirical risk minimization techniques. A framework for analyzing and implementing modal regression within the statistical learning context will be presented. Theoretical results concerning the generalization ability and approximation ability of modal regression estimators will be provided. We will then illustrate connections and differences of the proposed modal regression method with existing ones. Numerical examples will be given to show the effectiveness of the newly proposed modal regression method.

Equiangular tight frames that contain regular simplices

Matthew Fickus
Air Force Institute of Technology
Matthew.Fickus@gmail.com
Coauthors: John Jasper, Emily J. King, Dustin G. Mixon

An equiangular tight frame (ETF) is a type of optimal packing of lines in Euclidean space. A regular simplex is a special type of ETF in which the number of vectors is one more than the dimension of the space they span. In this talk, we consider ETFs that contain a regular simplex, that is, have the property that a subset of its vectors forms a regular simplex. As we explain, such ETFs are characterized as those that achieve equality in a certain well-known bound from the theory of compressed sensing. We then consider the so-called binder of such an ETF, namely the set of all regular simplices that it contains. In certain circumstances, we show this binder can be used to produce a particularly elegant Naimark complement of the corresponding ETF. We also apply these ideas to known constructions of ETFs, including harmonic ETFs.

Assimilating data to optimally compute quantities of interest

Simon Foucart
Texas A&M University
foucart@tamu.edu

If functions f from a certain model class are acquired through data taking the form of prescribed linear measurements, is there an optimal way to estimate a quantity of interest $Q(f)$? In case Q is a linear functional and the measurements are point evaluations, we give an affirmative answer to this optimal-recovery question for a novel model class inspired by parametric PDEs. In fact, we produce implementable linear algorithms that are optimal in the worst-case setting. We present some applications in atmospheric science and system identification along the way.

Sampling and reconstruction formulas for higher dimensional sampling

Christina Frederick
New Jersey Institute of Technology
christin@njit.edu

We consider sampling strategies for a class of multivariate functions that are bandlimited to the set $\Omega = \bigcup_{r \in I}[0,1]^d + r$, where I is a bounded subset of $\mathbb{Z}^d$. A standard approach for reconstruction involves inverting a Vandermonde system that relates periodic nonuniform samples to the unknowns. In higher dimensions, the invertibility of Vandermonde matrices is not guaranteed. We show that an iterative process can be used to overcome this limitation and guarantee a stable recovery. Furthermore, we provide explicit L^2 stability estimates involving the measure of Ω.

The Discretization Problem for Continuous Frames

Daniel Freeman
St Louis University
daniel.freeman@slu.edu
Coauthors: Darrin Speegle

Functions in $L2([0,1])$ can be analyzed continuously through the Fourier transform or discretely through Fourier series and sampling the Fourier transform at the integers. We consider what other continuous representations can be sampled to obtain discrete representations. Using the results of Marcus-Spielman-Srivastava in their solution of the Kadison-Singer problem, we give a complete characterization of when a continuous frame for a Hilbert space may be sampled to obtain a discrete frame. In particular, every bounded continuous frame may be sampled to obtain a discrete frame. This solves the discretization problem as posed by Ali, Antoine, and Gazeau in their physics textbook: Coherent States, Wavelets, and Their Generalizations.

Frames of exponentials with spectrum contained in a finite union of lattices.

Jean-Pierre Gabardo
McMaster University
gabardo@mcmaster.ca
Coauthors: Chun-Kit Lai (San Francisco State U.)

Suppose that $E \subset \mathbb{R}$ is a measurable subset with finite Lebesgue measure. For particular discrete sets of frequencies $\Lambda \subset \mathbb{R}$, we study conditions on E under which the associated collection of exponentials $\{e^{2\pi i\lambda x}\}_{\lambda\in\Lambda}$ forms a frame for $L^2(E)$. We are particularly interested in the case where Λ is a finite union of subgroups of the form $a_i\,\mathbb{Z}$, $a_i > 0$, $i = 1,\ldots,M$. We also consider discrete versions of this problem.

Approximation theory, Numerical Analysis and Deep Learning

Philipp Grohs
University of Vienna
philipp.grohs@univie.ac.at

The development of new classification and regression algorithms based on deep neural networks coined Deep Learning revolutionized the area of artificial intelligence, machine learning, and data analysis. More recently, these methods have been applied to the numerical solution of high dimensional partial differential equations with great success.

This talk will start with a brief introduction to machine learning and deep learning. Then we will show that the problem of numerically solving a large class of (high-dimensional) PDEs (such as linear Black-Scholes or diffusion equations) can be cast into a classical supervised learning problem which can then be solved by deep learning methods. Simulations suggest that the resulting algorithms are vastly superior to classical methods such as finite element methods, finite difference methods, spectral methods, or sparse tensor methods. In particular we empirically observe that these algorithms are capable of breaking the curse of dimensionality. In the last part of the talk we will present theoretical results which confirm this observation.

Centered reproducing kernels and their applications

Xin Guo
The Hong Kong Polytechnic University
`x.guo@polyu.edu.hk`
Coauthors: Ting Hu, Chendi Wang, Qiang Wu, and Ding-Xuan Zhou

We study the centering transformation of reproducing kernels, based on either a probability distribution, or the discrete uniform distribution on a sample of observations. The obtained kernel is also a reproducing kernel, with the associated reproducing kernel Hilbert space lying in the orthogonal complement of constant functions. We proved that the kernel space complexity in terms of effective dimensions is an asymptotic invariant to this transformation. We also obtained some relations between the integral operators before and after the kernel transformation. We applied the centered kernels to the regularized least squares scheme and the constant component is naturally separated from the regression function. We also explore the applications of centered kernels to the coordinate kernel polynomial models for variable selection.

CUR Decomposition and Subspace Clustering

Keaton Hamm
Vanderbilt University
`keaton.hamm@vanderbilt.edu`
Coauthors: Akram Aldroubi, Ahmet Bugra Koku, Ali Sekmen

The subspace clustering problem seeks to cluster data in a high-dimensional space that is drawn from the union of much smaller dimensional subspaces. One method of attack for this problem is to find a similarity matrix from the data which identifies the clusters. This talk will discuss an intriguing matrix decomposition method called CUR decomposition, and describe how many similarity matrix methods are special cases of this general decomposition, and how it ties this technique to other minimization problems used to find the clusters. In addition, applications to motion segmentation will be discussed.

Directional Tight and Quasi-tight Framelets with Applications

Bin Han

Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, Alberta, Canada

bhan@ualberta.ca

Coauthors: Chenzhe Diao, Qun Mo, Zhenpeng Zhao, Xiaosheng Zhuang

To achieve directionality of multivariate tight framelets for better performance in applications, we first introduce directional tensor product complex tight framelets (TPCTF) and explore their applications in image processing. It is important to construct compactly supported tight framelets with directionality or vanishing moments. Indeed we can construct directional TPCTFs with compact support. Owing to tensor product, directionality of TPCTFs is limited. This motivates us to study multivariate framelets with directionality or vanishing moments. However, construction of compactly supported multivariate tight or dual framelets, even without directionality or basic vanishing moments, is known to be a very challenging problem because it is linked to sum of squares and factorization of multivariate Laurent polynomials. To overcome this difficulty, we introduce the notion of a quasi-tight framelet, which is a dual framelet, but behaves almost like a tight framelet. From an arbitrary compactly supported refinable function (such as refinable box splines) with a general dilation matrix, we constructively prove that we can always derive a directional compactly supported quasi-tight framelet. If in addition all the coefficients of its low-pass filter are nonnegative, such a quasi-tight framelet becomes a directional tight framelet. Moreover, from an arbitrary refinable function, we can constructively derive a compactly supported quasi-tight framelet with the highest possible order of vanishing moments. Examples will be provided to illustrate our results. This talk is based on following joint work with Chenzhe Diao, Qun Mo, Zhenpeng Zhao, and Xiaosheng Zhuang:

1. B. Han, Framelets and Wavelets: Algorithms, Analysis, and Applications, in *Applied and Numerical Harmonic Analysis*, Birkhauser/Springer, (2017), 724 pages.

2. C. Diao and B. Han, Quasi-tight Framelets with Directionality or High Vanishing Moments Derived from Arbitrary Refinable Functions, preprint, (2018).

3. B. Han, Properties of Discrete Framelet Transforms, *Mathematical Modelling of Natural Phenomena*, **8** (2013), 18–47.

4. B. Han and Z. Zhao, Tensor Product Complex Tight Framelets with Increasing Directionality, *SIAM Journal on Imaging Sciences*, **7** (2014), 997–1034.

5. B. Han, Q. Mo, and Z. Zhao, Compactly Supported Tensor Product Complex Tight Framelets with Directionality, *SIAM Journal on Mathematical Analysis*, **47** (2015), 2464–2494.

6. B. Han, Q. Mo, Z. Zhao and X. Zhuang, Compactly Supported Directional Tensor Product Complex Tight Framelets with Applications to Image Processing, preprint, (2017).

Projective phase-retrievable representation frames

Deguang Han
University of Central Florida
`deguang.han@ucf.edu`

We will discuss some recent work and problems on phase-retrievable frames induced by projective unitary representations of finite abelian (and also non-abelian) groups.

Operator representations of frames: boundedness, duality, and stability

Marzieh Hasannasab
Technical University of Denmark
`mhas@dtu.dk`
Coauthors: Ole Christensen

In this talk we analyze frames $\{f_k\}_{k\in\mathbb{Z}}$ having the form $\{T^k f_0\}_{k\in\mathbb{Z}}$ for some linear operator $T : \mathrm{span}\{f_k\}_{k\in\mathbb{Z}} \to \mathrm{span}\{f_k\}_{k\in\mathbb{Z}}$. We characterize boundedness of the operator T in terms of shift-invariance of a certain sequence space. One of the consequences is a characterization of the case where the representation $\{f_k\}_{k\in\mathbb{Z}} = \{T^k f_0\}_{k\in\mathbb{Z}}$ can be achieved for an operator T that has an extension to a bounded bijective operator $\widetilde{T} : \mathcal{H} \to \mathcal{H}$. In this case, we also characterize all the dual frames that are representable in terms of iterations of an operator V;, in particular, we prove that the only possible operator is $V = (\widetilde{T}^*)^{-1}$. Finally, we consider the stability of the representation $\{T^k f_0\}_{k\in\mathbb{Z}}$; rather surprisingly, it turns out that the possibility to represent a frame on this form is sensitive towards some of the classical perturbation conditions in frame theory. Various ways of avoiding this problem will be discussed. Throughout the talk we will highlight the similarities and differences between operator representations index by $\mathbb{Z}$ versus $\mathbb{N}$.

Multiscale machine learning for many particle physics with wavelet scattering transforms

Matthew Hirn
Michigan State University
mhirn@msu.edu

Physical functionals are usually computed as solutions of variational problems or from solutions of partial differential equations, which may require huge computations for complex systems. Quantum many body problems, including those in quantum chemistry and materials science, are such examples. Machine learning algorithms do not simulate the physical system but estimate solutions by interpolating values provided by a training set of known examples. However, precise interpolations may require a number of examples that is exponential in the system dimension, and are thus intractable. Tractable algorithms compute interpolations in low dimensional approximation spaces, which leverage the underlying physical properties of the system. In this talk we present a type of multiscale, multilayer convolutional network, called a wavelet scattering transform, for the regression of potential energies in many body physics. Through a cascade of multiscale wavelet filters and nonlinearities, this transform encodes the appropriate physical invariants and regularity properties of the physical system, and obtains regression errors on the order of quantum mechanical simulations, but at a fraction of the cost.

Fourier Bases on the Skewed Sierpinski Gasket

Calvin Hotchkiss
Iowa State University
hotchkis@iastate.edu

We consider a certain iterated function system, whose invariant set is a skewed Sierpinski gasket, $\mathcal{S} = \{(x, y) \in \mathbb{R}^2 : x \in C_3, y \in C_3, x + y \in C_3\}$, where C_3 is the standard middle-thirds Cantor set. We show the existence of several sequences of exponentials which form orthonormal bases on $L^2(\mathcal{S})$, and discuss similar results for related fractals.

Frames Induced by the Action of Continuous Powers of an Operator

Longxiu Huang
Vanderbilt University
longxiu.huang@vanderbilt.edu
Coauthors: Akram Aldroubi, Armenak Petrosyan

We investigate systems of the form $\{A^t g : g \in \mathcal{G}, t \in [0, L]\}$ where $A \in B(\mathcal{H})$ is a normal operator in a separable Hilbert space $\mathcal{H}$, $\mathcal{G} \subset \mathcal{H}$ is a countable set, and L is a positive real number. The main goal of this work is to study the frame properties of $\{A^t g : g \in \mathcal{G}, t \in [0, L]\}$. Specifically, we show that under some mild conditions, $\{A^t g\}_{g \in \mathcal{G}, t \in [0,L]}$ is a frame system in $\mathcal{H}$ if and only if there exists a finite discretization $0 = t_0 < t_1 < \ldots < t_n < t_{n+1} = L$ of $[0, L]$ such that $\{A^{t_i} g\}_{g \in \mathcal{G}, i=\{0,1\ldots,n\}}$ is a frame in $\mathcal{H}$. Additionally, we also found that when A is an invertible self-adjoint linear operator in $\mathcal{H}$, then the frame properties of $\{A^t g\}_{g \in \mathcal{G}, t \in [0,L]}$ are independent of L.

Solving Jigsaw Puzzles by the Graph Connection Laplacian

Vahan Huroyan
University of Minnesota
huroy002@umn.edu

We propose a novel mathematical framework to address the problem of automatically solving large jigsaw puzzles. The latter problem assumes a large image, which is cut into equal square pieces that are arbitrarily rotated and shuffled and asks to recover the original image given the rotated and shuffled pieces. We propose a method for recovering the unknown rotations of the puzzle pieces by using the Graph Connection Laplacian associated with the puzzle. The Graph Connection Laplacian is also used to form a metric between puzzle pieces and this metric is more accurate than the common metric used. It thus results in better recovery of the unknown locations of the puzzle pieces. Numerical experiments demonstrate the competitive performance of the proposed method.

Accurate Quantization in Redundant Systems

Thang Huynh
UC San Diego
tlh007@ucsd.edu

We study quantization theory and analog-to-digital conversion methods in the settings of frame theory. In particular, we investigate the so-called "Synthesis Problem" of digitally approximating signals by linear combinations of frame vectors with binary coefficients. We obtain exponentially small bounds for the best achievable reconstruction error using random frames.

Sparse Harmonic Transforms: A New Class of Sublinear-Time Algorithms for Approximating Functions of Many Variables

Mark Iwen
Michigan State University
markiwen@math.msu.edu
Coauthors: Bosu Choi (MSU), and Felix Krahmer (TUM).

The development of sublinear-time compressive sensing methods for signals which are sparse in Tensorized Bases of Bounded Orthonormal Functions (TBBOFs) will be discussed. These new methods are obtained from CoSaMP by replacing its usual support identification procedure with a new faster one inspired by fast Sparse Fourier Transform (SFT) techniques. The resulting sublinearized CoSaMP method allows for the rapid approximation of TBBOF-sparse functions of many variables which are too hideously high-dimensional to be learned by other means. Both numerics and theoretical recovery guarantees will be presented.

Quantized Compressive Sensing with RIP Matrices: The Benefit of Dithering

Laurent Jacques
University of Louvain
laurent.jacques@uclouvain.be
Coauthors: Chunlei Xu

Quantized compressive sensing (QCS) deals with the problem of coding compressive measurements of low-complexity signals with quantized, finite precision representations, i.e., a mandatory process involved in any practical sensing model. While the resolution of this quantization clearly impacts the quality of signal reconstruction, there even exist incompatible combinations of quantization functions and sensing matrices that proscribe arbitrarily low reconstruction error when the number of measurements increases. This work shows that a large class of random matrix constructions known to respect the restricted isometry property (RIP) is "compatible" with a simple scalar and uniform quantization if a uniform random vector, or random dithering, is added to the compressive signal measurements before quantization. In the context of estimating low-complexity signals (e.g., sparse or compressible signals, low-rank matrices) from their quantized observations, this compatibility is demonstrated by the existence of (at least) one signal reconstruction method, the projected back projection (PBP), which achieves low reconstruction error, decaying when the number of measurements increases. Interestingly, given one RIP matrix and a single realization of the dithering, a small reconstruction error can be proved to hold uniformly for all signals in the considered low-complexity set. We confirm these observations numerically in several scenarios involving sparse signals, low-rank matrices, and compressible signals, with various RIP matrix constructions such as sub-Gaussian random matrices and random partial discrete cosine transform (DCT) matrices.

(– joint work with Chunlei Xu, University of Louvain, Belgium –)

Mean convergence of prolate spheroidail wave function expansions

Philippe Jaming
Universit de Bordeaux
philippe.jaming@math.u-bordeaux.fr
Coauthors: M. Boulsane, A. Souabni

The aim of this talk is to establish the range of p's for which the expansion of a function f in L p in a generalized prolate spheroidal wave function (PSWFs) basis converges to f in L p. Two generalizations of PSWFs are considered here, the circular PSWFs introduced by D. Slepian and the weighted PSWFs introduced by Wang and Zhang. Both cases cover the classical PSWFs for which the corresponding results has been previously established by Barcel and Cordoba. To establish those results, we prove a general result that allows to extend mean convergence in a given basis (e.g. Jacobi polynomials or Bessel basis) to mean convergence in a second basis (here the generalized PSWFs).

Equiangular tight frames from group divisible designs

John Jasper
South Dakota State University
john.jasper@sdstate.edu
Coauthors: Matthew Fickus

Given positive integers N ; D, an equiangular tight frame ETF(D,N) is a type of optimal packing of N lines in a real or complex Hilbert space of dimension D. In the complex setting, the existence of an ETF(D,N) remains unresolved for many choices of D and N. In this talk, we observe that the (D,N) parameters of many of the known constructions of ETFs are of one of two types. We further provide a new method for combining a given ETF of one of these two types with an appropriate group divisible design (GDD) in order to produce a larger ETF of the same type. By applying this method to known families of ETFs and GDDs, we obtain several new infinite families of ETFs. Our approach was inspired by a seminal paper of Davis and Jedwab which both unified and generalized McFarland and Spence difference sets. We provide combinatorial analogs of their algebraic results, unifying Steiner ETFs with hyperoval ETFs and Tremain ETFs.

Convergence of the randomized Kaczmarz method for phase retrieval

Halyun Jeong
University of British Columbia
hajeong@math.ubc.ca
Coauthors: Sinan Gunturk

The classical Kaczmarz iteration and its randomized variants are popular tools for fast inversion of linear overdetermined systems. This method extends naturally to the setting of the phase retrieval problem via substituting at each iteration the phase of any measurement of the available approximate solution for the unknown phase of the measurement of the true solution. Despite the simplicity of the method, rigorous convergence guarantees that are available for the classical linear setting have not been established so far for the phase retrieval setting. In this talk, I will talk about a convergence result for the randomized Kaczmarz method for phase retrieval when the number of measurements and the ambient signal dimension are of the same order. The convergence is exponential and comparable to the linear setting.

Adaptive Synchrosqueezing Transform with a Time-varying Parameter for Signal Separation

Qingtang Jiang
University of Missouri at St. Louis
jiangq@umsl.edu
Coauthors: Lin Li, Haiyan Cai and Hongbing Ji

Recently the synchrosqueezing transform (SST) has been developed for signal separation and a sharp time-frequency representation of a non-stationary signal by assigning the scale variable of the signal's continuous wavelet transform to the frequency variable by a phase transformation. In this talk we will discuss the adaptive SST with a time-varying parameter for signal separation. We will address the separation condition for a multicomponent non-stationary signal with the adaptive SST and discuss the selection of the time-varying parameter.

On the design of multi-dimensional compactly supported Parseval Framelets with directional characteristics.

Nikolaos Karantzas
University of Houston
nickos@math.uh.edu
Coauthors: Nikolaos Atreas, Manos Papadakis, Theodoros Stavropoulos.

In this paper, we propose a new method for the construction of multi-dimensional, wavelet-like families of affine frames, commonly referred to as framelets, with specific directional characteristics, small and compact support in space, directional vanishing moments (DVM), and axial symmetries or anti-symmetries. The framelets we construct arise from readily available refinable functions. The filters defining these framelets have few non-zero coefficients, custom-selected orientations and can act as finite difference operators.

Negative Cliques in Sets of Equiangular Lines

Emily King
University of Bremen
king@uni-bremen.de
Coauthors: Xiaoxian Tang, (Matt Fickus, John Jasper, and Dustin Mixon)

A negative clique is a set of equiangular lines such that the product of inner products of any three distinct vectors is a negative, real number. The structure of negative cliques contained in a set of equiangular lines gives a lot of information about the collection. In particular, the size of the largest negative clique in a set and the interaction of other vectors with such a clique form the foundation of a generalization of Lemmens and Seidel's pillar decomposition, which, in joint work with Xiaoxian Tang, is used in combination with semidefinite programming to improve the upper bounds on sizes of sets of equiangular lines in Euclidean space. Negative cliques of maximal size given the common angle between pairs of lines are called simplices. This talk will also tie in joint work with Matt Fickus, John Jasper, and Dustin Mixon, where the structure of such simplices in real or complex equiangular tight frames (called a binder) has connections to a number of topics in combinatorial design theory.

Applications of spatiotemporal sampling to problems in frame theory.

Keri Kornelson
University of Oklahoma
kkornelson@ou.edu
Coauthors: Fatma Bozkurt

If a signal evolves in a known way (via a bounded operator) over time, the signal may be reconstructed via samples taken both with respect time and space. This process is known as dynamical sampling. In this talk, we examine some of the connections between dynamical sampling and recent applications of frame theory.

On Unlimited Sampling

Felix Krahmer
Technical University of Munich
felix.krahmer@tum.de
Coauthors: Ayush Bhandari, MIT, and Ramesh Raskar, MIT

Shannons sampling theorem provides a link between the continuous and the discrete realms stating that bandlimited signals are uniquely determined by its values on a discrete set. This theorem is realized in practice using so called analogtodigital converters (ADCs). Unlike Shannons sampling theorem, the ADCs are limited in dynamic range. Whenever a signal exceeds some preset threshold, the ADC saturates, resulting in aliasing due to clipping. In this talk, we analyze an alternative approach that does not suffer from these problems. Our work is based on recent developments in ADC design, which allow for ADCs that reset rather than to saturate, thus producing modulo samples. An open problem that remains is: Given such modulo samples of a bandlimited function as well as the dynamic range of the ADC, how can the original signal be recovered and what are the sufficient conditions that guarantee perfect recovery? In this paper, we prove such sufficiency conditions and complement them with a stable recovery algorithm. Our results not limited to certain amplitude ranges, in fact even the same circuit architecture allows for the recovery of arbitrary large amplitudes as long as some estimate of the signal norm is available when recovering.

Characterization of lacunary functions in weighted BergmanBesovLipschitz spaces

Eddy Kwessi
Trinity University
ekwessi@trinity.edu
Coauthors: G. De Souza, A. Abebe, R. Aulaskari

We consider the the weighted Bergman-Besov-Lipschitz space of analytic function in the unit disc. We provide an analytic characterization of lacunary functions in this space under very mild conditions.

Alternating Projection Algorithm for Matrix Completion

Ming-Jun Lai
University of Georgia
mjlai@uga.edu
Coauthors: Abraham Varghese

We shall first explain the method of alternating projection (MAP) for matrix completion. Then we will show the convergence of the MAP between one affine linear space and another set of matrices of rank r. A linear convergence will be proved. Finally, we present numerical results which demonstrate an excellent performance.

An analogue of Slepian vectors for Boolean hypercubes

Joe Lakey
New Mexico State University
jlakey@nmsu.edu
Coauthors: Jeffrey A. Hogan

Slepian vectors are a finite-dimensional analogue of prolate spheroidal wave functions that are optimally concentrated in time among all bandlimited functions. N-dimensional Boolean cubes can be regarded as N-regular graphs, on one hand, and as N-fold products of the group of integers mod two. The structure enables one to develop aspects of time-frequency analysis analogous to the Euclidean setting. Here we address basic questions about which vertex functions on Boolean cubes are most concentrated among all bandlimited functions, and methods to compute these functions. This represents joint work with Jeff Hogan.

Uncertainty for Windows of Oversampled Parseval Gabor Frames.

Mark Lammers
UNCW
lammersm@uncw.edu
Coauthors: Kyle Bowman

Using oversampling, a Gabor system, $\mathcal{G}(g,a,b)$, can be produced that generates a frame for $L^2(\mathbb{R})$ with the window, g, of the system being well localized in time and frequency, i.e., $\|xg(x)\|^2+\|w\hat{g}(w)\|^2<\infty$. Said another way, oversampling can be used to "beat" the Balian-Low Theorem. Now that we know that this measure $\|xg(x)\|^2+\|w\hat{g}(w)\|^2$ can be finite, we ask what is the minimum value the measure can take for a window that generates a Parseval frame for a fixed oversampling rate. We explore this question for finite Gabor frames.

Phase Retrieval of Low-Rank Matrices

Kiryung Lee
Georgia Institute of Technology
kiryung@ece.gatech.edu
Coauthors: Sohail Bahmani; Yonina Eldar; Justin Romberg

Recovery of low-rank matrices from their magnitude-only measurements enables various practical applications including blind calibration in optical imaging, synchronization-free system identification, and subspace learning from streaming data. We present that if the matrices involved in the measurement process are random following certain distributions, the regualized anchored regression, which is a convex optimization formulation for low-rank phase retrieval without lifting, combined with a simple spectral initialization provides performance guarantees at near optimal sample rates.

Criteria for generalized translation-invariant frames

Jakob Lemvig
Technical University of Denmark
jakle@dtu.dk
Coauthors: Jordy van Velthoven

Generalized shift-invariant (GSI) systems in $L^2(\mathbb{R}^n)$ are structured and flexible function systems of the form $\{g_j(\cdot-\gamma)\}_{j\in J,\gamma\in\Gamma_j}$, where $\{\Gamma_j\}_{j\in J}$ and $\{g_j\}_{j\in J}$ are countable families of lattices in $\mathbb{R}^n$ and functions in $L^2(\mathbb{R}^n)$, respectively. GSI systems have since the beginning of the millennium been used as a unifying framework for the study of Gabor, wavelet, curvelet, shearlet, etc. systems, but they have only recently been identified as function systems of independent interest offering adaptive time-frequency and time-scale representations. As a consequence, many fundamental questions on GSI systems are still unanswered.

In this talk we focus on sufficient and necessary conditions for the frame property of generalized translation-invariant systems, which are a generalization of GSI systems to include continuous transforms. The conditions are formulated in the Fourier domain and consists of estimates involving the upper and lower frame bound. Contrary to known conditions of a similar nature, the estimates take the phase of the generating functions in consideration and not only their modulus. The possibility of phase cancellations makes these estimates optimal for tight frames. Our results are also shed new light on the case of sufficient and necessary conditions for the frame property of wavelet systems in $L^2(\mathbb{R}^n)$, e.g., we obtain Daubechies-Tchamitchian type estimates and a lower bound of the Calderón sum for almost all dilations $A \in \mathrm{GL}_n(\mathbb{R})$.

A Balian-Low Type Theorem for Gabor Schauder Bases

Sara Leshen
Vanderbilt University
sara.j.leshen@vanderbilt.edu
Coauthors: Alexander Powell

The Uncertainty Principle implies that a function and its Fourier transform cannot both be well-localized. The Balian-Low theorem is a form of the Uncertainty Principle for Riesz bases. We prove a new version of the Balian-Low theorem for Gabor Schauder bases generated by compactly supported functions. Moreover, we show that the classical Balian-Low theorem for Riesz bases does not hold for Schauder bases.

Sparse continuous wavelet transforms via a wavelet-Plancherel theory

Ron Levie
School of Mathematical Sciences, Tel Aviv University
ronlevie@post.tau.ac.il
Coauthors: Nir Sochen

It is well known that certain classes of signals can be effectively represented using a wavelet basis or a wavelet frame, keeping only a sparse number of coefficients. In this talk we extend sparse decomposition to the continuous realm, and introduce a sparse decomposition approach for continuous wavelet systems. To overcome the challenges in the continuous realm, we present an extension of the standard continuous wavelet theory, called the wavelet-Plancherel theory. Basing our sparse decomposition algorithm on the new theory, we improve the computational complexity of naive continuous sparse decomposition algorithms. Moreover, the computational complexity of the new method is equal to that of a discrete method, while squaring the sampling resolution in phase space.

Time-frequency Scattering Transforms: Theory and Applications

Weilin Li
University of Maryland
wl298@math.umd.edu
Coauthors: Wojciech Czaja and Ilya Kavalerov

Inspired by the success of deep learning, Mallat introduced the wavelet scattering transform and showed that it provides a useful representation of data. In contrast to his wavelet (time-scale) approach, we develop a Gabor (time-frequency) theory. To do this, we introduce the concept of a uniform covering frame, which is a generalization of traditional Gabor frames. When a uniform covering frame is incorporated into a scattering network, we obtain the Fourier scattering transform. This non-linear operator extracts time-frequency characteristics in a hierarchal manner by cascading convolutions with functions from a uniform covering frame and the complex modulus. It satisfies several provable properties that justify its use as a feature extractor for classification. We demonstrate how to use this for the classification of hyper-spectral data.

Multiscale methods for high-dimensional data with low-dimensional structures

Wenjing Liao
Georgia Institute of Technology
wliao60@gatech.edu
Coauthors: Mauro Maggioni, Johns Hopkins University Stefano Vigogna, Johns Hopkins University

Many data sets in image analysis and signal processing are in a high-dimensional space but exhibit a low-dimensional structure. We are interested in building efficient representations of these data for the purpose of compression and inference. In the setting where a data set in R^D consists of samples from a probability measure concentrated on or near an unknown d-dimensional manifold with d much smaller than D, we consider two sets of problems: low-dimensional geometric approximations to the manifold and regression of a function on the manifold. In the first case, we construct multiscale low-dimensional empirical approximations to the manifold and give finite-sample performance guarantees. In the second case, we exploit these empirical geometric approximations of the manifold and construct multiscale approximations to the function. We prove finite-sample guarantees showing that we attain the same learning rates as if the function was defined on a Euclidean domain of dimension d. In both cases our approximations can adapt to the regularity of the manifold or the function even when this varies at different scales or locations.

Analysis of Decimation on Finite Frames with Sigma-Delta Quantization

Kung-Ching Lin
University of Maryland–College Park
kclin@umd.edu

In Analog-to-digital (A/D) conversion on oversampled bandlimited functions, integration of samples in blocks before down-sampling, known as decimation, has been proven to greatly improve the efficiency of data storage while maintaining high accuracy. In particular, when coupled with the $\Sigma\Delta$ quantization scheme, the reconstruction error decays exponentially with respect to the bit-rate. Here, a similar result is proved for finite unitarily generated frames. Specifically, under certain constraints on the generator, decimation merely increases the reconstruction error estimate by at most a factor of $\pi/2$, independent of the block size. On the other hand, efficient encodings of samples are made possible from decimation, and thus the error decays exponentially with respect to total bits used for data storage. Moreover, the decimation on finite frames has the multiplicative structure that allows the process to be broken down into successive iterations of decimation with smaller blocks, which opens up the possibility for parallel computation and signal transmission through multiple nodes.

When do birds of a feather flock together? Kmeans, proximity and conic programming

Shuyang Ling
Courant Institute of Mathematical Sciences, New York University. 251 Mercer Street New York 10012 NY
sling@cims.nyu.edu
Coauthors: Xiaodong Li (UC Davis), Yang Li (UC Davis), Thomas Strohmer (UC Davis), Ke Wei (Fudan University, Shanghai)

Given a set of data, one central goal is to group them into clusters based on the similarity between the individual objects. One of the most popular and widely-used approaches is K-means despite its computational hardness to find the global minima. We take a different route by studying the convex relaxation of K-means by Peng-Wei and relating it to proximity condition, an idea introduced by Kannan-Kumar. Using conic programming, we present an improved proximity condition under which the Peng-Wei's relaxation recovers the underlying clusters exactly, and also provide a necessary lower bound for the separation of centers below which exact recovery is impossible for Peng-Wei's relaxation. This framework is not only deterministic and model-free but also comes with a clear geometric meaning which allows further analyses and generalization. Moreover, our framework can be conveniently adapted and easily applied to analyzing various data generative models such as stochastic ball models and Gaussian mixture models. With this method, we improve the current minimum separation bound for stochastic ball models and achieve the state-of-the-art results of learning Gaussian mixture models.

Path-Based Spectral Clustering: Guarantees, Robustness to Outliers, and Fast Algorithms

Anna Little
Michigan State University
littl119@egr.msu.edu
Coauthors: James Murphy, Mauro Maggioni

This talk will discuss new performance guarantees for robust path-based spectral clustering and an efficient approximation algorithm for the longest leg path distance (LLPD) metric, which is based on a sequence of multiscale adjacency graphs. LLPD-based clustering is informative for highly elongated and irregularly shaped clusters, and we prove finite-sample guarantees on its performance when random samples are drawn from multiple intrinsically low-dimensional clusters in high-dimensional space, in the presence of a large number of high-dimensional outliers. More specifically, we derive a condition under which the Laplacian eigengap statistic correctly determines the number of clusters for a large class of data sets, and prove guarantees on the number of points mislabeled by the proposed algorithm. Our methods are quite general and provide performance guarantees for spectral clustering with any ultrametric.

Quantization for Low Rank Matrix Recovery

Eric Lybrand
University of California, San Diego
elybrand@ucsd.edu
Coauthors: Rayan Saab

We study Sigma-Delta quantization methods coupled with appropriate reconstruction algorithms for digitizing randomly sampled low-rank matrices. We show that the reconstruction error associated with our methods decays polynomially with the oversampling factor, and we leverage our results to obtain root-exponential accuracy by optimizing over the choice of quantization scheme. Additionally, we show that a random encoding scheme, applied to the quantized measurements, yields a near-optimal exponential bit-rate. As an added benefit, our schemes are robust both to noise and to deviations from the low-rank assumption. In short, we provide a full generalization of analogous results, obtained in the classical setup of bandlimited function acquisition, and more recently, in the finite frame and compressed sensing setups to the case of low-rank matrices sampled with sub-Gaussian linear operators. Finally, we believe our techniques for generalizing results from the compressed sensing setup to the analogous low-rank matrix setup is applicable to other quantization schemes.

Deep Convolutional Neural Networks and Harmonic Analysis
Stephane Mallat
Collge de France, cole Normale Superieure
`stephane.mallat@ens.fr`

Supervised and unsupervised learning amount to approximating functions in high-dimensional spaces, given sample values. Deep convolutional networks have obtained outstanding results for complex classification and regression problems of highly diverse data. This includes images, speech, natural language and all kinds of physical measurements. Dimension reduction in deep neural networks relies on separation of scales, computation of invariants over groups of symmetries, and sparse representations. This could be called applied harmonic analysis. We shall analyze the construction of invariants through deep scattering networks computed with wavelet filters, and discuss open mathematical questions. Applications to image classification, quantum chemistry, and maximum entropy models of turbulences and textures will be shown.

Shearlets as multi-scale Radon transforms
Filippo De Mari
Universit di Genova
`demari@dima.unige.it`
Coauthors: F. Bartolucci, E. De Vito, F. Odone

We show that the shearlet transform can be computed by means of three classical transforms: the affine Radon transform, a 1D wavelet transform and a 1D convolution. This yields formulas that open new perspectives both for finding a new algorithm to compute shearlet coefficients and for the inversion of the Radon transform. Furthermore, the strong connection between shearlets and wavelets suggests an alternative proof of the wavefront set resolution properties of the shearlet transform.

Bi-stochastic kernels, Manifold Learning, and Diffusion Maps
Nicholas F. Marshall
Yale University
`nicholas.marshall@yale.edu`
Coauthors: Ronald R. Coifman

In this talk we answer the following question: what is the infinitesimal generator of the diffusion process defined by a kernel that is normalized such that it is bi-stochastic with respect to a specified measure? More precisely, under the assumption that data is sampled from a Riemannian manifold we determine how the resulting infinitesimal generator depends on the potentially nonuniform distribution of the sample points, and the specified measure for the bi-stochastic normalization. In a special case, we demonstrate a connection to the heat kernel. We consider both the case where only a single data set is given, and the case where a data set and a reference set are given. The spectral theory of the constructed operators is studied, and Nystrom extension formulas for the gradients of the eigenfunctions are computed. Applications to discrete point sets and manifold learning are discussed.

Féjer Polynomials in local Dirichlet spaces

Javad Mashreghi
Universite Laval
Javad.Mashreghi@mat.ulaval.ca

Taylor polynomials are not the most natural objects in polynomial approximation. However, in most cases Cesaro means help and the resulting sequence of Féjer polynomials are a good remedy. In the context of *local Dirichlet Spaces*, we show that the sequence of Taylor polynomials may (badly) diverge. However, and surprisingly enough, if we properly modify just the last coefficient in the Taylor polynomial, the new sequence becomes convergent. As a byproduct, this also leads to the convergence of Féjer polynomials and de la Vallée Poussin polynomials.

Smooth and symmetric convex sets have no orthogonal Gabor bases

Azita Mayeli
City University of New York
amayeli@gc.cuny.edu
Coauthors: Alex Iosevich

Let K be a convex and symmetric bounded set in $\mathbb{R}^d$, $d \geq 2$, with smooth boundary. Using a combinatorial approach, in this talk we show that for $d \neq 1(mod 4)$, the indicator function of K can not serve as an orthogonal Gabor window function for $L^2(\mathbb{R}^d)$, i.e., there is no countable set $S \subset \mathbb{R}^{2d}$ such that the Gabor family $\mathcal{G}(1_K, S) = \{e^{2\pi i x \cdot b} 1_K(x-a) : (a,b) \in S\}$ is an orthogonal basis for $L^2(\mathbb{R}^d)$.

Single Cluster Pursuit: A graph clustering algorithm using Compressive Sensing.

Daniel Mckenzie
University of Georgia
danmac29@uga.edu
Coauthors: Ming-Jun Lai

For any graph G on n vertices, let L denote its (normalized, random-walk) Laplacian. Let $\mathbf{1}_{C_i}$ denote the indicator vector of the i-th cluster in G. One can think of spectral clustering algorithms as attempting to approximate $\mathbf{1}_{C_i}$ by the eigenvectors associated to the lowest eigenvalues of L.

In this talk we observe that, as long as $|C_i| << n$, the vector $\mathbf{1}_{C_i}$ is sparse. Moreover it is approximately a solution to the linear system $L\mathbf{x} = 0$. Hence we may hope to recover $\mathbf{1}_{C_i}$ directly using techniques from Compressive Sensing. We develop this insight into a new, fast clustering algorithm, Single Cluster Pursuit (SCP). Our algorithm has the additional advantage of being able to find a single cluster, or all of them iteratively, depending on user preference.

Theoretical guarantees of success are provided for graphs drawn from a common model of random graph with clusters (the Stochastic Block Model) and numerical results are presented. Extensions to the semi- supervised setting and an online version of the algorithm are also discussed.

Binary block codes from random hyperplane tessellations of uniformly distributed Euclidean embeddings

Robert Mendez
University of Houston
rpmendez.math@gmail.com
Coauthors: Bernhard G. Bodmann, University of Houston

In this talk I describe the construction of a family of binary block codes via the random hyperplane tessellation of uniformly distributed embeddings of a linear code. The construction proceeds in two stages. First, an auxiliary ternary code is chosen which consists of vectors in the union of coordinate subspaces. The subspaces are selected so that any two vectors of different support have a sufficiently large distance. In addition, any two ternary vectors from the auxiliary codebook that have common support are at a guaranteed minimum distance. In the second stage, the auxiliary ternary code is converted to a binary code by random hyperplane tessellation.

Fast Illumination Normalization for Face and Object Detection

Nikolaos Mitsakos
Uiversity of Houston
mathmits@yahoo.gr
Coauthors: Sanat K. Upadhyay, Manos Papadakis

Numerous applications of computer vision, security and surveillance require an accurate, real-time method for illumination neutralization and contrast enhancement. We present a new, multiscale, mathematical framework, with a real time implementation, performing the task efficiently across multiple image modalities. Illumination variation across images is modeled as function in Campanato spaces, while image structures are modeled either through jump discontinuities or functions in micro-local spaces, characterizing local regularity. We demonstrate how our method assists state of the art Convolutional Neural Network (CNN) implementations dedicated to face and object detection.

Monte Carlo approximation certificates for k-means clustering

Dustin G. Mixon
The Ohio State University
mixon.23@osu.edu
Coauthors: Soledad Villar

Efficient algorithms for k-means clustering frequently converge to suboptimal partitions, and given a partition, it is difficult to detect k-means optimality. In this paper, we develop an a posteriori certifier of approximate optimality for k-means clustering. The certifier is a sub-linear Monte Carlo algorithm based on Peng and Wei's semidefinite relaxation of k-means. In particular, solving the relaxation for small random samples of the dataset produces a high-confidence lower bound on the k-means objective, and being sub-linear, our algorithm is faster than k-means++ when the number of data points is large. We illustrate the performance of our algorithm with both numerical experiments and a performance guarantee: If the data points are drawn independently from any mixture of two Gaussians over R^m with identity covariance, then with probability $1-O(1/m)$, our poly(m)-time algorithm produces a 3-approximation certificate with 99% confidence.

Representation, approximation, and optimization advances in restricted Boltzmann machines

Guido Montufar
University of California, Los Angeles
guidomontufar@gmail.com

The restricted Boltzmann machine is a probabilistic graphical model that has played a key role in the development of modern deep learning. It generalizes simpler graphical models with hidden variables and serves as building block of deep learning architectures. In this talk I collect recent advances on the theoretical analysis of these models. In particular, I discuss how Fourier analysis, algebraic correlation inequalities, and links to mixture models shed light on the representational power, approximation errors, optimization landscape, membership testing, and maximum likelihood estimation problems for the restricted Boltzmann machine.

Iterative projective approaches large-scale corrupted linear systems

Deanna Needell
UCLA
deanna@math.ucla.edu
Coauthors: Jamie Haddock

We consider solving large-scale systems of linear equations Ax = b that are inconsistent. We discuss and propose several classical and state of the art approaches including those motivated by the randomized Kaczmarz method and those put forth by Agmon, Motzkin et al. We discuss the tradeoffs of these various approaches as well as a hybrid approach that offers significant improvements in the convergence rate. In particular, we focus on settings when the measurement vector b is corrupted with arbitrarily large errors. We provide analytical justification for thedr approaches as well as experimental evidence on real and synthetic systems.

The Balian-Low theorem in the finite dimensional setting

Shahaf Nitzan
Georgia Tech
shahaf.n.h@gmail.com
Coauthors: Jan-Fredrik Olsen

The classical Balian-Low theorem states that if both a function and it's Fourier transform decay too fast then the Gabor system generated by this function (i.e. the system obtained from this function by taking integer translations and integer modulations) cannot be an orthonormal basis or a Riesz basis.

Though it provides for an excellent 'thumbs-rule' in time-frequency analysis, the Balian-Low theorem is not adaptable to many applications. This is due to the fact that in realistic situations information about a signal is given by a finite dimensional vector rather then by a function over the real line. In this work we obtain an analog of the Balian-Low theorem in the finite dimensional setting, as well as analogs to some of its extensions. Moreover, we will note that the classical Balian-Low theorem, and its extensions, can be derived from these finite dimensional analogs.

The frame set of the $B-$spline.

Kasso Okoudjou
Department of Mathematics & Norbert Wiener Center, University of Maryland, College Park, MD 20742
kasso@math.umd.edu
Coauthors: Ganiou Atindehou, Yebeni Kouagou

The frame set of a function $g \in L^2(\mathbb{R})$ is the subset of all parameters $(a, b) \in \mathbb{R}^2_+$ for which the time-frequency shifts of g along $a\mathbb{Z} \times b\mathbb{Z}$ form a Gabor frame for $L^2(\mathbb{R})$. In this talk, we present some new results on the frame set of the frame set of the $2-$ spline.

Frames arising from solvable actions

Vignon Sourou Oussa
Bridgewater State University
voussa@bridgew.edu

In this presentation, we will provide a unified method which is exploited to construct reproducing systems arising from unitary irreducible representations of solvable Lie groups. In contrast to well-known techniques such as the coorbit theory and other discretization schemes, we do not assume the integrability or square-integrability of the representations of interest. Additionally, we will present various examples illustrating how our method handles a variety of groups relevant to wavelet theory and time-frequency analysis.

Sparsity for Real-time Illumination Neutralization for Visibility Restoration in Images and Video

Manos Papadakis
University of Houston
mpapadak@math.uh.edu
Coauthors: Sanat K. Upadhyay, Nikolaos Mitsakos

Atmospheric suspended particles, vapors often constrain visibility and make structural details practically invisible. A similar phenomenon is caused by suspended particulates in water causing turbidity commonly observed in murky rivers, coastal and lake waters. In these conditions a lot of useful image information is concealed by the veil these particulates create. To address this problem we introduce illumination neutralization where we aim in extracting from the original image a derivative image containing all the useful structural information of the scene in a way that the derivative image is an approximate illumination invariant of the scene. Technically, this problem fits into the framework of the more general and old problem of blind deconvolution. We show how to use multiscale sparse representation combined with non-linear transforms to solve this problem in a deterministic way that overcomes the computational complexity of other solutions such as the optimization-based L1-Retinex.

We demonstrate both the speed and the visual improvement capabilities on still images and video and we show how this can improve the performance of state-of-the-art object detectors.

Minimizing the p-frame potential on unit balls

Josiah Park
Georgia Institute of Technology
j.park@gatech.edu
Coauthors: D. Bilyk, R. Matzke, and O. Vlasiuk,

It has been known that when an equiangular tight frame (ETF) of size $|\Phi| = N$ exists, $\Phi \subset F^d$ (real or complex), for $p > 2$ the p-frame potential $\sum_{i \neq j} | < \phi_j, \phi_k > |^p$ achieves its minimum value on an ETF over all N sized collections of vectors. We are interested in minimizing a related quantity: $1/N^2 \sum_{i,j=1} | < \phi_j, \phi_k > |^p$. In particular we ask when there exists a configuration of vectors for which this quantity is minimized over all sized subsets of the real or complex sphere of a fixed dimension. Also of interest is the structure of minimizers over all unit vector subsets of F^d of size N. We shall present some results for p in (2,4) along with numerical results and conjectures. Portions of this talk are based on recent work of D. Bilyk, R. Matzke, and O. Vlasiuk.

Phase Retrieval from Windowed Fourier Measurements via Wigner Deconvolution and Angular Synchronization with Associated Lower Bounds

Michael Perlmutter
Michigan State University
perlmut6@msu.edu
Coauthors: Mark Iwen, Sami Merhi

We will discuss phase retrieval from locally supported STFT magnitude measurements of a vector x based on a two-step approach: First, a modified Wigner distribution deconvolution approach is used to solve for a portion of the lifted rank-one signal xx^*. Second, an angular synchronization approach is used to recover x from the known portion of xx^*. We will also discuss lower bounds for the Lipschitz continuity of these measurements based off of the size of the support of our measurement masks. These lower bounds are independent of our reconstruction algorithm and give insight into the best possible performance of any such method.

Joint sparse recovery via manifold optimization

Armenak Petrosyan
Oak Ridge National Laboratory
petrosyana@ornl.gov

By appropriate matrix factorization we reformulate the l_0 minimization problem for joint sparse recovery into an equivalent problem on matrix manifolds. We then further relax this problem to a manifold optimization problem and numerically demonstrate its advantages.

Frame Properties of Operator Orbits

Friedrich Philipp
KU Eichsttt
fmphilipp@gmail.com
Coauthors: Ole Christensen, Marzieh Hasannasab

Given a bounded operator T in a Hilbert space $\mathcal{H}$ and a vector $f_0 \in \mathcal{H}$, we consider the orbit $\{T^n f_0 : n \in I\}$ of T with respect to f_0. Here, the index set I is either $I = \mathbb{N}$ or $I = \mathbb{Z}$ (only if T is invertible, of course). As a special instance of the Dynamical Sampling framework, we characterize those pairs (T, f_0) for which the corresponding orbit is a frame for $\mathcal{H}$. More precisely, in both cases $I = \mathbb{N}$ and $I = \mathbb{Z}$ we provide prototypes for such frame orbits in a model space and prove that each frame orbit is similar to one of these prototypes. As an interesting consequence of our results, it turns out that an overcomplete Gabor or wavelet frame can never be written in the form $\{T^n f_0 : n \in \mathbb{N}\}$ with a bounded operator T.

Computation of adaptive Fourier series by sparse approximation of exponential sums

Gerlind Plonka
University of Goettingen
plonka@math.uni-goettingen.de
Coauthors: Vlada Pototskaia

In this talk, we want to study adaptive Fourier expansions of real-valued 2π;-periodic functions using a generalized Fourier basis. We introduce the so-called Takenaka-Malmquist system which is also an orthonormal system with regard to the L_2-norm. It is completely determined by a sequence of "zeros", and the classical Fourier basis is obtained as a special case. We want to study the question, how well a given function f can be approximated by its generalized partial Fourier sum of length N if the zeros determining the Takenaka-Malmquist system are taken in an (almost) optimal way. Using theoretical results on rational approximation in Hardy spaces and on the decay of singular values of special infinite Hankel matrices we provide asymptotic estimates showing that the decay of Fourier sums can be strongly improved using the adaptive basis. Further, we give a constructive algorithm for computing the (almost) optimal adaptive Fourier basis for a given length of the Fourier sum. Our numerical results show, that the significantly better convergence behavior of adaptive Fourier sums for optimally chosen basis elements can also be achieved in practice.

Sampling with totally-positive functions

Jose Luis Romero
University of Vienna
jose.luis.romero@univie.ac.at
Coauthors: Karlheinz Groechenig and Joachim Stoeckler

I will present sampling theorems for shift-invariant spaces generated by totally-positive functions. These are formulated in terms of Beurling's density and match exactly the irregular sampling theorem for bandlimited functions. As an application, we characterize the lattice parameters that yield time-frequency (Gabor) expansions with totally-positive functions.

Joint work with Karlheinz Groechenig and Joachim Stoeckler.

New and Improved Binary Embeddings of Data (and Quantization for Compressed Sensing with Structured Random Matrices)

Rayan Saab
University of California San Diego
rsaab@ucsd.edu
Coauthors: Thang Huynh

We discuss two related problems that arise in the acquisition and processing of high-dimensional data. First, we consider distance-preserving fast binary embeddings. Here we propose fast methods to replace points from a set $\mathcal{X} \subset \mathbb{R}^N$ with points in a lower-dimensional cube $\{\pm 1\}^m$, which we endow with an appropriate function to approximate Euclidean distances in the original space. Second, we consider a problem in the quantization (i.e., digitization) of compressed sensing measurements. Here, we deal with measurements arising from the so-called bounded orthonormal systems and partial circulant ensembles, which arise naturally in compressed sensing applications. In both these problems we show state-of-the art error bounds, and to our knowledge, some of our results are the first of their kind.

How can we naturally sort and organize graph Laplacian eigenvectors?

Naoki Saito
University of California, Davis
saito@math.ucdavis.edu

When attempting to develop wavelet transforms for graphs and networks, some researchers have used graph Laplacian eigenvalues and eigenvectors in place of the frequencies and complex exponentials in the Fourier theory for regular lattices in the Euclidean domains. This viewpoint, however, has a fundamental flaw: on a general graph, the Laplacian eigenvalues cannot be interpreted as the frequencies of the corresponding eigenvectors. In this talk, we discuss this important problem further and propose a new method to organize those eigenvectors by defining and measuring 'natural' distances between eigenvectors using the Ramified Optimal Transport Theory followed by embedding the resulting distance matrix into a low-dimensional Euclidean domain for further grouping and organization of such eigenvectors. We demonstrate its effectiveness using a synthetic graph as well as a dendritic tree of a retinal ganglion cell of a mouse.

Inverse problems for PDEs via infinite dimensional compressed sensing

Matteo Santacesaria
University of Helsinki, Finland
matteo.santacesaria@helsinki.fi
Coauthors: Giovanni S. Alberti (University of Genoa, Italy)

In this talk I will discuss how ideas from applied harmonic analysis, in particular sampling theory and compressed sensing (CS), may be applied to inverse problems in PDEs. I will present new results concerning generalization of CS in the framework of Hilbert spaces: in particular, the measurement operator does not need to be an orthonormal transformation and the unknown is assumed to be sparse in a frame. Applications to linear and nonlinear inverse problems for PDEs, such as electrical impedance tomography, will be discussed in detail.

A Generalized Kaczmarz Algorithm

Anna Seitz
Iowa State University
acseitz@iastate.edu
Coauthors: Emelie Curl, Steven Harding, Mary Vaughan

The Kaczmarz Algorithm is a method for reconstructing vectors in a Hilbert Space, $\mathcal{H}$. Using inner products with a so-called " effective sequence," the Kaczmarz Algorithm can be applied at any vector $x \in \mathcal{H}$ to generate a sequence of approximations which converges to x in norm. Although a useful class of sequences, effective sequences are generally intolerant towards perturbation. To obtain more flexibility, we explore the idea of " effective pairs," two sequences that work together to analyze and synthesize in a generalized Kaczmarz Algorithm. After the pattern of effective sequences, we seek a complete characterization of these pairs via a partial isometry condition on an associated operator, U.

Pairwise orthogonal frames generated by regular representations of LCA groups

Niraj K. Shukla
Indian Institute of Technology Indore, India
nirajshukla@iiti.ac.in
Coauthors: Anupam Gumber

Having potential applications in multiplexing techniques and in the synthesis of frames, orthogonality (or strongly disjointness) plays a significant role in frame theory (e.g. construction of new frames from existing ones, constructions related with duality, etc). In this article, we study orthogonality of a pair of frames over locally compact abelian (LCA) groups. We start with the investigation of the dual Gramian analysis tools of Ron and Shen through a pre-Gramian operator over the set-up of LCA groups. Then we fiberize some operators associated with Bessel families generated by unitary actions of co-compact (not necessarily discrete) subgroups of LCA groups. Using this fiberization, we study and characterize a pair of orthogonal frames generated by the action of a unitary representation ρ of a co-compact subgroup $\Gamma \subset G$ on a separable Hilbert space $L^2(G)$, where G is a second countable LCA group. Precisely, we consider frames of the form $\{\rho(\gamma)\psi : \gamma \in \Gamma, \psi \in \Psi\}$ for a countable family Ψ in $L^2(G)$. We pay special attention to this problem in the context of translation-invariant space by assuming ρ as the action of Γ on $L^2(G)$ by left-translation. The representation of Γ acting on $L^2(G)$ by (left-)translation is called the (left-)regular representation of Γ. Further, we apply our results on co-compact Gabor systems over LCA groups. At this juncture, it is pertinent to note that the resulting characterization can be useful for constructing new frames by using various techniques including the unitary extension principle by Ron and Shen[Affine systems in $L^2(\mathbb{R}^d)$: the analysis of the analysis operator, J. Funct. Anal., 148 (1997) 408-447] and its recent extension to LCA groups by Christensen and Goh [The unitary extension principle on locally compact abelian groups, Appl. Comput. Harmon. Anal., (2017)].

Markov Chains and Generalized Wavelet Multiresolutions

Myung-Sin Song
Southern Illinois University Edwardsville
myungsin.song@gmail.com
Coauthors: Palle E. T. Jorgensen

We develop some new results for a general class of transfer operators, as they are used in a construction of multi-resolutions, provide a criterion for a family of Cuntz isometries to be an orthonormal basis. We then proceed to give explicit and concrete applications. We further discuss the need for such a constructive harmonic analysis/dynamical systems approach to fractals.

The wavelet existence problem

Darrin Speegle
Saint Louis Univeristy
speegled@slu.edu
Coauthors: Marcin Bownik, University of Oregon

The wavelet existence problem asks for which pairs (A, Γ), where A is an invertible matrix and Γ is a full rank lattice, there exists a function ψ such that

$$\{|A|^{j/2}\psi(A^j x + k) : j \in Z, k \in \Gamma\}$$

is an orthonormal basis for $L^2(R^n)$. The problem is solved in the one-dimensional case, but remains open in dimensions two and higher.

In this talk, I will provide an overview of progress on the wavelet existence problem, culminating with some recent results that provide an intriguing connection to the geometry of numbers.

Generalized Exponential Prony Method

Kilian Stampfer
University of Goettingen
Kilian.Stampfer@mathematik.uni-goettingen.de
Coauthors: Gerlind Plonka

It was shown by Peter&Plonka that the classical Prony method can be easily generalized to linear operators and used for the reconstruction of associated linear combinations of eigenfunctions. In this talk, a general approach is introduced to derive families of eigenfunction expansions of associated strongly continuous semi-groups. Furthermore, two classes of realizable sampling-schemes for the reconstruction of such structured functions are presented. Additionally, it is demonstrated how this approach already summerizes all of the former and new examples under the Generalized Exponential Prony (GEP) method.

Universal constructions in dynamical sampling

Sui Tang
Johns Hopkins University
stang@math.jhu.edu

Dynamical sampling is a new area in sampling theory that deals with processing signals that evolve over time. It is motivated by spatiotemporal sampling problem in a diffusion field in which we want to recover the initial distribution function from its coarsely sampled snapshots at multiple time instances. In this talk, I will give a complete characterization of universal spatiotemporal sampling sets in discrete linear diffusion fields and show the dynamical sampling problem is a dual version of compressed sensing.

Sparse non-negative super-resolution: simplified and stabilized

Jared Tanner
University of Oxford and Alan Turing Institute
tanner@maths.ox.ac.uk
Coauthors: Armin Eftekhari (University of Edinburgh and Alan Turing Institute), Andrew Thompson (University of Oxford) Bogdan Toader (University of Oxford) Hemant Tyagi (University of Edinburgh and Alan Turing Institute)

Super-resolution is a technique by which one seeks to overcome the inherent accuracy of a measurement device by exploiting further information. Applications are very broad, but in particular these methods have been used to great effect in modern microscopy methods and underpin recent Nobel prizes in chemistry. This topic has received a renewed theoretical interest starting in approximately 2013 where notions from compressed sensing were extended to this continuous setting. The simplest model is to consider a one dimensional discrete measure $\mu = \sum_{j=1}^{k} \alpha_j \delta_{t_j}$ which models k discrete objects at unknown locations t_j and unknown amplitudes α_j (typically with non-negative amplitudes). The measurement device can be viewed as a burring operator, where each discrete spike is instead replaced a function $\psi(s, t_j)$ such as a Gaussian $\exp(-\sigma|s - t_j|)$, in which case one can make measurements of the form $y(s) = \psi(s, t) \star \mu = \sum_{j=1}^{k} \alpha_j \psi(s, t_j)$. Typically one measures $m > 2k + 1$ discrete values; that is $y(s_i)$ for $i = 1, \ldots, m$. The aim is then to recover the $2k$ parameters ${t_j}_{j=1}^{k}$ and ${\alpha_j}_{j=1}^{k}$ from the m samples and knowledge of $\psi(s, t)$. In this talk we extend recent results by Schiebinger, Robava, and Recht to show that the the $2k$ parameters are uniquely determined by their $2k+1$ samples, and that any solution consistent with the measurement within τ is proportionally consistent with the original measure.

Group Representations and Higher Dimensional Wavelet-like Transforms

Keith F Taylor
Dalhousie University
keith.taylor@dal.ca

We will review the role of group representation theory in the existence of generalized continuous wavelet transforms and present some of the transforms in three dimensions arising from the recent classification of admissible subgroups of $GL_3(\mathbb{R})$.

Finite Balian-Low Theorems and Applications of the Quantitative BLT

Michael Northington V
Georgia Tech
mcnv3@gatech.edu
Coauthors: Josiah Park

The Balian-Low Theorem states that the generator of a Gabor Riesz basis for $L^2(\mathbb{R})$ must have poor localization in either time or frequency. Recently, Nitzan and Olsen have shown that Balian-Low type results exist for Gabor systems in the finite dimensional spaces $\ell^2(\mathbb{Z}_d)$. We first extend these results to their higher dimensional analogs in $\ell^2(\mathbb{Z}_d^n)$. Next, we show how many (finite dimensional and continuous) Balian-Low type theorems follow from a quantitative version of the BLT. In particular, we will discuss nonsymmetric verisons of the Balian-Low Theorem in $L^2(\mathbb{R}^n)$.

Frame construction and approximation of functions via relevant sampling of the STFT

Gino Angelo Velasco
University of the Philippines Diliman
gamvelasco@math.upd.edu.ph
Coauthors: Arvin Lamando

We consider the relevant sampling of the STFT of functions that are localized on a compact region in the time-frequency plane. The relevant sampling corresponds to a frame-like inequality that holds uniformly for such time-frequency localized functions in the region. We present an approximate reconstruction of these functions from the local relevant samples. In the spirit of quilted Gabor frames, we also obtain a global frame from a collection of time-frequency systems, each satisfying a local frame-like inequality, and we illustrate the results with numerical examples.

Speed of entanglement generation in quantum systems

Anna Vershynina
University of Houston
anna@math.uh.edu

Entanglement is one of the counterintuitive quantum phenomena that is still poorly understood. Entanglement allows distant particles to “feel” each other in a way that no classical interaction can offer. One of the most notorious (and widely popularized) protocols - quantum teleportation - relies on the entanglement to destroy a quantum state in one place and perfectly recreate it in another distant location. While entanglement became one of the most

valuable resources in quantum theory, many attributes of its behavior remain unknown. One such question is how fast the entanglement can be generated in a general system. We investigate the maximal rate at which entanglement can be generated in quantum systems. The goal is to upper bound this rate. The problem heavily depends on the entanglement measure considered. I will review the problem in closed system with von Neumann entanglement entropy taken as an entanglement measure, generalize the problem to the open systems. And will finish with the recent work, where a large class of entanglement measures can be considered, including Renyi and Tsallis entropies.

Stable denoising with generative networks and spherical harmonics

Soledad Villar
New York University
`soledad.villar@nyu.edu`
Coauthors: Dustin G. Mixon

It has been experimentally established that deep neural networks can be used to produce good generative models for real world data. It has also been established that such generative models can be exploited to solve classical inverse problems like compressed sensing and super resolution. In this work we focus on the classical signal processing problem of image denoising. We propose a theoretical setting that uses spherical harmonics to identify what mathematical properties of the activation functions will allow signal denoising with local methods.

Phase Retrieval from Local Measurements: Deterministic Measurement Constructions and Efficient Recovery Algorithms

Aditya Viswanathan
University of Michigan - Dearborn
`adityavv@umich.edu`
Coauthors: Mark Iwen, Brian Preskitt, Rayan Saab

Certain imaging applications such as x-ray crystallography and ptychography require the recovery of a signal from phaseless (or magnitude-only) measurements - a problem commonly referred to as Phase Retrieval. This is a challenging (and non-linear) inverse problem since the phase encapsulates a significant amount of structure in the underlying signal. In this talk, we will discuss a framework for solving the discrete phase retrieval problem from deterministic local measurements. While convex relaxation methods are commonly used to solve this problem, we summarize a recently introduced fast (essentially linear-time) and robust phase retrieval algorithm based on solving highly structured (block-circulant) linear systems to infer relative phase information, followed by an eigenvector based approach to learning individual phases from relative phase estimates. Theoretical recovery guarantees as well as numerical results demonstrating the method's speed, accuracy and robustness to measurement errors will be provided.

Multiple rank-1 lattice sampling and high-dimensional sparse FFT

Toni Volkmer
TU Chemnitz
toni.volkmer@mathematik.tu-chemnitz.de
Coauthors: Lutz Kmmerer, Daniel Potts

We consider the approximate reconstruction of a high-dimensional (e.g. d=10) periodic function from samples using trigonometric polynomials. As sampling schemes, we use so-called multiple rank-1 lattices. Assuming that we know the locations of the approximately largest Fourier coefficients of a function under consideration, we can efficiently construct a suitable multiple rank-1 lattice and compute approximants using few 1-dimensional fast Fourier transforms (FFTs). For functions from Sobolev Hilbert spaces of generalized mixed smoothness, error estimates are presented where the sampling rates are optimal up to an offset slightly larger than one half in the exponent. We present numerical results which confirm the theoretical estimates. These sampling errors are almost a factor of two better up to the mentioned offset compared to single rank-1 lattice sampling. For the case where we do not know the locations of important Fourier coefficients, we present a method which approximately reconstructs high-dimensional sparse periodic signals in a dimension-incremental way based on 1-dimensional FFTs. This is joint work with Lutz Kmmerer and Daniel Potts.

Convergence rate of the Douglas-Rachford method for finding best approximating pairs

Irne Waldspurger
CNRS, Universit Paris Dauphine
waldspurger@ceremade.dauphine.fr
Coauthors: Stefanie Jegelka

The problem of finding best approximating pairs consists, given two closed sets in a metric space, in finding two points, one in each set, such that the distance between the points is minimal. This problem arises in many convex as well as non-convex settings. We will discuss the case where the sets are convex polyhedrons in finite dimension. In this situation, several algorithms are known. The simplest one is alternating projections, and its convergence speed is relatively well understood. However, in practice, another algorithm, Douglas-Rachford, often seems to perform on par or better than alternating projections.

We will discuss the convergence speed of this second algorithm, globally as well as locally. This is a joint work with Stefanie Jegelka.

Tight framelets and fast framelet filter bank transforms on manifolds

Yu Guang Wang
ICERM, Brown University and UNSW Sydney
yuguang.wang@unsw.edu.au
Coauthors: Quoc T. Le Gia, Ian H. Sloan, Robert S. Womersley (UNSW Sydney) Houying Zhu (University of Melbourne) Xiaosheng Zhuang (City University of Hong Kong)

Data in practical application with some structure can be viewed as sampled from a manifold, for instance, data on a graph and in astrophysics. A smooth and compact Riemannian manifold $\mathcal{M}$, including examples of spheres, tori, cubes and graphs, is an important geometric structure. In this work, we construct a type of tight framelets using quadrature rules on $\mathcal{M}$ to represent the data (or a function) and to exploit the derived framelets to process the data (for example, image and signal processing on the sphere or graphs).

One critical computation for framelets is to compute, from the framelet coefficients for the input data (which are assumed at the highest level), the framelet coefficients at lower levels, and also to evaluate the function values at new nodes using the framelet representation. We design an efficient computational strategy, which we call fast framelet filter bank transform (FMT), to compute the framelet coefficients and to recover the function. Assuming the fast Fourier transform (FFT) and using polynomial-exact quadrature rules on the manifold $\mathcal{M}$, the FMT has the same computational complexity as the FFT. Numerical examples illustrate the efficiency and accuracy of the algorithm for the framelets.

Group SLOPE model with application to genomic data analysis

Yu-Ping Wang
Tulane University
wyp@tulane.edu
Coauthors: Alexej Gossmann

The Sorted L-One Penalized Estimation (SLOPE) is a sparse regression method recently introduced, which can be used to identify significant predictor variables in a linear model. When the correlations between predictor variables are small, the SLOPE method is shown to successfully control the false discovery rate at a user specified level. In this work, we extend SLOPE in the spirit of Group LASSO to Group SLOPE, a method that can handle group structures between the predictor variables, which are ubiquitous in many applications. Both our theoretical and simulation results show that Group SLOPE can control the group-wise false discovery rate in many settings. As an illustration of the merits of this method, an application of Group SLOPE to genomic data analysis is presented.

Learning the learning rate in gradient descent methods

Rachel Ward
University of Texas at Austin
rward@math.utexas.edu
Coauthors: Xiaoxia Wu, Leon Bottou

Finding a proper learning rate in stochastic optimization is an important problem. Choosing a learning rate that is too small leads to painfully slow convergence, while a learning rate that is too large can cause the loss function to fluctuate around the minimum or even to diverge. In practice, the learning rate is often tuned by hand for different problems at hand. Several methods have been proposed recently for automatic adjustment of the learning rate according to gradient data that is received along the way. We review these methods, and propose a simple method, inspired by reparametrization of the loss function in polar coordinates. We prove that the proposed method achieves optimal oracle convergence rates in batch and stochastic settings, but without having to know certain parameters of the loss function in advance.

The Kaczmarz Algorithm and Harmonic Analysis of Singular Measures

Eric Weber
Iowa State University
esweber@iastate.edu

The Kaczmarz algorithm is an iterative method for solving (finite) systems of linear equations. Kwapien and Mycielski have proven that the algorithm converges to the solution in infinite-dimensional Hilbert spaces under the conditions that the "rows of the matrix" form a stationary sequence, and the spectral measure of this stationary sequence is singular. This remarkable result has provided a new tool for understanding the harmonic analysis of singular measures, including the existence of Fourier series expansions, boundary representations for certain subspaces of the Hardy space, and Paley-Wiener like characterizations of entire functions with singular spectra. We shall present an overview of these results that are consequences of the Kaczmarz algorithm.

Geometric Approach to Medical Time Series Challenges

Hau-tieng Wu
Duke University
hauwu@math.duke.edu

Unsupervised feature extraction from massive datasets is at the core of modern data analysis. In this talk, the particular interest is extracting hidden dynamics from a single channel time series composed of multiple oscillatory signals, which could be viewed as a single-channel blind source separation problem. We consider signals consists of non-sinusoidal oscillations, with time varying amplitude/frequency, and by the heteroscedastic nature of the noise. I will discuss recent progress in solving this kind of problem by combining the cepstrum-based nonlinear time-frequency analysis and manifold learning techniques. Results of motivative medical problem, the extraction of a fetal ECG signal from a single lead maternal abdominal ECG signal will be discussed.

Learning Theory of Distributed Kernel Regression

Qiang Wu
Middle Tennessee State University
qwu@mtsu.edu

Distributed learning provides effective tools for big data processing. An effective non-interactive approach for distributed learning is the divide and conquer method. It first partitions a big data set into multiple subsets, then a base algorithm is applied to each subset, and finally the results from these subsets are pooled together. In the context of nonlinear regression analysis, regularized kernel methods usually serve as efficient base algorithms for the second stage. In this talk, I will discuss the minimax optimality of several kernel based regression algorithms in distributed learning.

Generalized phase retrieval

Zhiqiang Xu
Academy of Mathematics and System Science, Chinese Academy of Sciences
xuzq@lsec.cc.ac.cn
Coauthors: Yang Wang

Phase retrieval is an active topic recently. In this talk, we will introduce the generalized phase retrieval which includes as special cases the standard phase retrieval as well as the phase retrieval by orthogonal projections. We first explore the connections among generalized phase retrieval, low-rank matrix recovery and nonsingular bilinear form. Motivated by the connections, we present results on the minimal measurement number needed for generalized phase retrieval. Our work unifies and enhances results from the standard phase retrieval, phase retrieval by projections and low-rank matrix recovery and also explore the connection among phase retrieval, nongsingular bilinear and topology.

Tensor network ranks

Ke Ye
Chinese Academy of Sciences
keyk@amss.ac.cn
Coauthors: Lek-Heng Lim

Tensor network states are originally invented by quantum physicsts, which can be used to analyze and simulate quantum systems. Popular tensor network states include tensor trains (TT), matrix product states (MPS), projected entangled-pair states (PEPS) and so on. A lot of numerical algorithms concerning tensor network states have been discovered in the past 20 years. However, tensor network states are not well-understood mathematically. In this talk, we will first review the mathematical formalism of tensor network states. After that we will define the notion of tensor network rank which describes the complexity of a tensor network state and we will discuss some properties of tensor network ranks. If time permits, we will also present computational results on tensor network ranks of some particular tensors such as matrix multiplication tensor, W-states and GHZ states. This talk is based on joint works with Lek-Heng Lim.

Near-optimal sample complexity for convex tensor completion

Ozgur Yilmaz
University of British Columbia
oyilmaz@math.ubc.ca
Coauthors: Navid Ghadermarzy and Yaniv Plan

We study the problem of estimating a low-rank tensor when we have noisy observations of a subset of its entries. A rank-r, order-d, $N \times N \times \cdots \times N$ tensor where $r = O(1)$ has $O(dN)$ free variables. On the other hand, prior to our work, the best sample complexity that was achieved in the literature is $O(N^{\frac{d}{2}})$, obtained by solving a tensor nuclear-norm minimization problem. In this talk, we consider the "M-norm", an atomic norm whose atoms are rank-1 sign tensors. We also consider a generalization of the matrix max-norm to tensors, which results in a quasi-norm that we call "max-qnorm". We prove that solving an M-norm constrained least squares problem results in nearly optimal sample complexity for low-rank tensor completion. A similar result holds for max-qnorm as well. Furthermore, we show that these bounds are nearly minimax rate-optimal. This is joint work with Navid Ghadermarzy and Yaniv Plan.

Stochastic Optimization for AUC Maximization in Machine Learning

Yiming Ying
SUNY at Albany
yying@albany.edu
Coauthors: Michael Natole, Longyin Wen and Siwei Lyu

AUC (Area Under the ROC Curve) is a widely used performance measure for binary classi

cation and bipartite ranking in machine learning. This measure is particularly suitable for applications where the labels are imbalanced. A speci

c challenge in developing stochastic and online AUC maximization algorithms is that the objective function is usually de

fined over a pair of training examples from opposite classes while one usually receives individual examples sequentially. In this talk, I will present novel stochastic optimization algorithms for AUC maximization, which can sequentially process the data. Existing online AUC algorithms have expensive space and time complexities which are quadratic $O(d^2)$ where d is the dimensionality of the data. In contrast, our new algorithms have a linear space and per-iteration complexity $O(d)$. I will also present the theoretical results about the convergence of the proposed algorithm. Encouraging experimental results will be presented. This talk is based on a joint work with Michael Natole, Siwei Lyu and Longyin Wen from SUNY Albany

Some smooth compactly supported tight wavelet frames with vanishing moments

Richard Zalik
Auburn University
zalikri@auburn.edu
Coauthors: Angel San Antolin

Let $A \in \mathbb{R}^{d \times d}$, $d \geq 1$ be a dilation matrix with integer entries and $|\det A| = 2$. We construct several families of compactly supported Parseval framelets associated to A having any desired number of vanishing moments. The first family has a single generator and its construction is based on refinable functions associated to Daubechies low pass filters and a theorem of Bownik. For the construction of the second family we adapt methods employed by Chui and He and Petukhov for dyadic dilations to any dilation matrix A. The third family of Parseval framelets has the additional property that we can find members of that family having any desired degree of regularity. The number of generators is $2^d + d$ and its construction involves some compactly supported refinable functions, the Oblique Extension Principle and a slight generalization of a theorem of Lai and Stöckler. For the particular case $d = 2$ and based on the previous construction, we present two families of compactly supported Parseval framelets with any desired number of vanishing moments and degree of regularity. One of the families has only two generators, whereas the other family has only three generators.

Mathematical analysis of deep CNNs and distributed learning
Ding-Xuan Zhou
City University of Hong Kong
mazhou@cityu.edu.hk

In this talk we present some mathematical analysis for deep convolutional neural networks (CNNs) and distributed learning algorithms. We show that deep CNNs with the rectified linear unit activation function are universal as the depth increases. We demonstrate error analysis for some distributed learning algorithms presented in reproducing kernel Hilbert spaces. Our approach is based on machine learning, approximation theory, and wavelets.

Multiscale Data Analysis: Framelets, Manifolds and Graphs
Xiaosheng Zhuang
City University of Hong Kong
xzhuang7@cityu.edu.hk

While Big Data are high-volume, high-dimensional, and high complexity, they are typically concentrated on low-dimensional manifolds or can be represented by graphs, digraphs, etc. Sparsity is the key to the successful analysis of data in various forms. Multiscale representation systems provide efficient and sparse representation of various data sets. In this talk, we will discuss the characterizations, construction, and applications of framelets on manifolds and graphs. We shall demonstrate that tight framelets can be constructed on compact Riemannian manifolds or graphs, and fast algorithmic realizations exist for framelet transforms on manifolds and graphs. Explicit construction of tight framelets on the sphere and graphs as well as numerical examples will be shown.