## CS 4260 and CS 5260 Vanderbilt University

## Lecture on Uniformed Search

This lecture assumes that you have

- Read Chapter 2 through section 2.3 of ArtInt (15 pages)
- Read Chapter 3 through section 3.5.2 of ArtInt (20 pages) and
- Watched Doug's iterative deepening search video playlist (material from section 3.5.3 optional reading)

ArtInt: Poole and Mackworth, Artificial Intelligence 2E
at http://artint.info/2e/html/ArtInt2e.html
and some slides at https://artint.info/2e/slides/index.html
Iterative Deepening playlist:
https://www.youtube.com/watch?v=7QcoJjSVT38\&list=PLXAjOiPf89kPs82cbS6j9PR3t7ZzixnaO

## Learning Objectives

At the end of the class you should be able to:

- define a directed graph
- represent a problem as a state-space graph
- explain how a generic searching algorithm works


## Searching

- Often we are not given an algorithm to solve a problem, but only a specification of what is a solution - we have to search for a solution.
- A typical problem is when the agent is in one state, it has a set of deterministic actions it can carry out, and wants to get to a goal state.
- Many AI problems can be abstracted into the problem of finding a path in a directed graph.
- Often there is more than one way to represent a problem as a graph.


## State-space Search

- flat or modular or hierarchical
- explicit states or features or individuals and relations
- static or finite stage or indefinite stage or infinite stage
- fully observable or partially observable
- deterministic or stochastic dynamics
- goals or complex preferences
- single agent or multiple agents
- knowledge is given or knowledge is learned
- perfect rationality or bounded rationality


## Directed Graphs

- A (directed) graph consists of a set $N$ of nodes and a set $A$ of ordered pairs of nodes, called arcs.
- Node $n_{2}$ is a neighbor of $n_{1}$ if there is an arc from $n_{1}$ to $n_{2}$. That is, if $\left\langle n_{1}, n_{2}\right\rangle \in A$.
- A path is a sequence of nodes $\left\langle n_{0}, n_{1}, \ldots, n_{k}\right\rangle$ such that $\left\langle n_{i-1}, n_{i}\right\rangle \in A$.
- The length of path $\left\langle n_{0}, n_{1}, \ldots, n_{k}\right\rangle$ is $k$.
- Given a set of start nodes and goal nodes, a solution is a path from a start node to a goal node.


## Example Problem for Delivery Robot

The robot wants to get from outside room 103 to the inside of room 123.


Search tree for Figure 3.1 of Poole and Mackworth, assuming robot starts at 0103, with a goal of being in r123, with all arcs bidirectional (e.g., doors open in both directions, which is a different assumption than text's)


## Partial Search Space for a Video Game

Grid game: Rob needs to collect coins $C_{1}, C_{2}, C_{3}, C_{4}$, without running out of fuel, and end up at location ( 1,1 ):


## Partial Search Space for a Video Game

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State:
$\langle X$-pos, $Y$-pos,Fuel,C1,C2,C3,C4〉

Goal:
$\langle 1,1, ?, t, t, t, t\rangle$

## Problem Solving by Graph Searching



## Graph Search Algorithm

Input: a graph,
a set of start nodes,
Boolean procedure goal( $n$ ) that tests if $n$ is a goal node.
frontier $:=\{\langle s\rangle: s$ is a start node $\}$
while frontier is not empty:
select and remove path $\left\langle n_{0}, \ldots, n_{k}\right\rangle$ from frontier
if goal( $\left.n_{k}\right)$
return $\left\langle n_{0}, \ldots, n_{k}\right\rangle$
for every neighbor $n$ of $n_{k}$ add $\left\langle n_{0}, \ldots, n_{k}, n\right\rangle$ to frontier
end while

## Basic Search methods of a tree

Initial state


Leaves are terminal states with no successors (or neighbors).

## Depth-first search (DFS) of a tree

State Stack (Frontier):
$\{\mathbf{A}\} \backsim$ Pop A, check if A is a goal, $\{\mathbf{B}, \mathrm{D})^{\text {Push successors of A }}$
$\{\mathbf{B}, \mathrm{C}, \mathrm{D}\}$
$\{\mathbf{E}, \mathrm{F}, \mathrm{C}, \mathrm{D}\}$
$\{\mathbf{K}, \mathrm{L}, \mathrm{F}, \mathrm{C}, \mathrm{D}\}$
$\{\mathbf{L}, \mathrm{F}, \mathrm{C}, \mathrm{D}\}$
$\{\mathbf{F}, \mathrm{C}, \mathrm{D}\}$
$\{\mathbf{M}, \mathrm{C}, \mathrm{D}\}$
\{C, D \}
$\{\mathbf{G}, \mathrm{H}, \mathrm{D}\}$
$\{\mathbf{N}, \mathrm{H}, \mathrm{D}\}$
$\{\mathbf{H}, \mathrm{D}\}$
$\{\mathbf{O}, \mathrm{P}, \mathrm{D}\}$
$\{\mathbf{P}, \mathrm{D}\} \quad$ Pop P , check if P a goal, stop, return $(\mathrm{P},(\mathrm{A}, \mathrm{C}, \mathrm{H}))$ or $(\mathrm{A} \rightarrow \mathrm{C} \rightarrow \mathrm{H} \rightarrow \mathrm{P})$
Assume each state in stack also includes the path used to reach it. So, $A$ is really $(A,()), B$ is really $(B,(A))$, $E$ is really ( $\mathrm{E},(\mathrm{AB})$ ), $K$ is really $(\mathrm{K},(\mathrm{A}, \mathrm{B}, \mathrm{E})$ ), L is really $(\mathrm{L},(\mathrm{A}, \mathrm{B}, \mathrm{E})), \mathrm{F}$ is $(\mathrm{F},(\mathrm{A}, \mathrm{B})$ ), etc. This convention differs very slightly from textbook's.

Breadth-first search (BFS) of a tree
State Queue (Frontier):
$\{\mathbf{A}\} \rightarrow$ Dequeue $A$, check if $A$ is a goal
$\left\{B, C, D^{\text {Enqueue successors of } A}\right.$
$\{\mathbf{B}, \mathrm{C}, \mathrm{D}\}$
\{C, D, E, F \}
$\{\mathbf{D}, \mathrm{E}, \mathrm{F}, \mathrm{G}, \mathrm{H}\}$
$\{\mathbf{E}, \mathrm{F}, \mathrm{G}, \mathrm{H}, \mathrm{I}, \mathrm{J}\}$
$\{\mathbf{F}, \mathrm{G}, \mathrm{H}, \mathrm{I}, \mathrm{J}, \mathrm{K}, \mathrm{L}\}$
$\{\mathbf{G}, \mathrm{H}, \mathrm{I}, \mathrm{J}, \mathrm{K}, \mathrm{L}, \mathrm{M}\}$
$\{\mathbf{H}, \mathrm{I}, \mathrm{J}, \mathrm{K}, \mathrm{L}, \mathrm{M}, \mathrm{N}\}$
$\{\mathbf{I}, \mathrm{J}, \mathrm{K}, \mathrm{L}, \mathrm{M}, \mathrm{N}, \mathrm{O}, \mathrm{P}, \mathrm{K} \mathrm{L} \mathrm{M}$
$\{\mathbf{J}, \mathrm{K}, \mathrm{L}, \mathrm{M}, \mathrm{N}, \mathrm{O}, \mathrm{P}, \mathrm{Q}\}$
$\{\mathbf{K}, \mathrm{L}, \mathrm{M}, \mathrm{N}, \mathrm{O}, \mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S}\}$
$\{\mathbf{L}, \mathrm{M}, \mathrm{N}, \mathrm{O}, \mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S}\}$
$\{\mathbf{M}, \mathrm{N}, \mathrm{O}, \mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S}\}$
$\{\mathbf{N}, \mathrm{O}, \mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S}\}$
$\{\mathbf{O}, \mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S}\}$
$\{\mathbf{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S}\}$ Dequeue P , check if Pa goal, stop, return $(\mathrm{P},(\mathrm{A}, \mathrm{C}, \mathrm{H}))$ or $(\mathrm{A} \rightarrow \mathrm{C} \rightarrow \mathrm{H} \rightarrow \mathrm{P})$

Iterative deepening DFS of a tree

State Stack (Frontier):
1] $\{\mathbf{A}\},\{ \}$

$2]\{A\},\{B, C, D\},\{C, D\},\{D\},\{ \}$
$3]\{A\},\{B, C, D\},\{E, F, C, D\},\{F, C, D\},\{C, D\},\{\mathbf{D}, \mathrm{D}, \mathrm{B}, \mathrm{D}, \mathrm{D}, \mathrm{D},\{\mathrm{B}, \mathrm{D}\}$, $\{\mathrm{D}\},\{\mathrm{I}, \mathrm{J}\},\{\mathrm{J}\},\{ \}$
4] \{A\}, \{B, C, D\}, \{E, F, C, D\}, \{K, L, F, C, D\}, \{L, F, C, D\}, \{F, C, D\}, $\{\mathrm{M}, \mathrm{C}, \mathrm{D}\},\{\mathrm{C}, \mathrm{D}\},\{\mathrm{G}, \mathrm{H}, \mathrm{D}\},\{\mathrm{N}, \mathrm{H}, \mathrm{D}\},\{\mathrm{H}, \mathrm{D}\},\{\mathrm{O}, \mathrm{P}, \mathrm{D}\}$, $\{P, D\} \quad$ Pop $P$, check if Pa goal, stop, return $(P,(A, C, H)$ or $(A \rightarrow C \rightarrow H \rightarrow P)$

Iterative widening BFS of a tree


State Queue:
1] $\{\mathbf{A}\},\{B, G, D\},\{E\},\{K\},\{ \}$
2] $\{\mathbf{A}\},\{B, C\},\{C, E\},\{E, G\},\{G, K\},\{K, N\},\{N\},\{ \}$
3] $\{A\},\{B, C, D\},\{C, D, E\},\{D, E, G\},\{E, G, I\},\{G, I, K\},\{I, K, N\}$, $\{\mathrm{K}, \mathrm{N}, \mathrm{Q}\},\{\mathrm{N}, \mathrm{Q}\},\{\mathrm{Q}\}$ Dequeue Q , check if Q a goal, stop, return $(\mathrm{Q},(\mathrm{A}, \mathrm{D}, \mathrm{I})$ ) or $(\mathrm{A} \rightarrow \mathrm{D} \rightarrow \mathrm{I} \rightarrow \mathrm{Q})$

Search tree for Figure 3.1 of Poole and Mackworth, assuming robot starts at 0103, with a goal of being in r123, with all arcs bidirectional (a different assumption than text's)


## Searching a graph with DFS

Successors not replaced on Open (already on

## Open or Path to TOS State Stack $\begin{array}{ll}\text { Open or } & \text { Path th } \\ \text { Explored) } & \text { State }\end{array}$ (Frontier)



## Explored States

|  |  | $\{\mathrm{A}\}$ | $\}$ |
| :--- | :--- | :--- | :---: |
|  | $(\mathrm{A})$ | $\{\mathrm{B}, \mathrm{C}\}$ | $\{\mathrm{A}\}$ |
| A | $(\mathrm{A}, \mathrm{B})$ | $\{\underline{\mathrm{F}, \mathrm{D}, \mathrm{C}\}}$ | $\{\mathrm{A}, \mathrm{B}\}$ |
| B | $(\mathrm{A}, \mathrm{B}, \mathrm{F})$ | $\{\mathrm{G}, \mathrm{H}, \mathrm{D}, \mathrm{C}\}$ | $\{\mathrm{A}, \mathrm{B}, \mathrm{F}\}$ |
| F H | $(\mathrm{A}, \mathrm{B}, \mathrm{F}, \mathrm{G})$ | $\{\mathrm{L}, \mathrm{I}, \mathrm{H}, \mathrm{D}, \mathrm{C}\}$ | $\{\mathrm{A}, \mathrm{B}, \mathrm{F}, \mathrm{G}\}$ |
| G I | (A,B,F,G) | $\{\mathrm{I}, \mathrm{H}, \mathrm{D}, \mathrm{C}\}$ | $\{\mathrm{A}, \mathrm{B}, \mathrm{F}, \mathrm{G}, \mathrm{L}\}$ |
| G L | (A,B,F,G,I) | $\{\mathrm{J}, \mathrm{H}, \mathrm{D}, \mathrm{C}\}$ | $\{\mathrm{A}, \mathrm{B}, \mathrm{F}, \mathrm{G}, \mathrm{L}, \mathrm{I}\}$ |
| H I | (A,B,F,G,I,J | $\{\mathrm{M}, \mathrm{H}, \mathrm{D}, \mathrm{C}\}$ | $\{\mathrm{A}, \mathrm{B}, \mathrm{F}, \mathrm{G}, \mathrm{L}, \mathrm{I}, \mathrm{J}\}$ |
|  |  | M is goal, return $(\mathrm{M},(\mathrm{A}, \mathrm{B}, \mathrm{F}, \mathrm{G}, \mathrm{I}, \mathrm{J})\}$ |  |

## Tennessee Supreme Court



Searching a graph with BFS


Repeated
Successors
Path

|  | ( ) | \{ A \} |  |
| :---: | :---: | :---: | :---: |
|  | (A) | $\{\mathrm{B}, \mathrm{C}\}$ | \{ A \} |
| A | (A) | $\{\mathrm{C}, \mathrm{D}, \mathrm{F}\}$ | \{A,B \} |
| A,D | (A,B) | $\{\mathrm{D}, \mathrm{F}, \underline{\mathrm{E}}\}$ | $\{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$ |
| B,C,E | (A,B) | \{F,E\} | $\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}\}$ |
| B | (A,C) | \{E,G,H\} | \{A,B,C,D,F |
| C,D,H | (A,B,F) | \{G,H \} | \{A,B,C,D,F,E \} |
| F,H | (A,B,F) | \{H,L, I \} | \{A,B,C,D,F,E,G\} |
| E,F,G | (A,B,F,G) | \{L,I,J,K $\}$ | \{A,B,C,D,F,E,G,H\} |
| G,I | (A,B,F,G) | $\{\mathrm{I}, \mathrm{J}, \mathrm{K}\}$ | \{A,B,C,D,F,E,G,H,L\} |
| G,L,J | (A,B,F,H) | \{J,K\} | \{A,B,C,D,F,E,G,H,L,I $\}$ |
| H,I | (A,B,F,H) | \{K, $\underline{\mathrm{M}}$ \} | \{A,B,C,D,F,E,G,H,L,I,J \} |
| H,M | (A,B,F,H,J) | $\{\mathrm{M}\} \mathrm{M}$ | turn (M, (A,B,F,H,J)) |

## Searching a graph with costs (Lowest Cost First Search)

Repeated
 Successors

|  | () | $\{\mathrm{A}(0)\}$ |  |
| :--- | :--- | :--- | :--- |
|  | $(\mathrm{A})$ | $\{\mathrm{B}(1), \mathrm{C}(3)\}$ | $\mathrm{A}(0)$ |
| $\mathrm{A}(1)$ | $(\mathrm{A}, \mathrm{B})$ | $\{\mathrm{F}(2), \mathrm{C}(3), \mathrm{D}(6)\}$ | $\mathrm{A}(0), \mathrm{B}(1)$ |

continue the example

