CS 4260 and CS 5260
Vanderbilt University

Optimization and Constraints
Special Topics

Douglas H. Fisher

Applications of Constraint Satisfaction

Constraint satisfaction problems: Algorithms and applications
by Sally C. Brailsford, Chris N. Potts, Barbara M. Smith
https://www.sciencedirect.com/science/article/pii/S0377221798003646

"*7.1. Location*
A well-known problem in OR involves the location of facilities, such as warehouses, to supply demand to customers. A set {1,…,m} of potential facility locations is given, and a fixed cost fi is incurred if a facility is established at location i (i=1,…,m). There is a set of customers {1,…,n}, and the cost of supplying customer j (j=1,…,n) from a facility at location i is cij. The location problem is to choose a subset of the potential locations at which to establish facilities, and then to assign each customer to one of these facilities, so that the total cost is minimized."

- Transportation hubs

"*7.2. Scheduling*
There are many problems arising in production industries that require jobs to be scheduled on machines. One of the most general and challenging is the job shop scheduling problem in which n jobs are to be scheduled on m machines. Each job comprises a set Oj of operations that must be executed in a specified order. The machine on which any operation o must be performed is mo and the corresponding processing time is po. Since machine capacity constraints prevent any machine from processing more than one operation at the same time, a sequence of operations must be specified for each machine. A common objective is to find a schedule that minimizes the makespan, which is the completion time of the last job."

"7.3. Car sequencing
The car sequencing problem is one of the classical problems in the CSP literature, although it is not widely known in the OR community. The problem arises in the car production industry. After the basic model has been manufactured, various options (for example air-conditioning, metallic paint, ABS brakes, etc.) are added. Different versions of the car require different combinations of options. The cars are placed on a moving assembly line and pass through a number of workstation areas, where these options are installed. Depending on the total time the cars spend within each workstation area, and the set-up and installation times for each option, there are limitations on the rates at which the workstations can handle cars. These are expressed in the form of a ratio: "at most r out of every s consecutive cars can require this option". Given a set of N cars, each requiring a known set of options, the problem is to sequence the cars on the assembly line so that no workstation capacity is exceeded."

- Other manufacturing

- Rostering (for work crews, course schedules, …)
- Vehicle Routing (car pooling, bicycle allotments, …)

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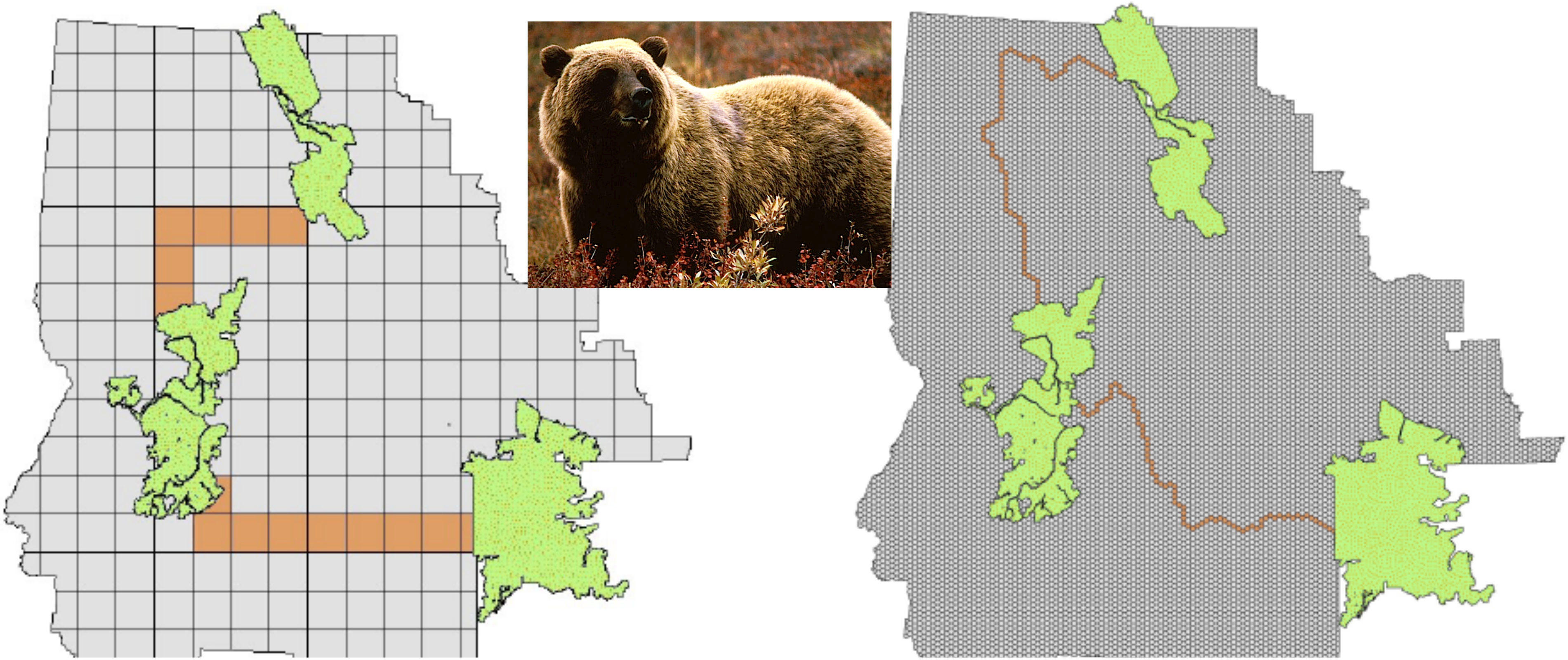
Many problems are a combination of hard and soft constraints (aka optimization under constraints)



Corridor design: connect existing habitat reserves (e.g., shown in green); each parcel of land has a ***cost*** (e.g., purchase price) and a ***utility*** (e.g., habitability). ***maximize the total (additive) utility of the corridor (soft constraint, optimization)***, ***without exceeding a fixed (total cost) budget (hard constraint, constraint satisfaction)***

Incorporating Economic and Ecological Information into the Optimal Design of Wildlife Corridors, Conrad, Gomes, van Hoeve, Sabharwal, and Suter. URI: http://hdl.handle.net/1813/17053.

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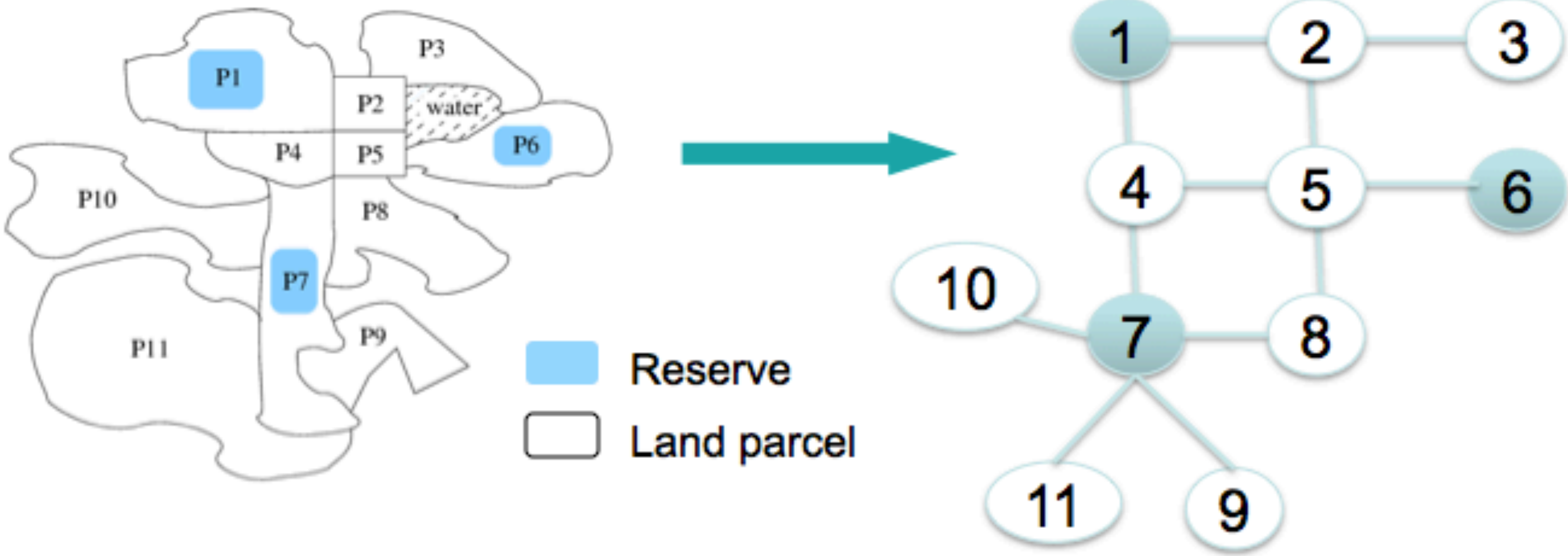
Modeling Wildlife Corridors as a Network



Courtesy of Carla Gomes

The Connection Subgraph Problem (Decision Version)

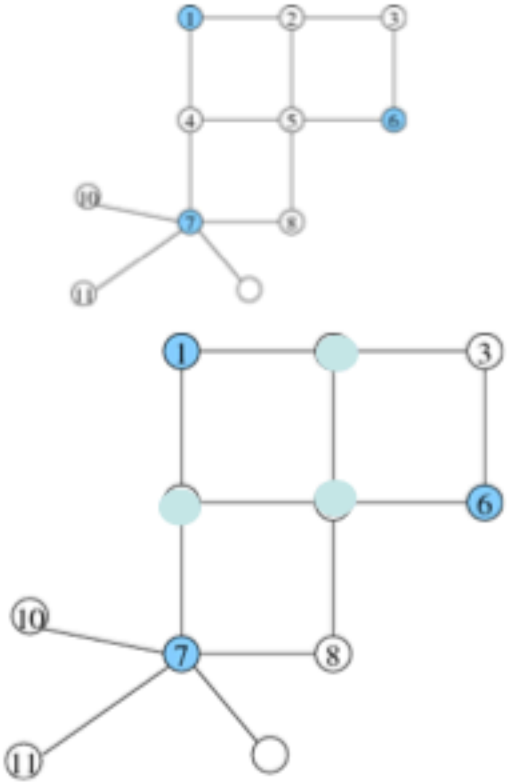
Input

- An undirected graph G = (V,E)
- Terminal vertices $T \subseteq V$
- Vertex cost function: c(v)
- Vertex utility function: u(v)
- Cost bound / budget C; desired utility U

Question

Is there a subgraph H of G such that

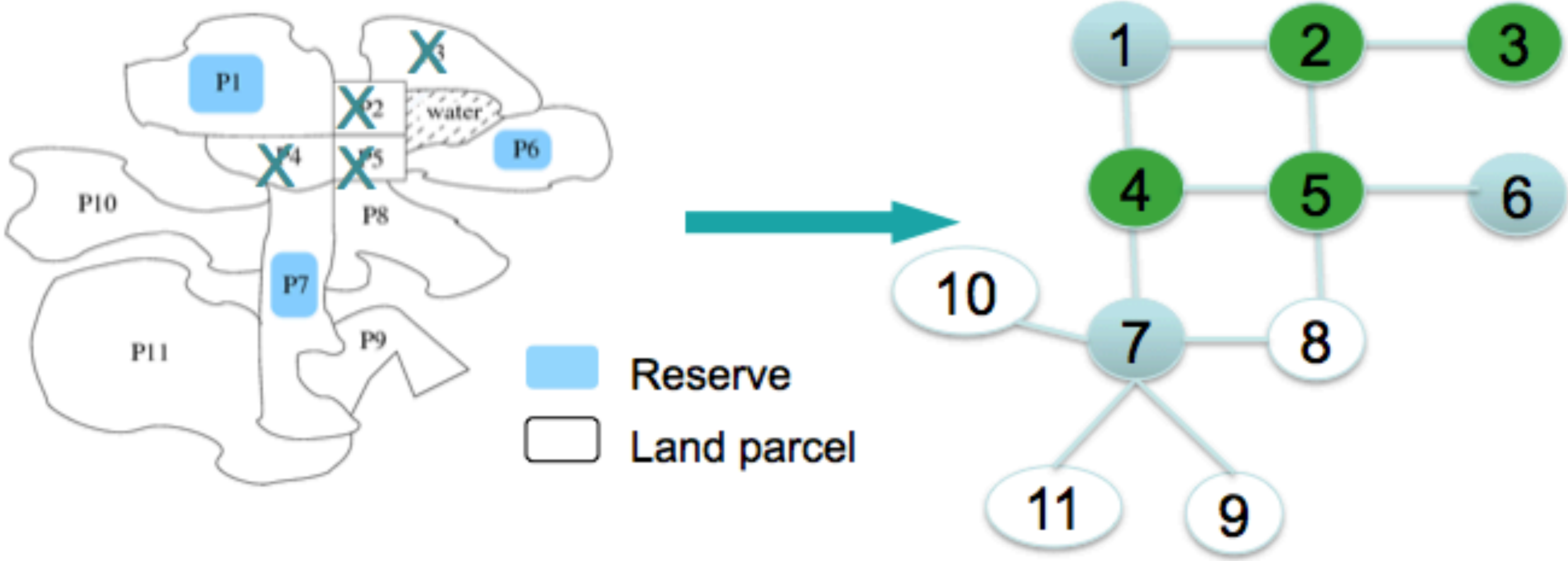
- H is connected and contains T
- $\text{cost}(H) \le C$; $\text{utility}(H) \ge U$?



Courtesy of Carla Gomes

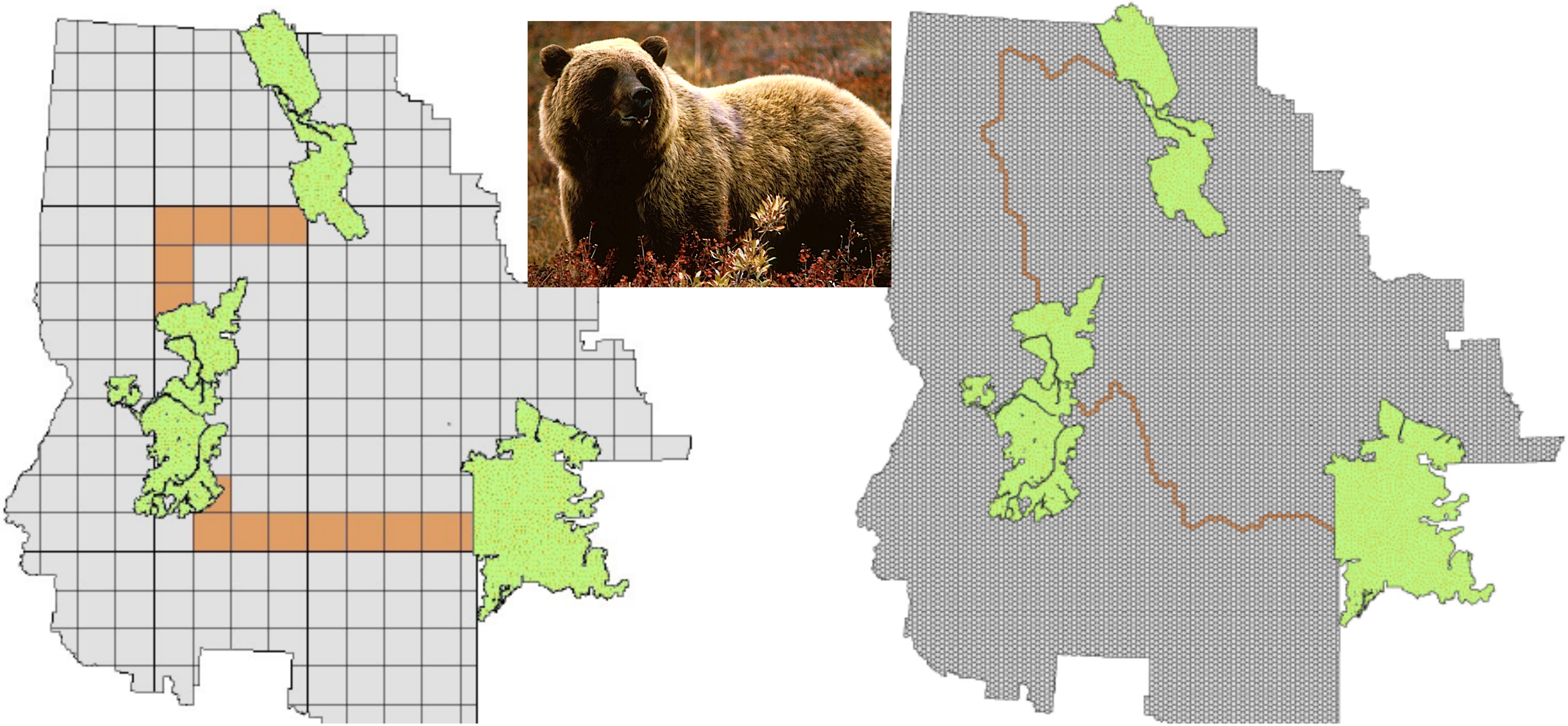
Modeling Wildlife Corridors as a Network

Corridor connecting the reserves



Courtesy of Carla Gomes

NP Hard in worst case … but structure in habitat problems often facilitates fast, near optimal solutions



For example, a greedy approach that seeks min-cost solution, followed by second phase that adds additional vertices for additional utility (e.g., according to U/C ratio and/or heuristic distance)

When would backtracking occur (i.e., what is a dead end)?

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Network Models for Wildlife Conservation
Elaborating one research theme

Motivation: Biodiversity Loss

Mechanism: Wildlife Reserves and Corridors (mitigating land fragmentation)

Constraints: Limited budget, fixed habitability

Computational approaches of optimization and optimization under constraints

- Maximizing utility
- Minimizing Cost
- Maximizing utility without exceeding budget
- Minimizing cost without dropping below threshold utility

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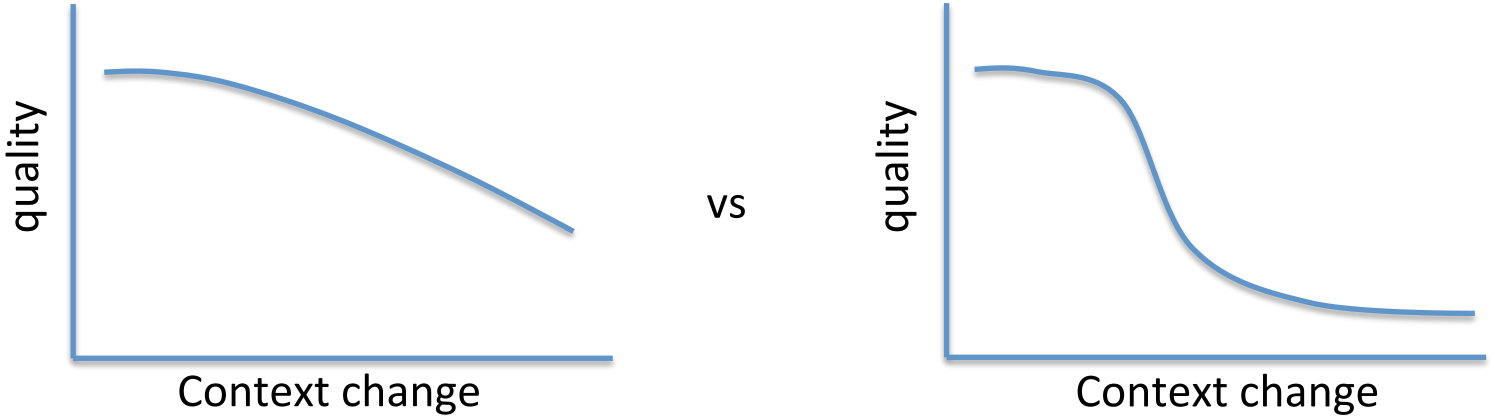
Variations: Examples of sustainability driving computing abstractions

Utilities and costs can change (e.g., based on neighboring states)

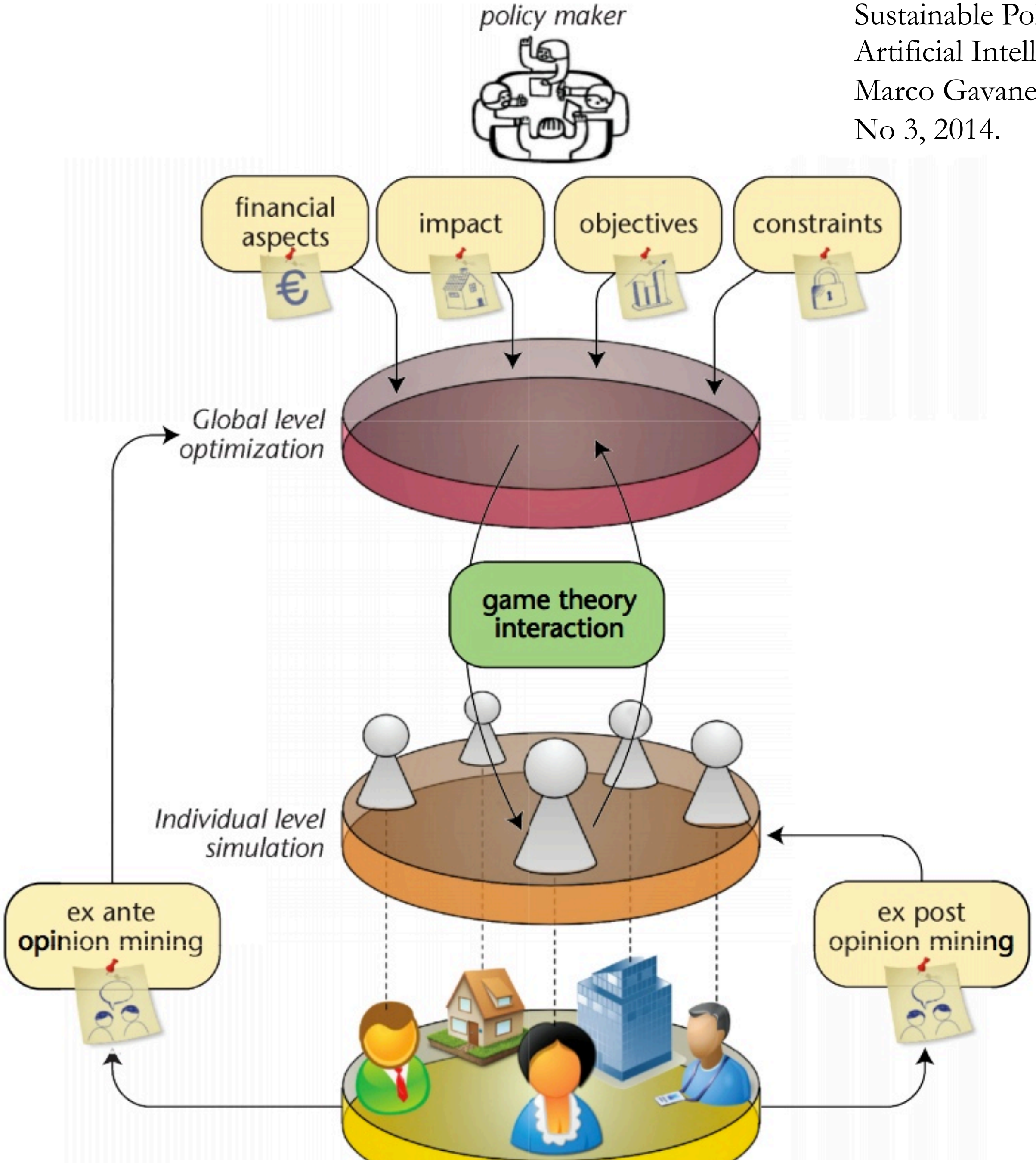
Available budget unfolds piecemeal over time and under uncertainty

D. Golovin et al., “Dynamic Resource Allocation in Conservation Planning,” *Proc. AAAI Conf. Artificial Intelligence*, AAAI, 2011, pp. 1331–1336; www.aaai.org/ocs/index.php/AAAI/AAAI11/paper/view/3617.

Robust Optimization



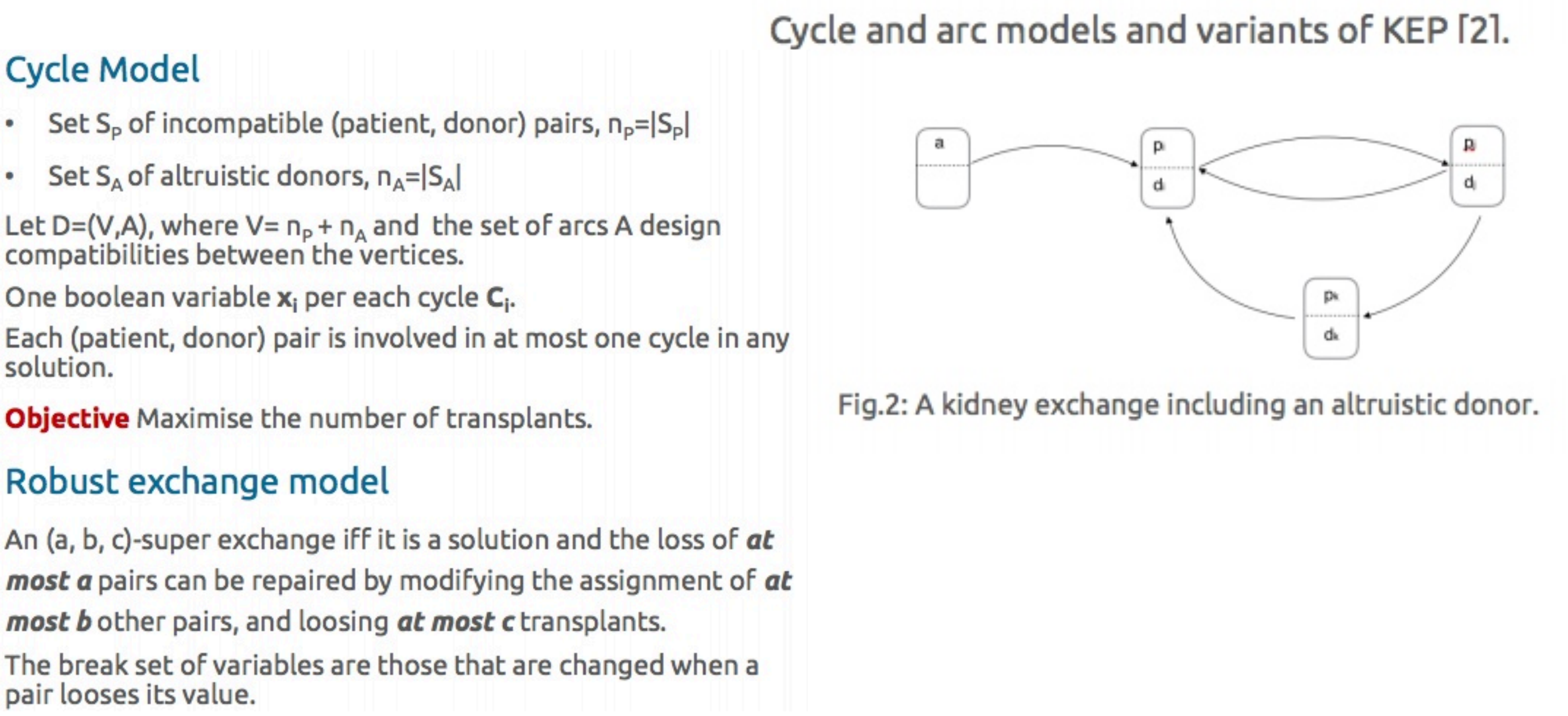
Sustainable Policy Making: A Strategic Challenge for Artificial Intelligence by Michela Milano, Barry O'Sullivan, Marco Gavanelli in AI Magazine, Vol 35, No 3, 2014.



■ *Policy making is an extremely complex process occurring in changing environments and affecting the three pillars of sustainable development: society, economy and the environment. Each political decision in fact implies some form of social reactions, it affects economic and financial aspects and has substantial environmental impacts. Improving decision making in this context could have a huge beneficial impact on all these aspects. There are a number of Artificial Intelligence techniques that could play an important role in improving the policy-making process such as decision support and optimization techniques, game theory, data and opinion mining and agent-based simulation. We outline here some potential use of AI technology as it emerged by the European Union (EU) EU FP7 project ePolicy: Engineering the Policy Making Life Cycle, and we identify some potential research challenges.*

https://www.aaai.org/ojs/index.php/aimagazine/article/view/2534

Robust Kidney Exchange
Sorina Chisca, Michela Milano, and Barry O’Sullivan
https://www.ucc.ie/en/media/academic/computerscience/documents/pdfdocuments/phd-mscposters/Chisca,Sorina.pdf



Cycle and arc models and variants of KEP [2].

Cycle Model

- Set S_P of incompatible (patient, donor) pairs, $n_P=|S_P|$
- Set S_A of altruistic donors, $n_A=|S_A|$

Let D=(V,A), where V= $n_P + n_A$ and the set of arcs A design compatibilities between the vertices.

One boolean variable $\mathbf{x_i}$ per each cycle $\mathbf{C_i}$.

Each (patient, donor) pair is involved in at most one cycle in any solution.

Objective Maximise the number of transplants.

Robust exchange model

An (a, b, c)-super exchange iff it is a solution and the loss of ***at most a*** pairs can be repaired by modifying the assignment of ***at most b*** other pairs, and loosing ***at most c*** transplants.

The break set of variables are those that are changed when a pair looses its value.

Fig.2: A kidney exchange including an altruistic donor.

Drawing Congressional Boundaries

- See for yourself: Interactive congressional maps show why gerrymandering is hard to avoid

http://www.mcall.com/news/nationworld/pennsylvania/mc-nws-gerrymandering-interactive-tool-20180214-story.html

- Autoredistrict (using genetic algorithms)

- Five Thirty Eight https://projects.fivethirtyeight.com/redistricting-maps/

"To explore how subtle (and not-so-subtle) changes to district lines can affect the makeup of the U.S. House, we embarked on a project to redraw each state's boundaries based on different priorities. We used a web-based application created by programmer Dave Bradlee and drew new maps six different ways:

- *To maximize the number of usually Democratic districts*
- *To maximize the number of usually Republican districts*
- *To make the partisan breakdown of states' House seats proportional to the electorate*
- *To promote highly competitive elections*
- *To maximize the number of districts in which one minority group makes up the majority of the voting-age population in the district (what we'll refer to as a majority-minority district)*
- *To be compact while splitting as few counties as possible*

Additionally, we explored an algorithmic approach to optimizing district compactness developed by programmer Brian Olson."

https://www.cmu.edu/news/stories/archives/2017/november/i-cut-you-choose-cake-cutting-protocol-inspires-solution-to-gerrymandering.html