CS 4260 and CS 5260
Vanderbilt University

Lecture on First-Order Logic

This lecture assumes that you have
- Read Chapter 13, through 13.2 of ArtInt

ArtInt: Poole and Mackworth, Artificial Intelligence 2E
at http://artint.info/2e/html/ArtInt2e.html

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Recall Propositional Logic

- Propositions represent facts (declarative language)
- Propositional logic is compositional ($\wedge$, $\vee$, ˜, $\rightarrow$)
- Context-independent and unambiguous
- Rudimentary uncertainty (e.g., using disjunction)
- Well-defined inference (proof) procedures

But propositions are atomic (e.g., Bill_is_happy, Bill_is_in_love)

Adapted from ArtInt

$light\_l_1.$

$light\_l_2.$

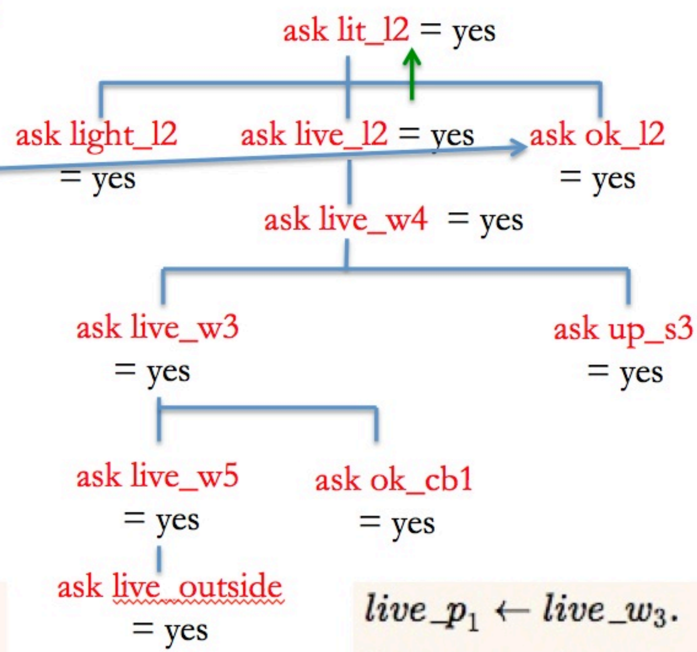
$ok\_l_1.$

$ok\_l_2.$

$ok\_cb_1.$

$ok\_cb_2.$

$live\_outside.$



$down\_s_1.$

$up\_s_2.$

$up\_s_3.$

$live\_l_1 \leftarrow live\_w_0.$

$live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2.$

$live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2.$

$live\_w_1 \leftarrow live\_w_3 \wedge up\_s_1.$

$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1.$

$live\_l_2 \leftarrow live\_w_4.$

$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3.$

$live\_p_1 \leftarrow live\_w_3.$

$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$

$live\_p_2 \leftarrow live\_w_6.$

$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$

$live\_w_5 \leftarrow live\_outside.$

$lit\_l_1 \leftarrow light\_l_1 \wedge live\_l_1 \wedge ok\_l_1.$

$lit\_l_2 \leftarrow light\_l_2 \wedge live\_l_2 \wedge ok\_l_2.$

Douglas H. Fisher

First Order Logic
(aka First Order Predicate Calculus)

Take best of propositional logic and natural language

Facts are not atomic, but are expressed as relations between objects

Objects (Constants): Mary, Bill, Hua, Ananya, Fido, Library, Restaurant, Class, Home

Relations (Predicate): Likes, At, Human, Mortal, …

Knowledge Base:
- Likes(Mary, Ananya),
- Likes(Hua, Mary),
- Likes (Mary, Hua),
- Likes(Ananya, Bill),
- At(Mary, Library)
- Human(Mary)
- Human(Hua)
- Human(Bill)
- Human(Ananya)
- ~Human(Fido)
- Mortal(Bill)

First Order Logic

Take best of propositional logic and natural language

Facts are not atomic, but are expressed as relations between objects

Objects (Constants): Mary, Bill, Hua, Ananya, Fido, Library, Restaurant, Class, Home

Relations (Predicate): Likes, At, Human, Mortal, …

Knowledge Base:

- Likes(Mary, Ananya),
- Likes(Hua, Mary),
- Likes (Mary, Hua),
- Likes(Ananya, Bill),
- At(Mary, Library)
- Human(Mary)
- Human(Hua)
- Human(Bill)
- Human(Ananya)
- ~Human(Fido)
- Mortal(Bill)

Variables allow for general statements (which are true or false)

implication

Human(X) → Mortal(X) (or Mortal(X) ← Human(X))

$\forall_X$ (Human(X) → Mortal(X)) ForAll$_X$ (Human(X) → Mortal(X))

A quantifier

equivalent to

Human(Mary) → Mortal(Mary) $\wedge$
Human(Bill) → Mortal(Bill) $\wedge$
Human(Hua) → Mortal(Hua) $\wedge$ *conjunction*
Human(Ananya) → Mortal(Ananya) $\wedge$
Human(Fido) → Mortal(Fido) $\wedge$
Human(Library) → Mortal(Library) $\wedge$
Human(Restaurant) → Mortal(Restaurant) $\wedge$

….

First Order Logic

Take best of propositional logic and natural language

Facts are not atomic, but are expressed as relations between objects

Objects (Constants): Mary, Bill, Hua, Ananya, Fido, Library, Restaurant, Class, Home

Relations (Predicate): Likes, At, Human, Mortal, …

Knowledge Base:

- Likes(Mary, Ananya),
- Likes(Hua, Mary),
- Likes (Mary, Hua),
- Likes(Ananya, Bill),
- At(Mary, Library)
- Human(Mary)
- Human(Hua)
- Human(Bill)
- Human(Ananya)
- ~Human(Fido)
- Mortal(Bill)

Variables allow for general statements (which are true or false)

Human(X)

$\forall_X$ (Human(X))

ForAll$_X$ (Human(X))

equivalent to

Human(Mary) $\wedge$
Human(Bill) $\wedge$
Human(Hua) $\wedge$
Human(Ananya) $\wedge$
Human(Fido) $\wedge$
Human(Library) $\wedge$
Human(Restaurant) $\wedge$
….

In propositional representation, each would have to be represented as an atomic proposition (e.g., human-mary, human-bill, … ~human-restaurant …)

First Order Logic

Take best of propositional logic and natural language

Facts are not atomic, but are expressed as relations between objects

Objects (Constants): Mary, Bill, Hua, Ananya, Fido, Library, Restaurant, Class, Home

Relations (Predicate): Likes, At, Human, Mortal, …

Knowledge Base:

- Likes(Mary, Ananya),
- Likes(Hua, Mary),
- Likes (Mary, Hua),
- Likes(Ananya, Bill),
- At(Mary, Library)
- Human(Mary)
- Human(Hua)
- Human(Bill)
- Human(Ananya)
- ~Human(Fido)
- Mortal(Bill)

Another quantifier

Variables allow for general statements (which are true or false)

Human(X)

$\exists_X$ (Human(X))

Exists$_X$ (Human(X))

equivalent to

Human(Mary) $\vee$
Human(Bill) $\vee$
Human(Hua) $\vee$
Human(Ananya) $\vee$
Human(Fido) $\vee$
Human(Library) $\vee$
Human(Restaurant) $\vee$
….

disjunction

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First Order Logic

Vocabularies can vary, and may seem arbitrary, but choices depend on needs of the AI and need to be consistent (?) within an application

Knowledge Base:

- Likes(Mary, Ananya), or Feeling(Mary, Ananya, Likes)
- Likes(Hua, Mary),
- Likes (Mary, Hua),
- Likes(Ananya),
- At(Mary, Library)
- Human(Mary) or Species(Mary, Human) or Species(Mary, HomoSapien)
- Human(Hua)
- Human(Bill)
- Human(Ananya)
- ~Human(Fido)
- Mortal(Bill)

First Order Logic

Represent that Mary is a parent of Jim

- Parent(Mary, Jim)

Represent that Jim is a child of Mary

- Child(Jim, Mary)

Represent that if X is a parent of Y then Y is a child of X

First Order Logic

Represent that Mary is a parent of Jim

- Parent(Mary, Jim)

Represent that Jim is a child of Mary

- Child(Jim, Mary)

Represent that if X is a parent of Y then Y is a child of X

- $\text{ForAll}_{X,Y}(\text{Parent}(X, Y) \rightarrow \text{Child}(Y, X))$

First Order Logic

Represent that Mary is a parent of Jim

- Parent(Mary, Jim)

Represent that Jim is a child of Mary

- Child(Jim, Mary)

Represent that if X is a parent of Y then Y is a child of X, and vice versa

- $\text{ForAll}_{X,Y}(\text{Parent}(X, Y) \rightarrow \text{Child}(Y, X))$
- $\text{ForAll}_{X,Y}(\text{Parent}(X, Y) \leftarrow \text{Child}(Y, X))$

$\text{ForAll}_{X,Y}(\text{Parent}(X, Y) \leftrightarrow \text{Child}(Y, X))$

First Order Logic

Represent that Mary is a parent of Jim

- Parent(Mary, Jim)

Represent that Jim is a child of Mary

- Child(Jim, Mary)

Represent that if X is a parent of Y then Y is a child of X, and vice versa

- $\text{ForAll}_{X,Y}(\text{Parent}(X, Y) \rightarrow \text{Child}(Y, X))$
- $\text{ForAll}_{X,Y}(\text{Parent}(X, Y) \leftarrow \text{Child}(Y, X))$

Represent that every X has a parent, Y

First Order Logic

Represent that Mary is a parent of Jim

- Parent(Mary, Jim)

Represent that Jim is a child of Mary

- Child(Jim, Mary)

Represent that if X is a parent of Y then Y is a child of X, and vice versa

- $\text{ForAll}_{X,Y}(\text{Parent}(X, Y) \rightarrow \text{Child}(Y, X))$
- $\text{ForAll}_{X,Y}(\text{Parent}(X, Y) \leftarrow \text{Child}(Y, X))$

Represent that every X has a parent, Y

- $\text{ForAll}_X\text{Exists}_Y(\text{Parent}(Y, X))$ or $\text{ForAll}_X\text{Exists}_Y(\text{Child}(X, Y))$ or
  $\text{ForAll}_Y\text{Exists}_X(\text{Parent}(X, Y))$ or
  $\text{ForAll}_Y\text{Exists}_X(\text{Child}(Y, X))$

Represent that every human X has a parent, Y

First Order Logic

Represent that Mary is a parent of Jim

- Parent(Mary, Jim)

Represent that Jim is a child of Mary

- Child(Jim, Mary)

Represent that if X is a parent of Y then Y is a child of X, and vice versa

- $\mathrm{ForAll}_{X,Y}(\mathrm{Parent}(X, Y) \rightarrow \mathrm{Child}(Y, X))$
- $\mathrm{ForAll}_{X,Y}(\mathrm{Parent}(X, Y) \leftarrow \mathrm{Child}(Y, X))$

~~Represent that every X has a parent, Y~~

- $\cancel{\mathrm{ForAll}_{X}\mathrm{Exists}_{Y}(\mathrm{Parent}(X, Y))}$

Represent that every human X has a parent, Y

- $\mathrm{ForAll}_{X}\mathrm{Exists}_{Y}(\mathrm{Human}(X) \rightarrow \mathrm{Parent}(Y, X))$ or ….

Represent that a grandparent is the parent of a parent

First Order Logic

Represent that Mary is a parent of Jim

- Parent**Of**(Mary, Jim)

Represent that Jim is a child of Mary

- Child**Of**(Jim, Mary)

Represent that if X is a parent of Y then Y is a child of X, and vice versa

- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$
- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \leftarrow \text{ChildOf}(Y, X))$

Changed because I may want a unary Parent and Child test too

Represent that every human X has a parent, Y

- $\text{ForAll}_{X}\text{Exists}_{Y}(\text{Human}(X) \rightarrow \text{ParentOf}(Y, X))$

Represent that a grandparent is the parent of a parent

- $\text{ForAll}_{X}\text{Exists}_{Y}\text{Exists}_{Z}(\text{Grandparent}(X) \rightarrow (\text{ParentOf}(X, Y) \land \text{ParentOf}(Y, Z)))$

In the video, I don't show the consequent delineated by ( ), and I should have (here and elsewhere). In general, we will clearly specify operator precedence with parentheses.

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First Order Logic

Represent that Mary is a parent of Jim

- ParentOf(Mary, Jim)

Represent that Jim is a child of Mary

- ChildOf(Jim, Mary)

Represent that if X is a parent of Y then Y is a child of X, and vice versa

- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$
- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \leftarrow \text{ChildOf}(Y, X))$

Represent that every human X has a parent, Y

- $\text{ForAll}_{X}\text{Exists}_{Y}(\text{Human}(X) \rightarrow \text{ParentOf}(Y, X))$

Represent that a grandparent is the parent of a parent, and vice versa

- $\text{ForAll}_{X}\text{Exists}_{Y}\text{Exists}_{Z}(\text{Grandparent}(X) \rightarrow (\text{ParentOf}(X, Y) \wedge \text{ParentOf}(Y, Z)))$
- $\text{ForAll}_{X}\text{ForAll}_{Y}\text{ForAll}_{Z}(\text{Grandparent}(X) \leftarrow (\text{ParentOf}(X, Y) \wedge \text{ParentOf}(Y, Z)))$

First Order Logic Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)

Human(Steve)

- ChildOf(Jim, Mary)

- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$
- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \leftarrow \text{ChildOf}(Y, X))$

Through inference (universal instantiation and modus ponens in this case)

ParentOf(Mary, Jim), $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$ |- ChildOf(Jim, Mary)
ParentOf(Mary, Steve), $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$ |- ChildOf(Steve, Mary)
ParentOf(Jim, Gene), $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$ |- ChildOf(Gene, Jim)
ParentOf(Hua, Jia), $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$ |- ChildOf(Jia, Hua)

- $\text{ForAll}_{X}\text{Exists}_{Y}(\text{Human}(X) \rightarrow \text{ParentOf}(Y, X))$

Through inference (universal instantiation and modus ponens in this case)

Human(Steve), $\text{ForAll}_{X}\text{Exists}_{Y}(\text{Human}(X) \rightarrow \text{ParentOf}(Y, X))$ |- $\text{Exists}_{Y}\text{Parent}(Y, \text{Steve})$

- $\text{ForAll}_{X}\text{Exists}_{Y}\text{Exists}_{Z}(\text{Grandparent}(X) \rightarrow (\text{ParentOf}(X, Y) \wedge \text{ParentOf}(Y, Z)))$
- $\text{ForAll}_{X}\text{ForAll}_{Y}\text{ForAll}_{Z}(\text{Grandparent}(X) \leftarrow (\text{ParentOf}(X, Y) \wedge \text{ParentOf}(Y, Z)))$

Through inference (universal and existential instantiation and modus ponens in this case)

ParentOf(Mary, Jim), ParentOf(Jim, Gene), $\text{ForAll}_{X}\text{ForAll}_{Y}\text{ForAll}_{Z}(\text{Grandparent}(X) \leftarrow (\text{ParentOf}(X, Y) \wedge \text{ParentOf}(Y, Z)))$ |- Grandparent(Mary)

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First Order Logic
Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)
- ChildOf(Jim, Mary)
- Human(Steve)
- $ForAll_{X,Y}(ParentOf(X, Y) \rightarrow ChildOf(Y, X))$
- $ForAll_{X,Y}(ParentOf(X, Y) \leftarrow ChildOf(Y, X))$
- $ForAll_{X}Exists_{Y}(Human(X) \rightarrow ParentOf(Y, X))$
- $ForAll_{X}Exists_{Y}Exists_{Z}(Grandparent(X) \leftrightarrow (ParentOf(X, Y) \wedge ParentOf(Y, Z)))$

Predicate names match

$ParentOf(Mary, Jim), ForAll_{X,Y}(ParentOf(X, Y) \rightarrow ChildOf(Y, X))$

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First Order Logic Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)
- ChildOf(Jim, Mary)
- Human(Steve)
- ForAll$_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X))
- ForAll$_{X,Y}$(ParentOf(X, Y) ← ChildOf(Y, X))
- ForAll$_X$Exists$_Y$(Human(X) → ParentOf(Y, X))
- ForAll$_X$Exists$_Y$Exists$_Z$(Grandparent(X) ←→ (ParentOf(X, Y) ∧ ParentOf(Y, Z)))

Predicate names match

ParentOf(Mary, Jim), ForAll$_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X))

Set Y to Jim

Set X to Mary

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First Order Logic
Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)
- ChildOf(Jim, Mary)
- Human(Steve)
- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$
- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \leftarrow \text{ChildOf}(Y, X))$
- $\text{ForAll}_{X}\text{Exists}_{Y}(\text{Human}(X) \rightarrow \text{ParentOf}(Y, X))$
- $\text{ForAll}_{X}\text{Exists}_{Y}\text{Exists}_{Z}(\text{Grandparent}(X) \leftrightarrow (\text{ParentOf}(X, Y) \wedge \text{ParentOf}(Y, Z)))$

Predicate names match

$\text{ParentOf}(\text{Mary}, \text{Jim}),\ \text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$

Set X to Mary

Set Y to Jim

This process is called unification
(or instantiation)

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First Order Logic Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)
- ChildOf(Jim, Mary)
- Human(Steve)
- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$
- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \leftarrow \text{ChildOf}(Y, X))$
- $\text{ForAll}_{X}\text{Exists}_{Y}(\text{Human}(X) \rightarrow \text{ParentOf}(Y, X))$
- $\text{ForAll}_{X}\text{Exists}_{Y}\text{Exists}_{Z}(\text{Grandparent}(X) \leftrightarrow (\text{ParentOf}(X, Y) \land \text{ParentOf}(Y, Z)))$

$$\text{ParentOf(Mary, Jim)}, \text{ForAll}_{X,Y}(\text{ParentOf}(\overset{\text{Mary}}{\cancel{X}}, \underset{\text{Jim}}{\cancel{Y}}) \rightarrow \text{ChildOf}(\underset{\text{Jim}}{\cancel{Y}}, \overset{\text{Mary}}{\cancel{X}}))$$

Substitute variable values throughout

First Order Logic
Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)
- ChildOf(Jim, Mary)
- Human(Steve)
- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \rightarrow \text{ChildOf}(Y, X))$
- $\text{ForAll}_{X,Y}(\text{ParentOf}(X, Y) \leftarrow \text{ChildOf}(Y, X))$
- $\text{ForAll}_{X}\text{Exists}_{Y}(\text{Human}(X) \rightarrow \text{ParentOf}(Y, X))$
- $\text{ForAll}_{X}\text{Exists}_{Y}\text{Exists}_{Z}(\text{Grandparent}(X) \leftrightarrow (\text{ParentOf}(X, Y) \wedge \text{ParentOf}(Y, Z)))$

$\text{ParentOf}(\text{Mary, Jim}),\ \cancel{\text{ForAll}_{X,Y}}(\text{ParentOf}(\text{Mary, Jim}) \rightarrow \text{ChildOf}(\text{Jim, Mary}))$

Modus Ponens now applies

$\text{ParentOf}(\text{Mary, Jim}),\ \cancel{\text{ForAll}_{X,Y}}(\text{ParentOf}(\text{Mary, Jim}) \rightarrow \text{ChildOf}(\text{Jim, Mary}))\ \vdash \text{ChildOf}(\text{Jim, Mary})$

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First Order Logic Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)
- ChildOf(Jim, Mary)
- Human(Steve)
- $\text{ForAll}_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X))
- $\text{ForAll}_{X,Y}$(ParentOf(X, Y) ← ChildOf(Y, X))
- $\text{ForAll}_{X}\text{Exists}_{Y}$(Human(X) → ParentOf(Y, X))
- $\text{ForAll}_{X}\text{Exists}_{Y}\text{Exists}_{Z}$(Grandparent(X) ←→ (ParentOf(X, Y) ∧ ParentOf(Y, Z)))

ParentOf(Mary, Jim), $\text{ForAll}_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X)) |- ChildOf(Jim, Mary)

Human(Steve), $\text{ForAll}_{X}\text{Exists}_{Y}$(Human(X) → ParentOf(Y, X)) |- Exists_{Y}Parent(Y, Steve)

ParentOf(Mary, Jim),

ParentOf(Jim, Gene),

$\text{ForAll}_{X}\text{ForAll}_{Y}\text{ForAll}_{Z}$(Grandparent(X) ← (ParentOf(X, Y) ∧ ParentOf(Y, Z)))

Unify antecedents with KB axioms

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First Order Logic Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)
- ChildOf(Jim, Mary)
- Human(Steve)
- $\mathrm{ForAll}_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X))
- $\mathrm{ForAll}_{X,Y}$(ParentOf(X, Y) ← ChildOf(Y, X))
- $\mathrm{ForAll}_{X}\mathrm{Exists}_{Y}$(Human(X) → ParentOf(Y, X))
- $\mathrm{ForAll}_{X}\mathrm{Exists}_{Y}\mathrm{Exists}_{Z}$(Grandparent(X) ←→ (ParentOf(X, Y) ∧ ParentOf(Y, Z)))

ParentOf(Mary, Jim), $\mathrm{ForAll}_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X)) |- ChildOf(Jim, Mary)

Human(Steve), $\mathrm{ForAll}_{X}\mathrm{Exists}_{Y}$(Human(X) → ParentOf(Y, X)) |- Exists_{Y}Parent(Y, Steve)

Unify antecedents with KB axioms

ParentOf(Mary, Jim),

ParentOf(Jim, Gene),

$$\mathrm{ForAll}_X\mathrm{ForAll}_Y\mathrm{ForAll}_Z(\mathrm{Grandparent}(\overset{\mathrm{Mary}}{\cancel{X}}) \leftarrow (\mathrm{ParentOf}(\overset{\mathrm{Mary}}{\cancel{X}}, \underset{\mathrm{Jim}}{\cancel{Y}}) \wedge \mathrm{ParentOf}(\underset{\mathrm{Jim}}{\cancel{Y}}, \overset{\mathrm{Gene}}{\cancel{Z}})))$$

First Order Logic Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)
- ChildOf(Jim, Mary)
- Human(Steve)
- $\text{ForAll}_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X))
- $\text{ForAll}_{X,Y}$(ParentOf(X, Y) ← ChildOf(Y, X))
- $\text{ForAll}_{X}\text{Exists}_{Y}$(Human(X) → ParentOf(Y, X))
- $\text{ForAll}_{X}\text{Exists}_{Y}\text{Exists}_{Z}$(Grandparent(X) ←→ (ParentOf(X, Y) ∧ ParentOf(Y, Z)))

ParentOf(Mary, Jim), $\text{ForAll}_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X)) |- ChildOf(Jim, Mary)

Human(Steve), $\text{ForAll}_{X}\text{Exists}_{Y}$(Human(X) → ParentOf(Y, X)) |- Exists_{Y}Parent(Y, Steve)

Unify antecedents with KB axioms

ParentOf(Mary, Jim),

ParentOf(Jim, Gene),

$\cancel{\text{ForAll}_X\text{ForAll}_Y\text{ForAll}_Z}(\text{Grandparent}(\overset{\text{Mary}}{\cancel{X}}) \leftarrow (\text{ParentOf}(\overset{\text{Mary}}{\cancel{X}}, \underset{\text{Jim}}{\cancel{Y}}) \wedge \text{ParentOf}(\underset{\text{Jim}}{\cancel{Y}}, \overset{\text{Gene}}{\cancel{Z}})))$

|- Grandparent(Mary)

First Order Logic Inference

Knowledge Base

- ParentOf(Mary, Jim)
- ParentOf(Mary, Steve)
- ParentOf(Jim, Gene)
- ParentOf(Hua, Jia)
- ChildOf(Jim, Mary)
- Human(Steve)
- $\text{ForAll}_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X))
- $\text{ForAll}_{X,Y}$(ParentOf(X, Y) ← ChildOf(Y, X))
- $\text{ForAll}_{X}\text{Exists}_{Y}$(Human(X) → ParentOf(Y, X))
- $\text{ForAll}_{X}\text{Exists}_{Y}\text{Exists}_{Z}$(Grandparent(X) ←→ (ParentOf(X, Y) ∧ ParentOf(Y, Z)))

ParentOf(Mary, Jim), $\text{ForAll}_{X,Y}$(ParentOf(X, Y) → ChildOf(Y, X)) |- ChildOf(Jim, Mary)

Human(Steve), $\text{ForAll}_{X}\text{Exists}_{Y}$(Human(X) → ParentOf(Y, X)) |- Exists_{Y}Parent(Y, Steve)

ParentOf(Mary, Steve),

~~ParentOf(Jim, Gene),~~

$\text{ForAll}_{X}\text{ForAll}_{Y}\text{ForAll}_{Z}(\text{Grandparent}(\overset{\text{Mary}}{\cancel{X}}) \leftarrow (\text{ParentOf}(\underset{\text{Mary}}{\cancel{X}}, \overset{\text{Steve}}{\cancel{Y}}) \wedge \text{ParentOf}(\overset{\text{Steve}}{\cancel{Y}}, Z)))$

No statement in KB unifies with this

Inference is a search, and some paths lead to dead ends

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Which of the choices below represent valid interpretations of

> "Every martian can fool some of the people all of the time (and these can be different subsets of people for each martian)"

Circle all valid interpretations.

a) Forall(x) [Martian(x) → Exists(y)[Person(y) AND ForAll(t)[Time(t) → Fools(x,y,t)]]]

b) Forall(x) [Martian(x) AND Exists(y)[Person(y) AND ForAll(t)[Time(t) → Fools(x,y,t)]]]

c) Exists(y) [Person(y) AND ForAll(x)[Martian(x) → ForAll(t)[Time(t) → Fools(x,y,t)]]]

d) Exists(y) ForAll(x) ForAll(t)[Person(y) AND (Martian(x) → (Time(t) → Fools(x,y,t)))]

e) ForAll(x) Exists(y) ForAll(t) [Martian(x) → (Person(y) AND (Time(t) → Fools(x,y,t))]

f) None of the above

Circle the subsets of the choices above (a-e) that have the same meaning (i.e. are equivalence sets):

i) a, b   ii) c, d   iii) a,b,e   iv) b, c, d   v) d, e   vi) None of these

Which of the choices below may represent valid interpretations of

"Every martian can fool some of the people all of the time (and these can be different subsets of people for each martian)"

Circle all valid interpretations.

(C&D are valid if it's the same people who are fooled by each martian)

a) Forall(x) [Martian(x) → Exists(y)[Person(y) AND ForAll(t)[Time(t) → Fools(x,y,t)]]]

b) Forall(x) [Martian(x) AND Exists(y)[Person(y) AND ForAll(t)[Time(t) → Fools(x,y,t)]]]

c) Exists(y) [Person(y) AND ForAll(x)[Martian(x) → ForAll(t)[Time(t) → Fools(x,y,t)]]]

d) Exists(y) ForAll(x) ForAll(t)[Person(y) AND (Martian(x) → (Time(t) → Fools(x,y,t)))]

e) ForAll(x) Exists(y) ForAll(t) [Martian(x) → (Person(y) AND (Time(t) → Fools(x,y,t))]

f) None of the above

a and e are equivalent, but not with b

Circle the subsets of the choices above (a-e) that have the same meaning (i.e. are equivalence sets):

i) a, b ii) c, d iii) a,b,e iv) b, c, d v) d, e vi) None of these

Exercise: Represent the following objects and relationships in first-order predicate calculus

- Students are people
- Instructors are people too
- Students take courses
- CS4260 is a course
- Courses have 0 or more prerequisites, which are also courses
- CS3250 and CS3251 are prerequisites of CS4260
- If a student takes a course, they must already have taken its prerequisites