Supplemental Online Appendix

This online appendix accompanies Sterba and Bauer (2010) "Matching method with theory in person-oriented developmental psychopathology research" *Development & Psychopathology, 22,* 239-244. Here we present model equations for each person-oriented method discussed in the manuscript. Here we also provide examples of how to test person-oriented principles listed in manuscript Table 1—where possible—using a given method. In each case *rejecting* the null hypothesis listed (H_o) constitutes support for the person-oriented principle.

Less-restrictive variable oriented methods

Latent growth model (LGM). Like all structural equation models, the LGM consists of a measurement model and a structural model. Let $\mathbf{y}_i$ be a *p* x 1 vector of repeated measures for person *i*. In Figure 1 Panel A, *p*=5. The measurement model is

$$\mathbf{y}_i = \boldsymbol{\upsilon} + \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \text{ where often } \boldsymbol{\varepsilon}_i \sim N(0, \boldsymbol{\Theta}). \quad (1)$$

The structural model is

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha} + \boldsymbol{\varsigma}_i \text{ where } \boldsymbol{\varsigma}_i \sim N(0, \boldsymbol{\Phi}). \quad (2)$$

$\boldsymbol{\upsilon}$ is a *p* x 1 vector of item intercepts fixed to 0 (not shown in Figure 1 Panel A). $\boldsymbol{\Lambda}$ is a *p* x *q* matrix of factor loadings (fixed to 1, 1, 1, 1, 1 for the first column and 0, 1, 2, 3, 4 for the second column in Figure 1 Panel A to define intercept and slope growth factors). $\boldsymbol{\eta}_i$ is a *q* x 1 vector of latent growth factor scores, and in Figure 1 Panel A *q*=2. $\boldsymbol{\varepsilon}_i$ is a *p* x 1 vector of time-specific residuals. $\boldsymbol{\Theta}$ is a typically-diagonal *p* x *p* covariance matrix of $\boldsymbol{\varepsilon}_i$. $\boldsymbol{\alpha}$ is a *q* x 1 vector of growth factor means (not shown in Figure 1 Panel A). $\boldsymbol{\varsigma}_i$ is a *q* x 1 vector of individual deviations from those growth factor means. $\boldsymbol{\Phi}$ is a typically-unstructured *q* x *q* covariance matrix of $\boldsymbol{\varsigma}_i$. Manuscript Table 2 lists which person-oriented principles are testable with LGM. Next we give examples of how these principles could be tested.

(1) *Interindividual differences/intraindividual change principle.* Assuming *pattern summarization and pattern parsimony* principles are invalid, an example of testing the

interindividual differences/intraindividual change principle is $H_0 : \phi_{11} = 0$ in Figure 1 Panel A (i.e. no slope variability).

(2) *Individual specificity principle.* Under the same assumption, an example of testing the *individual specificity principle* is $H_0 : \mathbf{\Phi} = 0$ (i.e. no variance or covariance in growth factors).

(3) *Complex-interactions principle.* Under the same assumption, an example of testing the *complex-interactions principle* in the LGM in Figure 1 Panel A is to expand the structural model in Equation (2) to regress growth factors on a vector of person-level predictors. An example vector of person-level predictors is $\mathbf{x}_i = [x_{1i},\ x_{2i},\ x_{1i}x_{2i}]'$, though more predictors and interaction terms could certainly be included. This yields:

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha} + \mathbf{\Gamma}\mathbf{x}_i + \boldsymbol{\varsigma}_i \tag{3}$$

where $\boldsymbol{\eta}_i$, $\boldsymbol{\varsigma}_i$, and $\boldsymbol{\alpha}$ are 2 x 1, $\mathbf{x}_i$ is 3 x 1, and $\mathbf{\Gamma}$ is a 2 x 3 matrix of regression coefficients. Then, we can, for example, test: $H_0 : \gamma_{23} = 0$ (*time* by x_{1i} by x_{2i} interaction).

(4) *Holism principle.* Limited testing of the *holistic principle* in the LGM is possible by expanding the univariate Equation (1)-(2) to include one or more parallel growth processes. Supposing the original growth process (labeled (*a*)) and additional growth process (labeled (*b*)) each had *p*=5 and *q*=2, this would entail stacking the vectors of repeated measures, intercepts, and residuals for the two processes $\mathbf{y}_i = \begin{bmatrix} \mathbf{y}_i^{(a)} \\ \mathbf{y}_i^{(b)} \end{bmatrix}$, $\boldsymbol{\upsilon} = \begin{bmatrix} \boldsymbol{\upsilon}^{(a)} \\ \boldsymbol{\upsilon}^{(b)} \end{bmatrix}$, and $\boldsymbol{\varepsilon}_i = \begin{bmatrix} \boldsymbol{\varepsilon}^{(a)} \\ \boldsymbol{\varepsilon}^{(b)} \end{bmatrix}$ such that $\mathbf{y}_i$, $\boldsymbol{\upsilon}$, and $\boldsymbol{\varepsilon}_i$ are now each 10 x 1. We would also stack vectors of growth factor scores, growth factor means and mean deviations for the two processes, $\boldsymbol{\eta}_i = \begin{bmatrix} \boldsymbol{\eta}_i^{(a)} \\ \boldsymbol{\eta}_i^{(b)} \end{bmatrix}$, $\boldsymbol{\alpha} = \begin{bmatrix} \boldsymbol{\alpha}^{(a)} \\ \boldsymbol{\alpha}^{(b)} \end{bmatrix}$, and $\boldsymbol{\varsigma}_i = \begin{bmatrix} \boldsymbol{\varsigma}_i^{(a)} \\ \boldsymbol{\varsigma}_i^{(b)} \end{bmatrix}$ such that $\boldsymbol{\eta}_i$, $\boldsymbol{\varsigma}_i$, and $\boldsymbol{\alpha}$ are now each 4 x 1. We would expand $\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}^{(a)} & 0 \\ 0 & \mathbf{\Lambda}^{(b)} \end{bmatrix}$ to be 10 x 4 and block diagonal with 5 x 2 blocks and expand $\mathbf{\Theta} = \begin{bmatrix} \mathbf{\Theta}^{(a)} & \mathbf{\Theta}^{(a,b)} \\ \mathbf{\Theta}^{(a,b)} & \mathbf{\Theta}^{(b)} \end{bmatrix}$ to be 10 x 10 where $\mathbf{\Theta}^{(a)}$, $\mathbf{\Theta}^{(b)}$, and $\mathbf{\Theta}^{(a,b)}$ are each 5 x 5 diagonal matrices. Finally, we would expand $\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}^{(a)} & \mathbf{\Phi}^{(a,b)} \\ \mathbf{\Phi}^{(a,b)} & \mathbf{\Phi}^{(b)} \end{bmatrix}$ to be 4 x 4 where $\mathbf{\Phi}^{(a)}$, $\mathbf{\Phi}^{(b)}$, and $\mathbf{\Phi}^{(a,b)}$ are each 2 x 2 and unstructured. Testing the interdependency aspect of the holism principle could involve seeing if growth factors track

together over time, i.e. $H_o : \mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}^{(a)} & 0 \\ 0 & \mathbf{\Phi}^{(b)} \end{bmatrix}$. Testing the reciprocity aspect of the holism principle could involve instead seeing if growth factors predict each other by reparameterizing the model so that $\mathbf{\Phi}$ is block diagonal with 2 x 2 blocks, and then expanding the structural model into Equation (4):

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha} + \boldsymbol{\beta}\boldsymbol{\eta}_i + \boldsymbol{\varsigma}_i \tag{4}$$

Here again $\boldsymbol{\eta}_i$, $\boldsymbol{\varsigma}_i$, and $\boldsymbol{\alpha}$ are 4 x 1, and $\boldsymbol{\beta}$ is 4 x 4. Then, an example of testing the reciprocity aspect of the *holism principle* would be $H_0 : \boldsymbol{\beta} = 0$.

Classification methods

Latent class growth model (LCGM). The measurement model for *p* repeated measures on person *i* in latent class *k* is:

$$\mathbf{y}_{ik} = \boldsymbol{\upsilon}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{ik} + \boldsymbol{\varepsilon}_{ik} \text{ where often } \boldsymbol{\varepsilon}_{ik} \sim N(0, \boldsymbol{\Theta}_k) \tag{5}$$

The structural models are

$$\boldsymbol{\eta}_{ik} = \boldsymbol{\alpha}_k \tag{6}$$

$$\pi_k = \frac{\exp(\nu_k)}{\sum_{k=1}^{K} \exp(\nu_k)}$$

Here $\mathbf{y}_{ik}$ is a *p* x 1 vector of repeated measures for person *i* in class *k*, where *p*=4 in Figure 1 Panel B. There are a total of *K* classes. $\boldsymbol{\upsilon}_k$ is a *p* x 1 vector of class-specific item intercepts fixed to 0 (not shown in Figure 1 Panel B). $\boldsymbol{\Lambda}_k$ is a *p* x q_k matrix of loadings of repeated measures on q_k growth parameters in class *k*. $\boldsymbol{\eta}_{ik}$ is a q_k x 1 vector of class-specific latent growth parameters. $\boldsymbol{\varepsilon}_{ik}$ is a *p* x 1 vector of time-specific residuals for class *k*. $\boldsymbol{\Theta}_k$ is a typically-diagonal *p* x *p* covariance matrix of $\boldsymbol{\varepsilon}_{ik}$. $\boldsymbol{\alpha}_k$ is a q_k x 1 vector of class-specific means (not shown in Figure 1 Panel B). Finally π_k is the probability of membership in class *k* which is calculated from a no-predictor multinomial logistic regression with intercept ν_k.

Manuscript Table 2 lists which person-oriented principles are testable with LCGM. Next we give examples of how these principles could be tested.

(1) *Pattern parsimony*. Assuming that the *pattern summary principle* is valid, testing the *pattern parsimony principle* in the LCGM could involve comparing the fit of K=2, 3…class models and ascertaining whether the optimally fitting number of classes is $\leq$ a predefined 'small' number.
(2) *Complex interactions principle*. Under the same assumption, testing the *complex interactions principle* in the LCGM could involve adding a vector of person-level predictor(s) of class k membership such as $\mathbf{x}_{ik} = [x_{1ik}, \ x_{2ik}, \ x_{1ik}x_{2ik}]'$ in Equation (7)

$$(\pi_k)_i = \frac{\exp(\nu_k + \boldsymbol{\delta}_k \mathbf{x}_{ik})}{\sum_{k=1}^{K} \exp(\nu_k + \boldsymbol{\delta}_k \mathbf{x}_{ik})} \tag{7}$$

Where here $\boldsymbol{\delta}_k$ is 1 x 3 and here $\mathbf{x}_{ik}$ is 3 x 1. Then testing implicitly for interactions in the prediction of growth parameter values could entail, $H_0 : \boldsymbol{\delta}_k = \boldsymbol{\delta}$.
(3) *Holism principle.* Limited testing of the *holism principle* is possible by expanding the univariate equation (5)-(6) to also model, for example, a second longitudinal behavior, having j=1…J trajectory classes. For each of j classes in the second growth process, a $\boldsymbol{\Lambda}_j$ would need to be specified and $\boldsymbol{\Theta}_j$ and $\boldsymbol{\alpha}_j$ would need to be estimated. Finally, the two growth processes would be longitudinally linked by estimating $\pi_{k|j}$, the conditional probability of membership in class k of process 1 given membership in class j of process 2 (see Nagin & Tremblay, 2001). Given that π_j was estimated from the first process in Equation (6) and $\pi_{k|j}$ was estimated from the second process, both of these quantities can be used to solve for: $\pi_{j|k}$, the conditional probability of membership in class j of process 1 given membership in class k of process 2, and π_{jk} ,the joint probability of membership in class j and k. Then, testing the interdependency aspect of the *holism principle* could involve H_0 :"$\pi_{k|j}$ not different than chance or $\pi_{j|k}$ not different than chance or π_{jk} not different than chance" and testing its reciprocity aspect could involve H_0 :"$\pi_{k|j}$ not different than chance and $\pi_{j|k}$ not different than chance."

Latent Markov model. The latent Markov model for a response pattern on one binary variable measured at 4 timepoints (e.g. 1,0,1,1 or 0,0,0,1 or 1,1,0,0), as shown in Figure 2 Panel C, is

$$P(y) = \sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O}\delta_k \rho_k \tau_{m|k} \rho_m \tau_{n|m} \rho_n \tau_{o|n} \rho_o \tag{8}$$

Here, δ and ρ 's are scalar, measurement model parameters and τ 's are scalar, structural model parameters. Here also there are *K* latent statuses at time 1, *M* at time 2, *N* at time 3 and *O* at time 4. δ_k are initial latent status probabilities, which sum to 1 across *K*. ρ_k is the probability of item endorsement at timepoint 1 given membership in latent status *k* at timepoint 1. ρ_m is the probability of item endorsement at timepoint 2 given membership in latent status *m* at timepoint 2. ρ_n is the probability of item endorsement at timepoint 3 given membership in latent status *n* at timepoint 3. ρ_o is the probability of item endorsement at timepoint 4 given membership in latent status *o* at timepoint 4. (Note that if there were not one but *J* measures per timepoint, as in a latent transition model, we would simply replace $\rho_k, \rho_m, \rho_n, \rho_o$ with $\left(\prod_{j=1}^{J}\rho_{j|k}\right), \left(\prod_{j=1}^{J}\rho_{j|m}\right), \left(\prod_{j=1}^{J}\rho_{j|n}\right), \left(\prod_{j=1}^{J}\rho_{j|o}\right)$ in Equation (8). Note also that the latent Markov model requires $\rho_k = \rho_m = \rho_n = \rho_o$ but the latent transition model does not.) $\rho_k, \rho_m, \rho_n, \rho_o$ each sum to 1 across their respective binary response categories. τ 's are scalar transition probabilities from a particular latent status at a prior timepoint to a particular latent status at the current timepoint. Hence, $\tau_{m|k}$ denotes the probability of transitioning to membership in status *m* at timepoint 2 given membership in status *k* at timepoint 1 (there are a *K* x *M* such probabilities). $\tau_{n|m}$ denotes the probability of transitioning to membership in status *n* at timepoint 3 given membership in status *m* at timepoint 2 (there are a *M* x *N* such probabilities). Finally, $\tau_{o|n}$ denotes the probability of transitioning to membership in status *o* at timepoint 4 given membership in status *n* at timepoint 3 (there are *N* x *O* such probabilities).

Manuscript Table 2 lists which person-oriented principles are testable with latent Markov model. Next we give examples of how these principles could be tested.

(1) *Pattern parsimony principle.* Assuming that the *pattern summary principle* is valid, testing the *pattern parsimony principle* in the latent Markov model could involve comparing the fit of *K*=2, 3…statuses, *M*=2, 3…statuses, *N*=2, 3…statuses, *O*=2, 3…statuses and ascertaining whether the optimally fitting number of statuses/timepoint is $\leq$ a predefined ‘small’ number.

(2) *Complex interactions principle.* Under the same assumption, testing the *complex interactions principle* in the latent Markov model could involve, for example, adding a vector, $\mathbf{x}_i$, of person-level predictor(s) of latent transition probabilities. This would entail including a multinomial logistic regression to predict latent transition probabilities:

$$(\tau_{m|k})_i = \frac{\exp(\alpha_m + \beta_{km} + \boldsymbol{\gamma}_m \mathbf{x}_i)}{\sum_{m=1}^{M} \exp(\alpha_m + \beta_{km} + \boldsymbol{\gamma}_m \mathbf{x}_i)} \tag{9}$$

In Equation (9), β_{km} denotes the difference in log odds of being in class *m* vs. the reference class at time 2 for persons in class *k* at time 1—compared to the reference class. The $\boldsymbol{\gamma}_m$ allows the effect of $\mathbf{x}_i$ on *k*-to-*m* transition probabilities to differ across latent statuses *m* (see Nylund, 2007 for examples). Testing for a *status* by $\mathbf{x}_i$ interaction could be accomplished by $H_0 : \boldsymbol{\gamma}_m = \boldsymbol{\gamma}$.

(3) *Holism principle.* Limited testing of the *holism principle* in the latent Markov model would be possible if the longitudinal sequence of another, entirely different, behavior were modeled simultaneously (not multiple indicators of the same repeated construct as in latent transition analysis). This is called an associative latent Markov model (Flaherty, 2008). Suppose the second behavior had *V* latent statuses at time 1, *W* at time 2, *X* at time 3, and *Z* latent statuses at time 4. Then, in the associative latent Markov model, δ_k would be estimated as in Equation (8), but now transition probabilities would be conditional on current status on the second behavior as well (i.e. $\tau_{k|m,v}$, $\tau_{m|n,w}$, and $\tau_{n|o,x}$), and response probabilities for each item would be conditional on current latent statuses for *both* behaviors (i.e. $\rho_{k,v}$, $\rho_{m,w}$, $\rho_{n,x}$,and $\rho_{o,z}$). As well, initial status for the *second* behavior would be conditional on initial status of the first behavior (i.e. $\delta_{v|k}$), and transition probabilities for the second behavior would be conditional on prior *and* current latent status for the first behavior (i.e., $\tau_{w|v,k,m}$, $\tau_{x|w,m,n}$, $\tau_{z|x,n,o}$; see Flaherty's 2008 Appendix for similar model). Then, testing the reciprocity aspect of the *holism principle* from timepoint 1 to 2, for example, could involve evaluating: $H_0 : \tau_{w|v,k,m} = \tau_{w|v}$ and $\tau_{k|m,v} = \tau_{k|m}$ (i.e. that transition probabilities on one behavior do not depend on current and/or prior latent status membership in the other behavior). Testing the interdependency aspect of the *holism principle* at timepoint 1, for example, could involve evaluating $H_0 : \delta_{v|k} = \delta_v$ (i.e. that initial latent status probabilities on the second behavior do not depend on initial latent status probabilities on the first behavior). (Note:

although such models can in principle be fit in structural equation modeling programs, estimation problems can arise with increasing numbers of states/timepoint and timepoints.)

Hybrid classification methods

Growth mixture model (GMM). The measurement model for *p*=4 repeated measures on person *i* in latent class *k* from Figure 1 Panel D is:

$$\mathbf{y}_{ik} = \boldsymbol{\upsilon}_k + \boldsymbol{\Lambda}_k \boldsymbol{\eta}_{ik} + \boldsymbol{\varepsilon}_{ik} \text{ where often } \boldsymbol{\varepsilon}_{ik} \sim N(0, \boldsymbol{\Theta}_k) . \quad (10)$$

The structural models are

$$\boldsymbol{\eta}_{ik} = \boldsymbol{\alpha}_k + \boldsymbol{\varsigma}_{ik} \text{ where } \boldsymbol{\varsigma}_{ik} \sim N(0, \boldsymbol{\Phi}_k) \quad (11)$$

$$\pi_k = \frac{\exp(\nu_k)}{\sum_{k=1}^{K} \exp(\nu_k)}$$

All notation is as defined in the LCGM except for $\boldsymbol{\varsigma}_{ik}$, which is a q_k x 1 vector of class-specific individual deviations from growth factor means and $\boldsymbol{\Phi}_k$, which is a typically-unstructured $q_k \text{ x } q_k$ variance-covariance matrix of $\boldsymbol{\varsigma}_{ik}$ for class *k*.

Manuscript Table 2 lists which person-oriented principles are testable with the GMM. Next we give examples of how these principles could be tested.

(1) *Pattern summary.* Assuming that trajectory classes represent population subgroups, testing the *pattern summary principle* in GMM could entail $H_0 : K = 0$, but see qualifications/cautions in the text.

(2) *Pattern parsimony*. Under the same assumption, testing the *pattern parsimony principle* could entail $H_0 : K \leq$ 'predefined small number', but see qualifications in the text.

(3) *Interindividual differences/intraindividual change.* Under the same assumption, an example of testing whether there is remaining interindividual variability in change, over and above that which was accounted for by $\boldsymbol{\alpha}_k$ differences would be $H_0 : (\phi_{11})_k = 0$, in Figure 1 Panel D.

(4) *Individual specificity principle*. Under the same assumption, an example of testing whether there is remaining individual specificity, after accounting for $\boldsymbol{\alpha}_k$ differences, is $H_0 : \boldsymbol{\Phi}_k = 0$.

(5) *Complex interactions principle.* Under the same assumption, testing *the complex-interactions principle* in the GMM in Figure 1 Panel D could involve both adopting strategies employed for

detecting explicit interactions in the prediction of growth factors from LGM (i.e. including a vector of person-level predictor(s) $\mathbf{x}_{ik} = [x_{1ik},\ x_{2ik},\ x_{1ik}x_{2ik}]'$ of growth factors within-class):

$$\boldsymbol{\eta}_{ik} = \boldsymbol{\alpha}_k + \boldsymbol{\Gamma}_k \mathbf{x}_{ik} + \boldsymbol{\varsigma}_{ik} \tag{12}$$

and strategies employed for detecting implicit interactions in the prediction of growth parameter values from LCGM (i.e. including a vector of person-level predictor(s) $\mathbf{x}_{ik}$ of class membership):

$$(\pi_k)_i = \frac{\exp(\nu_k + \boldsymbol{\delta}_k \mathbf{x}_{ik})}{\sum_{k=1}^{K} \exp(\nu_k + \boldsymbol{\delta}_k \mathbf{x}_{ik})} \tag{13}$$

Then we could, for example, test $H_0 : (\gamma_{23})_k = 0$ (i.e. that there is no *time* by x_{1i} by x_{2i} interaction) from Equation (12) and test $H_0 : \boldsymbol{\delta}_k = \boldsymbol{\delta}$ from Equation (13).

(6) *Holism principle.* Finally, in theory, limited testing of the *holism principle* in GMM could be possible using the same procedures discussed for the LCGM model.

Mixed latent Markov model. The mixed Latent Markov model for a response pattern on one binary variable measured at 4 timepoints (e.g. 1,0,1,1 or 0,0,0,1 or 1,1,0,0), as shown in Figure 2 Panel E, is

$$P(y) = \sum_{c=1}^{C}\sum_{k=1}^{K}\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O} \pi_c \delta_{k|c} \rho_{k|c} \tau_{k|m,c} \rho_{m|c} \tau_{m|n,c} \rho_{n|c} \tau_{n|o,c} \rho_{o|c} \tag{14}$$

Here, there are *C* latent chains which allow for across-chain heterogeneity in longitudinal status-to-status behavioral sequences. The proportion of membership in chain *c* is denoted π_c and all other model parameters are as defined in the latent Markov model—except that they are now conditioned on chain membership also. Note that, in this model, parameters are often constrained equal across chain or fixed in one chain / free the other.

Manuscript Table 2 lists which person-oriented principles are testable with the mixed latent Markov model. Next we give examples of how these principles could be tested.

(1) *Pattern parsimony principle.* Assuming that the *pattern summary principle* is valid, testing the *pattern parsimony principle* in the mixed latent Markov model could involve comparing the fit of *K*=2, 3…statuses/chain, *M*=2, 3…statuses/chain, *N*=2, 3…statuses/chain, *O*=2, 3…statuses/chain ascertaining whether the optimally fitting number of statuses/timepoint in each chain is $\leq$ a predefined ‘small’ number.

(2) *Individual specificity principle.* Under the same assumption, testing the *individual specificity principle* could involve H_0: C=1 .

(3) *Interindividual differences/intraindividual change principle.* Under the same assumption, testing the *interindividual differences/intraindividual change principle* could involve the more specific hypothesis that the transition probabilities are the same across chain:

$H_0: \ \tau_{k|m,c} = \tau_{k|m}; \ \tau_{m|n,c} = \tau_{m|n}; \ \tau_{n|o,c} = \tau_{n|o}$.

(4) *Complex interactions principle.* Under the same assumption, testing the *complex interactions principle* in the mixed latent Markov model could involve adding person-level predictors of within-chain latent transition probabilities (much like in Equation (9)).

(5) *Holism principle.* Finally, although limited testing of the *holism principle* using similar procedures to those described in the latent Markov section is in principle possible, in practice it is unlikely that multiple chains and multiple Markov processes within chain would be estimable.

Single subject methods

P-technique factor model. The measurement model for p variables on t occasions for one person is

$$\mathbf{y}_t = \mathbf{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \text{ where often } \boldsymbol{\varepsilon}_t \sim N(0, \mathbf{\Theta}). \tag{15}$$

The structural model is

$$\boldsymbol{\eta}_t = \boldsymbol{\varsigma}_t \text{ where } \boldsymbol{\varsigma}_t \sim N(\mathbf{0}, \mathbf{\Phi}). \tag{16}$$

Note that the conventional p-technique model has no mean structure. Here $\mathbf{y}_t$ is a p x 1 vector of observed variables, where in Figure 1 Panel F p=20 . q=number of process-factors, which in Figure 1 Panel F is 2. $\mathbf{\Lambda}$ is a p x q matrix of process-factor loadings, where q is the number of process-factors. $\boldsymbol{\eta}_t$ is a q x 1 vector of process-factor scores that vary across timepoints t. $\boldsymbol{\varepsilon}_t$ is a p x 1 vector of residuals. $\mathbf{\Theta}$ is a typically-diagonal p x p covariance matrix of $\boldsymbol{\varepsilon}_t$. $\boldsymbol{\varsigma}_t$ is a q x 1 vector of time-specific deviations from process factor means; (these means are assumed to be 0). $\mathbf{\Phi}$ is a typically-unstructured q x q covariance matrix of $\boldsymbol{\varsigma}_t$.

We only describe testing person-oriented principles with respect to the dynamic factor model below, as the *p*-technique model was only presented as an intermediate step to build up to the dynamic factor model.

Dynamic factor model. The measurement model for *p* variables on *t* occasions for one person and for only 1 lag (as in Figure 2 Panel G) is

$$\mathbf{y} = \boldsymbol{\Lambda}\boldsymbol{\eta} + \boldsymbol{\varepsilon} \text{ where } \boldsymbol{\varepsilon} \sim N(0, \boldsymbol{\Theta}). \tag{17}$$

The structural model allowing for mean trend (Molenaar, de Gooijer, & Schmitz, 1992) (not shown in Figure 1 Panel G) is:

$$\boldsymbol{\eta} = \boldsymbol{\gamma}\tau + \boldsymbol{\varsigma} \text{ where } \boldsymbol{\varsigma} \sim N(0, \boldsymbol{\Phi}). \tag{18}$$

This particular dynamic factor model is often called a white noise factor model with nonstationarity of means or a shock factor model with nonstationarity of means (Browne and Nesselroade, 2005). However, this model still requires that there be no systematic trend in variances/covariances of the repeated measures, or that such a trend has been removed. Here $\mathbf{y} = \begin{bmatrix} \mathbf{y}_t \\ \mathbf{y}_{t-1} \end{bmatrix}$ is a 2*p* x 1 vector which contains $\mathbf{y}_t$, a *p* x 1 vector of lag-0 measured variables, stacked on top of $\mathbf{y}_{t-1}$, a *p* x 1 vector of lag-1 measured variables. In Figure 2 Panel G $\mathbf{y}$ would be of dimension 40 x 1, as there are 20 lag-0 measures constituting the vector $\mathbf{y}_t$ and 20 lag-1 counterparts constituting the vector $\mathbf{y}_{t-1}$. *q* is the number of process factors, where in Figure 2 Panel G *q*=2. $\boldsymbol{\Lambda}$ is dimension 2*p* x 3*q* and contains lag-0 *p* x *q* factor loading matrix $\boldsymbol{\Lambda}^{(0)}$ and lag-1 *p* x *q* factor loading matrix $\boldsymbol{\Lambda}^{(1)}$ in the following pattern:

$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{\Lambda}^{(0)} & \boldsymbol{\Lambda}^{(1)} & 0 \\ 0 & \boldsymbol{\Lambda}^{(0)} & \boldsymbol{\Lambda}^{(1)} \end{bmatrix}$. $\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{\eta}_t \\ \boldsymbol{\eta}_{t-1} \\ \boldsymbol{\eta}_{t-2} \end{bmatrix}$ is a 3*q* x 1 vector which contains factor scores for process factors at lag-0, i.e. $\boldsymbol{\eta}_t$ and lag-1, i.e. $\boldsymbol{\eta}_{t-1}$ and lag-2 $\boldsymbol{\eta}_{t-2}$. Note that $\boldsymbol{\eta}_{t-2}$ are included even though this is only a 1-lag model because they are needed to specify the initial condition/history of the two processes prior to the first measurement occasion. $\boldsymbol{\varepsilon} = \begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\varepsilon}_{t-1} \end{bmatrix}$ is a 2*p* x 1 vector of residuals.

$\boldsymbol{\Theta} = \begin{bmatrix} \boldsymbol{\Theta}^{(0)} & \boldsymbol{\Theta}^{(1)} \\ \boldsymbol{\Theta}^{(1)} & \boldsymbol{\Theta}^{(0)} \end{bmatrix}$ is a 2*p* x 2*p* covariance matrix of the $\boldsymbol{\varepsilon}$'s, and has a specialized (block-

Toeplitz) form such that $\boldsymbol{\Theta}^{(0)}$ =COV ($\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_t$) =COV($\boldsymbol{\varepsilon}_{t-1}, \boldsymbol{\varepsilon}_{t-1}$), which is p x p and diagonal, and $\boldsymbol{\Theta}^{(1)}$ =COV($\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_{t-1}$), which is p x p and diagonal. This allows residuals to be correlated across but not within lag. $\boldsymbol{\gamma}$ is a 3q x 1 vector of slopes relating process factors to time. τ is a scalar time variable denoting the occasion. $\boldsymbol{\varsigma} = \begin{bmatrix} \varsigma_t \\ \varsigma_{t-1} \\ \varsigma_{t-2} \end{bmatrix}$ is a 3q x 1 vector of stochastic terms. $\boldsymbol{\Phi}$ is a 3q x 3q block-diagonal covariance matrix of the $\boldsymbol{\varsigma}$, with equal blocks, where $\boldsymbol{\varsigma}$'s have variances of 1 and are allowed to be correlated only within lag. Fitting this model using structural equation modeling software requires first adding a time variable τ (e.g. with values 1 to 71 if there were 71 occasions) to the occasion by variables data matrix and then converting this matrix into a Block Toeplitz form; (SAS Macro for doing so is available from Wood & Brown, 1994). See Hershberger (1998) for code for fitting this model.

Manuscript Table 2 lists which person-oriented principles are testable with the dynamic factor model. Next we give examples of how these principles could be tested.

(1) *Pattern summary principle.* If we had p variables on t occasions for more than one person, we could denote this as

$$\mathbf{y}_i = \boldsymbol{\Lambda}_i \boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \text{ where } \boldsymbol{\varepsilon}_i \sim N(\mathbf{0}, \boldsymbol{\Theta}_i). \tag{19}$$

$$\boldsymbol{\eta}_i = \boldsymbol{\gamma}_i \tau + \boldsymbol{\varsigma}_i \text{ where } \boldsymbol{\varsigma}_i \sim N(\mathbf{0}, \boldsymbol{\Phi}_i). \tag{20}$$

Then we could test whether there is evidence of measurement invariance of intra-individual processes across persons. That is, we could test (a) H_o :" the same number of process factors q is best-fitting across persons." If so, we could test (b) $H_o : \boldsymbol{\Lambda}_i = \boldsymbol{\Lambda}$, (i.e. that the magnitude of lag-0 and lag-1 loadings in $\boldsymbol{\Lambda}_i$ is equal across persons). If so, we could test (c) $H_o : \boldsymbol{\Theta}_i = \boldsymbol{\Theta}$, (i.e. residual variances in $\boldsymbol{\Theta}_i$ are equal across persons). If the above three hypotheses (a)-(c) were supported within *groups* of persons, but not across groups of persons, this yields evidence for the *pattern summary principle.*

(2) *Pattern parsimony principle.* The *pattern-parsimony principle* is supported to the extent that the number of groups (i.e. number different best-fitting models) is much less than the number of persons whose data were modeled.

(3) *Individual-specificity principle.* We may further test whether structural parameters, for example, still vary across persons within each group, i.e. $H_o : \mathbf{\Phi}_i = \mathbf{\Phi}$ and $H_o : \boldsymbol{\gamma}_i = \boldsymbol{\gamma}$, which would be indicative of some remaining *individual-specificity*. Also, if certain individuals have their own unique best-fitting dynamic factor model, this too supports the *individual specificity principle*.

(4) *Interindividual differences/intraindividual change.* Although no variance trends are allowed here, if mean trends were found and included in the model, we could test whether these intraindividual mean changes had interindividual variability with $H_o : \boldsymbol{\gamma}_i = \boldsymbol{\gamma}$.