On Spanning Trees with few Branch Vertices

Warren Shull Emory University Joint work with Ron Gould

May 21, 2017

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• Leaf of a tree: degree 1

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- Leaf of a tree: degree 1
- Branch vertex of a tree: degree \geq 3

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- Leaf of a tree: degree 1
- Branch vertex of a tree: degree \geq 3
- Hamiltonian paths are a special kind of spanning tree

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- Leaf of a tree: degree 1
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 - Max degree 2 (except K_2 and K_1)

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In the next few slides, spanning trees are more "desirable" the fewer branch vertices they have.

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In the next few slides, spanning trees are more "desirable" the fewer branch vertices they have.

• What conditions might lead to a desirable spanning tree?

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• Independent sets may have many outgoing edges.

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Given the right parameters, there is either a desirable spanning tree or a large independent set with few outgoing edges.

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What are the best possible parameters?

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Let k be a non-negative integer and let G be a connected claw-free graph. Then G contains *either* a spanning tree with at most k branch vertices, or an independent set of 2k + 3 vertices.

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Let k be a non-negative integer and let G be a connected claw-free graph. Then G contains *either* a spanning tree with at most k branch vertices, or an independent set of 2k + 3 vertices.

Conjecture (Matsuda, Ozeki, Yamashita 2012)



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Let k be a non-negative integer and let G be a connected claw-free graph. Then G contains *either* a spanning tree with at most k branch vertices, or an independent set of 2k + 3 vertices.

Conjecture (Matsuda, Ozeki, Yamashita 2012)

Let k be a non-negative integer and let G be a connected claw-free graph. Then G contains *either* a spanning tree with at most k branch vertices,

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Let k be a non-negative integer and let G be a connected claw-free graph. Then G contains *either* a spanning tree with at most k branch vertices, or an independent set of 2k + 3 vertices with at most |V(G)| - 3 outgoing edges.

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Let k be a non-negative integer and let G be a connected claw-free graph. Then G contains *either* a spanning tree with at most k branch vertices, or an independent set of 2k + 3 vertices. This is best possible.

Conjecture (Matsuda, Ozeki, Yamashita 2012)

Let k be a non-negative integer and let G be a connected claw-free graph. Then G contains *either* a spanning tree with at most k branch vertices, or an independent set of 2k + 3 vertices with at most |V(G)| - 3 outgoing edges. This is best possible.

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Connected and claw-free

Warren Shull Emory University Joint work On Spanning Trees with few Branch Vertice

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Warren Shull Emory University Joint work On Spanning Trees with few Branch Vertice

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Any spanning tree must have a branch vertex in this triangle...

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Any spanning tree must have a branch vertex in this triangle...

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Any spanning tree must have a branch vertex in this triangle...

...and each of these others...

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Any spanning tree must have a branch vertex in this triangle...

...and each of these others...

... for a minimum of k + 1 branch vertices.

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|V(G)| =

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$$|V(G)| =$$

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$$|V(G)| = m(k+3)$$

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$$|V(G)| = m(k+3)$$

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$$|V(G)| = m(k+3)+2k$$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

independent set

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

independent set X

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

X

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$
$$|X| \leq$$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

 $|X| \le k+3$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

 $|X| \le k+3+k$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

 $|X| \le k+3+k = 2k+3$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

 $|X| = k+3+k = 2k+3$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

 $|X| = k + 3 + k = 2k + 3$
 $\sum_{x \in X} \deg(x) \ge$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$
$$|X| = k+3+k = 2k+3$$
$$\sum_{x \in X} \deg(x) \ge$$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

$$|X| = k+3+k = 2k+3$$

$$\sum_{x \in X} \deg(x) \ge (k+3)(m-1)$$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

$$|X| = k+3+k = 2k+3$$

$$\sum_{x \in X} \deg(x) \ge (k+3)(m-1)$$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

$$|X| = k+3+k = 2k+3$$

$$\sum_{x \in X} \deg(x) \ge (k+3)(m-1) + 3k$$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

$$|X| = k+3+k = 2k+3$$

$$\sum_{x \in X} \deg(x) \ge (k+3)(m-1) + 3k$$

$$= mk - k + 3m - 3 + 3k$$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

$$|X| = k+3+k = 2k+3$$

$$\sum_{x \in X} \deg(x) \ge (k+3)(m-1) + 3k$$

$$= mk - k + 3m - 3 + 3k$$

$$= mk + 3m + 2k - 3$$

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$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

$$|X| = k+3+k = 2k+3$$

$$\sum_{x \in X} \deg(x) \ge (k+3)(m-1) + 3k$$

$$= mk - k + 3m - 3 + 3k$$

$$= mk + 3m + 2k - 3 = |V(G)| - 3$$

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Theorem (Matsuda, Ozeki, Yamashita 2012)

Let k be a non-negative integer and let G be a connected claw-free graph. Then G contains *either* a spanning tree with at most k branch vertices, or an independent set of 2k + 3 vertices. This is best possible.

Conjecture (Matsuda, Ozeki, Yamashita 2012)

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Corollary

Let G be a connected claw-free graph. Then G contains either a spanning tree with at most 2 leaves (0 branch vertices), or an independent set of 3 vertices with at most |V(G)| - 3 outgoing edges.

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Theorem (Matsuda, Ozeki, Yamashita 2012)

Let G be a connected claw-free graph. Then G contains either a spanning tree with at most 1 branch vertex, or an independent set of 5 vertices with at most |V(G)| - 3 outgoing edges.

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- By the theorem of Kano et. al. above (with k = 4), G has a spanning tree with at most 6 leaves.

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Among spanning trees with at most 6 leaves, choose a tree T such that: (T1) T has as few branch vertices as possible.



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Among spanning trees with at most 6 leaves, choose a tree T such that: (T1) T has as few branch vertices as possible. (T2) T has as few leaves as possible, subject to (T1).

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(T1) T has as few branch vertices as possible.

(T2) T has as few leaves as possible, subject to (T1).

(T3) TBA

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- (T1) T has as few branch vertices as possible.
- (T2) T has as few leaves as possible, subject to (T1).(T3) TBA
- (T4) The parts of *T* in-between branch vertices are as small as possible.

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- (T1) T has as few branch vertices as possible.
- (T2) T has as few leaves as possible, subject to (T1).
- (T3) TBA
- (T4) The parts of T in-between branch vertices are as small as possible.
- How many different structures could T possibly have?

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First case: T has only 5 leaves (the fewest possible):

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First case: T has only 5 leaves (the fewest possible):



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(T3) If choosing between trees of these two types, we always choose one of the first type.

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Fourth and fifth cases: T has 4 branch vertices (and therefore 6 leaves)

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Fourth and fifth cases: T has 4 branch vertices (and therefore 6 leaves)



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Fourth and fifth cases: T has 4 branch vertices (and therefore 6 leaves)



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- Consider one part
- Show certain neighbor sets must be disjoint



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 $(X = \{I_1, I_2, I_3, I_4, I_5, b_1, b_2\})$

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$$egin{aligned} N_G(b_j) \cap V(M_1) & j \in \{1,2\} \ N_G(l_i) \cap V(M_1) & i
eq 1 \ (N_G(l_1) \cap V(M_1))^- \end{aligned}$$

 $\sum_{v \in X} |N_G(v) \cap V(M_1)| \qquad (X = \{l_1, l_2, l_3, l_4, l_5, b_1, b_2\})$

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$$egin{aligned} N_G(b_j) \cap V(M_1) & j \in \{1,2\} \ N_G(l_i) \cap V(M_1) & i
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 $\sum_{v \in X} |N_G(v) \cap V(M_1)| \qquad (X = \{l_1, l_2, l_3, l_4, l_5, b_1, b_2\})$

$$= \sum_{i=1}^{5} |N_{G}(I_{i}) \cap V(M_{1})| + \sum_{j=1}^{2} |N_{G}(b_{j}) \cap V(M_{1})|$$

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$$egin{aligned} N_G(b_j) \cap V(M_1) & j \in \{1,2\} \ N_G(l_i) \cap V(M_1) & i
eq 1 \ (N_G(l_1) \cap V(M_1))^- \end{aligned}$$

 $\sum |N_G(v) \cap V(M_1)|$ $(X = \{l_1, l_2, l_3, l_4, l_5, b_1, b_2\})$ $v \in X$

$$= \sum_{i=1}^{5} |N_{G}(I_{i}) \cap V(M_{1})| + \sum_{j=1}^{2} |N_{G}(b_{j}) \cap V(M_{1})|$$
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$$= |N_G(I_1) \cap V(M_1)| + \sum_{i \neq 1} |N_G(I_i) \cap V(M_1)| + \sum_{j=1} |N_G(b_j) \cap V(M_1)|$$

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$$egin{aligned} N_G(b_j) \cap V(M_1) & j \in \{1,2\} \ N_G(l_i) \cap V(M_1) & i
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$$= |(N_G(I_1) \cap V(M_1))^-| + \sum_{i \neq 1} |N_G(I_i) \cap V(M_1)| + \sum_{j=1}^2 |N_G(b_j) \cap V(M_1)|$$

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$$egin{aligned} N_G(b_j) \cap V(M_1) & j \in \{1,2\} \ N_G(l_i) \cap V(M_1) & i
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$$= |(N_G(I_1) \cap V(M_1))^-| + \sum_{i \neq 1} |N_G(I_i) \cap V(M_1)| + \sum_{j=1}^2 |N_G(b_j) \cap V(M_1)|$$

$$\leq |V(M_1)|$$

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$$\sum_{v\in X} |N_G(v)\cap V(M_1)| \leq |V(M_1)|$$

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 $\sum_{v \in X} |N_G(v) \cap V(M_h)| \leq |V(M_h)| \qquad h \neq 3$

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 $\sum_{v \in X} |N_G(v) \cap V(M_h)| \leq |V(M_h)| \qquad h \neq 3$

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 $\sum_{v \in X} |N_G(v) \cap V(M_h)| \leq |V(M_h)| \qquad h \neq 3$

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 $\sum_{v \in X} |N_G(v) \cap V(M_h)| \leq |V(M_h)| \qquad h \neq 3$

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$$\sum_{v \in X} |N_G(v) \cap V(M_h)| \leq |V(M_h)| \qquad h \neq 3$$
$$\sum_{v \in X} |N_G(v) \cap V(M_3)| \leq |V(M_3) \setminus \{z^-\}|$$

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$$\sum_{v \in X} |N_G(v) \cap V(M_h)| \leq |V(M_h)| \qquad h \neq 3$$
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$$\sum_{v \in X} |N_G(v) \cap V(M_h)| \leq |V(M_h)| \qquad h \neq 3$$
$$\sum_{v \in X} |N_G(v) \cap V(M_3)| \leq |V(M_3) \setminus \{z^-\}|$$

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$$\begin{split} &\sum_{v \in X} |N_G(v) \cap V(M_h)| &\leq |V(M_h)| &h \neq 3 \\ &\sum_{v \in X} |N_G(v) \cap V(M_3)| &\leq |V(M_3) \setminus \{z^-\}| \\ &\sum_{v \in X} |N_G(v) \cap V(Q_j)| &\leq |V(Q_j) \setminus \{w_j\}| &j \in \{1,2\} \end{split}$$

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$$\begin{split} &\sum_{v \in X} |N_G(v) \cap V(M_h)| &\leq |V(M_h)| &h \neq 3 \\ &\sum_{v \in X} |N_G(v) \cap V(M_3)| &\leq |V(M_3) \setminus \{z^-\}| \\ &\sum_{v \in X} |N_G(v) \cap V(Q_j)| &\leq |V(Q_j) \setminus \{w_j\}| &j \in \{1,2\} \end{split}$$

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$$\sum_{v \in X} |N_G(v) \cap V(M_h)| \leq |V(M_h)| \qquad h \neq 3$$

$$\sum_{v \in X} |N_G(v) \cap V(M_3)| \leq |V(M_3) \setminus \{z^-\}|$$

$$\sum_{v \in X} |N_G(v) \cap V(Q_j)| \leq |V(Q_j) \setminus \{w_j\}| \qquad j \in \{1, 2\}$$

$$\sum_{v \in X} |N_G(v)| \leq |V(G)| - 3$$

 $v \in X$ Warren Shull Emory University Joint work On Spanning Trees with few Branch Vertice

Second and third cases: T has 6 leaves, but only 3 branch vertices.



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Second and third cases: T has 6 leaves, but only 3 branch vertices.



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(T5) P is as short as possible, subject to (T1)-(T4).

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(T5) P is as short as possible, subject to (T1)-(T4).

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• We think we've proven the entire conjecture (currently editing).



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- We think we've proven the entire conjecture (currently editing).
- Algorithmically, we suspect we can guarantee either the spanning tree or the low-degree independent set in polynomial time.

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• Open question once this is done:

- We think we've proven the entire conjecture (currently editing).
- Algorithmically, we suspect we can guarantee either the spanning tree or the low-degree independent set in polynomial time.

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• Open question once this is done: Reduce the degree of this polynomial.

- We think we've proven the entire conjecture (currently editing).
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• Open question once this is done: Reduce the degree of this polynomial (will likely be in the teens).

- We think we've proven the entire conjecture (currently editing).
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- Open question once this is done: Reduce the degree of this polynomial (will likely be in the teens).
- This algorithm might not find the tree with the fewest branch vertices.

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- Open question once this is done: Reduce the degree of this polynomial (will likely be in the teens).
- This algorithm might not find the tree with the fewest branch vertices.
 - Can it be done for certain classes of graphs?

- We think we've proven the entire conjecture (currently editing).
- Algorithmically, we suspect we can guarantee either the spanning tree or the low-degree independent set in polynomial time.
- Open question once this is done: Reduce the degree of this polynomial (will likely be in the teens).
- This algorithm might not find the tree with the fewest branch vertices.
 - Can it be done for certain classes of graphs? And/or within some margin?

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Thank you for your attention!



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